Macroeconomic Dynamics, **23**, 2019, 3483–3508. Printed in the United States of America.
doi:10.1017/S1365100518000111

A NOTE ON NOMINAL GDP TARGETING AND MACROECONOMIC (IN)STABILITY

Shu-Hua Chen
National Taipei University

Benhabib and Farmer show that in a laissez-faire one-sector real business cycle model under aggregate increasing returns, under sufficiently high degrees of productive externality, the demand-side effect in the labor market triggered by agents' expectations about the economy's future outweighs the supply-side effect, making agents' expectations become self-fulfilling. This paper analytically demonstrates that the conduct of monetary policy under nominal gross domestic product (GDP) targeting reinforces the supply-side effect in the labor market, thereby making belief-driven aggregate fluctuations more difficult to occur. This reinforcement effect on labor supply is absent under nominal consumption targeting and inflation targeting. Hence, under these two monetary regimes, the necessary and sufficient conditions for the economy to display equilibrium indeterminacy and sunspot fluctuations are identical to those in Benhabib and Farmer's laissez-faire economy. The results are robust to an endogenous growth extension of the model, implying that targeting the nominal GDP growth rate is more desirable than targeting the nominal consumption growth rate or the inflation rate in terms of macroeconomic stability.

Keywords: Nominal GDP Targeting, Inflation Targeting, Equilibrium (In)determinacy, Business Cycles

1. INTRODUCTION

Nominal gross domestic product (GDP) targeting, first proposed by Meade (1978), has attracted the research interest of eminent economists including Tobin (1980), Bean (1983), Taylor (1985), McCallum (1985), Frankel (1995), and McCallum and Nelson (1999), among many others. In the 1980s, due to vulnerability to velocity shocks of monetary aggregates, the literature proposed that nominal GDP targeting can be a good candidate to succeed monetary targeting as an intermediate target. Nevertheless, nominal GDP targeting was not adopted by any central banker.

The nominal GDP targeting proposal has recently come back to the forefront of monetary policy discussions. Its proponents point out that nominal GDP

The author would like to thank William A. Barnett (Editor) and two anonymous referees for very helpful comments and suggestions which substantially improved the paper. Address correspondence to: Shu-Hua Chen, Department of Economics, National Taipei University, 151 University Road, San Shia District, New Taipei City 23741, Taiwan; e-mail: shchen@mail.ntpu.edu.tw.

 1365-1005/18

targeting (i) is able to deliver sufficient monetary stimulus in the aftermath of the recent global financial crisis; (ii) provides a clear near-term target criterion, while maintaining the credibility of the central bank's commitment to its medium-term inflation target; (iii) stabilizes demand and does not have the problem of excessive tightening in response to adverse supply shocks; and (iv) can respond appropriately to terms of trade shocks [see, e.g., Carney (2012), Frenkel (2012, 2013), and Woodford (2012, 2013)].

In response to the debate on the efficacy and desirability of nominal GDP targeting as an intermediate target, this paper builds upon Benhabib and Farmer's (1994) indeterminate one-sector real business cycle (RBC) model with aggregate increasing returns to scale and examines the theoretical consequence of adopting the monetary regime of nominal GDP targeting on the economy's macroeconomic stability properties.[1] I analytically demonstrate that nominal GDP targeting raises the requisite level of the degree of productive externalities for equilibrium indeterminacy, and hence reduces the possibility for the occurrence of business cycle fluctuations driven by "animal spirits" of agents. By contrast, under both nominal consumption targeting and inflation targeting, the necessary and sufficient conditions for the economy to display equilibrium indeterminacy and belief-driven aggregate fluctuations are identical to those in Benhabib and Farmer's (1994) laissez-faire economy.[2]

To understand the intuition behind the above indeterminacy result, it is helpful to start with the mechanism that productive externalities produce belief-driven business fluctuations in the Benhabib–Farmer real economy. Start from the model's steady-state and consider a slight deviation caused by agents' optimistic anticipation about an expansion in future economic activities. Acting upon this belief, the representative household will reduce consumption and raise investment today, which in turn leads to another trajectory with higher future output, consumption, and factor employment. It turns out that when the degree of productive externality exceeds a requisite threshold such that the equilibrium wage-hours locus becomes positively sloped and steeper than the labor supply curve, the increase in labor demand is sufficiently higher than the decline in labor supply caused by the income effect. As a result of a sufficient increase in hours worked, the return on investment can be increasing along this alternative transitional path. Thus, agents' initial rosy expectations on the economy's future are validated as a self-fulfilling equilibrium.

