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ARTICLE

The anatomy of government bond yields synchronization in the Eurozone

Claudio Barbieri[1,2], Mattia Guerini[3,4,2,6], and Mauro Napoletano[2,5,6]

[1]European Central Bank, Frankfurt am Main, Germany
[2]Université Côte d'Azur, CNRS, GREDEG, Valbonne, France
[3]Deparment of Economics and Management, University of Brescia, Brescia, Italy
[4]Fondazione ENI Enrico Mattei, Milano, Italy
[5]Sciences Po, OFCE, Paris, France
[6]Institute of Economics, Scuola Superiore Sant'Anna, Pisa, Italy
Corresponding author: Mattia Guerini; Email: mattia.guerini@unibs.it

Abstract

We investigate the synchronization of the Eurozone's government bond yields at different maturities. For this purpose, we combine principal component analysis with random matrix theory. We find that synchronization depends on yield maturity. Short-term yields are not synchronized. Medium- and long-term yields, instead, were highly synchronized early after the introduction of the Euro. Synchronization then decreased significantly during the Great Recession and the European Debt Crisis, to partially recover after 2015. We interpret our empirical results using portfolio theory, and we point to divergence trades as a source of the self-sustained yield asynchronous dynamics. Our results envisage synchronization as a requirement for the smooth transmission of conventional monetary policy in the Eurozone.

Keywords: Synchronization; bond yields; factor models; random matrix theory; monetary policy

1. Introduction

Since the introduction of the common currency, the Eurozone has been characterized by a common monetary policy authority and distinct fiscal policy authorities. Hence, government bonds yields are allowed to differ to reflect country-specific characteristics and financial traders can create portfolios comprising the different bonds. In this context, idiosyncratic shocks affect the composition of optimal portfolios and, as a consequence, bond yields. This might give rise to asynchronous yield movements, hindering the transmission mechanism of conventional monetary policy within the union. The Great Recession and the subsequent European Debt Crisis have highlighted this possibility, even endangering the stability of the whole European architecture. A precise assessment of yields' synchronization in the Eurozone is therefore crucial for identifying periods in which conventional monetary policies are ineffective and unconventional ones required.

In this paper, we empirically investigate synchronization in government bonds' yields by employing a rich dataset retrieved from the Bloomberg platform at daily frequency, covering the period 2003–2019, for all Eurozone economies and 11 different bond's maturities. We apply a procedure based on random matrix theory [RMT, see e.g., Onatski (2009, 2010)] that allows us to select the number of statistically significant factors estimated via principal component analysis (PCA). Compared to traditional static factor analysis, our procedure does not require the introduction of penalty functions and/or *ad hoc* truncation strategies to identify the number of significant factors [see e.g., Ludvigson and Ng (2007); Guo et al. (2018)]. It only needs a comparison between empirically estimated eigenvalues and the distribution of eigenvalues that is generated by a Gaussian random model with only spurious correlations. Our approach to factor selection is guided by the consideration that a factor when used as a regressor in a forecasting model might turn out to be significant even if the information it contains is indistinguishable from the one obtained from a spurious relationship obtainable with a finite number of observations.

Furthermore, the eigenvalues and the eigenvectors selected through RMT provide more precise and accurate information about the synchronization of bond yields compared to the information obtained from a mere comparison of correlation coefficients.[1] In addition, it is possible to theoretically interpret significant eigenvalues as optimal portfolios and the associated eigenvectors as the weights of the assets therein [see Avellaneda and Lee (2010)]. Recent results in portfolio theory [see Avellaneda and Lee (2010); Bouchaud and Potters (2015)] show indeed the equivalence between PCA-estimated factors and the relative variance of mutually independent portfolios. In that, the components of eigenvectors associated with each factor correspond to the weights of assets within one portfolio. On these grounds, finding that eigenvector components diverge in sign indicates the presence of an optimal trading strategy wherein some bonds are held on a long position while some others are on a short one. These "divergence trades" [Avellaneda and Lee (2010)] might cause significant asymmetries in government bonds yields within a monetary union and pose debt sustainability problems for some countries.

By employing the above methodology, we first characterize the Eurozone government bond yields dynamics. We then provide a formal interpretation of observed patterns based on the above-mentioned portfolio theory. We find that synchronization in yields is scarce at 1-year maturity, as the most relevant factor explains around 30% of the total variance, and it is not statistically significant at specific time windows. The corresponding eigenvector components are also heterogeneous over the whole 2003–2019 period. The situation is different for 5-year and 10-year government bond yields. One factor explains between 75% and 80% of the total variance of those yields until 2008, around 40% between 2008 and 2014, and about 60% from 2015 to 2019. The associated eigenvector components contribute in equal amount to the dynamic of the first factor until 2008, indicating a strong synchronization between 2003 and 2008. Starting from 2008, instead, the eigenvector components of some "core" countries keep contributing to the dynamic of the first factor, while the eigenvector components of the second group of countries (the "periphery") contribute only to a low extent. Moreover, a second factor becomes significant during the crisis, accounting for a share between 20% to 30% of the total variance. Its eigenvector components constantly differ in sign across core and peripheral countries.

The presence of two significant factors for bonds at 5- and 10-year maturities between 2008 and 2014 indicates the presence of *divergence trades* and *flight-to-quality* effects during the sovereign debt crisis. One explanation for the results is that two bond portfolios might have been profitable at the same time: a first portfolio holding long positions toward all the Eurozone, although in different shares, and a second portfolio holding long positions for core countries and short positions for the periphery. According to this interpretation, the bonds of the core countries became akin to a safe asset during the crisis. In addition, the incentives to invest in a diverging portfolio amplified the effects of the crisis across countries and for the whole system.

Our results deliver new insights for the Eurozone policy debate. It is well known that the heterogeneous response of different countries' yields to the same ECB interest rate policy is one of the main fragilities for the Euro area. Our procedure allows one to quantify asynchronicity in yields' movements and, being based on daily data, it can be useful to policymakers to timely understand whether a conventional policy is hampered by divergence or not. Under diverging government bond yields, the conventional monetary policy results in asymmetric effects across the Eurozone and the unconventional monetary policy should be used accordingly.

The paper is organized as follows. Section 2 summarizes results from previous studies that are related to our work. The data are presented in Section 3, together with a preliminary statistical descriptive analysis. Section 4 describes the econometric procedure we use in the paper. Baseline results for 1-year, 5-year, and 10-year yields are presented in Section 5. A series of robustness checks, controlling for sample selection, model parameters, as well as non-Gaussian null models are discussed in Section 6. Section 7 discusses the results and their policy implications. Section 8 concludes.

2. Some methodological roots

Our work is related to two empirical research strands. First, we refer to the bulk of studies that have investigated macroeconomic time series by using factor models and PCA. The work by Stock and Watson (2002) is among the first studies that estimate static factors through PCA, showing that the obtained estimators are consistent and efficient as the number of time series and their length grow to infinity.[2] Several works have also proposed methods to select the number of factors. For instance, Bai and Ng (2002, 2008) introduce factor selection based on penalty functions, to consistently estimate the optimal number of factors that shall then be included in a structural vector autoregressive model. Ludvigson and Ng (2010) estimate eight macroeconomic factors and interpret them based on the marginal R-squared obtained by regressing all the variables of their dataset of the factors taken one at a time. Onatski (2010) and Kapetanios (2010) provide an alternative selection criteria of factors based on applications of RMT. In particular, Onatski (2010) shows that selection based on RMT performs better when the variance attributed to the factors is small relative to the variance caused by the idiosyncratic components. Kapetanios (2010) relaxes some of the RMT assumptions and finds that the method is still robust to these violations. This suggests not only that RMT performs better, but also that its assumptions are not necessarily stricter than the ones required by alternative static factors estimators.

PCA has also been extensively applied to portfolio analysis in finance. For instance, Avellaneda and Lee (2010) show the existence of a direct relation between optimal portfolio theory and principal components. They compare the Sharpe ratio performances of exchange-traded funds (ETF) and portfolios weighted by principal components—that is, the so-called *eigenporfolios*.[3] The study is conducted on the US equity market from 1997 to 2007. The authors find that the portfolios that maximize the Sharpe ratio are given by considering either 15 ETF or the first 15 principal components, or a variable number of principal components accounting for approximately 55% of the total variance.[4] Following a similar approach, Guo et al. (2018) propose a selection criteria to maximize Sharpe ratio eigenportfolios. They use a LASSO-based factor selection, in which the threshold for the truncation is computed by introducing a function for the maximum tolerance for the relative approximation error in the estimation of the empirical correlation matrix.[5]

Second, our work is related to the large literature that has investigated interest rates convergence by using various econometric procedures.[6] Vajanne (2007) studies the integration of retail banking rates in the Eurozone between 2003 and 2006 using a monthly data panel and adopts beta- and gamma-convergence as measures of convergence. Its findings are in favor of some convergence, notwithstanding important substantial cross-country differences. Furthermore, Arouri et al. (2013) consider interbank interest rates for France, the UK, and the USA between 2004 to 2010 and use the Geweke Contemporaneous Feedback Measure [see Geweke (1982)] as a measure of convergence.[7] Such a measure consists of a log-likelihood ratio test statistic, which tests whether a model with interdependent interest rates is significantly different from a model wherein interest rates depends only on their past values. The study finds evidence of convergence among the countries examined, with changes in US interest rates being slightly leading for those in France and the UK. In addition, the work by Wälti (2011) estimates the effect of a common factor (capturing trade and financial integration) on stock market returns cross-correlation.[8] The work considers yearly data for 15 developed countries from 1975 to 2006, and it shows that trade and financial integration have a positive effect on stock market correlation. Nevertheless, for Eurozone members, the above effect is significant only for countries that were already tightly connected before the introduction of the common currency. Finally, Dow et al. (2012) analyze convergence in interest rates across Italian regions. The authors employ quarterly data on short- and long-term interest rates on loans and deposits from 1998 to 2008. Methodologically, convergence is examined through different unit root tests, where some evidence of convergence is considered if the cointegration hypothesis is not rejected.

We contribute to the above two streams of literature by performing an analysis of synchronization in Eurozone government bond yields that combines PCA with a factor selection method based on RMT. Our procedure exploits the comparison of eigenvalues of correlation matrices of the time series of interest with alternative null models, and it selects the number of factors that should be included in a model *ex ante,* that is, before the estimation of a factor model. This is a significant difference with respect to the previous works [see e.g. Bai and Ng (2002, 2008); Ludvigson and Ng (2010)] that have employed *ex post* selection methods employing penalty functions or the extremes of a truncation procedure after a model has been estimated.

3. Data description and preliminary analysis

We retrieve from the Bloomberg platform the time series of government bonds yields at a daily frequency for 11 Eurozone countries and at 11 maturities. Our data include time series of bonds with maturity at 3 and 6 months as well as bonds at 1, 2, 3, 5, 7, 10, 15, 20, and 30 years maturity. The selected Eurozone countries are Austria, Belgium, Finland, France, Greece, Germany, Ireland, Italy, the Netherlands, Portugal, and Spain. Luxembourg is missing because of data unavailability problems in the Bloomberg platform. All countries are member states of the Eurozone since its creation except for Greece, which joined the Euro area in 2001.[9] The dataset covers the period from 2003-01-01 to 2019-02-21. We focus our analysis on government bond yields with 1-year, 5-year, and 10-year maturities. These are available for all countries (except for the Netherlands for 1-year bonds) over the whole sample period. The other maturities are used as a robustness check to complement our understanding of the problem (see Section C.1).

We focus on nominal rather than real interest rates. Several reasons are underlying such a choice. First, there is a time mismatch between the reactions of nominal interest rate and inflation rate to monetary policy shocks, with inflation adjusting at a slower pace [Christiano et al. (1999)]. Second, nominal interest rates are directly affected by monetary policy shocks, while inflation depends on a wider range of factors, also beyond monetary policy [see e.g. Reis and Watson (2010); Coibion and Gorodnichenko (2015); Blanchard (2016)]. Our core dataset comprises the time series for three maturities of government bond yields for $N = 11$ countries and $T > 4000$ periods (see Appendix A for further details).

Augmented Dickey–Fuller (ADF) tests for the presence of unit roots suggest that all the time series in our sample are at least $I(1)$. Thus, before applying the RMT procedure, we perform a prewhitening of the data by taking a first difference transformation. The series resulting from each transformation are all stationary, and no structural break can be identified.[10]

4. Methodology

Our econometric strategy to investigate synchronization in bond yields exploits RMT to select the number of statistically significant principal components based on the comparison between the empirically estimated correlations of bond yields and theoretical correlations generated by a random normal model [Laloux et al. (1999, 2000)].

4.1. Principal component analysis and eigenportfolios

We perform a PCA on overlapping rolling windows of our stationary time series. A rolling window $\underset{K\times N}{X}$ is a section of *length* $K < T$ of our stationary time-series dataset. When moving from a

Table 1. Windows dimensions for daily Bloomberg time series of government yields

Number of observations	Window's time span
22	Month
65	Quarter
130	Semester
261	Year
522	Two years
783	Three years

rolling window to the next one, observations are updated by a *step* S, that is, S observations are discarded at the beginning of the window and S observations are added at the end.

By construction, the smaller is the window the more accurate is the tracking of synchronization. At the same time, windows' length should also be sufficiently large to ensure statistical significance of the correlation coefficients to be computed and, since we employ daily data, to obtain results that are not driven by daily outliers. We fix the length of each window to one semester (i.e. 130 daily observations) and the length of each step of the windows to 1 month (i.e. 22 daily observations, see Table 1 for the number of observations for each window's period). As a result, two subsequent rolling windows overlap for 5 months out of 6. Such a choice allows us to closely track the evolution of the principal components and their dynamics over time, by replacing only a few pieces of information at each step. However, in Section C.2, we also check the robustness of our main results to several alternative values of the length and step parameters of the rolling windows.

For each window, we compute the matrix of Pearson's correlation coefficients of bond yields, E as:

$$E = \frac{1}{K}\tilde{X}^T\tilde{X} \tag{1}$$

where $\underset{K\times N}{\tilde{X}}$ is the demeaned and standardized version of $\underset{K\times N}{X}$ and $(\cdot)^T$ is the transpose operator. The matrix E has size $N \times N$. Its element $e_{i,j}$ is the correlation coefficient between the government bonds yields of countries i and j in a specific window $w \in \{1, \ldots W\}$, where W is the maximum number of windows. From this positive semi-definite matrix, it is possible to compute the eigenvalues, which are all nonnegative and distinct, and their associated eigenvectors. Each eigenvalue λ_i corresponds to a principal component, which is a linear combination of the original series that explains a specific portion of the variance contained in the data.

Furthermore, the eigenvalues of the above correlation matrices have a financial interpretation. More precisely, consider a portfolio $\Pi_i = \tilde{X}u_i$, where the normalized returns $\tilde{X}$ are weighted by the eigenvector u_i. The variance of such a portfolio is equal to:

$$Var(\Pi_i) = Var(\tilde{X}u_i) = \mathbb{E}[\tilde{X}u_i]^2 = \frac{1}{K}u_i^T(\tilde{X}^T\tilde{X})u_i \tag{2}$$

By using the spectral decomposition, we can write equation (1) as:

$$E = u_i\lambda_i u_i^T \tag{3}$$

and by substituting it into equation (2), it follows that

$$\frac{1}{K}u_i^T(\tilde{X}^T\tilde{X})u_i = u_i^T E u_i = \lambda_i \tag{4}$$

Equation (4) thus shows that each eigenvalue λ_i corresponds to the variance of returns—and thus to the risk—of the *eigenportfolio* Π_i, composed by the N different government bonds. The weights of each bond in the portfolio are returned by the elements of the corresponding eigenvector [see Bouchaud and Potters (2015)]. In particular, a larger eigenvalue corresponds to a higher risk of the corresponding eigenportfolio. It follows that the eigenportfolio associated with the dominant eigenvalue of the correlation matrix is also the one that maximizes risk.[11]

4.2. Random matrix theory and factor selection

RMT allows one to select only the principal components that contain information that is not reducible to spurious volatility.[12] For spurious volatility here, we do not refer to the correlation of two $I(1)$ variables, but to the idea that even two stationary *i.i.d* variables with a finite number of observations can display some degree of correlation. The important question thus becomes how to distinguish whether the distribution of the observed empirical correlations is different from the one that would have been observed for finite *i.i.d* variables. RMT solves this problem by comparing the empirical correlation matrix with the correlation matrix of a randomly generated normal model.

