Focus on the Visual

(The 2022 Presidential Address)

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Introduction

There are visual components of mathematics that are vital to the learning of the subject and should be a focus for its teaching. One aspect is the impressive capacity of the brain to visualise, an important element in the discovery process of some prominent mathematicians and scientists. The pedagogical importance of diagrams in mathematics is considered, with examples showing their role in problem-solving, elucidation, persuasion and decision making, concluding with some particularly stimulating and revealing diagrams for use in the classroom.

Visualisation

Imagine being asked the following three questions: Can you play chess? Could you play three games simultaneously? Could you play chess blindfolded? For many the answer to the first would be 'yes' and perhaps the response to the second would be 'I'd have a go' but for the vast majority the answer to the last would be either 'no' or 'perhaps for three or four moves'. Yet in 1783, François-André Philidor entered the St James's Street Chess Club in London and played three opponents simultaneously whilst blindfolded, two of the opponents considered the best in London. He won two games and drew the third. Philidor's feat has since been trumped and trumped again. In 2016, the Uzbeki-American grandmaster, Timur Garayev, played 48 games simultaneously whilst blindfolded and riding an exercise bike. He won 35 games, drew seven and lost just six.

We visualise either by interpreting what is seen (external visual thinking) or by pure creation (internal visual thinking). Humans have the ability to visualise, even in an exceptional way. A study of taxi drivers by UCL researchers in 2000 delivered staggering results. It took place before the advent of satnav and before the appearance of Uber, at a time when taxi drivers were tested on what they called 'The Knowledge'. Scans of their brains showed unusual physical development of the posterior part of the hippocampus where spatial processes are carried out. The volume of this part of the brain was roughly proportional to the number of years on the job [1].

Many mathematicians have the capacity to picture two-dimensional geometrical diagrams in their minds, some to imagine and rotate three-dimensional figures. A few years ago, I was copied into an email from Douglas Hofstadter to Sir Michael Atiyah and with the consent of both parties I published it in *Mathematics in School* [2]. Hofstadter made reference to the Swiss geometer, Jacob Steiner, who:

> 'insisted on teaching all of his geometry classes literally in the dark … He would cover all the windows of his room and would force his students to think in the dark'.

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Hofstadter went on:

> 'teaching geometry in a pitch-dark room is a wonderful exercise both for teacher and for students, and it … forces one to ponder, "What is visual imagery, if it is not seen by the eye?" It makes one realize that blind people have visual imagery every bit as rich as that of sighted people, and that indeed the eyes are just the entry channels for visual imagery for sighted people but that the actual imagery transcends vision and has to do with how space and shapes in space are represented in the brain'.

He tried to emulate Steiner by pulling down the blinds in his lecture theatre and working through Morley's theorem with his students in the dark. Morley's theorem states that if the angles of an arbitrary triangle are trisected, the rays produced intersect at points that define an equilateral triangle, thus creating symmetry out of asymmetry (see Figure 1). The proof is far from straightforward and so it is not surprising that Hofstadter's experiment was not an unmitigated success.

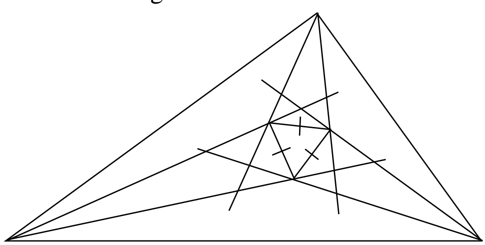

FIGURE 1: Morley's theorem

Steiner is not alone amongst mathematicians and scientists in emphasising the importance of visualisation. The German organic chemist August Kekulé first envisaged the ring structure of benzene as an ouroboros, a snake devouring its own tail. Lawrence Bragg, Nobel Laureate in physics in 1915, claimed his new ideas always came to him in the form of visual images. (In the quotes that follow, the emphases are mine.) René Descartes wrote [3] that

> '*Imagination* will chiefly be of great use in solving a problem by several deductions, the results of which need to be coordinated after a complete enumeration. *Memory* is necessary to retain the data of the problem if we do not use them all from the beginning. We should risk forgetting them if the *image* of the objects under consideration were not constantly present to our mind and did not offer all of them to us at each instant.'

