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Domains of modulation parameter in the interaction of finite Airy–Gaussian laser beams with plasma

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Abstract

In this paper, self-focusing of finite Airy–Gaussian (AiG) laser beams in collisionless plasma has been investigated. The source of nonlinearity considered herein is relativistic. Based on the Wentzel–Kramers–Brillouin (WKB) and paraxial-ray approximations, the nonlinear coupled differential equations for beam-width parameters in transverse dimensions of AiG beams have been established. The effect of beam's modulation parameter and linear absorption coefficient on the self-focusing/defocusing of the beams is specifically considered. It is found that self-focusing/defocusing of finite AiG beams depends on the range of modulation parameter. The extent of self-focusing is found to decrease with increase in absorption.

Introduction

The study of high-power laser–plasma interaction has paved the way in various directions due to its large applications (Kaw, 2017) such as laser electron acceleration, optical harmonic generation, laser-driven fusion, generation of X rays, etc. For many of these applications, it is necessary that the optical beam is intense and propagates for extended distances without divergence. In this respect, the propagating distance is limited to approximately Rayleigh diffraction length in the absence of optical guiding mechanism. Such propagating distance is strongly affected by nonlinear self-focusing (Akhmanov *et al.*, 1968) at sufficient high-power and intensity of laser. In plasma, three main mechanisms of self-focusing (Sodha *et al.*, 1976), namely relativistic, ponderomotive, and thermal, have been pointed out. The latter two require finite time to set up, while relativistic self-focusing arises instantly and requires very high laser intensity. The analytical theory of relativistic self-focusing of laser in plasma has been included in the numerical treatment by Hora and co-workers (Hora, 1975; Hauser *et al.*, 1988). By considering the arbitrary magnitude of beam intensity, relativistic self-focusing of Gaussian laser beam in plasma has been discussed in different situations (Asthana *et al.*, 2000; Feit *et al.*, 2001; Khanna and Baheti, 2001; Varshney *et al.*, 2006; Hasson *et al.*, 2010; Sharma and Kourakis, 2010; Patil *et al.*, 2011, 2013*a*, 2016, 2018*a*). In the same context, the field of relativistic self-focusing of laser has received a considerable bonus in some stimulated scattering processes (Mahmoud and Sharma, 2001), acceleration of electrons (Habara *et al.*, 2006), generation of high harmonics (Sharma *et al.*, 2019), etc. from plasmas. By exploring classical Hamiltonian formalism, relativistic self-focusing of ultra-intense lasers in underdense plasmas have been analyzed to determine the limit between geometrical optics and wave optics considerations (Curcio *et al.*, 2018). An analytic theory has been described for the formation of a self-focusing structure of Gaussian laser beam in plasma with relativistic nonlinearity (Kovalev and Bychenkov, 2019).

On the other hand, considerable interest has been also elevated in the relativistic self-focusing of some modified Gaussian beams such as cosh-Gaussian beams (Vhanmore *et al.*, 2017, 2018*a;* Kumar *et al.*, 2018), Hermite–Gaussian beams (Kant *et al.*, 2012), Hermite-cosh-Gaussian beams (Patil *et al.*, 2007; Nanda *et al.*, 2013; Vhanmore *et al.*, 2019, 2020; Gavade *et al.*, 2020), Hermite-cosine-Gaussian beams (Wani and Kant, 2016), elliptic Gaussian beams (Kumar and Aggarwal, 2018), quadruple Gaussian beams (Aggarwal *et al.*, 2018), *q*-Gaussian beams (Vhanmore *et al.*, 2018*b*; Kashyap *et al.*, 2019), Bessel–Gaussian beams (Patil *et al.*, 2019), and Laguerre–Gaussian beams (Dwivedi *et al.*, 2019), due to their definite characteristics in comparison to that of Gaussian laser beam. In contrast to the traditional plasma physics, some works in the literature on relativistic self-focusing of Gaussian laser beam in quantum plasmas have already been properly discussed (Hefferon *et al.*, 2010; Patil *et al.*, 2013*b*, 2018*b*; Zare *et al.*, 2015; Kumar *et al.*, 2016; Aggarwal *et al.*, 2017*a*, 2017*b*, 2019). It has been realized that in comparison with the classical relativistic box of situation, the quantum effects have a key part in better focusing of Gaussian laser beam in plasmas. It has been observed that early and strong relativistic self-focusing is observed in case of the cosh-Gaussian laser beam in cold quantum plasma (Nanda *et al.*, 2018). Such enhanced focusing is found to occur earlier and strongest for the case of thermal quantum plasma in comparison with cold quantum plasma (Patil and Takale, 2013, 2014; Mahajan *et al.*, 2018).

