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Excitation of surface cyclotron O modes by charged-particle beams

V. O. GIRKA

Department of Physics and Technology, Kharkiv National University,
Svobody sq. 4, Kharkiv 61077, Ukraine
(girka@pem.kharkov.ua)

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Abstract. A theoretical investigation of the interaction between a charged-particle beam and ordinarily polarized surface waves at the first two harmonics of ion and electron cyclotron frequencies propagating across an external steady magnetic field is carried out. The case of weak plasma spatial dispersion when the Larmor radii of plasma particles are much less than the penetration depth of these waves into the plasma is considered. The indicated waves are eigenmodes of the planar plasma–vacuum–metal waveguide structure, which is exposed to a steady magnetic field oriented parallel to the plasma surface. It is supposed that a cold charged-particle beam propagates over the plasma surface in the vacuum region. The beam density is less than the plasma density. Excitation of these O modes at the first and second harmonics of electron and ion cyclotron frequencies caused by the both resonant beam and dissipative instabilities is studied analytically. The dependence of their growth rates on parameters of the considered waveguide structures is also examined.

1. Introduction

It is known that two types of bulk cyclotron waves (X and O modes) can propagate in unbounded plasmas (see e.g. [1]). Their theoretical description is quite well developed [2]. That is why bulk cyclotron waves are actively utilized in practice. For example, ion cyclotron resonance heating (CRH) [3] and electron CRH [4] are widely used methods for additional heating of magnetically confined fusion plasmas. Cyclotron resonance is also applied to the generation of powerful electromagnetic emission [5].

The study of surface waves is of great importance in various branches of plasma physics [6] and solid state physics [7,8] because of their numerous applications. For instance, surface waves are widely used in radiofrequency and microwave gas discharges, which can be utilized for processing of solids, plasma production, etc. [9,10]. Nevertheless, until now, there has only been a theory of surface cyclotron X modes, which can propagate along such interfaces as magnetoactive plasma–dielectric and magnetoactive plasma–metal interfaces. The dispersion properties of these surface X modes (STXM) in a nonuniform plasma-filled metal waveguide with dielectric coating have been studied in [11]. It was found that ion and electron STXM have different dispersion properties and, in the case of a plasma–metal interface, they can propagate only in mutually opposed directions [12]. The directions of their propagation are determined by the orientation of the Larmor rotation of the corresponding charged plasma particles near the metal interface. Interaction between charged-particle beams and STXM has been studied in [13]. Parametric instability of the STXM caused by a nonmonochromatic pumping electric field was examined in [14]. However, up to the present time, there has been no theory of surface cyclotron O-mode (SCOM) propagation and excitation. The first work devoted to study of their dispersion is reported in [15]. Therefore investigation of SCOM excitation caused by beam–plasma interaction is relevant.

The principles of beam–plasma interaction are described in [16–18]. The physical aspects of technical applications of charged-particle beams are studied, for example, in [5, 18, 19]. The possibility of transfer of charged-particle beam energy into electromagnetic power emission is investigated there for various types of high-frequency electron devices. The features of their operation are also examined there. The monograph [20] is devoted mostly to theoretical analysis of surface wave excitation by charged-particle beams. However, there is no theoretical description of the interaction between surface cyclotron waves and charged-particle beams. That is why this paper is devoted to examining the possibility of SCOM excitation by charged-particle beams under the regimes of resonant beam and dissipative instabilities and also for studying the effect of the considered waveguide parameters on the values of their growth rates.

The paper is organized as follows. Section 2 presents the formulation of the problem and the basic assumptions and equations. Section 3 presents results of the analytical investigation of the interaction between the SCOM and charged-particle beams. A summary of the results obtained is presented in the Section 4.