The theorem stated in Marčenko and Pastur (1967)—Marčenko–Pastur theorem henceforth—plays a central role in RMT. This theorem states that the probability density function of the eigenvalues of a random correlation matrix obtained from normal independent and identically distributed series distributed with variance σ^2 is distributed according to the Marčenko–Pastur distribution [see Marčenko and Pastur (1967); Laloux et al. (1999); Guerini et al. (2022)]. More formally

Theorem—Marchenko–Pastur law. *Given N normal independent and identically distributed series of length T, for $N, T \to \infty$ and $Q = \frac{T}{N} \to a \in (1, +\infty)$, the density function of the eigenvalues of the correlations matrix $\hat{\Sigma}$ is given by:*

$$\rho\hat{\Sigma}(\lambda) = \begin{cases} \frac{Q}{2\pi\sigma^2}\frac{\sqrt{(\lambda_{max}^{RMT}-\lambda)(\lambda-\lambda_{min}^{RMT})}}{\lambda} & \text{for } \lambda \in \left(\lambda_{min}^{RMT}, \lambda_{max}^{RMT}\right) \\ 0 & \text{oherwise} \end{cases} \tag{5}$$

where $\lambda_{max\,/\,min}^{RMT} = \sigma^2\left(1 \pm \sqrt{\frac{1}{Q}}\right)^2$ are the upper/lower bounds of the eigenvalues associated with a random matrix with the same variance σ^2 and the same Q of the empirical series.

The above theorem implies that any empirical eigenvalue lying within the boundaries of the theoretical Marčenko–Pastur distribution, which read $\lambda_{max\,/\,min}^{rmt} = \sigma^2\left(1 \pm \sqrt{\frac{1}{Q}}\right)^2$, explains a fraction of the variance σ^2 that is comparable to the one generated by a purely random model where correlations are only spurious. In contrast, eigenvalues lying outside such boundaries contain some pure information about the underlying stochastic process governing co-movements of the variables of interest.

We exploit thisn result in our application of RMT. Accordingly, we do not compare the density functions of the eigenvalues, as it suffices to compare the eigenvalues with the $\lambda_{max\,/\,min}^{rmt}$ Marčenko–Pastur theoretical bounds.

As stated by the theorem above, the Marčenko–Pastur theorem holds under two asymptotic assumptions: (i) $N, T \to \infty$, and (ii) $Q = \frac{T}{N} > 1$ as $N, T \to \infty$. Concerning the second hypothesis, in our study we have that $T = 130$ as we set our time window equal to 6 months of working days and $N = 11$ countries so that $Q = 11.81 > 1$ satisfies the condition. Turning to the first assumption, our dataset is limited in the country dimension (small N). In addition to the theoretical bounds, therefore, we additionally compute the Marčenko–Pastur boundaries using Monte Carlo simulations and we compare our empirically estimated eigenvalues to the simulated ones.

Furthermore, we carry out several robustness checks on the Marčenko–Pastur boundaries. Specifically, we focus on two robustness tests that account for violations of the assumption underlying the Marčenko–Pastur theorem: (i) we apply a rotational random shuffling procedure to our time series to account for the possible autocorrelations present in the stationary time series (see Section 6.1); (ii) we adopt a heavy-tailed random i.i.d. modes which accounts also for the possibility of extreme values in the yields series (see Section 6.2). Additional robustness checks are presented in Appendix C.

Algorithmic representation of the Random Matrix Theory

- *Time series preparation:* standardize the stationary time series so that they have zero mean and unitary variance.
- *Rolling windows:* select the time width and the time step parameters and build the rolling windows.
- *Correlation matrix:* for each rolling window, compute the correlation matrix E and its eigenvalues;
- *Null model:* compute the RMT upper and lower bounds $\lambda^{rmt}_{max\,/\,min} = \sigma^2 \left(1 \pm \sqrt{\frac{1}{Q}}\right)^2$ where $Q = \frac{T}{N}$;[13]
- *Selection:* compare the empirical eigenvalues with the RMT bounds and select only the significant ones.

4.3. Measuring synchronization

An additional measure that summarizes the co-movements that stand out relative to a purely random model is the *inverse participation ratio* (IPR) associated with each eigenvalue i. It formally reads as:

$$\mathrm{IPR}_i = \sum_{j=1}^{N} u_i(j)^4 \tag{6}$$

where $u_i(j)$ stands for the jth element of the eigenvector associated with the ith eigenvalue.[14] The IPR_i is a concise measure for the number of significant variables contributing to the eigenvalue for it is continuous and bounded in the interval $\frac{1}{N} \leq \mathrm{IPR}_i \leq 1$. In particular, we have that:

$$\mathrm{IPR}_i = \begin{cases} 1/N & \iff \quad u_i(j)^2 = \frac{1}{N} \quad \forall j \\ 1 & \iff \quad u_i(j)^2 = 0 \quad \forall j \text{ but one.} \end{cases}$$

In other words, if the $\mathrm{IPR}_i = 1/N$, the N variables equally contribute to the ith eigenvalue. This corresponds to a state of perfect synchronization between countries. At the other extreme, when $\mathrm{IPR}_i = 1$, only one variable contributes to the ith eigenvalue. This would correspond to a state of extreme asynchronicity, according to which the co-movement between variables reflects in reality the movement of only one variable. It is unlikely that in real applications these two extreme cases will emerge, even though they offer theoretical handles to evaluate the degree of synchronization.

5. Results

Let us now present the main results of our empirical analysis of government bond yields in the Eurozone. We first present some statistics about the dynamics of bond yields correlation in our

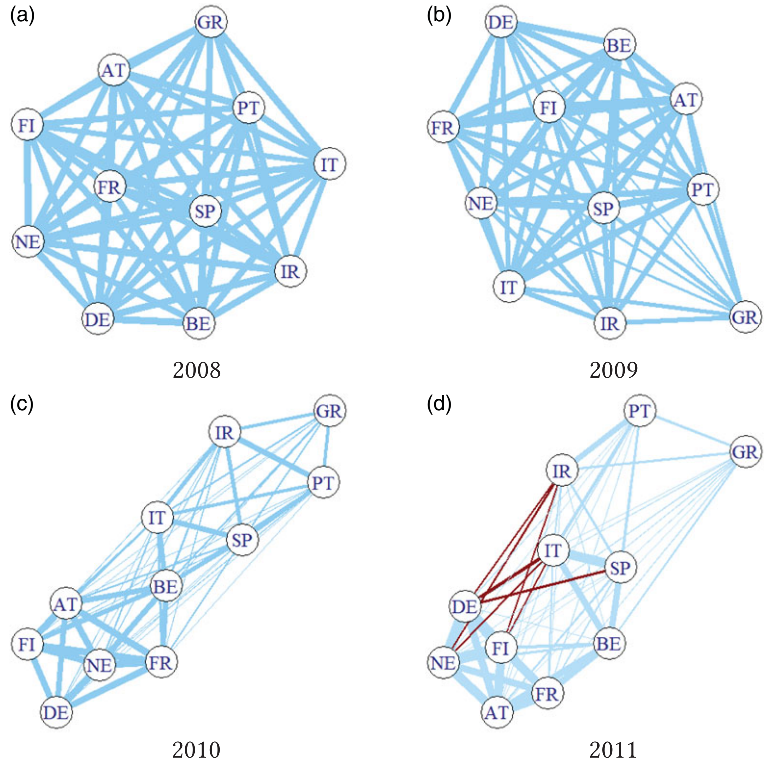


Figure 1. The structure of the bond yields correlation networks from 2008 to 2011. Government bonds with 10-year maturity. The thickness of the links is proportional to the magnitude of the correlations. Positive correlations are light-colored (sky blue), and negative correlation are dark-colored (red). Countries: AT = Austria, BE = Belgium, FI = Finland, FR = France, DE = Germany, GR = Greece, IR = Ireland, IT = Italy, NE = Netherlands, Pt = Portugal, SP = Spain.

sample (Section 5.1). These statistics deliver first insights about the nature of yields co-movements in the Euro area. Yet, they are only an imperfect measure of yields synchronization because they do not account for the presence of spurious correlation. In Section 5.2, we then present the results obtained using our preferred measure of synchronization that combines PCA and RMT, and which allows us to filter out the spurious (noisy) information.

5.1. The network of government bond yields correlations

The empirical correlation matrix among government bond yields E that we analyze (See Equation (1) above) is a combination of a nonspurious correlation component and of a spurious idiosyncratic component—that is, $e_{i,j} = c_{i,j} + \chi_{i,j}$. Following Tumminello et al. (2010) and Diebold and Yılmaz (2014), this matrix can be interpreted as a weighted undirected graph where the vertices represent the stationary normalized yields and the link weights measure their correlations [see also Diebold and Yılmaz (2014)].

Figure 1 visualizes the structure of some of the above-defined correlation networks (from 2008 to 2011) for bonds with 10 years of maturity. The thickness of the links in the networks is proportional to the magnitude of correlations. Negative correlations are depicted in red.

Overall, strong and positive correlations among all yields were present up until 2008. From 2009 onward, instead, the average correlation in yields decreases and it becomes even negative in some cases. Two clusters of countries can be identified in this period. The first group is composed of "core" countries, and it mainly includes Northern countries like Germany, the Netherlands, Austria, and Finland. These countries have strong positive correlations among their yields. In contrast, the bond yields of the second group of countries (the "periphery"), including Greece, Ireland, Portugal, Spain, and Italy, displayed weak or even negative correlations with all the other yields. It must be stressed that negative yields correlations disappeared after 2015 (not shown in the figure) and that the magnitude of yields correlation generally increased, though not recovering the high levels displayed before 2008.[15]

We also quantify how much the structure of correlation networks has changed over time. For this purpose, and following Münnix et al. (2012), we build a similarity index that measures how much the correlation adjacency matrix E in a specific time window w_1 is similar to the correlation matrix of another window w_2. This similarity index is defined as:

$$\mathcal{S}(w_1, w_2) = 1 - \mathbb{E}|E(w_1) - E(w_2)| \tag{7}$$

it represents the average Euclidean distance between two correlation matrices. The closer the index is to 1, the higher is the similarity between the network structures of two time windows. Figure 2 plots the evolution of the similarity index, normalized for the first window, that is, the one of 2003, which we select as the reference window.[16]

The network of 1-year yields (Panel a Figure 2) does not display much variation over time. The similarity index is always very high, between 0.75 and 0.9. In contrast, the network similarity of 5-year and 10-year bond yields (Panel b and c Figure 2) decreased steadily since 2003 hitting the lowest value of about 0.5 between 2011 and 2013. This is following the previous results about the emergence of a core–periphery structure during the Great Recession and the European Debt Crisis.[17]

The above network representation could be affected by endogeneity problems [see e.g. Manski (1993)]. For instance, the link between the yields of countries i and j might be affected by the fact that they are in reality correlated with the yield of a third country m. To control for this problem, we employ the club convergence procedure introduced by Phillips and Sul (2007).[18] The procedure was originally developed for low-frequency data, and the main technical issue concerns the selection of the baseline period for the ordering of the time series. Our time series have more than 4000 daily observations. Accordingly, the selection of clubs might be sensible to the selection of the reference period. We tested the algorithm over different daily selections. We find that two clubs of countries are robustly present over the 2008–2012 period in line with the results discussed above. Nevertheless, the composition of the two clubs is not constant across time.

5.2. Detecting significant synchronization in government bond yields

In the previous section, we highlighted that the network of government bond yields correlations in the Eurozone changed structure during the 2003–2019 period, especially during the Great Recession and the European Debt Crisis. This resulted in divergent bond yields dynamics and the appearance of a core–periphery structure in the network. We now move to a more systematic analysis of bond yields synchronization by employing the methodology presented in Section 4.

Figure 3 shows the evolution of the eigenvalues $\lambda(w)$ obtained from the spectral decomposition of each window-specific correlation matrix $E(w)$, where $w \in \{1, \ldots, W\}$ represents a specific time window. The three plots in the figure refer to the different maturities we consider (1 year, 5 years, and 10 years). Each plot also shows the different boundaries resulting from our selection procedures based on RMT, which allow one to detect significant components. Finally, each plot also allows one to compare the dynamics of eigenvalues with the timing of important events related either to an external shock (e.g. Lehman Brothers' bankruptcy, which represents the start of the Great Recession) or to policy announcements (e.g. the famous "Whatever it Takes" speech by the ECB president Mario Draghi in July 2012 during the European Debt crisis) that have likely had an impact on bond yields (see also Section 7 for a discussion of the policy implications of our results).

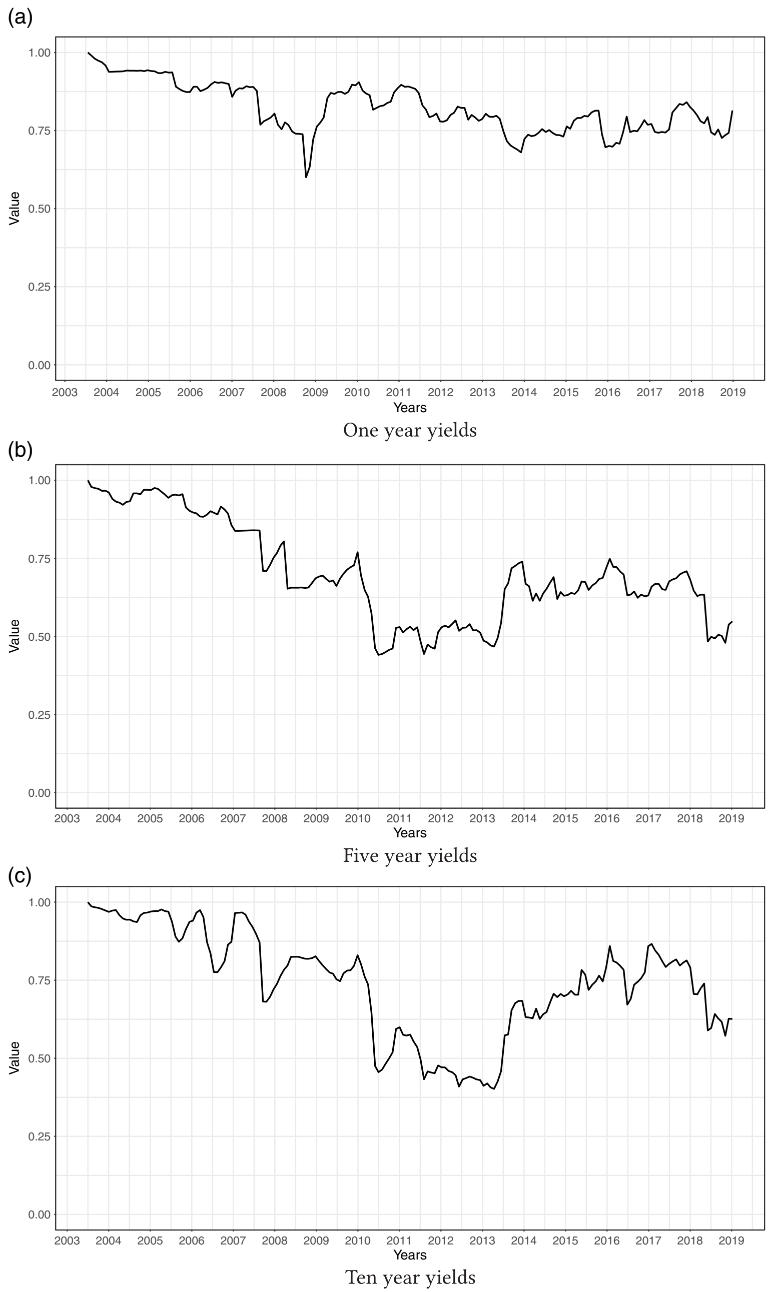


Figure 2. The evolution of the similarity index for bond yields correlation networks at different maturities.