Besides, a significant interest has been gained in a new family of paraxial light beams, known as Airy beams (Siviloglou and Christodoulides, 2007). Unlike normal optical beams, Airy beams transversely accelerate (self-bend) throughout propagation. This exotic behavior is possible even in entirely homogeneous media. Remarkably, the intensity peaks of Airy beams follow parabolic trajectories much like the ballistics of projectiles (Polynkin *et al.*, 2009*a*). Such finite Airy beams have potential applications in plasma channel generation (Polynkin, *et al.*, 2009*b*), laser-driven acceleration (Li *et al.*, 2010), optical trapping (Zheng *et al.*, 2011), etc. By considering general nonlinear media, the propagation of Airy beams has been studied in detail (Deng and Li, 2012; Chen *et al.*, 2015). Using the Wentzel–Kramers–Brillouin (WKB) approximation, relativistic self-focusing of finite Airy–Gaussian (AiG) beams in plasma has been presented (Ouahid *et al.*, 2018*a*). They have also extended the same under the combined effects of relativistic and ponderomotive nonlinearities in a plasma (Ouahid *et al.*, 2018*b*). It is found that the modulation parameter plays an important role in the self-focusing. In the present work, we have emphasized analytically to set the numerical domain of modulation parameter of finite AiG beams propagation in plasma.

The present paper is devoted to investigate the domains of modulation parameter of finite AiG beams propagating through plasma. Using the ansatz for the electric field in the wave equation, a mathematical formulation for the beam-width parameters in plasma is obtained through the parabolic equation approach (Akhmanov *et al.*, 1968) under paraxial and WKB approximations. By considering the relativistic nonlinearity, the evolution of the beam-width parameter is introduced in the distance of propagation. The present work is structured as follows: In the "Self-focusing" section, evolution equations in governing beam-width parameters in transverse dimensions of finite AiG beams have been established by using the parabolic equation approach under WKB and paraxial approximations. The detailed discussion of results is presented in the context of domains of the modulation parameter in the "Numerical results and discussion" section. A brief conclusion is added in the "Conclusion" section.

Self-focusing

We begin by considering the propagation of finite AiG laser beams along the z-direction. The initial electric field distribution of the beams is expressed as follows (Ouahid *et al.*, 2018*a*):

$$E(x,y,0) = E_0 \mathrm{Ai}\left(\frac{x}{r_0}\right)\exp\left(\frac{a_0 x}{r_0}\right)\exp\left(\frac{-x^2}{r_0^2}\right)\mathrm{Ai}\left(\frac{y}{r_0}\right)\exp\left(\frac{a_0 y}{r_0}\right)\exp\left(\frac{-y^2}{r_0^2}\right) \tag{1}$$

where E_0 is the constant amplitude of electric field, $\mathrm{Ai}(\cdot)$ is the Airy function of the first kind, r_0 is the initial beam-width, and a_0 is the modulation parameter also called the aperture coefficient.