Albert Einstein commented [4] that

> 'The psychical entities which seem to serve as elements in thought are certain signs and more or less clear *images* which can be "voluntarily" reproduced and combined…but taken from a psychological viewpoint,

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> this combinatory play seems to be the essential feature in productive thought — before there is any connection with logical construction in words or other kinds of signs which can be communicated to others.'

And in an interview [5] with Melvyn Bragg, Sir Roger Penrose said that

> 'I was very much on the *visual* side … other people could do it but it wasn't their primary way of thinking, the variation in the ways people thought about things was quite striking … I thought about the subject [relativity] very much in *pictures* … rather than equations.'

Clearly, examples abound of mathematicians and scientists attesting to the impact of the visual on their thinking. But perhaps we should not be surprised. After all, the equating of seeing and understanding is embedded in the English language. A dictionary definition of

- *perception* is 'understanding fuelled by the senses, especially sight';
- *insight* is 'a clear, deep, and sometimes sudden understanding of a complicated problem';
- *clarity* is 'seeing in high definition or full understanding'.

Indeed, Sir Michael Atiyah in his 1982 Presidential Address to The Mathematical Association [6], argued that

> '… the commonest way to indicate that you have understood an explanation is to say "I see". This indicates the *enormous power of vision* in mental processes, the way in which the brain can analyse and sift what the eye sees.'

The impact of diagrams

In a sense, the discussion thus far is a preamble to my main arguments about the importance and impact of diagrams in mathematics and elsewhere. So, consider what happens when the brain is confronted with a diagram. The diagram is received by the occipital lobe, our visual processor. Instantaneous spatial synthesis of the information occurs in the left parietal-occipital region and, specifically, the hippocampus handles spatial maps and orientations. Meanwhile, the temporal lobe is supervising short-term memory and the frontal lobe is overseeing the whole process of planning, creating and problem-solving.

It is my contention that diagrams help to:

- present information in a concise and appealing way;
- draw out the key features of a problem;
- reveal the connections between pieces of information;
- provide a setting for challenging or unusual problems;
- support the explanation of conclusions;

and consequently have the capacity to

- shape decisions, inform policy, persuade, and ultimately bring about change.

That contention comes from my teaching experience. There was barely a topic or a skill for which I could not roll out a diagram to support or clarify an explanation. And when someone was stuck on a problem, I would routinely ask "Have you drawn a diagram?"– enough in itself on many an occasion to get that student back on track. 'Diagrams are essential' became a mantra for me and for my students.

It is reassuring that, over the years, mathematics educationalists have promoted the use of diagrams. In *How to Solve It*, George Pólya gave us a four-step plan to solving mathematical problems [7]. The first step concerns 'Understanding the Problem' and includes the key tactic: Draw a figure. Richard Skemp explained that when we draw a figure as an aid to problem-solving, 'by leaving out quite a lot of the visual properties of an object we can abstract at a higher level, while still representing the resulting concepts visually' [8]. Ideally, the diagram is pared to the bone, so only the essentials are retained. He uses Figure 2 as an example.

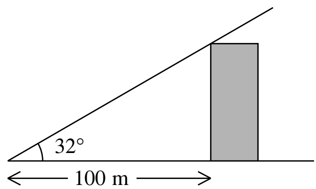

FIGURE 2: example of pared diagram

No doubt, teachers would be able to work backwards, suggesting the topic in which it might arise, noting the level of difficulty and assigning it to a particular age or stage. The question itself, or something akin to it, could easily be constructed from the diagram. Perhaps, teachers could ask students to work in pairs, one drawing a diagram and the other trying to write a problem to go with it, rather than always working the other way round?

Diagrams that bring clarity

An early example of how diagrams can be used for elucidation is provided by Leonhard Euler, that most prolific of mathematicians. For part of his career Euler was employed by the Prussian enlightened despot, Frederick the Great, and beginning in 1760 he was charged with the science education of the king's niece, Princess Frederike Charlotte. Euler's lessons have been preserved in a series of letters to his pupil and among them we find his explanation of syllogisms [9, 10]. He gives the four primitive syllogisms in diagrammatic form (Figure 3):

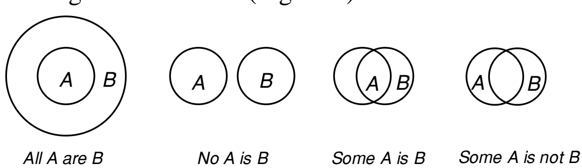

FIGURE 3: original Euler diagrams

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While we now recognise the possibility for misinterpreting such 'Euler diagrams' – hence their supersession by Venn, Carroll and Grünbaum diagrams – Euler's heuristics are clear. He believed at the time that any difficulty in understanding syllogisms can be overcome using 'these round figures, or rather spaces', writing that 'the mysteries of which we boast in logic and show with great difficulty are immediately apparent by means of these figures'.