As explained in Section 4.1, the higher is the component, the higher is the fraction of the time-series bond yields variance that it can explain. The fraction of explained variance by each principal component is also called the *absorption rate*. These absorption rates are shown in Figure 4.[19]

The analysis of the plot in the top panel of Figure 3 indicates that only one nonspurious principal component explains the dynamic of 1-year bond yields. Indeed, only the largest eigenvalue is statistically significant according to RMT bounds, and only at specific moments like the Great

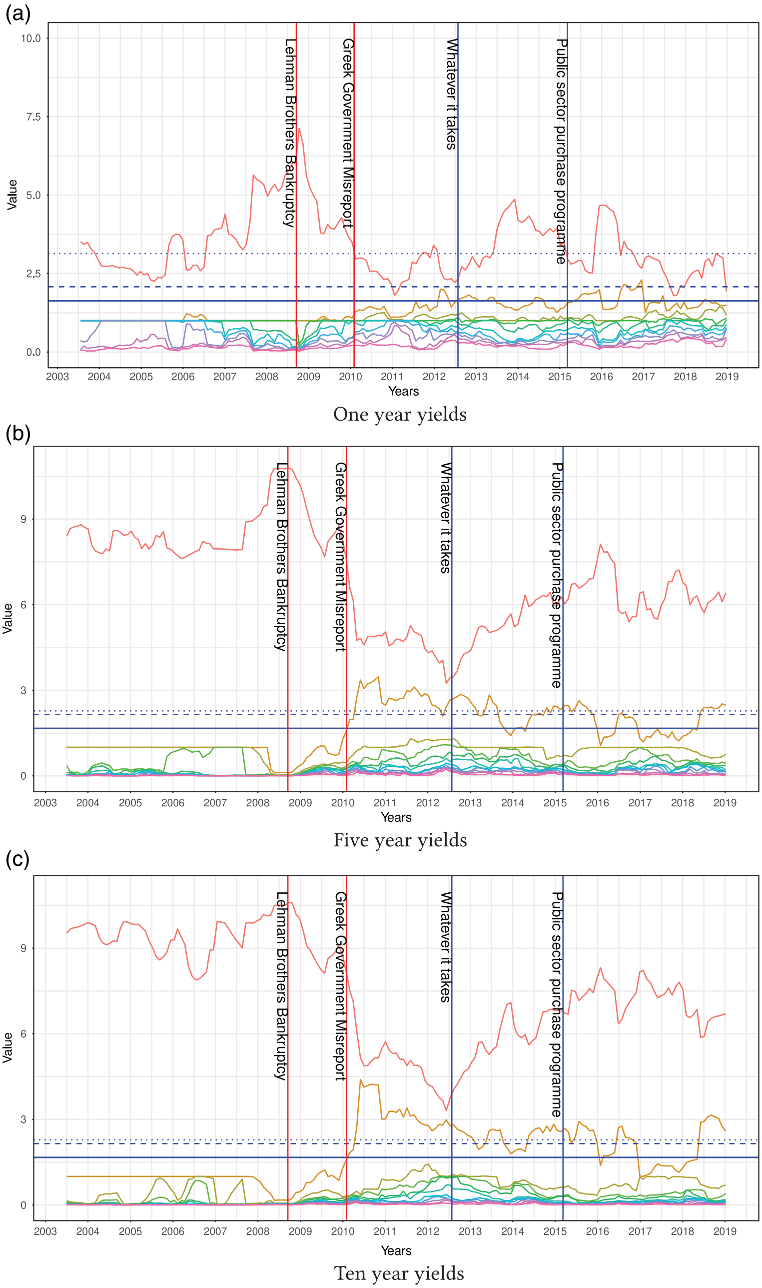


Figure 3. Eigenvalues evolution and multiple theoretical bounds.
Notes: the horizontal solid line indicates the Marčenko–Pastur theoretical upper bound; the horizontal dotted line indicates the Monte Carlo simulated upper bound; the horizontal thinner dotted line indicates the rotational random shuffling upper bound. For both the Monte Carlo simulated model and the rotational random shuffling, 300 Monte Carlo simulations were run. Each window of the RMT exercise contains 130 time observations with steps of 22 observations each.

Recession of 2008–2010, and the Eurozone debt crisis (that started with the Greek government misreport in 2009 and lasted at least until 2015).[20] In addition, the dominant eigenvalue explains about 30% of the total variance except during the Great Recession period, where the share of explained variance is about 50%.

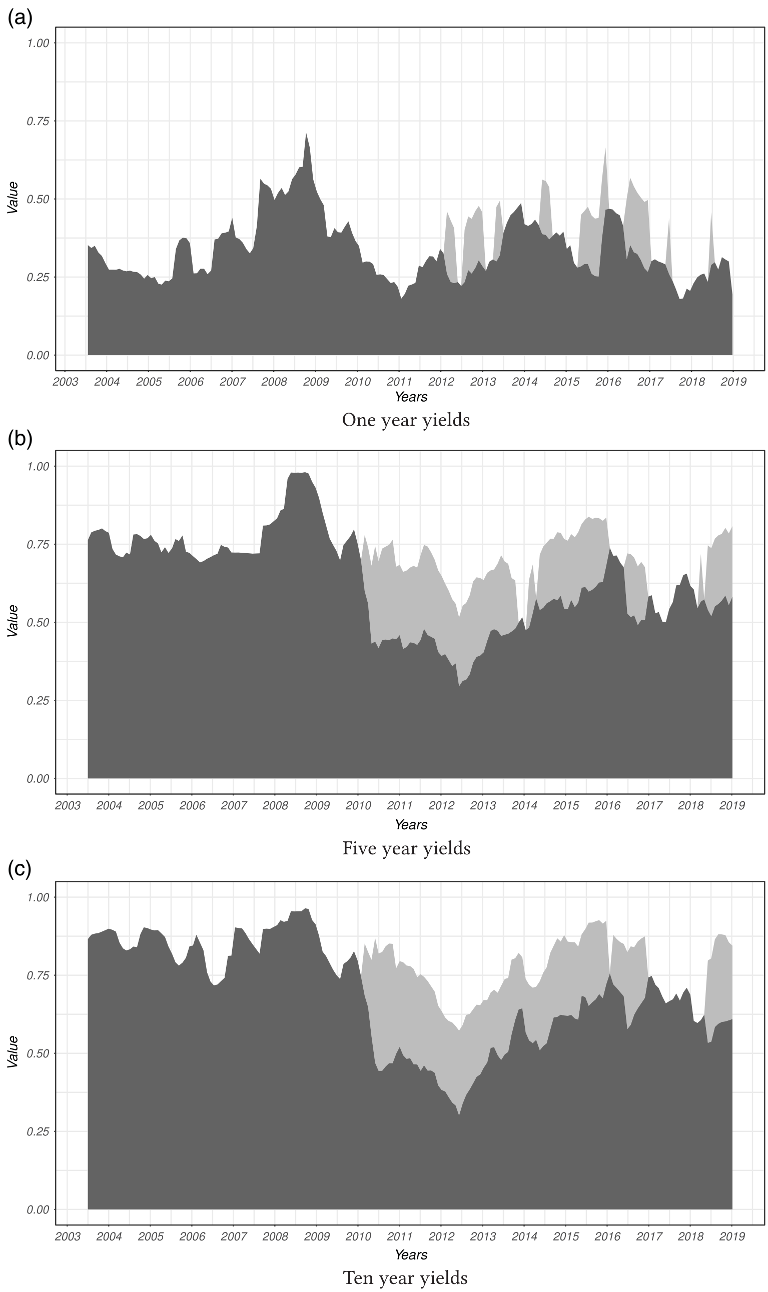


Figure 4. Absorption ratios for significant eigenvalues according to the RMT Marčenko–Pastur theoretical upper bound. First largest eigenvalue in dark gray, and second largest eigenvalue in gray (eigenvalues normalized between 0 and 1).

In contrast to 1-year bonds, the largest eigenvalue of 5-year and 10-year government bond yields is always significant (cf. Panels b and c of Figure 3). This first principal component accounts, on average, for about 75% and 80% of the total variance before 2008, reaching a peak of more than 90% in 2008. Nevertheless, the portion of variance explained by the dominant eigenvalue falls abruptly after 2008 to recover only in 2014. A second principal component becomes statistically significant in that period, and it accounts for a share between 20% and 30% of the total variance.

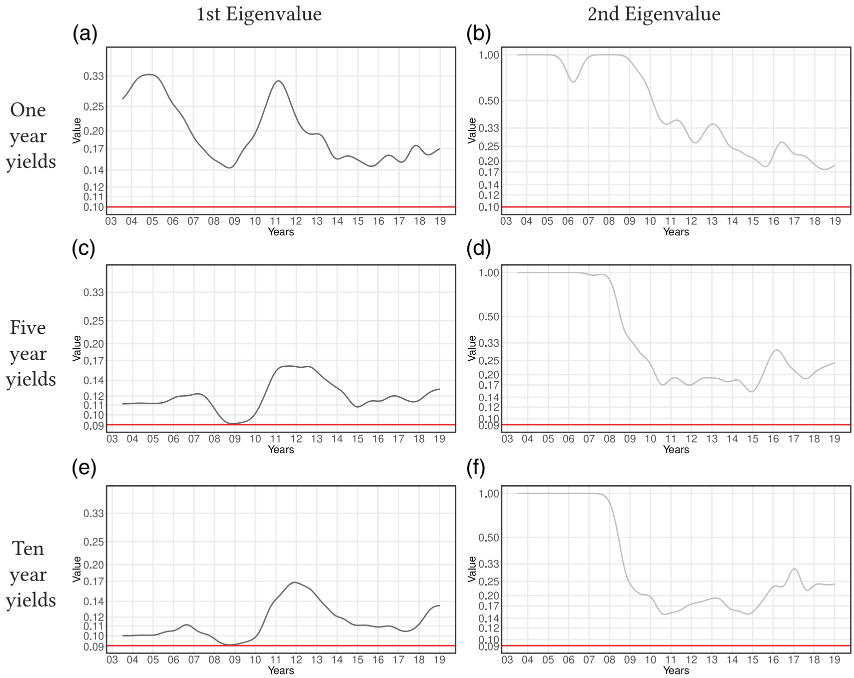


Figure 5. Inverse participation rate.

Let us now discuss the IPR to quantitatively summarize our first result. As explained in Section 4.3, the IPR of each eigenvalue is bounded between $\frac{1}{N}$ and 1, where the two extremes correspond respectively to the highest and the lowest possible synchronization. Figure 5 shows the IPR of the first- and second-largest eigenvalues. Considering 1-year yields the IPR of the largest eigenvalue shows (Figure 5, panel a) that the highest degree of synchronization is reached during 2009 and 2015 and, periods in which the first factors can account for about 7 countries out of 10.[21] Between 2009 and 2014, differently, the co-movement is sustained on average by half of the countries. The IPR of the largest eigenvalues for the 5-year (Figure 5, panel c) and 10-year yields (Figure 5, panel e) are similar: most countries co-move until 2008. Next, right after the financial crisis, in 2009, the IPR almost reaches its lowest value due to the sudden fall of all yields simultaneously. Then, between 2010 and 2012 the synchronicity in yields gradually disappears and the IPR reaches its highest values and the main factor only accounts for half of the countries' variances. Lastly, from 2012 onward, synchronization reappears, but only in a milder form to the pre-crisis levels. Indeed, the IPR converges to a stable value which is higher than the pre-2008 one. The dynamics of the second-largest eigenvalues, instead, are similar for all the three different maturities under investigation (Figure 5, panels b, d, f). As the second eigenvalue is not significant before the crisis, its IPR is close to 1. During the financial crisis period, around 2008 and 2009, however, it rapidly reaches low values which imply a co-movement sustained by around a half of the Eurozone countries. The second IPR remains stable even after the crisis. The analysis of the IPR, therefore, confirms the evidence of two distinct groups of countries that have been generated during the crisis. This division, however, has remained significant even after the end of the crisis.

5.3. Detecting the synchronization within the co-movements

Finding that one principal component explains a large fraction of the total variance of bond yields is the first condition for synchronization. Yields, however, can contribute in different—and

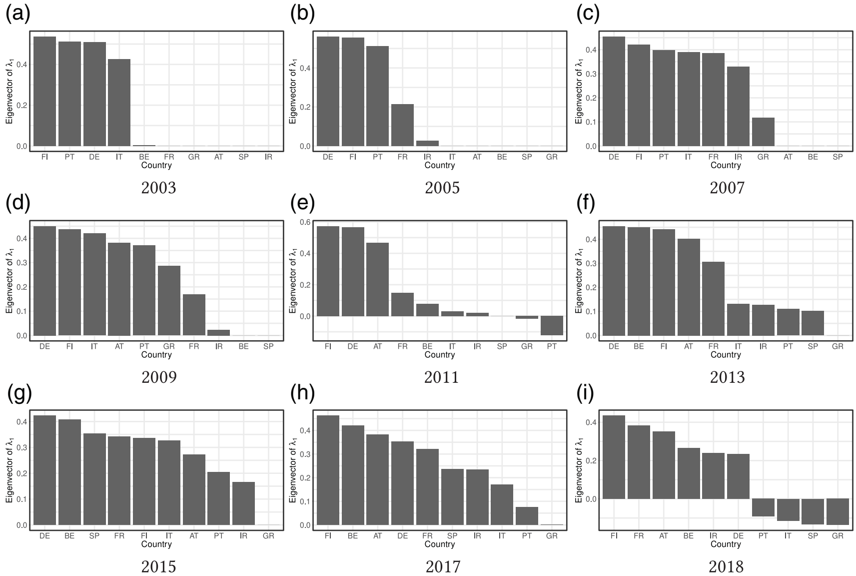


Figure 6. One-year bonds. Yearly average of eigenvector elements associated with the first principal component. Countries: AT = Austria, BE = Belgium, FI = Finland, FR = France, DE = Germany, GR = Greece, IR = Ireland, IT = Italy, NE = Netherlands, Pt = Portugal, SP = Spain.

possibly opposite—directions to a component dynamic. For this reason, we also investigate in detail the components of the eigenvectors associated with each significant eigenvalue.

As we mentioned in Section 4, our approach to bond yields synchronization includes the analysis of the eigenvector components associated with each significant eigenvalue as a second condition for synchronization. To this end, Figure 6 shows the evolution of the elements of the eigenvector associated with the largest eigenvalue of 1-year bond yields. The dynamics of these eigenvector components is very fragmented. No group dynamics can be detected, and eigenvector elements evolve heterogeneously throughout the whole sample period. We conclude that synchronization in short-term yields is much weaker than synchronization in medium- and long-term yields.

Let us now turn to the eigenvectors of bond yields at 5 and 10 years, which had at least one significant component being over the whole sample period. The bar plots in Figures 7 and 8 show the evolution of the elements of the eigenvector associated with the largest eigenvalue of 5-year and 10-year yields, respectively, between 2008 and 2014. Eigenvector elements are all very similar until 2008, documenting a high level of synchronization in bond yields. The pattern changes dramatically afterward, and eigenvector elements start to diverge since 2009. In this respect (and like in Section 5.1), we can identify two groups of countries. The eigenvector components of the first group of *core* countries (mainly corresponding to Northern economies in the Eurozone) all display high values until 2014. In contrast, the values of the elements of the other group, the *periphery* (Italy, Ireland, Spain Portugal, and Greece), steadily decrease after 2009. Eigenvector elements even become of opposite sign for some countries like Portugal at some specific windows. The above group dynamics in bond yields is even more evident when we analyze the elements of the eigenvector associated with the second principal component, which is significant between 2008 and 2014 (see Figures 9 and 10). In particular, the eigenvector elements corresponding to yields of Southern economies like Greece, Portugal, Spain, and Italy are all of opposite sign between 2011 and 2015 relatively to the first group of countries. Interestingly, between 2011 and 2013—that is, amid the European Debt Crisis—also eigenvector elements of French and Austrian bond yields switch sign.[22]

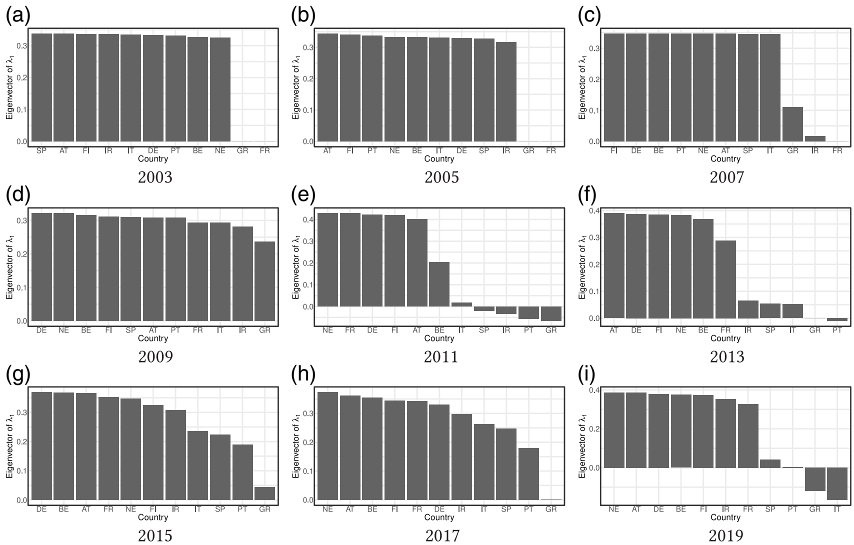


Figure 7. Five-year bonds. Yearly average of eigenvector elements associated with the first principal component. Countries: AT = Austria, BE = Belgium, FI = Finland, FR = France, DE = Germany, GR = Greece, IR = Ireland, IT = Italy, NE = Netherlands, Pt = Portugal, SP = Spain.