The propagation of finite AiG laser beam in plasma is characterized by the dielectric function which can, in general, be expressed as follows (Sodha *et al.*, 1976):

$$\varepsilon = \varepsilon_0 + \Phi(\mathrm{EE}^*) - i\varepsilon_i \tag{2}$$

where $\varepsilon_0 = 1 - \omega_p^2/\omega^2$ is the linear part of the dielectric function. In the relativistic regime, the usual expression for nonlinear part $\Phi(\mathrm{EE}^*)$ of the dielectric function for the plasma is written as follows (Sharma and Kourakis, 2010):

$$\Phi(\mathrm{EE}^*) = \frac{\omega_p^2}{\gamma\omega^2}(\gamma - 1) \tag{3}$$

where ω is the angular frequency of laser beam, ω_p is the plasma frequency given by $\omega_p = (4\pi n e^2/m)^{1/2}$, n_0 is the density of plasma electrons in the absence of the beam, γ is the relativistic factor expressed as $\gamma = (1 + \alpha \mathrm{EE}^*)^{1/2}$. Here, $\alpha = e^2/m^2\omega^2c^2$ with e is the charge on electron and m is the rest mass of electron. We limit ourselves to the case when ε_i is field independent and $\varepsilon_i \ll \varepsilon_0$.

The wave equation governing the electric vector of the beam in plasma with the dielectric function given by Eq. (2) can be written as follows:

$$\nabla^2 E - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \tag{4}$$

In writing Eq. (4), the term $\nabla(\nabla \cdot E)$ has been neglected which has been justified when $(c^2/\omega^2)|\nabla(\ln\varepsilon)/\varepsilon| \ll 1$. For the convenience of field distribution given by Eq. (1), we have adopted the Cartesian coordinate system. Within the framework of WKB approximation, the solution of Eq. (4) can be written as follows:

$$E = A(x,y,z)\exp[i(\omega t - kz)] \tag{5}$$

Substituting for E and ε from Eqs. (5) and (2) in Eq. (4), one obtains

$$2ik\frac{\partial A}{\partial z} = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{k^2\Phi(\mathrm{AA}^*)}{\varepsilon}A \tag{6}$$

Equation (6) is known as the parabolic wave equation that describes the evolution of beam envelope in plasma.

We may express A as

$$A = A_0(x,y,z)\exp[-ikS(x,y,z)] \tag{7}$$

where A_0 and S are real functions of x, y, and z. Here, S is the eikonal of the beam which determines convergence or divergence of the beam. Equation (7) is valid when the polarization of the beam does not change with propagation. Substituting for A from Eq. (7) in Eq. (6) and separating real and imaginary parts, we get

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial x^2} + \frac{\partial^2 A_0}{\partial y^2}\right) + \frac{\Phi(\mathrm{A_0A_0^*})}{\varepsilon_0} \tag{8a}$$

and

$$\frac{\partial A_0^2}{\partial z} + \left(\frac{\partial S}{\partial x}\right)\left(\frac{\partial A_0^2}{\partial x}\right) + \left(\frac{\partial S}{\partial y}\right)\left(\frac{\partial A_0^2}{\partial y}\right) + \left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2}\right)A_0^2 - \left(\frac{k\varepsilon_i}{\varepsilon_0}\right)A_0^2 = 0 \tag{8b}$$

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Following the approach given by Akhmanov *et al.* (1968) and Sodha *et al.* (1976), the solutions corresponding to Eq. (8) can be written as follows:

$$S = \beta_1(z)\frac{x^2}{2} + \beta_2(z)\frac{y^2}{2} + \varphi(z) \tag{9a}$$

and

$$A_0^2 = \frac{E_0^2}{f_1 f_2}\left(\mathrm{Ai}\left(\frac{x}{r_0 f_1}\right)\mathrm{Ai}\left(\frac{y}{r_0 f_2}\right)\right)^2 \exp\left(\frac{2a_0 x}{r_0 f_1} + \frac{2a_0 y}{r_0 f_2} - \frac{2x^2}{r_0^2 f_1^2} - \frac{2y^2}{r_0^2 f_2^2} - k_i z\right) \tag{9b}$$

where $\beta_1(z) = (1/f_1(z))(\partial f_1(z)/\partial z)$ and $\beta_2(z) = (1/f_2(z))(\partial f_2(z)/\partial z)$ are the inverse of radius of curvatures of the beam along the x and y directions, respectively, $\varphi(z)$ is the axial phase, $f_1(z)$ and $f_2(z)$ are the dimensionless beam-width parameters along the x and y directions, respectively, and k_i is the absorption coefficient.