2. Basic assumptions and equations

Let us consider a semibounded plasma that occupies the half-space $x \geqslant 0$ and is bounded by vacuum in the region $-a < x < 0$. A metal wall is situated in the region $x \leqslant -a$. An external steady magnetic field $\mathbf{B}_0$ is parallel to the plasma boundary and is oriented along the Z axis. The electromagnetic fields of the waves are described by the Maxwell equations. Their dependence on the Y coordinate and time t is chosen in the form $E, H \sim f(x) \exp[i(k_2 Y - \omega t)]$, where ω is the frequency and k_2 is the component of the SCOM wavevector along the Y axis. There is no dependence on the Z coordinate. Then the Maxwell equations can be divided into two independent sets. One of them describes the STXM with components E_x, E_y, and H_z. The other describes ordinarily polarized waves with components H_x, H_y, and E_z. Their eigenfrequency depends weakly on the value of the external magnetic field. These are the SCOM.

Plasma-particle motion is governed by the kinetic Vlasov–Boltzmann equation in a nonrelativistic approach. Their unperturbed state is described by a Maxwell ion distribution function. The spatial plasma dispersion along the X axis, perpendicular to the plasma interface, is supposed to be weak, i.e. the inequality $k_1 \rho_\alpha \ll 1$ is valid, where $\rho_\alpha = v_{T\alpha}(|\omega_\alpha|)^{-1}$ is the Larmor radius of electrons ($\alpha = e$) or ions ($\alpha = i$), $v_{T\alpha}$ is the mean value of thermal velocity, k_1 is the perpendicular component of the SCOM wavevector, and ω_α is the cyclotron frequency.

Using the Fourier method, in which

$$E_3(k_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-ik_1 x)\, E_z(x)\, dx, \tag{1}$$

one can write down the following expressions for the SCOM fields:

$$kH_1 = k_2E_3, \tag{2a}$$

$$kH_2 = -k_1E_3, \tag{2b}$$

$$E_3(k_2^2 - k^2\varepsilon_{33}) = kk_1H_2 + ikH_y(0)\pi^{-1}, \tag{2c}$$

where $H_y(0)$ is the magnitude of the SCOM tangential component of magnetic field on the plasma interface, $kc = \omega$, c is the speed of light, and ε_{33} is the component of the plasma permittivity tensor obtained in the nonrelativistic approach for the case of an unbounded magnetoactive plasma with finite temperature [1].

In fact, under the indicated conditions, the relation between the Fourier coefficients of the Z component of the electron current density and the corresponding component of the electric field differs from that in the case of an unbounded plasma (i.e. from the relation $j_3(k_1) = \sigma_{33}(k_1)E_3(k_1)$, where σ_{33} is the component of the plasma conductivity tensor obtained in the kinetic approach for a magnetoactive unbounded plasma). This is explained by the necessity to take into account the interaction between a definite group of plasma particles and the plasma boundary (see e.g. [21]). That is why the additional contribution caused by this interaction can be included in this relation. The value of this contribution is determined by the nondifferential part of the integration kernel of the plasma conductivity tensor σ_{33}. Therefore it determines the collisionless damping rate of the considered surface modes. It should also be pointed out that this damping is analogous to collisionless Landau damping and that there is no damping caused by the Doppler mechanism of wave–particle interaction, because we have taken a nonrelativistic approach and supposed that SCOM propagate strictly across the external magnetic field. In any case, the problem of SCOM damping is outside the scope of the paper; thus we do not take this contribution into account. Thus the expression for the component ε_{33} of plasma permittivity tensor coincides with that in the case of an unbounded magnetoactive plasma.