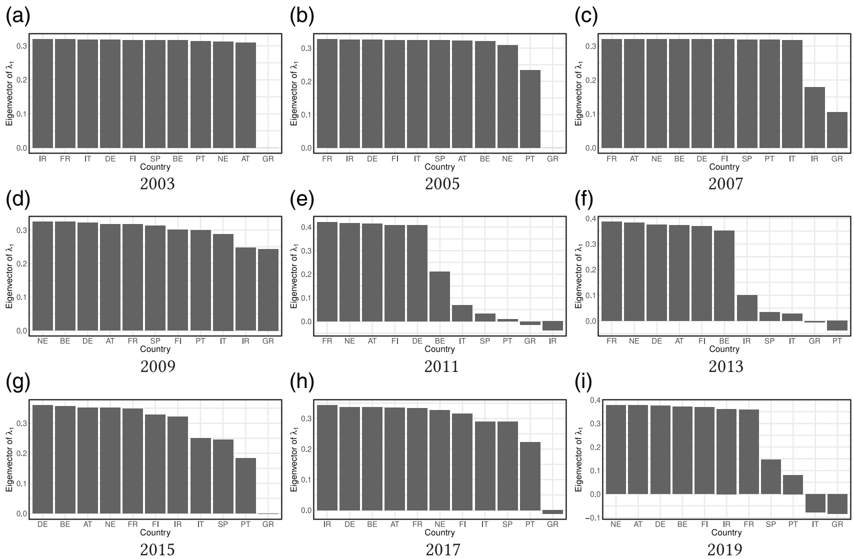


Figure 8. Ten-year bonds. Yearly average of eigenvector elements associated with the first principal component. Countries: AT = Austria, BE = Belgium, FI = Finland, FR = France, DE = Germany, GR = Greece, IR = Ireland, IT = Italy, NE = Netherlands, Pt = Portugal, SP = Spain.

6. Robustness analyses

The results presented in the previous section indicate that movements in short-term government bond yields are pretty much asynchronous in the Eurozone. Medium- and long-term yields were instead very synchronized until the Great Recession hit in 2008 and began to move asynchronously afterward. This yields asynchronicity became dramatic between 2011 and 2014 with the emergence of a clear divide between Northern and Southern countries in the Eurozone.

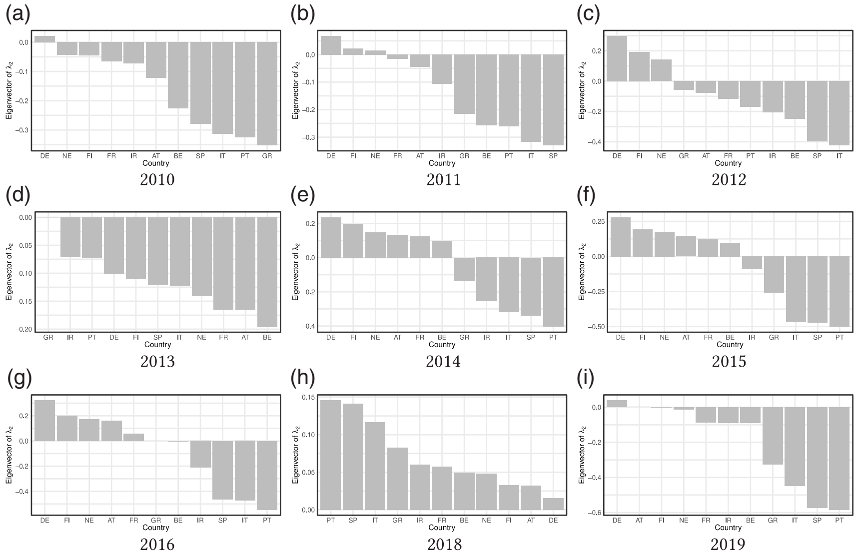


Figure 9. Five-year bonds. Yearly average of eigenvector elements associated with the second principal component. Only years wherein the second principal component is significant are displayed. Countries: AT = Austria, BE = Belgium, FI = Finland, FR = France, DE = Germany, GR = Greece, IR = Ireland, IT = Italy, NE = Netherlands, Pt = Portugal, SP = Spain.

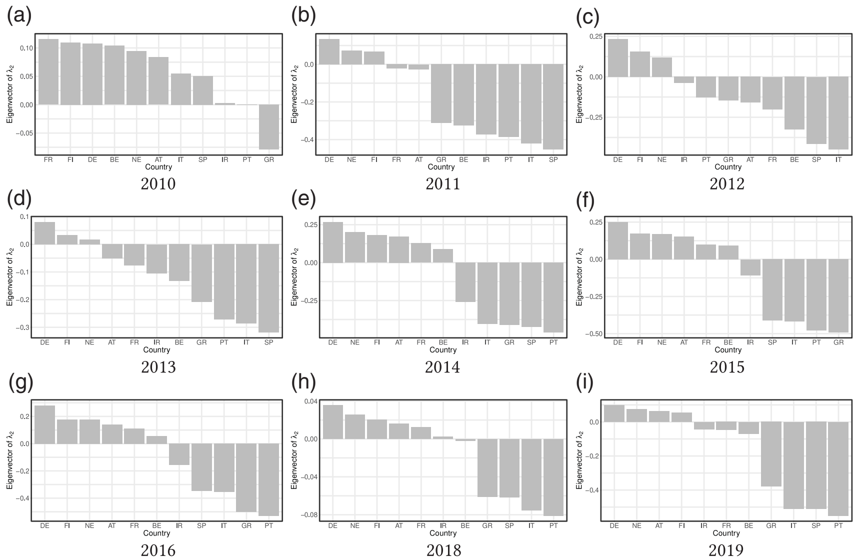


Figure 10. Ten-year bonds. Yearly average of eigenvector elements associated with the second principal component. Only years wherein the second principal component is significant are displayed. Countries: AT = Austria, BE = Belgium, FI = Finland, FR = France, DE = Germany, GR = Greece, IR = Ireland, IT = Italy, NE = Netherlands, Pt = Portugal, SP = Spain.

Our evidence could however be biased by at least five problems. First, the patterns we uncovered could be limited to the specific bond maturities we considered (1 year, and 5 and 10 years). Bonds at alternative maturities could exhibit very different dynamics from the one documented in the previous section. Second, our results could be affected by the length and step used to build rolling windows, as well as by the filtering technique employed (first differencing). Third, the statistical significance of principal components could be biased by the presence of autocorrelation in the filtered series. Finally, the statistical significance of principal components could be biased upwardly if the distributions of the underlying time series are heavy-tailed. In the remainder of this section, we present the results of robustness checks that address all the above issues in detail.[23]

6.1. Controlling for autocorrelation in the data

Previous works using RMT have shown that the empirical eigenvalue distribution becomes fat-tailed when time series are autocorrelated [see Aoyama et al. (2017)]. A fat-tailed distribution of eigenvalues implies that more principal components might turn significant according to the Marčenko–Pastur upper bound, even if these do not truly reflect a higher explained variance of the cross-correlations. To control for this problem, we perform a random rotational shuffling Monte Carlo exercise of the kind suggested by Aoyama et al. (2017) and Iyetomi et al. (2011). In particular, we let

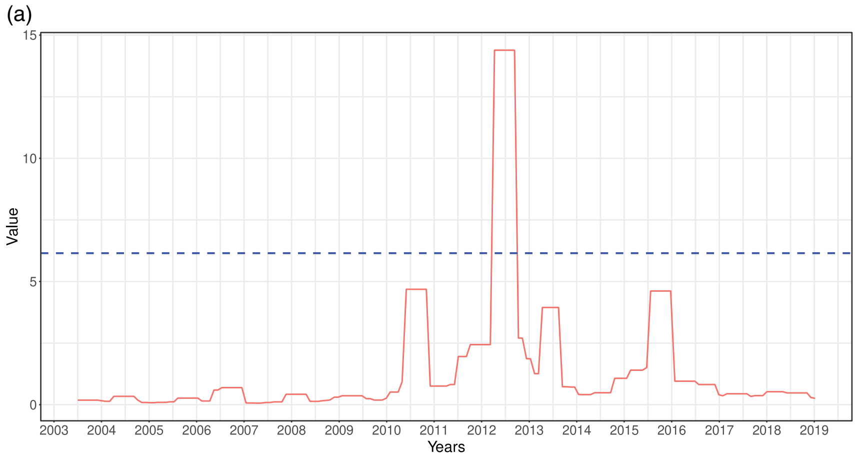
$$x_n(t_i) \to \begin{cases} x_n\left(t_{T-|i-\tau|}\right) & \text{if } i \leq \tau \\ x_n\left(t_{|i-\tau|}\right) & \text{if } i > \tau \end{cases} \tag{8}$$

where $x_n(t_i)$ is the observation of the time series of the bond yields of the nth country at time t_i with $i \in [1, T]$, and τ is a random number drawn from the interval $[1, T]$. The rotational random shuffling procedure divides the series into two parts: from 1 to τ and from $\tau + 1$ to T. The two parts are then switched. The procedure is identical to a shift of the series by a random number, where the observations falling out of the length of the series are repositioned at its beginning. Since every series is rotated by a different number, this rotation breaks the cross-correlation between the series, while maintaining the autocorrelation within the series (the observations of the series do not change order within the two parts).[24] We perform multiple Monte Carlo iterations of the above rotational shuffling exercise within each rolling window. For each window, we then compute the average of the maximum eigenvalues of the simulated series and the correspondent confidence intervals. The latter average constitutes a new upper bound that can be used to test the statistical significance of empirical eigenvalues. Eigenvalues lying above the random rotational shuffling bound contain a fraction of variance that is beyond the one explained by autocorrelation of the series and that it can then only be related to systematic cross-correlation. Notice that such an upper bound is larger and thus more stringent than the ones obtained through the procedure described in Section 4.

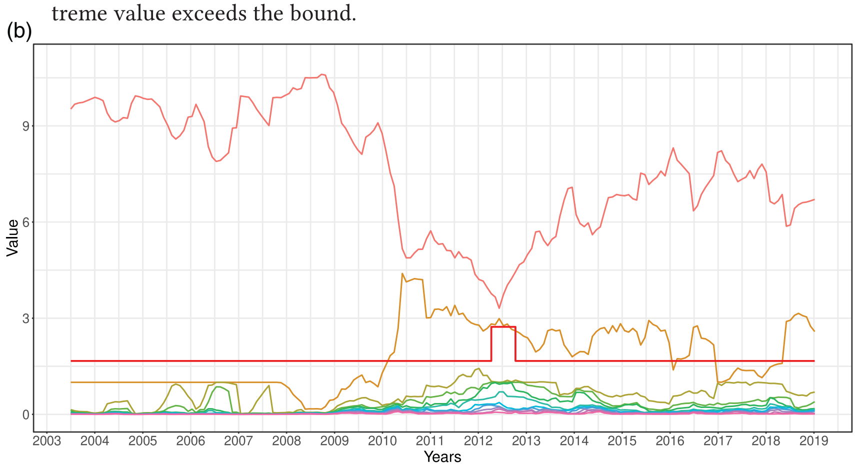
The upper bound stemming from the above-described procedure corresponds to the thin dotted lines in Figures 3 and C3. All the components that were significant according to the standard RMT bounds are also significant according to the upper bounds based on rotational random shuffling. Our main results are thus robust to this additional restrictive test. In particular, the lack of synchronization for short-term yields is even more evident, as the largest component becomes significant only in very few windows. In contrast, the largest component of medium- and long-term yields is always above the new bound and is thus always significant once we control for autocorrelation in the series. Finally, the significance of the second-largest component is somehow reduced but not eliminated.

6.2. Controlling for heavy tails in the data

Besides the bias introduced by autocorrelation and discussed in the previous section, the presence of outliers in the filtered data might in general suggest that a Gaussian null model is not a correct benchmark for the selection of significant components. The work by Biroli et al. (2007) considers the case where the distribution of the observations is heavy-tailed. It also proposes a procedure to adapt the upper bounds of the Marčenko–Pastur law when the most extreme observation in a specific window overcomes a threshold that depends on the number of series and their



Comparison between the $(NT)^{\frac{1}{4}}$ limit and the extreme observation S. The upper bound of the random matrix theory should be adjusted only if the extreme value exceeds the bound.



Eigenvalues with the heavy-tailed adjusted upper bound.

Figure 11. Heavy-tailed random null model for yields with 10-year maturity.

length. More precisely, let S be the maximum standardized yield (i.e. the most extreme value) in a specific window of observations X. Then, assuming that the distribution of X follows a heavy-tailed distribution, the RMT adjusted upper bound for the eigenvalues $\lambda_{\max}$ can be computed as follows:

$$\lambda_{\max} = \begin{cases} \sigma^2 \left(1 + \sqrt{\frac{1}{Q}}\right)^2, & \text{if } S \leq (NT)^{\frac{1}{4}} \\ \left(\frac{1}{Q} + \frac{S^2}{T}\right)\left(1 + \frac{T}{S^2}\right), & \text{if } S > (NT)^{\frac{1}{4}} \end{cases} \tag{9}$$

where $Q = T/N$ and $\sigma^2 = 1$ because series are standardized. Figure 11 shows the results of our robustness check on 10-year yields using the procedure in Biroli et al. (2007). The new adjusted bound applies only for some windows in 2012. Even at this new level of significance, the first component remains strongly significant, whereas the second one loses significance only in one window in 2012. In the case of short-term yields, the adjustment of the upper bound applies for some windows during the period 2011–2012 as well as in 2016. The result is that the largest component becomes even less significant. In the case of 5-year yields, the new restriction applies for some windows in 2010, 2014, and 2017. The value of the new limit in some of the cases is significantly high, and it is mostly explained by the extreme values that some yields, especially Greek ones, reached during that periods. The periods of extreme values are nonetheless very limited, and even the introduction of such high limits does not undermine the validity of our main results.

7. Discussion and implications

The evidence we presented so far robustly indicates that synchronization is not a statistically significant phenomenon for government bond yields in the Eurozone at short-term maturities. In contrast, yields at medium- and long-term maturities were very much synchronized until the arrival of the Great Recession and the European Debt Crisis. The strains induced by these two crises generated an abrupt fall in medium- and long-term yields synchronization. This is highlighted by the loss in the share of the variance explained by the largest principal component and by the emergence of a second significant component during the period corresponding to the above two crises, that is, between 2008 and 2014. Moreover, the elements of the eigenvectors associated with the above principal components were very heterogeneous during the periods corresponding to the two crises. Some of these eigenvector elements even took negative values during that period while others remained positive. Synchronization in medium- and long-term yields increased again since 2015, however without fully recovering the high levels observable before the Great Recession.