Diagrams that reduce complexity

The power of diagrams can also lie in their ability to convey a huge amount of information in a compact form, and a classic example of such concision is provided by Charles Joseph Minard's 'carte figurative' depicting the dramatic erosion of Napoleon's army during the Russian campaign [11, 12]. (For want of space it is not reproduced here but view it at https://en.wikipedia.org/wiki/Charles_Joseph_Minard before reading on.) From crossing into Russia from Poland in June 1812 with 442,000 men, Napoleon saw the strength of his army fall to just a quarter of that as it reached Moscow, with numbers declining further in retreat until only 10,000 survived to re-enter Poland.

The historian of visual graphics, Edward Tufte, commented that this diagram 'may be the best statistical graphic ever drawn' [13]. It has no fewer than six variables: the *strength* of the army is shown by the width (thickness) of the line, the *direction* of travel is indicated by the diminishing thickness and the switch of colour as the army advances on Moscow and then retreats, and the position (*latitude* and *longitude*), *time* and *temperature* are also indicated. On so many levels it is a *tour de force*, though to claim that Tolstoy takes over half a million words to reach the same effect in *War and Peace* is surely an overstatement.

Diagrams that persuade

Concision can also be combined with elucidation to produce something which is persuasive, even to the point of effecting policy change. A case in point is Florence Nightingale's polar area graph. The story of Florence Nightingale has been told by numerous subject writers and biographers but more so in recent times by those with a focus on statistics as well as on nursing and hospital administration [14, 15]. Her background was one of privilege, with social connections to politicians, including Sidney Herbert and her neighbour Lord Palmerston.

During the Crimean War in the mid-1850s, British soldiers fought in nightmarish conditions with extreme heat followed by extreme cold, not to mention flies, scurvy, diarrhoea and cholera. The sick and injured were transported to a hospital in Scutari that was poorly equipped and dirty, and where their treatment was badly organised. Nightingale was appointed by Herbert as Superintendent of Female Nurses in the East and she arrived with a team of nurses immediately before the Balaklava and Inkerman casualties came in. She addressed the pressing issues of sanitation, hygiene and ventilation but also instituted rigorous record keeping and data collection. The result of the new hospital régime was a dramatic

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reduction in deaths at Scutari and Nightingale captured the before-and-after situation in the polar area graph that is shown on the front cover of this journal. The circle is divided into sectors, one each for the twelve months, their areas proportional to the deaths recorded, with the outer blue shading representing preventable deaths. As our eye sweeps around the diagram clockwise from the '9 o'clock position' (1 April 1854), the improving situation is plain to see.

The diagram appeared in Nightingale's *Health of the British Army* (1858) but it is not known whether she had drawn it prior to her audience in 1856 with Queen Victoria and her statistically-savvy consort, Prince Albert. It is tempting to imagine her taking slices from twelve sponge cakes of different sizes and arranging them on a plate to dramatic effect. Anyway, with the Queen on board, with the support of her politician friends and armed with a wonderful diagram, Nightingale persuaded parliament to make organisational changes affecting the army and subsequently civilian hospitals, including an emphasis on accurate data collection and analysis.

Diagrams for posing problems

If diagrams are important in life outside the classroom, then they are equally important in lessons. For example they can be used as vehicles for challenging and engaging geometrical problems. A style of problem advanced in my books on the elementary mathematics of area is characterised by questions such as, 'What fractions of the area of each outer circle are taken up by the quadrant and by the sextant (sixth of a circle) in the diagrams in Figure 4? [16]

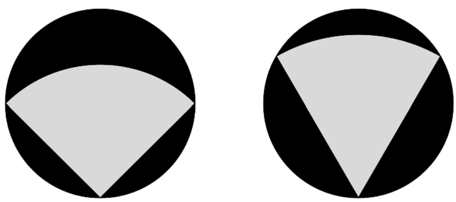

FIGURE 4: visually-attractive geometry problems

Becoming ever more popular are similar (yet subtly different) geometry problems created and disseminated by Catriona Agg and others via social media. The problem shown in Figure 5 is one of Catriona's. It is interesting to note that while the Twitterati are in awe, the popularity of the problems (albeit among an audience with a different profile) proves just as strong when they are published in printed form in *Mathematics in School*.