Substituting for S and A_0^2 from Eq. (9) in Eq. (8a) and using the paraxial approximation, we find

$$\frac{d^2 f_1(z)}{d\xi^2} = \frac{A}{f_1(z)^3} + \frac{BCe^{-2\xi k_i'}\left(1 + \frac{\alpha E_0^2 g_2^4 e^{-\xi k_i'}}{f_1(z) f_2(z)}\right)^{-3/2}}{f_1(z)^3 f_2(z)^2} \tag{10a}$$

$$\frac{d^2 f_2(z)}{d\xi^2} = \frac{A}{f_2(z)^3} + \frac{BCe^{-2\xi k_i'}\left(1 + \frac{\alpha E_0^2 g_2^4 e^{-\xi k_i'}}{f_1(z) f_2(z)}\right)^{-3/2}}{f_2(z)^3 f_1(z)^2} \tag{10b}$$

with

$$A = \left(4 + a_0 - \frac{2a_0 g_1^3}{g_2^3} + \frac{4g_1^2}{g_2^2}\right)$$

$$B = (g_1^2 g_2^2 - 4a_0 g_1 g_2^3 - 2g_2^4 + 2a_0^2 g_2^4)$$

$$C = \frac{\alpha E_0^2 g_2^4 r_0^2 \omega_p^2}{2c^2}$$

$$g_1 = \frac{1}{3^{1/3}\Gamma(1/3)}$$

and

$$g_2 = \frac{1}{3^{2/3}\Gamma(2/3)}$$

where $\xi = z/R_d$ the dimensionless distance of propagation, $R_d = kr_0^2$ is the Rayleigh diffraction length, and $k_i' = k_i R_d$ is the normalized absorption coefficient.

Equations (10a) and (10b) are the nonlinear coupled differential equations governing the variation of the beam-width parameters f_1 and f_2 with the distance of propagation ξ. The first term on the right-hand side corresponds to the diffraction divergence of the beam and the second term corresponds to convergence due to the nonlinearity. We now devote our efforts for obtaining and analyzing the domain of propagation of finite AiG beams through plasma.

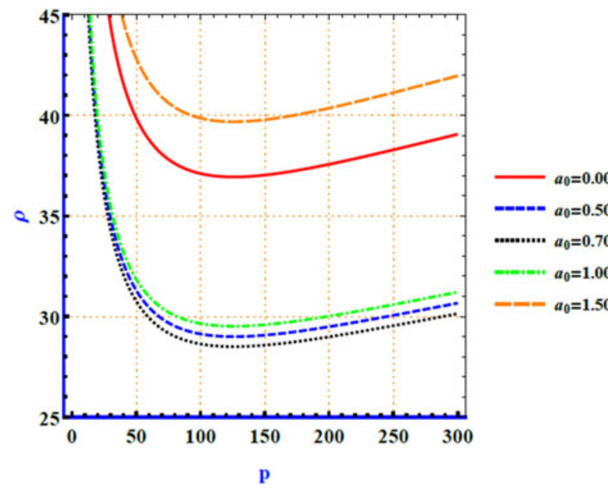

Fig. 1. Critical curves for different modulation parameters a_0.