There are several reasons why one should expect short-term yields to be less synchronized than long-term yields. Abreu and Brunnermeier (2002) suggest that rational traders under uncertainty incur holding costs while becoming aware of arbitrage opportunities. The synchronization risk, therefore, originates a delayed arbitrage, as rational traders price the time of uncertainty. Using this theory to interpret the specific case of government bonds, we argue that short-term bonds can be more exposed to delayed arbitrage than long-term ones. Furthermore, Hwang and Kim (2021) show that trade integration affects long-term rater than short-term co-movements, which is totally coherent with our findings. Finally, Hirata et al. (2013) finds that global house prices are synchronized, although they are not affected by global monetary policy shocks. This result is also in line with the fact that long-term maturities are highly syncronized across countries given that long-term yields are more relevant than short-term ones for house mortgages.

Against this background, we discuss the above patterns by using the financial interpretation of principal components outlined in Section 4. In portfolio theory, the normalized elements of the eigenvector associated with significant eigenvalues represent the weights to be assigned to the different bonds in one portfolio, also called the eigenportfolio, with risk measured by the eigenvalue. In addition, positive and negative eigenvector components correspond to long and short positions on the government bonds, identified up to a rotation. Thus, if some differential between the eigenvector components exists, investors update their positions by reallocating their wealth over the bonds. These operations sustain the price of some bonds to others. Speculation is feasible even if the components are of the same sign [see Avellaneda and Lee (2010)].

On these grounds, our results show the presence of *divergence trades* and *flight-to-quality* effects that introduced asynchronicity in bond yields dynamics and that contributed to amplifying the sovereign debt crisis in the Eurozone. More precisely, the fact that two eigenvalues were significant for bonds at 5- and 10-year maturities between 2008 and 2014 indicates that two eigenportfolios were likely to be traded in that period. A possible interpretation of the results is that the portfolios were characterized by a long position for core Eurozone economies (Germany, France, Netherlands, Austria, Finland, and Belgium) and low exposition or even short positions for the peripheral economies (Spain, Greece, Portugal, Italy, and Ireland). Following this interpretation, at the peak of the European Debt Crisis (i.e. between 2011 and 2013), the eigenportfolio with lower risk, that is, the one associated with the second largest eigenvalue, implied short positions on government bonds of all Eurozone countries but Germany, the Netherlands, and Finland. Accordingly, in those periods government bonds of those countries became a *safe asset* [see Caballero et al. (2017)] over which strong long positions were taken.[25]

The above-documented dynamics of bond yields has several implications for the conduct of policy in the Eurozone. Yields asynchronicity hampers the effectiveness of the common monetary policy via the interest rate channel. In particular, finding that eigenvector components of government bond principal components are heterogeneous and with differing signs indicates that common monetary policy actions by the European central bank affect asset returns differently across different countries. For interest rate policies to be more symmetric in the Eurozone, instead, government bond yields of the members should tend to move jointly, as if they were unique interest rate. Furthermore, government bonds are the main type of collateral used to secure loans in interbank markets and their value directly impact the ability of banks to collect liquidity. In that respect, the Eurozone differs from other monetary unions, as several bonds are associated with the same currency. As a result, if the dynamics of the interest rates on government bonds differ, liquidity is likely to move heterogeneously across Eurozone countries. Last but not least, excessive divergence in government bond yields may map into divergent dynamics of government debts within a monetary union, complicating fiscal and monetary policy interactions in response to economic shocks and even posing threats to the existence of the union itself.

Notice that the existence of spurious correlation would affect our monetary policy conclusions in three ways (see Appendix D for a comparative exercise in which the spurious correlation is not filtered out). First, it would underestimate synchronization by around 10%, suggesting a more urgent or strong policy reaction than what would be actually optimal. Second, around the sovereign debt crisis, the RMT-significant PCA is more precise in detecting the turning point of synchronization in 2012, while the average correlation appears smoother. Lastly, in addition to a possibly strong synchronization factor, we stress that synchronization is achieved if any divergence factor is not RMT-significant. The RMT, therefore, identifies the periods when policy is needed the most, that is, the periods when the second factor emerges and not necessarily those when the first factor is low. Such a distinction would not have been possible without the RMT method.

The importance of government bond yields synchronicity for the functioning of Eurozone monetary policy has also been acknowledged by members of the ECB board in the past [see e.g. Draghi (2012); Cœuré (2017)]. In addition, the presence of excessive divergence in sovereign bond premia has been used to support the implementation of unconventional monetary policies in the Eurozone. With this perspective in mind, Figure 3 discussed in Section 5.2 reveals that the turning point in bond yields synchronization coincides with the notorious "whatever it takes" statement by the ECB President Mario Draghi on 26 July 2012. Synchronization gained momentum after the announcement, as the variance explained by the first component (and measured by the value of the dominant eigenvalue) rapidly increased, while the importance of the second principal component diminished. These trends continued also after the start of the Public Security Purchase Program (PSPP), explicitly targeting the purchase of Eurozone government bonds in the secondary market (cf. Figure 3). Nevertheless, the second component remained significant in many windows also after the start of the PSPP, with eigenvector elements of some countries still being negative (see Figures 9 and 10 as well as Section 5.2). This suggests that divergence trades survived after the implementation of the latter program. The foregoing case study does not allow one to infer any causal impact of unconventional monetary policies. Nonetheless, it suggests the presence of a relationship between the chronology of QE events and a significant recovery of bond yields synchronization in the Eurozone. Moreover, the fact that the statistical significance of a second common factor persists also after the bond purchasing program poses concerns about a possible return of diverging yields dynamics once QE programs will be over.

8. Concluding remarks

We have employed a novel econometric procedure combining PCA and RMT to detect significant synchronization in the daily time series of Eurozone government bond yields at different maturities. We found that bond yields at short maturities are in general poorly synchronized. In contrast, bond yields at medium- and long-term maturities were highly synchronized until the onset of the Great Recession. This recession and the subsequent European Debt Crisis resulted in a disruption of synchronization in the government bond yields of Eurozone countries. This is evidenced by the loss of significance of the first principal component and by the emergence of a significant second component. In addition, we find that in the aforementioned periods, the eigenvector elements associated with significant components became highly heterogeneous and even displayed opposite signs across countries. We also provided a financial interpretation of our main results grounded on eigenportfolio theory [Avellaneda and Lee (2010)], and we discussed how our evidence indicates that yields asynchronicity can be related to the presence of divergence trades and flight-to-quality effects. Finally, we showed that our results are robust to the use of different maturities, the use of alternative windows to compute principal components, the use of different filtering techniques, and the presence of autocorrelation and heavy tails in the data.

Our study could be extended in several directions. First of all, we did not consider lagged correlations. Such an extension is possible either by implementing additional static factor models [Bouchaud and Potters (2015)] or by focusing directly on dynamic factor models. This second extension would allow one to precisely evaluate the pass-through of monetary policy across different yield maturities. Second, our study could be extended to encompass the main refinancing operation rate (set by the central bank) and different banks rates (the interbank market one as well as the ones of loans). An enlarged dataset might also include data on real economic variables as well as monetary ones to investigate synchronization between financial and real activities.

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Notes

1 For the sake of comparison, in Appendix D, we show that alternative synchronization measure based on average correlation lead to underestimate bond yields synchronization compared to our RMT-based metric. See also Section 7 for more discussion of the implications of this bias for monetary policy.

2 A notable application of static factors to the analysis of bond risk premia is the one by Cochrane and Piazzesi (2005). They regress the excess 1-year bond return on a factor constructed from a linear combination of five forward spreads and find that a single factor predicts 1-year excess returns on 1–5 year maturity bonds with up to 43%.

3 "An ETF is an investment vehicle, with a specific architecture that typically seeks to track the performance of a specific index" [see Lettau and Madhavan (2018)].

4 The article shows that a portfolio weighted by the ETF dominates the portfolio weighted by the principal components in some periods, while the opposite holds for some other periods. The performance of the two types of portfolios is nonetheless comparable, and the alternating dominance of one of the two is never striking. It can be observed, accordingly, that portfolios weighted by principal components do not outperform alternative portfolios strategies, but that principal components strategies are comparable and sometimes preferable in terms of Sharpe ratio maximization to alternative portfolio strategies.

5 PCA has also been applied to the analysis of systemic risk in financial markets and business cycle synchronization [see Guerini et al. (2022)]. In particular, Billio et al. (2012) include principal components extracted from financial series as measures of systemic risk and early warnings indicators. Kritzman et al. (2011) focus on the fraction of total variance explained by the main factors, extracted from PCA, as an index of systemic risk.

6 In this paper, we focus on synchronization. While convergence is defined as the tendency of different series to reach a unique value, synchronization only implies that the different series are moving together, for instance increasing or decreasing together, notwithstanding possible differences in absolute values. Synchronization might be seen in this sense as a milder form of convergence.
7 Data for France are European data from the ECB. In the study, therefore, France stands as a representative country for the Eurozone in general.
8 In his analysis, the common international factor consists of weekly averages of stock market indexes, the intensity of trade is estimated using a gravity model, the financial integration is represented by a measure of foreign assets that are domestically owned, and the exchange rate volatility, a dummy for being a member of the European Stability Mechanism, and a dummy for being a member of the European Monetary Union account for the monetary integration.
9 For some maturities, we postpone the beginning of the series due to a lack of observations before specific dates. In addition, not all bond maturities are available for all countries. See Appendix A for a detailed description of these two issues.
10 We also performed a cointegration analysis (by employing Johansen's procedure) to detect cointegration relationships among government bond yields of different countries at the same maturity. We also checked whether these relationships changed over time, by previously identifying relevant dates over which divide our time series based on clustered structural breaks analysis (K-Means Cluster Analysis and Ward Hierarchical Clustering). The results indicate that the order of cointegration increased after 2008 for all the three yield maturities considered in our sample. We conclude that the number of common trends has increased after the strains of the Great Recession and the European Debt crisis. However, this analysis does not allow us to interpret the cointegration relations as co-movements among time series. Indeed, common trends are mere linear combinations of our nonstationary time series. They do not provide information about whether the time series belonging to the same linear combination also evolve together. Nonetheless, the increasing number of common trends suggests already a possible diverging dynamic in bond yields after 2008.
11 This portfolio is also known in the literature as the "market mode" portfolio [see Bouchaud and Potters (2015)].
12 Since factor models can be estimated through principal components [Stock and Watson (2002); Onatski (2010)], our method can also be used for factor selection. The factor analysis is discussed in Appendix B.
13 Notice that the Marčenko–Pastur bounds are our baseline reference, but alternative null models can be considered (e.g. fat-tailed models and random rotational shuffling).
14 The fourth power is needed because for positive semi-definite symmetric matrices (like the covariance matrix), the sum of squares of the eigenvectors is always unitary, that is, $\sum_{j=1}^{N} u_i(j)^2 = 1$.
15 This is especially the case for Greece, whose yields still display a weak correlation with yields of other countries in the Eurozone.
16 It shall be noticed that, even if two windows are similar to the first one, they might be very different from one another. What one can say is only that they have a similar number of weights with positive and negative links, not whether the vertices of the networks of the two windows are correlated similarly in the two periods.
17 The similarity index can additionally be used to measure the network turbulence—that is, how fast changes in the network structure occur. Thus, we also calculated the similarity index between each window and the windows of 3, 6 months, and 1 year before. We find that the speed of the changes in the network structure increases before the periods of crisis, that is, around 2007, 2009, and 2017.
18 For this purpose, we use the R package `ConvergenceClubs` developed by Sichera and Pizzuto (2019).
19 The absorption rate of the ith component is formally defined as $\mathrm{AR}_i = \frac{\lambda_i}{\sum_{j=1}^{J} \lambda_j}$, with $i \in \{1, \ldots, J\}$. The absorption rates of statistically significant components according to RMT are shown in Figure 4.
20 The misreport of public finances data by the Greek government occurred in late 2009. However, it was officially announced by the European Commission in January 2010. The plots in Figure 3 use the latter date.
21 Notice that for 1-year yields, we are not able to include Greece in the sample. The number of countries contributing to the co-movement is computed as the inverse of the IPR.
22 For the identification of the signs of the components, see Appendix B.
23 In addition to the robustness checks addressing the above issues, we also repeated the analysis by computing cross-correlations using the Spearman rank correlation coefficient rather than the Pearson coefficient. This type of control has also been used in the literature on business cycles synchronization [see e.g. Belo (2001)]. The results we obtain match (qualitatively and quantitatively) those obtained by applying the Pearson correlation coefficient.
24 Consider, for instance, a time series of $T = 130$ observations where we draw a random number $\tau = 54$. The observations from 55 to 130 take the positions from the $|55 - 54| = 1$st to the $|130 - 54| = 76$th. The observations from 1 to 54 take the positions from the $130 - |1 - 54| = 77$th to the $130 - |54 - 54| = 130$th.
25 Notice that since we identify the signs of the components through the rotation that fits the time series the best, a different interpretation based on a different rotation would not be robust (See Appendix B). Our results, moreover, is in line with the literature reviewed in the paper.
26 This choice of loadings is common in the literature for its asymptotic properties [see e.g. Stock and Watson (2002)]. In our case, however, given $n = 11$ we cannot claim any particular asymptotic property so that the multiplication by $\sqrt{n}$ can be seen as a simple rescaling factor.

27 Unfortunately the 1-year yields does not allow a friendly visualization due to the extreme values of some yields, especially Greece. The high frequency of the data also makes visualization of the filtered time series and their factors useless. Finally, notice that already for the 5-year yields, the levels of the second factor are already influenced by the extreme values of some time series. It is worth remembering, however, that factors for the time series in levels are derived from the factors computed on the first differences by adding any scalar to the sequential sum of their values. The absolute values of the factors in levels matter, therefore, only for the graphical visualization, while the analysis of the correlations between the factors and the series considers only the slope of the factor.

28 Data for maturities lower than 1 year and above 10 years begin only in 2008 and 2006, respectively.

29 For the sake of brevity, we do not report the results using second-order differences. However, they are available from the authors upon request.

30 The window is kept as in the baseline analysis: 6 months window length (i.e. 130 daily observations) and 1-month step length (i.e. 22 daily observations). The combinations of lower and upper bounds are such that 2–22 selects only the frequencies between 2 days and 1 month, 2–65 between 2 days and one-quarter, 65–261 between the quarter and the year, and so on for every possible combination.

31 See, for example, "Frankel and Rose (1998) and almost all subsequent studies measure synchronization of business cycles of two countries as the bilateral correlation of some measure of (de-trended) real economic activity" [Inklaar et al. (2008), p. 648].

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A. Dataset

Table A1. Short-term daily data

Bloomberg ID	Maturity	Country	Begin	End
GTBEF3M Govt	3 months	BE	2006-07-13	2019-02-19
GTFRF3M Govt	3 months	FR	2006-07-13	2019-02-19
GTDEM3M Govt	3 months	DE	2006-07-13	2019-02-19
GTGRD3M Govt	3 months	GR	2007-10-22	2019-02-19
GTITL3M Govt	3 months	IT	2006-07-13	2019-02-19
GTNLG3M Govt	3 months	NE	2006-07-13	2019-02-19
GTPTE3M Govt	3 months	PT	2011-08-08	2019-02-19
GTESP3M Govt	3 months	SP	2006-07-13	2019-02-19
GTBEF6M Govt	6 months	BE	2006-07-13	2019-02-18
GTFRF6M Govt	6 months	FR	2006-07-13	2019-02-18
GTDEM6M Govt	6 months	DE	2006-07-13	2019-02-18
GTGRD6M Govt	6 months	GR	2007-10-22	2019-02-18
GTITL6M Govt	6 months	IT	2006-07-13	2019-02-18
GTNLG6M Govt	6 months	NE	2006-07-13	2019-02-18
GTPTE6M Govt	6 months	PT	2011-08-08	2019-02-18
GTESP6M Govt	6 months	SP	2006-07-13	2019-02-18
GTATS1Y Govt	1 year	AT	2008-04-03	2019-02-21
GTBEF1Y Govt	1 year	BE	2003-01-20	2019-02-21
GTFIM1Y Govt	1 year	FI	2003-01-20	2019-02-21
GTFRF1Y Govt	1 year	FR	2003-01-20	2019-02-21
GTDEM1Y Govt	1 year	DE	2003-01-20	2019-02-21
GTGRD1Y Govt	1 year	GR	2007-03-02	2019-02-21
GTIEP1Y Govt	1 year	IR	2003-01-20	2019-02-21
GTITL1Y Govt	1 year	IT	2003-01-20	2019-02-21
GTPTE1Y Govt	1 year	PT	2003-01-20	2019-02-21
GTESP1Y Govt	1 year	SP	2011-08-08	2019-02-21

Source: Bloomberg. Countries: AT = Austria, BE = Belgium, FI = Finland, FR = France, DE = Germany, GR = Greece, IR = Ireland, IT = Italy, NE = Netherlands, Pt = Portugal, SP = Spain.