Solving the set of equations (2), one can find the following expression for the SCOM E_z field in the plasma region with the aid of the inverse Fourier transform:

$$E_z(x) = \int_{-\infty}^{\infty} \frac{ikH_y(0)\exp(ik_1x)\,dk_1}{\pi[k_1^2 + k_2^2 - k^2\varepsilon_{33}(k_1)]}. \tag{3}$$

Under the adopted approach of weak plasma spatial dispersion along the X direction, one can employ in (3) the following approximate expression for ε_{33}:

$$\varepsilon_{33}(k_1) \approx 1 - \sum_\alpha \frac{\Omega_\alpha^2}{\omega^2}\left[1 + \sum_{S=-\infty}^{\infty} \frac{(k_1\rho_\alpha)^{2S}}{2^S h(\alpha, S)S!}\right], \tag{4}$$

where $h(\alpha, S) = 1 - S\omega_\alpha\omega^{-1}$, Ω_α is the plasma frequency of species α, and S is the number of the cyclotron harmonic.

When studying waves at harmonics of the corresponding cyclotron frequency, one can take into account only those contributions in the sum over the numbers of cyclotron harmonics that have resonant denominators, i.e. those $h(\alpha, S = S_0)$ that are much less than unity just for the chosen value of the cyclotron harmonic number S_0 (see e.g. [1]). Considering the sum of terms over the plasma species (subscript α), one can see that the ion and electron contributions appear in the expression for the ε_{33} symmetrically. That is why, from a mathematical point of view, there is no difference in principle between the cases of ion and electron SCOM, unlike the case of the STXM excitation studied in [11]. One can use the obtained analytical expressions for both cases, simply changing the subscript $\alpha = i$ to $\alpha = e$, with an accuracy of their mass ratio m_e/m_i.

Let us consider here the case of SCOM excitation at the first and second cyclotron harmonics, because it is these cases that are of the most practical interest. The dispersion relation that describes the interaction between the SCOM and the charged-particle beam can be obtained using the method of impedance relations. To find the ratio $E_z(0)/H_y(0)$ for the plasma region, one can calculate the integral expression (3). This integral over the wavenumber k_1 from $-\infty$ to ∞ can be changed to a contour integral in the upper half-space of the complex plane of the wavenumber k_1. It can be calculated using the theory of residues. In the case of cyclotron waves propagating at the S_0 harmonic of the cyclotron frequency, the integral equation (3) for $E_z(x)$ has just S_0 poles in this half-space. They are determined as complex roots of the following equation in the upper half-space:

$$k_1^2 + k_2^2 + \delta^2 + \frac{(\rho_\alpha k_1)^{2S_0}}{2^{S_0}\delta^2 h S_0!} = 0, \tag{5}$$

where $\delta = c\Omega_\alpha^{-1}$ for ion and electron terms, respectively. Thus it is necessary to calculate the residues for these values of k_1. To describe the SCOM field in the region $-a < x < 0$, one can use (2) but with the following change:

$$\varepsilon_{33} \to \varepsilon_b = 1 - \frac{\Omega_b^2}{\omega(\omega - k_2 V_0)},$$

where $\Omega_b^2 = \theta\Omega_\alpha^2$, $\theta = n_b/n_{pl}$ is the ratio of beam and plasma concentrations, and V_0 is the beam velocity. In this region, the SCOM fields decrease with distance from the plasma surface according to the exponential law: $\exp(|k_2|x)$. Thus it is easy to find the relation between the tangential components of the SCOM fields in this region:

$$\frac{E_z(0)}{H_y(0)} \approx \frac{ik}{k_2}\tanh(k_2 a). \tag{6}$$

To obtain the SCOM dispersion equation, one can equate the impedance of the plasma and the impedance of the region $0 > x > -a$ where the charged-particle beam is moving. Then one can write down the SCOM dispersion equation for the considered plasma-beam structure:

$$D_0 = D_b. \tag{7}$$

Here D_b describes the beam properties; it can be written as

$$D_b = \frac{\theta\omega}{k_2^2\delta^2(\omega - k_2 V_0)};$$

the left-hand side D_0 describes the contribution from the plasma region. In the absence of the beam ($\theta = 0$), the equation $D_0 = 0$ is the SCOM dispersion relation for the considered plasma–vacuum–metal structure. Let us write it down for the case of SCOM at the first (ion or electron) harmonic:

$$D_0 \approx (1 + k_2^{-2}\delta^{-2})\left[1 + \frac{0.25\delta^{-2}\rho_\alpha^2}{h(\alpha, S=1)}\right]\tanh^2(ak_2) - 1. \tag{8}$$