Numerical results and discussion

For an initial plane wavefront of the beam, the initial conditions on f_1 and f_2 are $f_1(\xi = 0) = f_2(\xi = 0) = 1$ and $df_1/d\xi = df_2/d\xi = 0$. When the two terms on the right-hand side of Eqs. (10a) and (10b) cancel each other at $\xi = 0$, $d^2f_1/d\xi^2 = d^2f_2/d\xi^2 = 0$ since $df_1/d\xi$ and $df_2/d\xi$ are also zero and $f_1 = f_2 = 1$ for all values of ξ. In other words, the beam propagates without convergence or divergence. The condition for self-trapping is therefore

$$\rho(a_0, p) = \sqrt{\frac{-2\left(4 + a_0 - \frac{2a_0 g_1^3}{g_2^3} + \frac{4g_1^2}{g_2^2}\right)(1 + pg_2^4)^{3/2}}{(g_1^2 g_2^2 - 4a_0 g_1 g_2^3 - 2g_2^4 + 2a_0^2 g_2^4) p g_2^4}} \tag{11}$$

where $\rho = r_0\omega_p/c$ is the dimensionless initial beam radius and $p = \alpha E_0^2$ is the initial intensity parameter. Equation (11) is definitely more amenable to mathematical manipulations. With the help of this equation, the critical values of a_0 and p pertaining to uniform propagation of finite AiG beams can be easily determined. We have determined the minimum value of ρ by minimizing it with respect to a_0 and p using the theorem on extremum values in two variable cases. This gives respective critical values of modulation and intensity parameters as $a_0 = a_{0c} = 0.70635$ and $p = p_c = 125.88654$. At these two values, one can get the minimum value of the critical beam radius as $\rho = \rho_c = 28.50137$. Now one may investigate the response of ρ against p around a_{0c} by using Eq. (11).

In Figure 1, we have plotted the dimensionless initial beam radius ρ as a function of the initial intensity parameter p for different values of a_0. Such variation of ρ against p is regarded as critical curves for finite AiG beams propagation in plasma. Each of these curves divides (p, ρ) plane into two regions. Initial points (p, ρ) lying above and below the each curve corresponds to self-focusing and divergence of finite AiG beams, respectively, which accords with earlier investigation (Sharma *et al.*, 2003). One should note from this figure that as a_0 increases, the critical

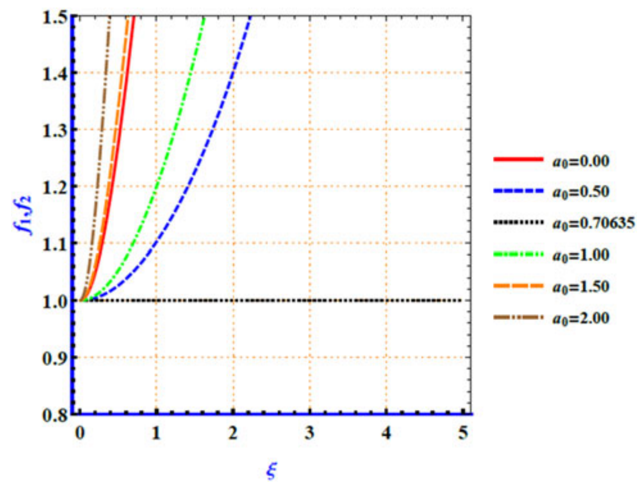

Fig. 2. Dependence of beam-width parameters f_1 and f_2 with dimensionless propagation distance ξ for different modulation parameters a_0. The other numerical parameters are ρ = 28.50137, p = 125.88654, and $k_i' = 0$.

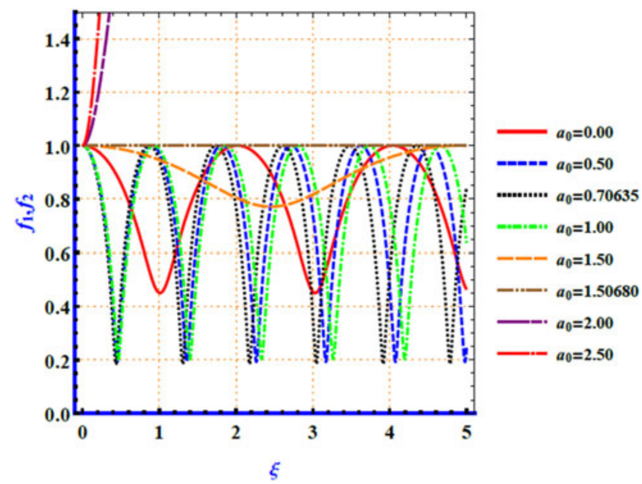

Fig. 4. Dependence beam-width parameters f_1 and f_2 with dimensionless propagation distance ξ for different modulation parameters a_0 with $k_i' = 0$. Other parameters are same as in Figure 3.