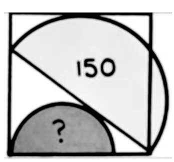

FIGURE 5: typical Catriona Agg problem

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Diagrams that move

Unfortunately, the paucity of time when delivering the Presidential Address precluded any detailed mention of the pedagogic value of the incorporation of dynamic elements, of movement. If the visual should have a significant part to play in mathematics education, then motion brings an extra vitality. Take, for example, television series for the young, such as *Numberblocks*, video games, including *NumBots* and *Minecraft*, and dynamic geometry software such as *Geometer's Sketchpad*, *Cabri*, *Desmos*, *GeoGebra* and *Autograph*. Among the facilities built into the software packages are that which allows a point to be grabbed and moved so that what is invariant in the diagram becomes apparent, and sliders that, for example, allow us to see that change of state (from chord, or secant, to tangent), when the limit is taken, illuminating the process of differentiation.

Diagrams and proof or demonstration

Justifying (or proving) key results in mathematics provides the subject's credibility and this is something that can hardly be overlooked by the teacher. Diagrams often enhance a proof or at the very least provide a *prima facie* case for its validity and hence acceptance. There are some particularly nice demonstrations that go under the heading 'proofs without words', and they are equally as appealing as Mendelssohn's 'songs without words' (*Lieder Ohne Worte*). Such demonstrations have been collected by Roger Nelsen in three books published by the Mathematical Association of America, and are highly recommended [17]. The following examples are taken from or adapted from Nelsen's compilations.

(1) Factorising a difference of squares and a difference of cubes

For the former identity, cut a square of side y from one corner of a square of side x to give the L-shaped diagram with area on the left of Figure 6. Then slice off a rectangle as shown and rotate it into position on the right-hand side of the larger rectangle. The rectangle produced by reattaching it has length $x + y$, width $x - y$ and hence area $(x + y)(x - y)$. Since area is conserved in such dissection, rotation and reconnection, we have established the standard difference of squares identity $x^2 - y^2 = (x + y)(x - y)$ via the diagram.

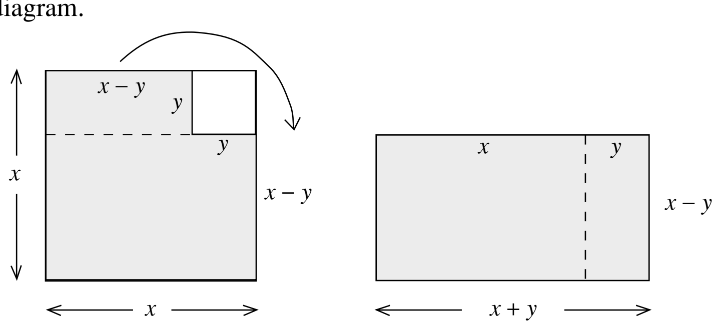

FIGURE 6: dissection demonstration of the difference of squares identity

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But there's more for the teacher here. The two pieces align on one side because they share a common side and the length of that side, $x - y$, is one of the factors of $x^2 - y^2$. Selecting where to cut is equivalent to identifying one of the factors. The geometry and the algebra go hand in glove and this is surely an aid to understanding. The approach can be extended to a difference of cubes (Figure 7). We begin with a small cube cut from the corner of a larger cube. Three cuts are made judiciously, so that the blocks created have one dimension in common (again $x - y$), and they are reconfigured into a prism. The volume of the original figure is $x^3 - y^3$ and that of the prism is $(x - y)\left(x^2 + xy + y^2\right)$.