Table A2. Medium-term daily data

Bloomberg ID	Maturity	Country	Begin	End
GTATS2Y Govt	2 years	AT	1999-11-02	2019-03-06
GTBEF2Y Govt	2 years	BE	1999-11-02	2019-03-06
GTFIM2Y Govt	2 years	FI	1999-11-02	2019-03-06
GTFRF2Y Govt	2 years	FR	1999-11-02	2019-03-06
GTDEM2Y Govt	2 years	DE	1999-11-02	2019-03-06
GTITL2Y Govt	2 years	IT	1999-11-02	2019-03-06
GTPTE2Y Govt	2 years	PT	1999-11-02	2019-03-06
GTESP2Y Govt	2 years	SP	1999-11-02	2019-03-06
GTATS3Y Govt	3 years	AT	1999-11-02	2019-02-21

Table A2. Continued

Bloomberg ID	Maturity	Country	Begin	End
GTBEF3Y Govt	3 years	BE	1999-11-02	2019-02-21
GTFIM3Y Govt	3 years	FI	1999-11-02	2019-02-21
GTFRF3Y Govt	3 years	FR	1999-11-02	2019-02-21
GTDEM3Y Govt	3 years	DE	1999-11-02	2019-02-21
GTGRD3Y Govt	3 years	GR	2007-03-01	2019-02-21
GTIEP3Y Govt	3 years	IR	1999-11-02	2019-02-21
GTNLG3Y Govt	3 years	NE	1999-11-02	2019-02-21
GTPTE3Y Govt	3 years	PT	1999-11-02	2019-02-21
GTESP3Y Govt	3 years	SP	1999-11-02	2019-02-21
GTATS5Y Govt	5 years	AT	1999-11-02	2019-02-21
GTBEF5Y Govt	5 years	BE	1999-11-02	2019-02-21
GTFIM5Y Govt	5 years	FI	2007-10-12	2019-02-21
GTFRF5Y Govt	5 years	FR	1999-11-02	2019-02-21
GTDEM5Y Govt	5 years	DE	1999-11-02	2019-02-21
GTGRD5Y Govt	5 years	GR	2007-03-02	2019-02-21
GTIEP5Y Govt	5 years	IR	1999-11-02	2019-02-21
GTITL5Y Govt	5 years	IT	1999-11-02	2019-02-21
GTNLG5Y Govt	5 years	NE	1999-11-02	2019-02-21
GTPTE5Y Govt	5 years	PT	1999-11-02	2019-02-21
GTESP5Y Govt	5 years	SP	1999-11-02	2019-02-21
GTATS7Y Govt	7 years	AT	2003-11-06	2019-03-06
GTBEF7Y Govt	7 years	BE	2003-11-06	2019-03-06
GTFIM7Y Govt	7 years	FI	2003-11-06	2019-03-06
GTFRF7Y Govt	7 years	FR	2003-11-06	2019-03-06
GTDEM7Y Govt	7 years	DE	2003-11-06	2019-03-06
GTGRD7Y Govt	7 years	GR	2007-03-02	2019-03-06
GTIEP7Y Govt	7 years	IR	2003-11-06	2019-03-06
GTITL7Y Govt	7 years	IT	2003-11-06	2019-03-06
GTPTE7Y Govt	7 years	PT	2003-11-06	2019-03-06
GTESP7Y Govt	7 years	SP	2003-11-06	2019-03-06

Source: Bloomberg. Countries: AT = Austria, BE = Belgium, FI = Finland, FR = France, DE = Germany, GR = Greece, IR = Ireland, IT = Italy, NE = Netherlands, Pt = Portugal, SP = Spain.

Table A3. Long-term daily data

Bloomberg ID	Maturity	Country	Begin	End
GTATS10Y Govt	10 years	AT	1999-11-02	2019-02-18
GTBEF10Y Govt	10 years	BE	1999-11-02	2019-02-18
GTFIM10Y Govt	10 years	FI	1999-11-02	2019-02-18
GTFRF10Y Govt	10 years	FR	1999-11-02	2019-02-18
GTDEM10Y Govt	10 years	DE	1999-11-02	2019-02-18
GTGRD10Y Govt	10 years	GR	2007-03-02	2019-02-18
GTIEP10Y Govt	10 years	IR	1999-11-02	2019-02-18

Table A3. Continued

Bloomberg ID	Maturity	Country	Begin	End
GTITL10Y Govt	10 years	IT	1999-11-02	2019-02-18
GTNLG10Y Govt	10 years	NE	1999-11-02	2019-02-18
GTPTE10Y Govt	10 years	PT	1999-11-02	2019-02-18
GTESP10Y Govt	10 years	SP	1999-11-02	2019-02-18
GTATS15Y Govt	15 years	AT	2002-10-18	2019-03-06
GTBEF15Y Govt	15 years	BE	2002-10-18	2019-03-06
GTFIM15Y Govt	15 years	FI	2002-10-18	2019-03-06
GTFRF15Y Govt	15 years	FR	2002-10-18	2019-03-06
GTDEM15Y Govt	15 years	DE	2002-10-18	2019-03-06
GTGRD15Y Govt	15 years	GR	2007-03-02	2019-03-06
GTIEP15Y Govt	15 years	IR	2002-10-18	2019-03-06
GTITL15Y Govt	15 years	IT	2002-10-18	2019-03-06
GTNLG15Y Govt	15 years	NE	2002-10-18	2019-03-06
GTPTE15Y Govt	15 years	PT	2002-10-18	2019-03-06
GTESP15Y Govt	15 years	SP	2002-10-18	2019-03-06
GTATS20Y Govt	20 years	AT	2005-05-19	2019-03-06
GTBEF20Y Govt	20 years	BE	2005-05-19	2019-03-06
GTFRF20Y Govt	20 years	FR	2005-05-19	2019-03-06
GTDEM20Y Govt	20 years	DE	2005-05-19	2019-03-06
GTGRD20Y Govt	20 years	GR	2012-03-13	2019-03-06
GTIEP20Y Govt	20 years	IR	2005-05-19	2019-03-06
GTITL20Y Govt	20 years	IT	2005-05-19	2019-03-06
GTPTE20Y Govt	20 years	PT	2014-01-17	2019-03-06
GTESP20Y Govt	20 years	SP	2005-05-20	2019-03-06
GTATS30Y Govt	30 years	AT	2006-03-16	2019-03-06
GTBEF30Y Govt	30 years	BE	2006-03-16	2019-03-06
GTFIM30Y Govt	30 years	FR	2012-06-29	2019-03-06
GTFRF30Y Govt	30 years	DE	2006-03-16	2019-03-06
GTDEM30Y Govt	30 years	GR	2006-03-16	2019-03-06
GTIEP30Y Govt	30 years	IR	2015-02-04	2019-03-06
GTITL30Y Govt	30 years	IT	2006-03-16	2019-03-06
GTNLG30Y Govt	30 years	NE	2006-03-16	2019-03-06
GTPTE30Y Govt	30 years	PT	2006-03-16	2019-03-06
GTESP30Y Govt	30 years	SP	2006-03-16	2019-03-06

Source: Bloomberg. Countries: AT = Austria, BE = Belgium, FI = Finland, FR = France, DE = Germany, GR = Greece, IR = Ireland, IT = Italy, NE = Netherlands, Pt = Portugal, SP = Spain.

B. Factor analysis

Our main procedure based on the detection of statistically significant principal components has strong relations with static factor models [Stock and Watson (2002); Onatski (2010)]. The reason is that static factors are typically estimated by using PCA. In this appendix, we complement our main analysis by estimating factors and by tracking their evolution over time. More precisely, a static factor model decomposes each time series in a component driven by a few common factors and an idiosyncratic component [see e.g. Stock and Watson (2002)]:

$$X_t = BF_t + e_t \tag{B.1}$$

where $\underset{N\times T}{X}$ is the matrix of the time series, $\underset{N\times N}{B}$ is the matrix of the loadings of each factor, $\underset{N\times T}{F}$ is the matrix of the factors, and $\underset{1\times T}{e}$ is the vector of the idiosyncratic components.

The matrices B and F are estimated via the PCA described in Section 4.1 above. The covariance matrix Γ_Y of the n time series Y can be factorized as:

$$\Gamma_Y = \mathbb{E}[X_T X_T'] = U\Lambda U^T \tag{B.2}$$

where $\underset{N\times N}{\Lambda}$ is the matrix with the ordered eigenvalues of the covariance matrix on the diagonal, while $\underset{N\times N}{U}$ is the orthogonal matrix of the corresponding eigenvectors. Once the first K eigenvalues and eigenvectors are selected via the random matrix procedure illustrated in Section 4.2, the factor model boils down to:

$$X_t = \tilde{B}\tilde{F}_t + e_t \tag{B.3}$$

where $\tilde{B}$ has dimension $N \times K$ and $\tilde{F}$ has dimension $K \times T$. If we define the factor loadings[26] as $\tilde{B} = \sqrt{n}\tilde{U}$, then the matrix $\tilde{F}$ can be estimated by ordinary least squares (OLS) such that:

$$\tilde{F} = (\tilde{B}^T\tilde{B})^{-1}\tilde{B}^T X \tag{B.4}$$

Lastly, factors can be identified up to a rotation:

$$\hat{F} = \Theta\tilde{F} \tag{B.5}$$

where $\Theta = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$ is the rotation matrix. Consider for instance the two models:

$$X_{(1)} = \tilde{B}\tilde{F} + e \tag{B.6}$$

$$X_{(2)} = \tilde{B}\Theta\tilde{F} + e \tag{B.7}$$

Assuming $\mathbb{E}(e) = 0$ and computing the covariance matrices, we can show that $\Gamma_{X_{(2)}} = \mathbb{E}[X_T X_T'] = (\tilde{B}\Theta\tilde{F} + e)'(\tilde{B}\Theta\tilde{F} + e) = \tilde{F}'\Theta'\tilde{B}'\tilde{B}\Theta\tilde{F} + e'e$. Using $\tilde{B} = \tilde{U}$, we have that $\tilde{B}'\tilde{B} = I$, while $\Theta'\Theta = I$ by definition of the rotation matrix. We thus have $\Gamma_{X_{(2)}} = \tilde{F}'\tilde{F} + e'e = \Gamma_{X_{(1)}}$.

For each window, we estimate the first two factors according to our selection procedure. More precisely, for each window, we compute the Spearman correlation of the factors with the filtered time series. Given the large number of windows (around 200), we report the correlation coefficients aggregated by year in Tables D1 and D2 for 1-year yields, in Tables D3 and D4 for 5-year yields, and in Tables D5 and D6 for 10-year yields. An inspection of the tables shows that the correlations of the peripheral countries tend to be weaker with the first factor in times of crisis and stronger in general with the second factor. To help a visual inspection, Figure B1 shows the factors on the graph in levels for 5- and 10-year yields.[27]

The analysis of the factors supports the conclusions reached in Section 5.2 that bond yields at 1-year maturity are poorly synchronized. In contrast, one factor has been driving all the series of 5 and 10 year bonds up to 2008. From 2008, some series are still strongly correlated with the factor, while others lose correlation. In some cases, such as the years 2011 and 2012, we have that the correlations even differ in sign between the two groups of countries. Notice that this dynamic and the signs of the correlations are robust for all three maturities and both filtered and raw data. The analysis also supports the interpretation of the second factor, which from the year 2008 on

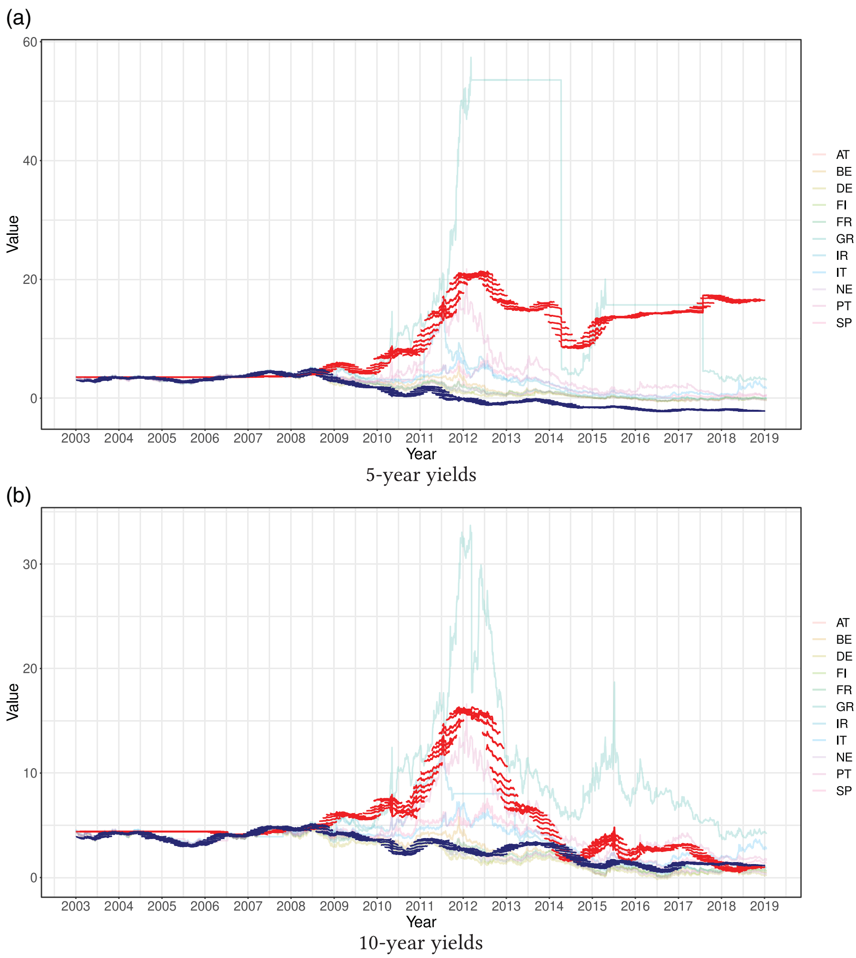

Figure B1. Factors evolution.
Notes: The first factor is in blue, while the second factor is in red.

robustly shows a higher correlation with the peripheral countries and generally a low correlation with the core countries.