It is convenient to find the solution of the dispersion equation (8) for the parameter

$h(\alpha, S)$. Thus one can find from (8) the following expression:

$$h(\alpha, S = 1) = \frac{\rho_\alpha^2 (1 + k_2^2 \delta^2) \sinh^2(ak_2)}{4\delta^2 [k_2^2 \delta^2 - \sinh^2(ak_2)]}. \tag{9}$$

Unfortunately, the expression for D_0 has a more cumbersome form in the case of the second cyclotron harmonics, which is why we do not give it here. The approximate solution of the SCOM dispersion equation in the case of a thick dielectric layer ($ak_2 \gg 1$) can be found for the wavenumber range determined by the inequality $ak_2 \gg \delta k_2 > 1$. Then the parameter $h(\alpha, S = 2)$ can be written in the form

$$h(\alpha, S = 2) \approx \tfrac{1}{2} \rho_\alpha^4 k_2^4 (1 + \delta^2 k_2^2). \tag{10}$$

3. Analytical study of interaction between the SCOM and the charged-particle beam

The SCOM are characterized by only one component of the electric field that is oriented along the applied external magnetic field. As a result, the SCOM can interact with a charged-particle beam only by the Cherenkov mechanism, but not by the Doppler one, in spite of the fact that they are waves at cyclotron harmonics. Let us study (7) first of all in the case of the fundamental ($S_0 = 1$) cyclotron harmonic. The beam density is supposed to be relatively small (which means that the inequality $\theta \ll 1$ is satisfied); then the right-hand side of (7) can be significant only under the condition for Cherenkov resonance: $\omega - k_2 V_0 = \gamma, |\gamma| \ll \omega$. It should be pointed out that this condition is contrary to that made in the case of surface X-mode excitation [13]. The value of the parameter D_b is inversely proportional to the value of γ:

$$D_b \approx \frac{\theta \omega}{\gamma \delta^2 k_2^2}. \tag{11}$$

Using the assumption of the resonant character of SCOM excitation at the fundamental cyclotron harmonic, one can transform (7) into the following:

$$\gamma(\gamma + i\nu_\alpha) \frac{(1 + k_2^2 \delta^2) \rho_\alpha^2 |\omega_\alpha|}{4 k_2^2 \delta^4 \omega^2 h^2 \coth^2(ak_2)} = \frac{\theta \omega}{k_2^2 \delta^2}, \tag{12}$$

where ν_α is the effective collision frequency between plasma particles. Considering the left-hand side of (12), one can see that it has different values in the cases of two possible types of beam–plasma instabilities: resonant beam instability (when the inequality $|\gamma| \gg \nu_\alpha$ is satisfied) and dissipative instability (when the opposite inequality $|\gamma| \ll \nu_\alpha$ is valid). Let us consider both cases for the SCOM at the first and second cyclotron harmonics. If $S_0 = 1$, then one can find the following expression for the SCOM growth rate in the case of resonant beam instability:

$$\mathrm{Im}(\gamma) \approx \frac{2\sqrt{\theta} \omega \delta |h(S = 1)|}{\rho_\alpha \sqrt{1 + \delta^2 k_2^2} \tanh(ak_2)}. \tag{13}$$

In the case of dissipative instability, $\mathrm{Im}(\gamma)$ can be written as

$$\mathrm{Im}(\gamma) \approx 4\omega^2 \theta \frac{\delta^2 h^2(S = 1) \coth^2(ak_2)}{\nu_\alpha \rho_\alpha^2 (1 + k_2^2 \delta^2)}. \tag{14}$$