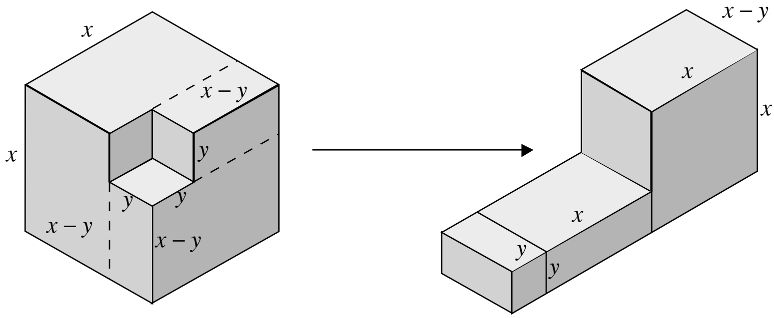

FIGURE 7: dissection demonstration of the difference of cubes identity

(2) The sum of the first n squares

Successive perfect squares can be represented by layers of unit cubes, each arranged in a square design. A sum of perfect squares can therefore be depicted as a stepped pyramid. In the limited representation on the left below, there are just four layers and so the number of unit cubes is

$$1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30.$$

But for the purposes of generalisation, we imagine that there are n layers. Take three such pyramids, as in Figure 8, rotate and reattach in a particular way, the one on the left joining at base level, that on the right joining one unit up.

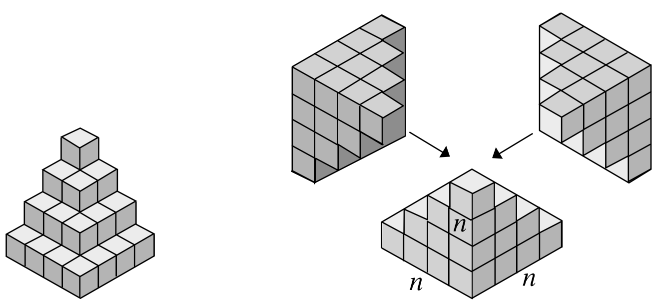

FIGURE 8: building and connecting three stepped pyramids

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This gives a block with an incomplete top layer (on the left in Figure 9); in fact, exactly half of the $(n + 1)$th layer is present. But if two such blocks are joined together – here I have rotated again for convenience – a cuboid of dimensions n, $n + 1$ and $2n + 1$ is created (Figure 9, right).

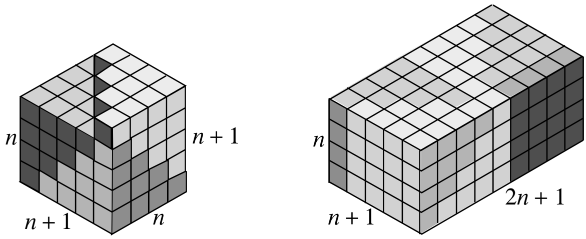

FIGURE 9: completing the demonstration for the sum of n squares

The volume of this cuboid is the product of these dimensions, but we must remember that this represents six times the sum of squares we require. Hence,

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n + 1)(2n + 1).$$

(3) The sum of the first n cubes

Consider one 1×1 square, two 2×2 squares, three 3×3 squares and so on, as in Figure 10.

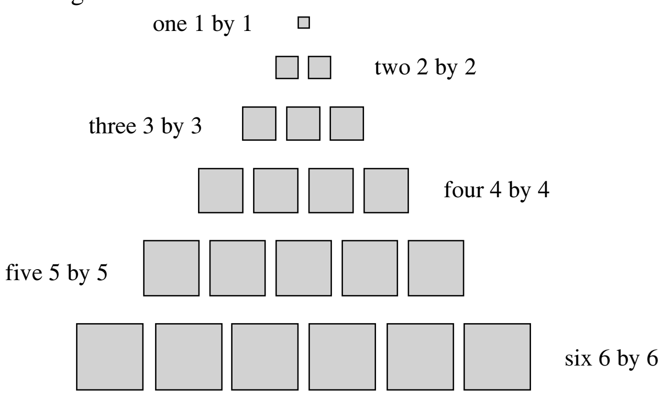

FIGURE 10: ‘building blocks’ for a sum of cubes demonstration

The three dimensions needed for cubes come about in a strange, some would say synthetic way. The side-length comes in twice as if for area and the number of occurrences of each square size provides the third. Together these squares of different sizes and different numbers of sizes can be fitted together to make an asymmetrical stepped pyramid, and four such pyramids make a square (Figure 11).