C. Additional robustness checks

C.1. Alternative yield maturities

We perform the random matrix theory analysis on all types of bond yields available in the dataset retrieved from the Bloomberg platform. We consider yields below 1-year maturity as short term, yields between 1 year and 10 years as medium term, and yields above 10-year maturity as long term (Figure C1).[28]

The analysis of these more comprehensive datasets confirms the main results presented in Section 5. Synchronization of short-term yields is limited, and the largest component is typically nonsignificant. Furthermore, synchronization is high for medium-term maturities until 2008, and it sharply falls between 2008 and 2014, to slowly recover after 2015. However, in none of the medium-term maturities considered, the degree of synchronization (see Panels b and c of Figure 3, and Panel b of Figure C1) recovered the pre-2008 levels. Finally, synchronization dynamics in

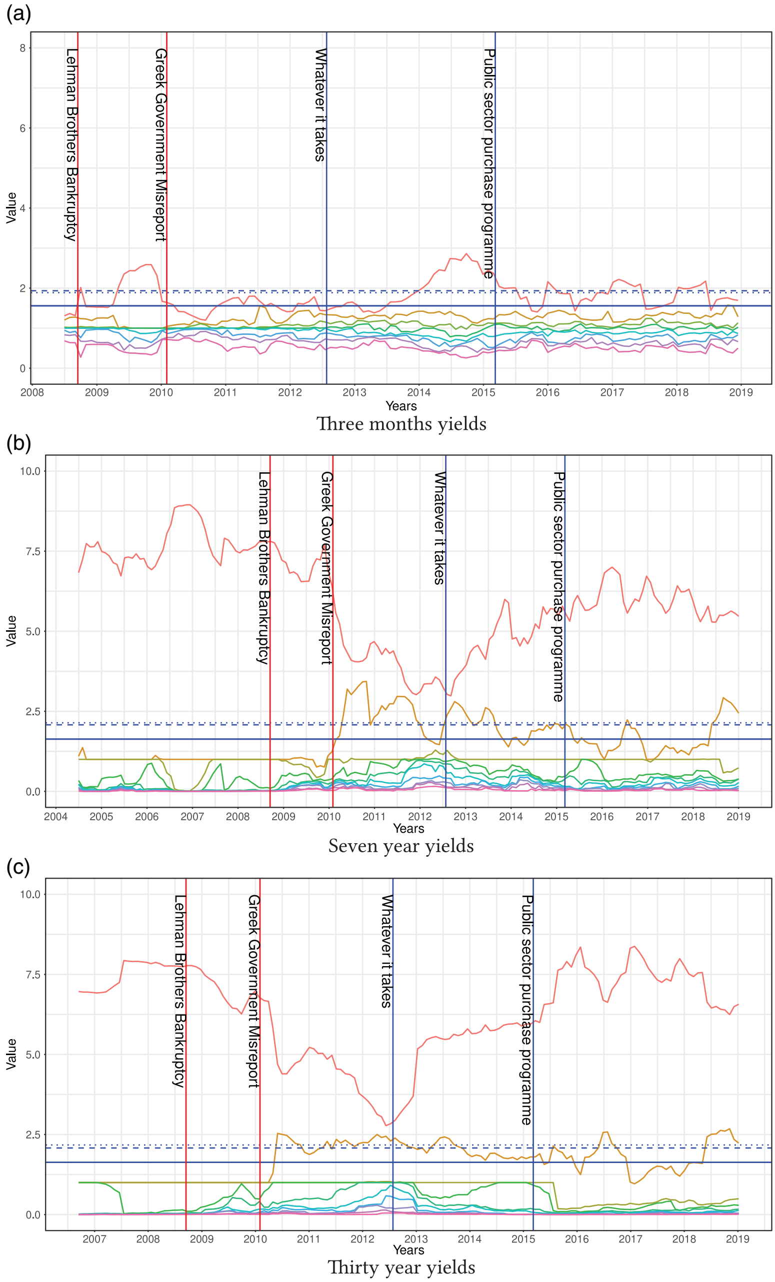

Figure C1. Eigenvalue evolution for 3-month, 7-year, and 30-year yields.
Notes: the solid line indicates the Marčenko–Pastur theoretical bound, the dotted line indicates the simulated random model, and the thin dotted line indicates the rotational bound. For both the simulated random model and rotational random shuffling, 300 Monte Carlo simulations have been run. The dimension of the windows of the random matrix theory are 130 observations and step 22 observations.

longer-term yields mimics the one observable for medium-term ones. However, we find that the degree of synchronization between 2016 and 2018 of bonds with 15, 20, and 30 years of maturity is very similar to the one observed in the 2006–2009 period. At such very long time-horizons yields, synchronization has fully regained the levels observed before the Great Recession.

Table C1. Optimal combinations of parameters at the 1% level of significance

Maturity	Length	Step	Average share of nonsignificant coefficients	Maximum share of nonsignificant coefficients
1-year	783	65	0.3401	0.7143
1-year	783	65	0.3401	0.7143
5-years	783	65	0.1640	0.4364
5-years	522	22	0.1591	0.4545
10-years	783	65	0.1070	0.4545
10-years	783	65	0.1071	0.4545

C.2. Alternative rolling windows

Our econometric procedure for the analysis of synchronization involves determining rolling windows, wherein correlations across bond yields are calculated. We, therefore, investigate the robustness of our results for changes in the values of (i) the length of the windows K and (ii) the step parameter S defining such rolling windows (see Section 4 for more details). In particular, we test all the possible combinations of parameters such that $K > S$, and such that subsequent windows do overlap for at least one observation. This sensitivity analysis also encompasses combinations of parameters with apparently little interest. For instance, very large values of both the length and step parameters deliver a too aggregate and static image of synchronization, and they are not useful to track synchronization through time. Accordingly, here we discuss only the results referring to alternatives with a plausible economic interpretation and selected according to a procedure minimizing the share of nonsignificant correlation coefficients $e_{i,j}$ along with all the windows.

Table C1 shows the combinations of the length and step parameters that minimize the average share of nonsignificant coefficients for the different maturities considered in our main analysis. The average percentage of nonsignificant coefficients at 5-year and 10-year yields is about 16% and 10%, respectively. It increases to 30% for 1-year yields. In addition, in most cases shown in Table C1, the optimal length of the window is the largest possible given the size of our sample: 783 observations (i.e. windows of 3-year length). This is also a statistical artifact since large windows have more observations and lead to higher levels of significance. The results of the synchronization analysis using the parameters of the window in Table C1 confirm our main results (see Figure C2).

C.3. Alternative filtering techniques

It is well known that econometric results are very sensitive to the filter used to de-trend the data. For instance, the first differencing filter we use in our main analysis might generate spurious correlation [see e.g. Uhlig (2009)]. We, therefore, carry out a sensitivity analysis of our main results also by taking second-order differences to eliminate residual spurious correlation. We then perform the PCA and RMT analysis, and we find that the results are comparable to those obtained by using the first difference.[29] Furthermore, Canova (1999) and Baxter and King (1999) remarked that the first difference filter might overweight high-frequency components in the time series. For this reason, we also perform a further battery of sensitivity analysis on data transformed by using the bandpass filter proposed in Christiano and Fitzgerald (2003). We employ this bandpass filter to select all the possible frequency bands combining a lower bound comprised in the interval {2, 22, 65, 130} and an upper bound in the interval {22, 65, 130, 261}, while keeping the dimension of the windows fixed.[30] We repeat our synchronization analysis on all the foregoing frequency bands. Figure C3 shows the results about the resulting dominant eigenvalue, whereas Figure C4 tracks the evolution of the second largest eigenvalue. The analysis of both principal components confirms the main results discussed in Section 5.2.

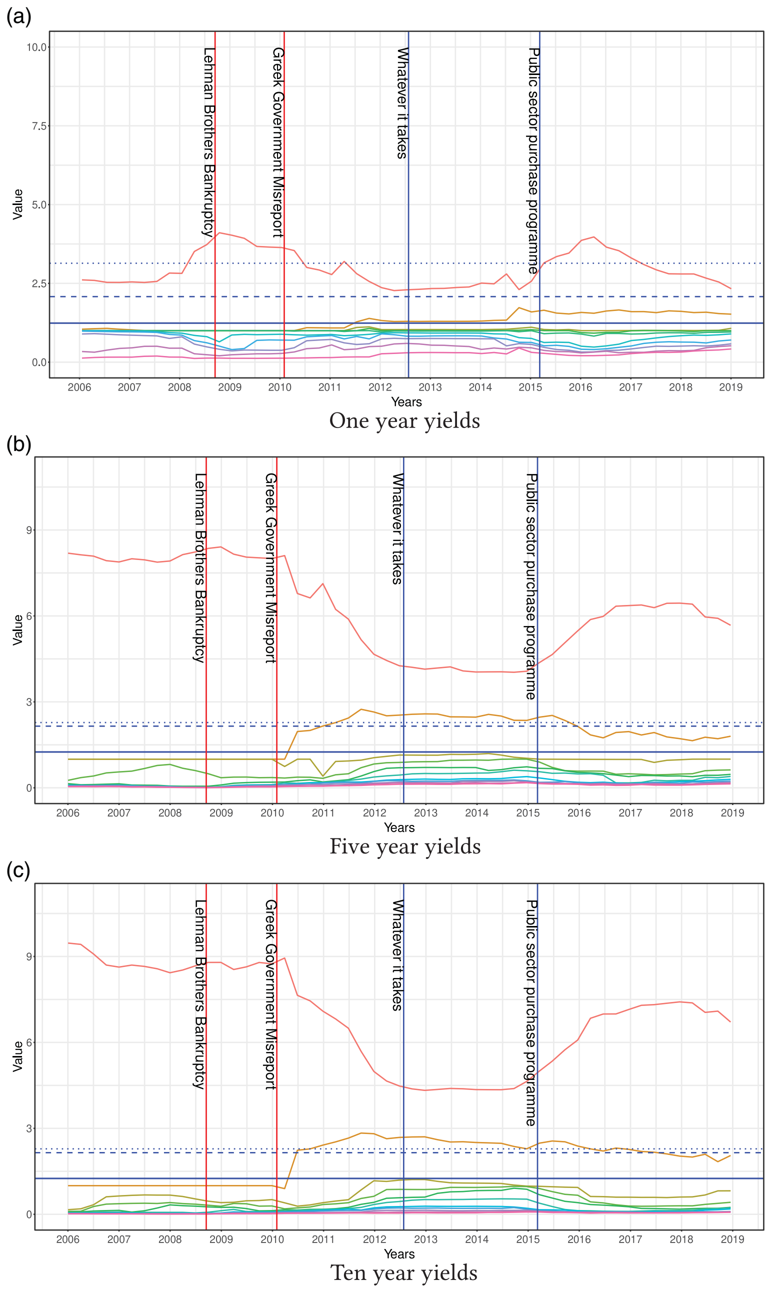

Figure C2. Eigenvalue evolution with optimal parameters for first difference.
Notes: the full line indicates the Marčenko–Pastur theoretical bound, the dotted line indicates the simulated random model, and the thin dotted line indicates the rotational bound. For both the simulated random model and rotational random shuffling, 300 Monte Carlo simulations have been run. The dimension of the windows of the random matrix theory are reported in Table C1 in Section C.2 (in short, windows' width of 783 observations, i.e. 3 years, and step of 65 observations, i.e. one-quarter).

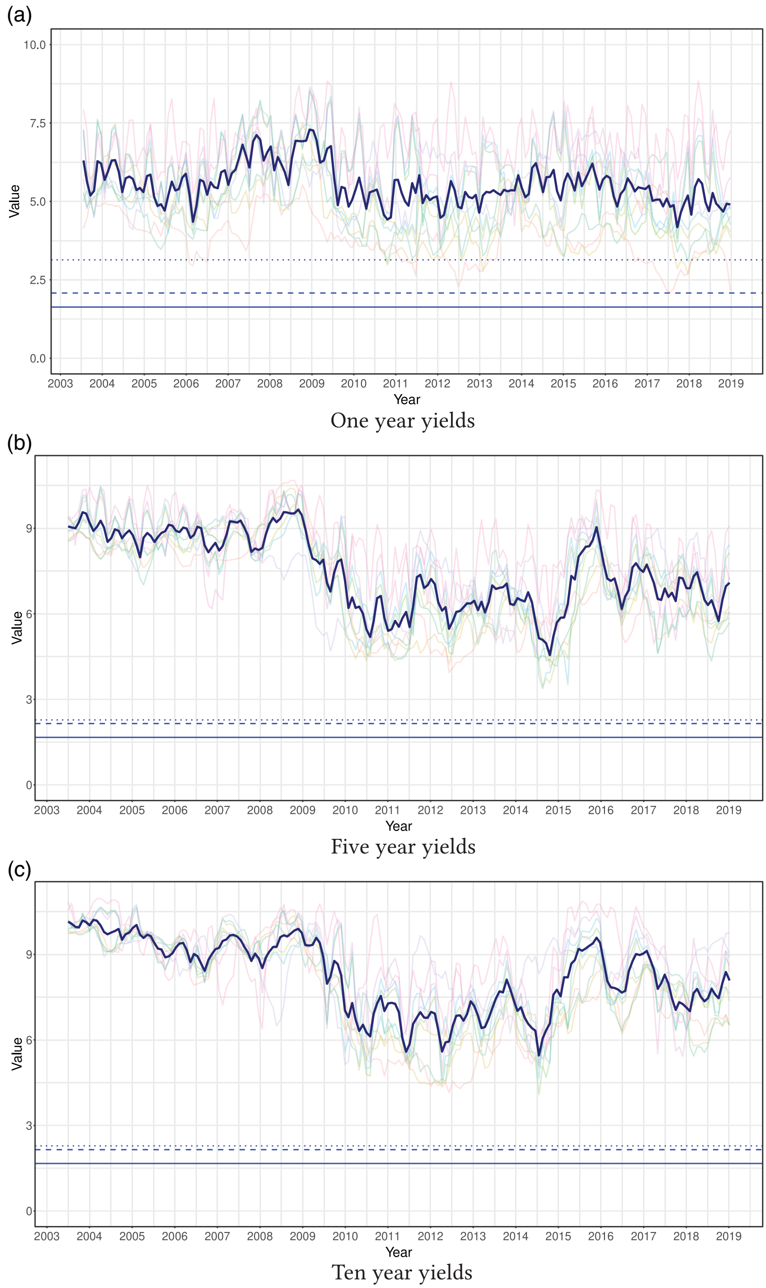

Figure C3. Evolution of the largest eigenvalue under different bandpass filters. The line in bold is the time average across the eigenvalues corresponding to the different frequency bands.
Notes: the solid line indicates the Marčenko–Pastur theoretical bound, the dotted line indicates the simulated random model, and the thin dotted line indicates the rotational bound. For both the simulated random model and rotational random shuffling, 300 Monte Carlo simulations have been run. The full dark blue line indicates the mean of the values given by the different frequencies selection. The selection of the frequencies is discussed in Section C.2.

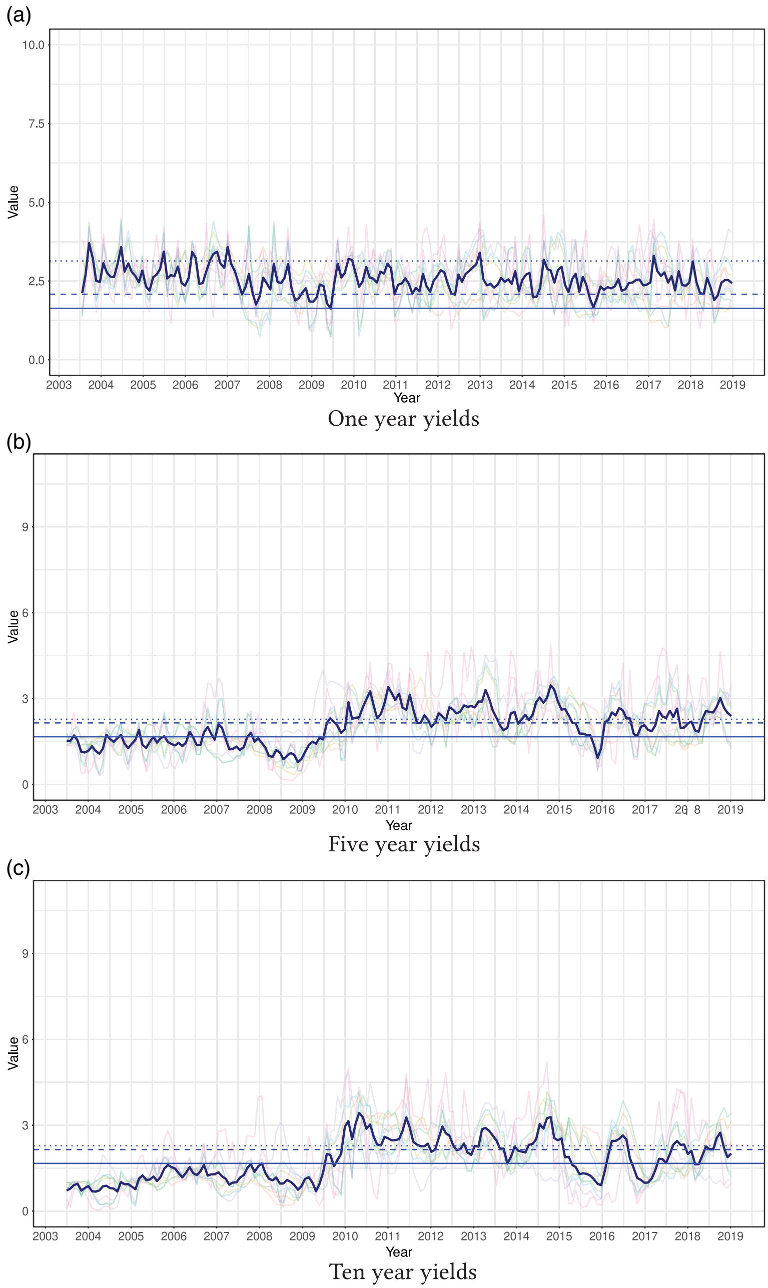

Figure C4. Evolution of the second largest eigenvalue under different bandpass filters. The line in bold is the time average across the eigenvalues corresponding to the different frequency bands.
Notes: the full line indicates the Marčenko–Pastur theoretical bound, the dotted line indicates the simulated random model, and the thin dotted line indicates the rotational bound. For both the simulated random model and rotational random shuffling, 300 Monte Carlo simulations have been run. The full dark blue line indicates the mean of the values given by the different frequencies selection. The selection of the frequencies is discussed in Section C.2.