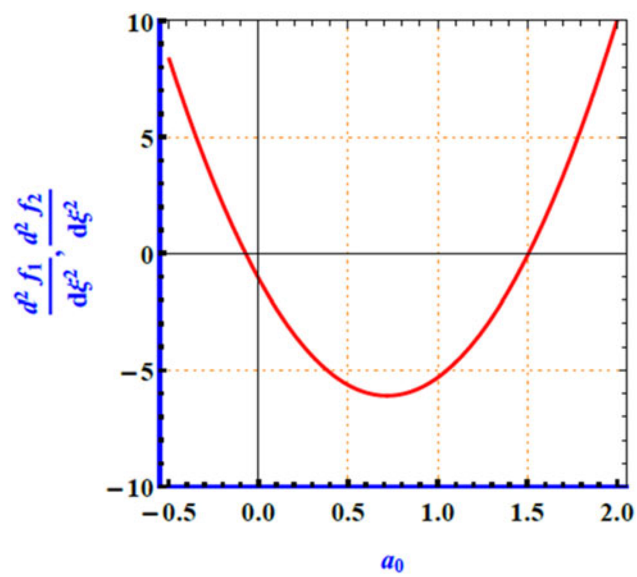

Fig. 3. Domains of modulation parameter a_0 with ρ = 40 and p = 125.88654.

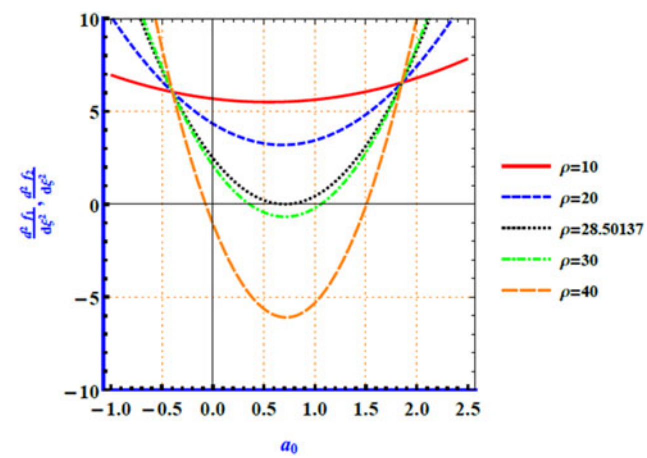

Fig. 5. Domains of modulation parameter a_0 for different ρ with p = 125.88654.

curve shifts downward till a_0 takes a critical value $a_{0c} = 0.70635$. However, with further increase in a_0, the curve shifts upward. Figure 2 shows the variation of beam-width parameters f_1 and f_2 with dimensionless distance of propagation ξ for same values of a_0 as varied in Figure 1 and $k_i' = 0$. We have chosen a representative point (p, ρ) which lie below the all critical curves of Figure 1, i.e., $\rho = \rho_c = 28.50137$ and $p = p_c = 125.88654$. It is evident from Figure 2 that as a_0 increases, AiG beams suffers with defocusing character of f_1 and f_2 with ξ up to a_{0c}. At $a_{0c} = 0.70635$, beam shows a stationary self-trapped mode. With further increase in a_0 above a_{0c} ($a_0 > a_{0c}$) causes defocusing of finite AiG beams. It is to be noted that for $\rho < \rho_c$, AiG beams always get defocused, although it has $p > p_c$.