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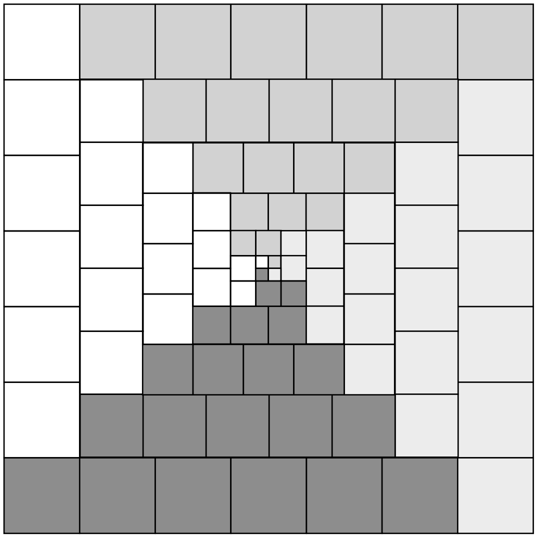

FIGURE 11: completing the demonstration for the sum of n cubes

As before, the representation is for a small number of terms (6 here) but we imagine it extended to n terms. Paying particular attention to the bottom edge of the figure, it is made up of n lengths of magnitude n (in the darker shading) and an extra length of magnitude n in the bottom right corner. So the base of the outer square has length $n(n + 1)$ and the square's area is $n^2(n + 1)^2$. Since this is four times what is required, the sum of the first n perfect cubes is

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n + 1)^2.$$

Consistency of form

Among all the diagrams that are available to the mathematics teacher, those that exhibit a consistency of form are especially powerful. Here is a model that explains multiplication in arithmetic and in algebra (Figures 12 and 13), whilst also helping to justify a key result in elementary calculus, the product rule (Figure 14). Notice in particular how the partial products appear in the diagrams and the written algorithms.

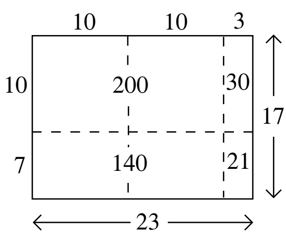

FIGURE 12

$$\begin{aligned} 23 \times 17 &= (20 + 3)(10 + 7) \\ &= 20 \times 10 + 20 \times 7 + 3 \times 10 + 3 \times 7 \\ &= 200 + 140 + 30 + 21 \\ &= 391 \end{aligned}$$

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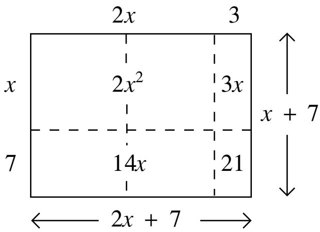
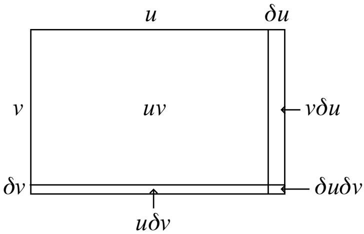


$$(2x + 3)(x + 7) = 2x(x + 7) + 3(x + 7)$$
$$= 2x^2 + 14x + 3x + 21$$
$$= 2x^2 + 17x + 21$$

FIGURE 13



u is increased by a small amount δu
v is increased by a small amount δv
The derivative of uv is the increase in its area
$u\delta v + v\delta u$
plus a vanishingly small quantity $\delta u \delta v$

FIGURE 14

Avoiding disaster

Finally, a salutary lesson about what could go wrong should we completely ignore the visual. It was delivered by the British statistician, Frank Anscombe, from his desk at Yale University in 1973 [17]. With unusual dexterity, Anscombe produced four sets of bivariate data with very strange properties (Table 1).

Set 1		Set 2		Set 3		Set 4	
x	y	x	y	x	y	x	y
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

TABLE 1: Anscombe's quartet

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The summary statistics for Set 1 are:

$n = 11$
Mean of x values = 9,
Mean of y values = 7.5,
Sample variance of x = 11,
Sample variance of y = 4.125,
Correlation coefficient = 0.816,
Equation of regression line: $y = \frac{1}{2}x + 3$.