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D. Random matrix theory and unfiltered correlations: a comparison

In this section, we describe how the random correlation could bias our results if not filtered through RMT. For the purpose, we compare the PCA to the average of the correlation coefficients [i.e. the average among the elements of the correlation matrix E excluding the diagonal, see equation (1)]. We argue that such a comparison is the most appropriate for a few reasons. First, the Pearson correlation is a well-known measure, and it is established as a methodological standard in the study of synchronization.[31]

Second, it computes the mean among the correlations from the same correlation matrix that we subsequently filter through RMT. The comparison is thus based on the same input, while different methodologies, such as a penalty function, would build on different process (e.g. the result of a regression). The use of average correlations is therefore directly comparable to our method in an visually intuitive way as shown in Figure D1.

Mere correlation coefficients would, in our case, underestimate the synchronization between countries up to around 10% of the variance. The lower bias implies that in times of higher synchronization (e.g. 2000–2008), the average correlation would appear milder than its true value. On the contrary, in times of higher divergence (e.g. 2010–2014), divergence would be overestimated, showing a decrease of synchronization down to 25% instead of around 35%. Looking at the average correlation, therefore, a policy aiming at reducing divergence would risk being eventually stronger than what could be sufficient to restore synchronization. Notice also that around the sovereign debt crisis, the RMT-significant PCA is more precise in detecting the turning point of synchronization in 2012.

In addition, the average correlation coefficient does not allow identifying different co-movements between the series, which can lead to significant policy implications. We observe that the average correlation approximates in our case a mean between the largest and the second largest PCA. As we stress in the paper, however, synchronization does only depend on a possibly strong synchronization factor but also on the absence of an RMT-significant diverging factor. Identifying the different co-movements behind the series is thus essential for our study. Ideally, if the second co-movement is nonsignificant, synchronization policies should focus on strengthening the first factor. On the contrary, we argue that the significance of a second co-movement reveals the presence of diverging market incentives, which imply a different policy focus.

In conclusion, using RMT allows us to measure more precisely synchronization with respect to the downward bias of the average correlation and to provide a description of the multiple co-movements of the series, which mere correlations do not consider. The higher precision and characterization of synchronization through RMT are in turn better suited for policy considerations.

Filtered 1-year yields

Table D1. Yearly average Spearman's correlation between the first factor and the 1-year yields filtered time series (first difference)

	AT	BE	FI	FR	DE	GR	IR	IT	PT	SP
2011	0.34	0.22	0.28	0.41	0.28	−0.60	−0.12	0.12	−0.28	0.15
2012	0.44	0.50	0.12	0.34	0.19	−0.56	−0.05	0.62	0.30	0.55
2016	0.32	0.34	0.39	0.33	0.36	0.13	0.29	0.14	−0.01	0.17
2017	0.67	0.31	0.69	0.32	0.59	0.01	0.62	0.10	−0.18	0.02
2018	0.33	0.12	0.45	0.21	0.54	−0.46	0.28	−0.78	−0.12	−0.25

Note: In the case of 1-year yields, many time series result being constant for long periods before 2010. In many case, therefore, it is not possible or meaningless to compute the Spearman correlation. We report the more complete results that we are able identify for this type of yields.

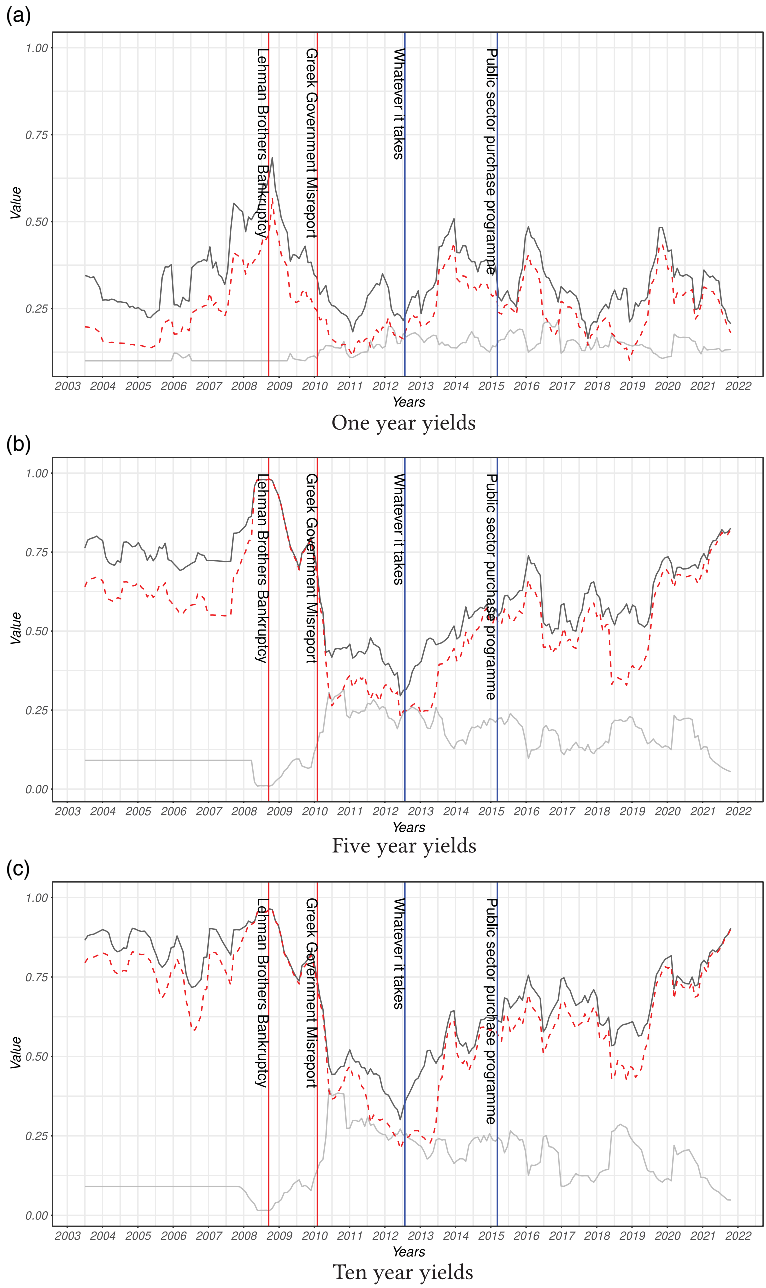

Figure D1. Fraction of variance explained by the first and second largest PCA versus the average correlation coefficient. The black line refers to the share of variance explained by the largest PCA, the gray line that of the second largest PCA, and the dotted red line is the average of the correlation coefficients (i.e. the coefficients in the variance–covariance matrix excluding the diagonal). Notice that we refer only to the first and the second largest PCA for our analysis shows that the remaining components are not RMT-significant.

Table D2. Yearly average Spearman's correlation between the second factor and the 1-year yields filtered time series (first difference)

	AT	BE	FI	FR	DE	GR	IR	IT	PT	SP
2011	−0.04	0.05	−0.08	0.02	−0.02	0.33	0.06	0.08	−0.03	0.10
2012	0.20	0.26	−0.16	0.15	−0.05	−0.55	0.04	0.70	0.16	0.68
2016	−0.03	−0.09	−0.06	−0.05	−0.07	0.05	−0.07	−0.20	−0.13	−0.21
2017	−0.10	−0.03	−0.09	−0.02	−0.12	−0.18	−0.12	−0.02	0.07	0.04
2018	−0.02	0.01	0.01	−0.06	0.10	−0.33	−0.15	−0.55	−0.06	−0.26

Note: In the case of 1-year yields, many time series result being constant for long periods before 2010. In many case, therefore, it is not possible or meaningless to compute the Spearman correlation. We report the more complete results that we are able identify for this type of yields.

Filtered 5-year yields

Table D3. Yearly average Spearman's correlation between the first factor and the 5-year yields filtered time series (first difference)

	AT	BE	FI	FR	DE	GR	IR	IT	NE	PT	SP
2003	0.99	0.98	0.00	0.98	0.97	0.00	0.88	0.98	0.97	0.98	0.98
2004	0.66	0.66	0.00	0.64	0.64	0.00	0.65	0.65	0.66	0.66	0.66
2005	0.66	0.66	0.00	0.64	0.65	0.00	0.52	0.64	0.65	0.66	0.64
2006	0.98	1.00	0.00	0.99	1.00	0.00	0.39	0.98	0.98	1.00	1.00
2007	0.99	0.99	0.37	1.00	0.99	0.98	0.78	0.98	0.99	0.99	1.00
2008	0.94	0.96	0.96	0.96	0.93	0.85	0.92	0.89	0.97	0.94	0.96
2009	0.86	0.85	0.84	0.86	0.81	0.47	0.62	0.72	0.88	0.73	0.81
2010	0.81	0.54	0.84	0.84	0.84	−0.20	0.09	0.19	0.84	−0.08	0.12
2011	0.66	0.28	0.59	0.64	0.53	−0.18	−0.20	−0.08	0.61	−0.24	−0.04
2012	0.81	0.81	0.34	0.78	0.13	−0.03	0.06	0.59	0.57	0.07	0.58
2013	0.92	0.89	0.92	0.92	0.86	0.07	0.48	0.73	0.92	0.41	0.59
2014	0.44	0.45	0.38	0.43	0.32	0.44	0.48	0.52	0.41	0.49	0.51
2015	0.90	0.92	0.86	0.92	0.79	0.04	0.84	0.74	0.88	0.62	0.71
2017	0.89	0.92	0.89	0.90	0.88	0.01	0.90	0.61	0.89	0.43	0.60
2018	0.90	0.85	0.92	0.87	0.93	−0.32	0.82	−0.40	0.89	−0.17	−0.04

Table D4. Yearly average Spearman's correlation between the second factor and the 5-year yields filtered time series (first difference)

	AT	BE	FI	FR	DE	GR	IR	IT	NE	PT	SP
2006	−0.07	−0.08	0.00	−0.07	−0.08	0.00	−0.01	−0.05	−0.04	−0.08	−0.09
2007	0.04	0.05	0.01	0.05	0.04	0.08	−0.12	0.09	0.05	0.06	0.05
2008	−0.12	−0.05	−0.12	−0.12	−0.22	0.35	0.05	0.20	−0.09	0.02	−0.06
2009	0.00	0.11	−0.07	−0.06	−0.14	0.46	0.18	0.26	−0.07	0.25	0.19
2010	0.14	0.52	−0.00	0.06	−0.08	0.74	0.20	0.70	0.01	0.78	0.75
2011	0.07	0.37	−0.07	0.10	−0.18	0.48	0.34	0.42	−0.08	0.41	0.42
2012	0.21	0.38	−0.48	0.24	−0.58	−0.06	0.24	0.86	−0.25	0.20	0.84
2013	0.05	0.09	0.05	0.05	−0.07	0.33	0.44	0.64	0.04	0.83	0.72
2014	0.07	0.08	−0.01	0.07	−0.10	0.83	0.44	0.64	0.03	0.74	0.64
2015	0.19	0.21	0.15	0.20	0.00	0.50	0.34	0.74	0.17	0.79	0.75
2017	−0.04	0.04	−0.07	0.11	−0.05	0.06	0.09	0.54	−0.07	0.57	0.54
2018	−0.01	−0.03	−0.01	−0.03	−0.01	−0.11	−0.03	−0.12	−0.01	−0.13	−0.11

Filtered 10-year yields

Table D5. Yearly average Spearman's correlation between the first factor and the 10-year yields filtered time series (first difference)

	AT	BE	FI	FR	DE	GR	IR	IT	NE	PT	SP
2003	0.96	0.99	0.99	0.98	0.99	0.00	0.99	0.99	0.98	0.98	0.98
2004	0.82	0.82	0.83	0.82	0.82	0.00	0.83	0.82	0.82	0.79	0.82
2005	0.94	0.99	0.99	0.99	0.99	0.00	0.99	0.98	0.92	0.74	0.98
2006	0.95	1.00	0.99	0.99	1.00	0.00	0.94	0.98	0.99	0.78	0.99
2007	0.40	0.40	0.40	0.40	0.40	0.20	0.20	0.40	0.40	0.40	0.40
2008	0.95	0.98	0.95	0.96	0.91	0.85	0.92	0.90	0.96	0.94	0.95
2009	0.86	0.86	0.85	0.86	0.79	0.50	0.61	0.75	0.87	0.72	0.80
2010	0.73	0.57	0.79	0.78	0.77	−0.07	0.01	0.32	0.77	0.09	0.21
2011	0.51	0.16	0.52	0.48	0.50	−0.12	−0.16	−0.11	0.51	−0.09	−0.11
2012	0.75	0.64	0.83	0.71	0.87	−0.70	−0.14	−0.46	0.83	−0.54	−0.42
2013	0.42	0.43	0.41	0.43	0.37	0.16	0.28	0.32	0.41	0.18	0.29
2014	0.69	0.71	0.67	0.71	0.59	0.29	0.68	0.63	0.67	0.50	0.63
2015	0.93	0.96	0.90	0.95	0.87	0.03	0.90	0.78	0.92	0.54	0.73
2016	0.92	0.94	0.91	0.94	0.81	0.01	0.92	0.84	0.90	0.66	0.79
2017	0.74	0.77	0.75	0.78	0.71	0.06	0.78	0.60	0.74	0.48	0.59
2018	0.97	0.94	0.98	0.96	0.96	−0.14	0.93	−0.06	0.97	0.18	0.29

Table D6. Yearly average Spearman's correlation between the second factor and the 10-year yields filtered time series (first difference)

	AT	BE	FI	FR	DE	GR	IR	IT	NE	PT	SP
2006	−0.04	−0.08	−0.08	−0.08	−0.08	0.00	−0.09	−0.08	−0.07	−0.18	−0.09
2007	−0.06	−0.03	−0.05	−0.06	−0.08	−0.17	0.08	−0.00	−0.05	−0.01	−0.04
2008	−0.15	−0.10	−0.19	−0.15	−0.26	0.20	−0.04	0.13	−0.13	0.03	−0.05
2009	0.13	0.12	−0.03	0.00	−0.15	0.64	0.49	0.36	0.02	0.40	0.24
2010	−0.11	0.01	−0.10	−0.09	−0.09	0.09	0.14	0.11	−0.12	0.10	0.12
2011	0.12	0.63	−0.17	0.17	−0.28	0.61	0.45	0.72	−0.17	0.52	0.70
2012	−0.06	0.09	−0.19	0.01	−0.30	0.58	0.37	0.77	−0.17	0.75	0.79
2013	−0.01	0.02	−0.04	0.01	−0.07	0.50	0.31	0.37	−0.04	0.43	0.41
2014	−0.02	0.04	−0.04	0.03	−0.17	0.88	0.44	0.64	−0.06	0.74	0.62
2015	−0.00	0.05	−0.06	0.03	−0.11	0.93	0.22	0.42	−0.03	0.59	0.44
2016	0.03	0.12	0.02	0.13	−0.09	0.27	0.27	0.46	0.01	0.54	0.45
2017	0.02	0.12	0.01	0.16	−0.03	0.20	0.17	0.55	0.01	0.64	0.56
2018	−0.01	0.05	−0.01	0.04	−0.05	0.28	0.05	0.39	−0.03	0.36	0.33

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