Figure 3 shows the variation of $d^2f_1/d\xi^2$ and $d^2f_2/d\xi^2$ with a_0 for ρ = 40 (> ρ_c) with $p = p_c = 125.88654$. Three domains of a_0 have been observed characterizing the nature of propagation of finite AiG beams as follows:

(i) Self-focusing
$$d^2f_1/d\xi^2 < 0 \text{ and } d^2f_2/d\xi^2 < 0 \text{ for } -0.07179 < a_0 < 1.50680.$$
(ii) Defocusing
$$d^2f_1/d\xi^2 > 0 \text{ and } d^2f_2/d\xi^2 > 0 \text{ for } -0.07179 > a_0 > 1.50680.$$
(iii) Self-trapping
$$d^2f_1/d\xi^2 = 0 \text{ and } d^2f_2/d\xi^2 = 0 \text{ for } -0.07179 = a_0 = 1.50680.$$

In Figure 4, we have displayed beam-width parameters f_1 and f_2 as a function of ξ for different values of a_0 with ρ = 40 and $k_i' = 0$. From this figure, we have observed exact propagation behaviors of f_1 and f_2 with ξ as per the domains of a_0 discussed in Figure 3. Consequently, by increasing the modulation parameter a_0, self-focusing of finite AiG beams becomes better and shifted toward lower values of propagation distance ξ as reported earlier in Ouahid *et al.* (2018a). However, such early and strong self-focusing trend of f_1 and f_2 is observed to be reversed beyond critical modulation parameter a_{0c}. As such finite AiG beams suffer more defocused character of f_1 and f_2. Figure 5 illustrates the three domains of a_0 for different values of ρ. The most striking feature of this figure is that the range of a_0 remain unchanged as discussed in Figure 3. However, the self-focusing region enhances with an increase in ρ values.