Amazingly, when the summary statistics for the other three sets are calculated, they come out the same in every detail. The lesson that Anscombe is teaching us is that these summary statistics hide the true nature of the data and that that true nature becomes apparent only through plotting the points (Figure 18). A diagram is absolutely essential to understanding what is going on.

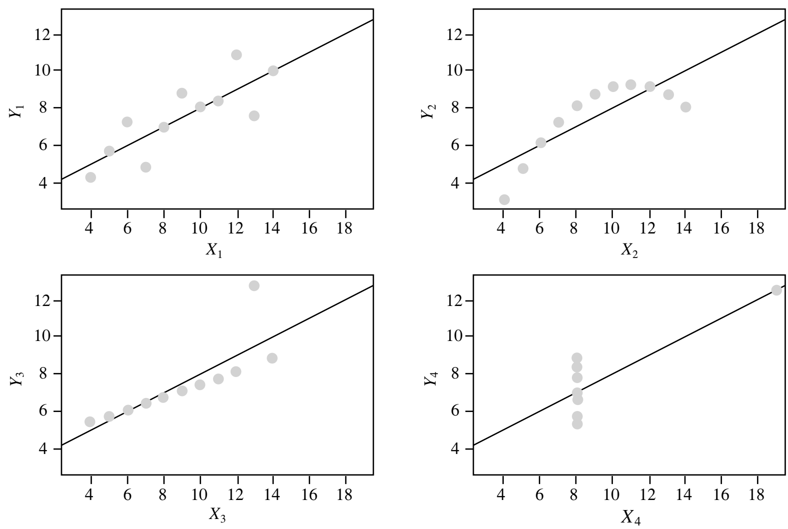

FIGURE 15: plots of Anscombe's quartet of bivariate data

The key is the appropriateness of adopting a linear model of regression. It appears totally justifiable for the first set (top-left) but for the second data set, represented top-right, the points lie on a parabola. A linear regression line is suitable for each of the other two sets, though a particular point has thrown things out. The outlier in Set 3 is dragging the gradient upwards, whereas the outlier in Set 4 is masking the fact that the correct line is vertical, which would indicate independence.

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Summary and conclusion

In summary,

1 We have a great capacity for visualisation, and that capacity can be increased.
2 Visual stimuli and the images constructed in the brain are important in learning mathematics, perhaps especially in problem-solving.
3 Pupils' understanding is bolstered by their own diagrams and by their teacher's diagrams, whether imagined or drawn by hand, whether static or dynamic, whether produced on a graphing calculator or in a software package, or in a spreadsheet, or by using an app.
4 There are advantages in having a consistency of visual models.
5 Diagrams are efficient in presenting information in a compact way and in an appealing way.
6 They can be used to convince others of the accuracy of a result and make a valuable contribution to decision making at all levels.
7 The lack of a diagram can lead to a failure to make progress on a problem or, in the worst-case scenario, to the drawing of false conclusions.

If there is a lesson to be learnt, it is that we should **focus on the visual**.

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The answers to the *Nemo* page from July 2022 on applied mathematics were:

1. Anthony Trollope The Small House at Allington Chapter 55
2. Anthony Powell Casanova's Chinese Restaurant Chapter 4
3. GK Chesterton Songs of education: the higher mathematics
4. James Clerk Maxwell A Vision of a Wrangler, of a University
5. Hilary Mantel The Mirror and the Light Part 3, Chapter 1
6. Thomas Carlyle The French Revolution Vol 3, Book 2, Chapter I

Congratulations to Martin Lukarevski and Henry Ricardo on tracking all of these down. I apologise for omitting Martin from the list of solvers for the 2021 page on polygons. This selection is more specific, focusing on pulleys (which are probably neither light nor frictionless, any more than the strings inextensible). Quotations are to be identified by reference to author and work. Solutions are invited to the Editor by 23rd February 2023.

1. He said no more, but signed to me to lift a heavy wooden corb with an iron loop across it, and sunk in a little pit of earth, a yard or so from the mouth of the shaft. I raised it, and by his direction dropped it into the throat of the shaft, where it hung and shook from a great cross-beam laid at the level of the earth. A very stout thick rope was fastened to the handle of the corb, and ran across a pulley hanging from the centre of the beam, and thence out of sight in the nether places.

Continued on page 442.