In the case of SCOM excitation at the second ($S_0 = 2$) cyclotron harmonic (for ions or electrons), one can find from (7) the following expressions for $\mathrm{Im}(\gamma)$:

$$\mathrm{Im}(\gamma) \approx \frac{\sqrt{\theta}\omega[8h^3(S=2)]^{1/4}k_2\delta^2}{(k_2^2\delta^2+1)^{1/4}\rho_\alpha \tanh(ak_2)} \tag{15}$$

$$\mathrm{Im}(\gamma) \approx \frac{\theta\omega^2\sqrt{8h^3(S=2)}k_2^2\delta^4}{\nu_\alpha\sqrt{1+k_2^2\delta^2}\rho_\alpha^2 \tanh^2(ak_2)}, \tag{16}$$

in the cases of resonant beam and dissipative instabilities, respectively. On analysing (13)–(16), one can see that decreasing the transverse size of the region occupied by the beam (i.e. $ak_2 \to 0$) leads to a decrease in the SCOM growth rates. This is explained by the fact that the magnitude of the SCOM electromagnetic field decreases with decreasing ak_2 (see (2) for the plasma region and (6) for the vacuum region). Thus the SCOM cannot exist in a plasma–metal structure.

As was found in [11], STXM dispersion depends on the plasma transverse size very weakly. That is why there is approximately no influence of this parameter on the STXM growth rates [13]. However, the SCOM dispersion properties differ sufficiently from the STXM dispersion properties. Thus let us consider briefly the influence of finite transverse size of the plasma region on the SCOM growth rates. Then, instead of the expression (7) that is valid for the SCOM at the first cyclotron harmonic in the case of a semibounded plasma, one can write down the following expression for D_0 in the case of a plane plasma layer model:

$$D_0 \approx (1+k_2^{-2}\delta^{-2})\left[1+\frac{\rho_\alpha^2}{4\delta^2 h(S=1)}\right]\frac{\tanh^2(ak_2)}{\tanh^2(\kappa a_{pl})} - 1, \tag{17}$$

where a_{pl} is the thickness of the plasma layer, and the SCOM penetration depth into the plasma region κ^{-1} is determined by the following expression:

$$\kappa = \frac{\delta^2 k_2 \coth(ak_2)}{k_2^2\delta^2+1}.$$

From a comparison of (8) and (17), one can draw the following conclusion: a decrease in the plasma-layer thickness leads to a decrease in the SCOM growth rates. On analysing (13)–(16), one can see that there is no dependence of the SCOM growth rates on the applied external magnetic field. This can be explained as follows. SCOM dispersion depends weakly on the external magnetic field, and interaction between these modes and charged-particle beams can occur only due to Cherenkov resonance, unlike the case of the STXM. The growth rate of the resonant beam instability is larger than that in the case of dissipative instability.

Let us make some estimates of the SCOM growth rate at the first ($S_0 = 1$) cyclotron harmonics. To calculate the growth rate in the case of resonant beam instability, one can apply the formula

$$\mathrm{Im}(\gamma) \approx \frac{\sqrt{\theta}\Omega_\alpha v_{T\alpha}}{2c}. \tag{18}$$

Thus $\mathrm{Im}(\gamma)$ increases with increasing concentration and thermal velocity of plasma particles. In the case of $n_{pl} = 10^{12}\ \mathrm{cm}^{-3}$, one finds $\mathrm{Im}(\gamma) \approx \sqrt{\theta}v_{T\alpha}$.

Comparing the SCOM growth rate under the condition of resonant beam instability with that for the surface cyclotron X modes [13], one finds the following ratio for the case of the second ($S_0 = 2$) electron cyclotron harmonic:

$$\frac{\mathrm{Im}(\gamma)_{STXM}}{\mathrm{Im}(\gamma)_{SCOM}} \sim \frac{0.2}{\delta^3 k_2^5 \rho_e^3}. \tag{19}$$

Therefore, under the condition $k_2\delta > 1$, the SCOM growth rate is greater than the STXM growth rate.