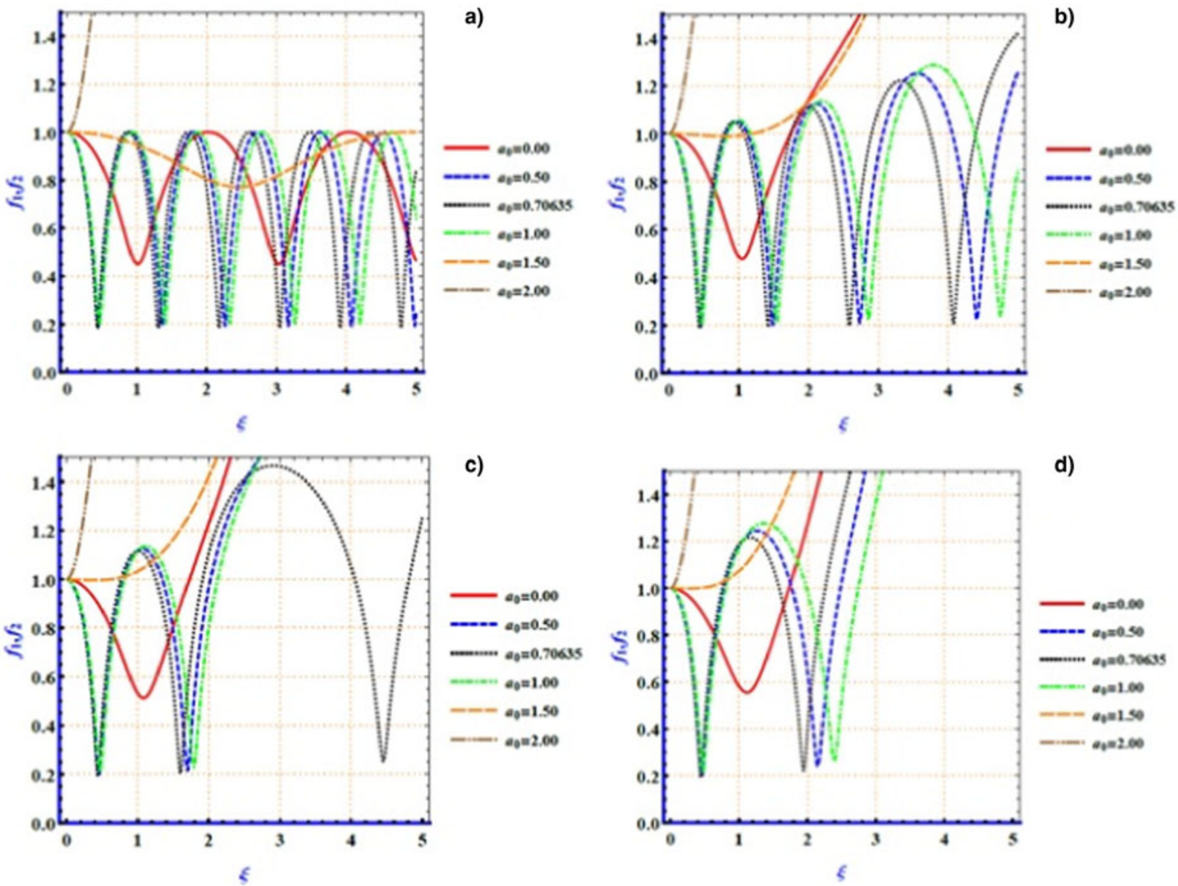

Fig. 6. Dependence beam-width parameters f_1 and f_2 with dimensionless propagation distance ξ for different modulation parameters a_0 with (a) $k_i' = 0.00$, (b) $k_i' = 0.04$, (c) $k_i' = 0.08$, and (d) $k_i' = 0.12$. Other parameters are same as in Figure 3.

It may be noted that critical beam radius ρ has an infinite value at two values of modulation parameter a_0. These are $a_0 = -0.39603$ $(= a_{01})$ and $a_0 = 1.85406$ $(= a_{02})$. Thus, it is of interest to find the range of a_0, within which the self-trapping of beam is valid which demands the following inequality to be satisfied.

(i) For $a_{01} < a_0 < a_{02}$, ρ is real.
(ii) For $a_{01} > a_0 > a_{02}$, ρ is imaginary.
(iii) For $a_{01} = a_0 = a_{02}$, ρ is undefined.

Hence, the range of a_0, for which self-trapping of finite AiG beams is valid, is $-0.39603 < a_0 < 1.85406$. The above three domains are independent on the power (p) of the AiG beam. To further elucidate the results for delineating the propagation of finite AiG beams through plasma, we numerically analyze the dependence of beam-width parameters f_1 and f_2 as a function of ξ for different values of normalized absorption coefficient k_i' when relativistic nonlinearity is taken into account. The results are depicted in the form of a set of graphs in Figure 6. This figure demonstrates that with an increase in a_0 for given k_i', the beam exhibits strong ad early self-focusing. But such focusing trends get reversed to sharp defocusing depending on the location of a_0 in the relevant domain as defined earlier. Further, at a given modulation parameter, an increase in k_i' causes a substantial reduction in self-focusing. This is because the weakening of self-focusing action takes place due to absorption and thus beam suffers sharp steady divergence for higher k_i' values.

Conclusions

Starting with the electric field distribution of finite Airy–Gaussian beams, nonlinear coupled differential equations in transverse dimensions of the beams has been established by using the parabolic equation approach under WKB and paraxial approximations. The existence equation for a self-trapped mode of laser has been obtained. Using the theorem on extremum values in two variable cases, the critical curve has been analyzed to obtain domains of the modulation parameter in the propagation of AiG beams through plasma taking into account relativistic nonlinearity. Following important conclusions are drawn from the present analysis:

- Self-focusing/defocusing of finite AiG beams depends on critical values modulation parameter.
- The range of modulation parameter for self-focusing remains unchanged with an increase in the initial beam radius.
- There is a range of modulation parameter within which the initial beam radius has a real value.
- Extent of self-focusing is found to decrease with increase in absorption.

We find that the study of finite Airy–Gaussian beams can be analyzed like Gaussian beam in plasma, but the modulation parameter and its range is found to play a key role in determining the nature of self-focusing/defocusing of the beams.

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