4. Conclusions

The results of a theoretical study of interaction between charged-particle beams and SCOM have been presented. The possibility of SCOM excitation by cold charged-particle beams propagating over the plasma surface has been shown. The SCOM growth rate under the conditions of resonant beam and dissipative instabilities has been calculated. It is found that it does not depend on the applied external magnetic field.

Unlike the case of STXM excitation, SCOM can be excited only through the Cherenkov mechanism of beam–wave interaction. In the case of resonant beam instability, the SCOM growth rate is proportional to the small factor $\sqrt{\theta} \ll 1$. In the case of dissipative instability, the SCOM growth rate is less than the collision frequency ν_α. Therefore it decreases as compared with the case of resonant beam instability, and proportional to the first power of the small factor mentioned above.

It is found that the SCOM growth rate is greater than it is for the case of STXM excitation. Decreasing both the transverse size of the plasma and the thickness of the dielectric coating at the waveguide metal wall leads to a decrease in the SCOM growth rate.

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References

[1] Krall, N. A. and Trivelpiece, A. W., *Principles of Plasma Physics*. New York: McGraw-Hill, 1973 (reprinted San Francisco Press, 1986).
[2] Stix, T. X., *Phys. Fluids* **B2**, 1729 (1990).
[3] Becoulet, A., *Plasma Phys. Contr. Fusion* **38**, 1 (1996).
[4] Laqua, H. P., The W7-AS team and the ECRH group, *Plasma Phys. Contr. Fusion* **41**, 273 (1999).
[5] Miller, R. B., *An Introduction to the Physics of Intense Charged Particle Beams*. New York: Plenum, 1982.
[6] Kondratenko, A. N., *Surface and Volume Waves in Bounded Plasmas*. Moscow: Energoatomizdat, 1985 (in Russian).
[7] Platsmann, P. M. and Wolf, P. A., *Waves and Interactions in Solid State Plasma*. New York: Academic Press, 1973.
[8] Boardman, A. D. and Egan, P. In: *Surface Waves in Plasmas and Solids* (ed. S. Vukovich). Singapore: World Scientific, 1986.
[9] Moisan, M. and Pelletier, J. (eds), *Microwave Excited Plasmas*. Amsterdam: Elsevier, 1992.
[10] Ferreira, C. M. and Moisan, M. (eds), *Microwave Discharges: Fundamentals and Applications*. New York: Plenum, 1993.
[11] Azarenkov, N. A., Girka, V. O. and Sporov, A. E., *Physica Scripta* **55**, 339 (1997).

[12] Azarenkov, N. A., Girka, V. O., Kondratenko, A. M. and Sporov, A. E., *Plasma Phys. Contr. Fusion* **39**, 375 (1997).
[13] Girka, V. O., *J. Plasma Phys.* **60**, 413 (1998).
[14] Girka, V. O., Lapshyn, V. I. and Puzyrkov, S. Yu., *Contrib. Plasma Phys.* **39**, 487 (1999).
[15] Girka, V. O., *Plasma Phys. Contr. Fusion* **42**, 999 (2000).
[16] Briggs, R. J., *Electron-Stream Interaction with Plasmas*. Cambridge, MA: MIT Press, 1964.
[17] Davidson, R. C., *Theory of Nonneutral Plasmas*. New York: Benjamin, 1974.
[18] Lawson, J. D., *The Physics of Charged-Particle Beams*. Oxford University Press, 1977.
[19] Siegman, A. E., *An Introduction to Lasers and Masers*. New York: McGraw-Hill, 1971.
[20] Kondratenko, A. N. and Kuklin, V. M., *Principles of Plasma Electronics*. Moscow: Energoatomizdat, 1988 (in Russian).
[21] Kondratenko, A. N., *Field Penetration into Plasmas*. Moscow: Atomizdat, 1979 (in Russian).