In this paper's monetary economy under the cash-in-advance (CIA) constraint, the shadow value of wealth and hence labor supply is affected not only by consumption, but also by the nominal interest rate. When the production of real output increases as a result of agents' anticipation of an expansion in future real activities, the nominal GDP targeting central banker will in response execute a contractionary monetary policy that causes a decline in the price level such that nominal GDP can be maintained at the target level. Complementarity between consumption and real balances under the CIA constraint subsequently leads the household to raise consumption purchases, which on one hand induce more labor demand, and on the other lower labor supply. Although the inflation rate falls, the real interest rate rises under sufficient aggregate increasing returns. The consequential higher nominal interest rate further raises the shadow value of wealth and therefore further suppresses labor supply. Thus, by generating a reinforcement effect on labor supply, the conduct of monetary policy under nominal GDP targeting makes belief-driven business fluctuations harder to occur. It thus requires a higher degree of increasing returns for equilibrium indeterminacy to arise.

The reasons why the Benhabib–Farmer condition for indeterminacy holds under both nominal consumption targeting and inflation targeting are as follows. First, when consumption is constrained by the CIA constraint, a constant level of nominal consumption is attained by a constant stock of nominal money balances. Given that the central bank does not respond to sunspot shocks and that the nominal interest rate is constant over time, the implementation of nominal consumption targeting does not change the Benhabib–Farmer condition for indeterminacy. Second, under inflation targeting, the fall in the inflation rate caused by agents' animal spirits induces the central bank to engage in monetary expansion so as to raise the inflation rate to the target level. Complementarity between consumption and real money balances under the CIA constraint thus implies a reduction in consumption, which in turn raises labor supply and reduces the derived labor demand. On the other hand, the increased nominal interest rate by reducing the shadow value of wealth lowers labor supply. As it turns out, the net effect of the expansionary monetary policy on equilibrium labor hours is nil. The Benhabib–Farmer condition for indeterminacy therefore holds under inflation targeting.

The remainder of this paper is organized as follows. Section 2 describes a CIA extension of the Benhabib–Farmer model and analyzes its equilibrium conditions. Section 3 investigates the theoretical consequence of implementing the nominal GDP targeting rule on the model's local stability properties. Section 4 examines the robustness of Section 3's results under a circumstance where productive externalities are strong enough to generate sustained economic growth. Section 5 instead adopts the money-in-the-utility approach, which allows for more general interrelations between consumption and real balances. Section 6 concludes.

2. THE ECONOMY

I incorporate into Benhabib and Farmer's (1994) laissez-faire one-sector RBC model a monetary authority that adopts the nominal GDP targeting rule. Households live forever, and derive utilities from consumption and leisure. The production side consists of a social technology that displays increasing returns-to-scale due to positive productive externalities from aggregate capital and labor inputs. I assume that there are no fundamental uncertainties present in the economy.

2.1. Firms

There is a continuum of identical competitive firms, with the total number normalized to one. The representative firm produces real output y_t according to a

Cobb–Douglas production function:

$$y_t = x_t k_t^{\alpha} h_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where k_t and h_t are capital and labor inputs, respectively, and x_t represents positive productive externalities that are taken as given by each individual firm. As in Benhabib and Farmer (1994), I postulate that externalities take the form:

$$x_t = \bar{k}_t^{\alpha\theta} \bar{h}_t^{(1-\alpha)\theta}, \quad \theta \geq 0, \tag{2}$$

where $\bar{k}_t$ and $\bar{h}_t$ denote the economy-wide levels of capital and labor services, respectively.

Under the assumption that factor markets are perfectly competitive, the firm's profit maximization conditions are given by

$$r_t = \alpha \frac{y_t}{k_t}, \quad \text{and} \quad w_t = (1-\alpha)\frac{y_t}{h_t}, \tag{3}$$

where r_t is the capital rental rate, and w_t is the real wage. In addition, α and $1-\alpha$ represent the capital and labor share of national income, respectively.

In a symmetric equilibrium, all firms make the same decisions such that $k_t = \bar{k}_t$ and $h_t = \bar{h}$, for all t. As a result, (2) can be substituted into (1) to obtain the following aggregate increasing returns-to-scale production function for total output y_t[3]:

$$y = k_t^{\alpha(1+\theta)} h_t^{(1-\alpha)(1+\theta)}, \tag{4}$$

where I consider the case of $\alpha(1+\theta) < 1$ such that externalities are not strong enough to generate sustained economic growth.

2.2. Households

The economy is also populated by a unit measure of identical infinitely lived households. Each household is endowed with one unit of time and maximizes:

$$\int_0^{\infty} \left(\log c_t - A \frac{h_t^{1+\gamma}}{1+\gamma} \right) e^{-\rho t} dt, \quad A > 0, \tag{5}$$

where c_t is consumption, $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply, and $\rho > 0$ is the subjective rate of time preference.

Money holdings are required for real purchases of the consumption good [Clower (1967), Lucas (1980)]. The representative household therefore faces the following CIA or liquidity constraint:

$$c_t \leq m_t, \tag{6}$$

where m_t denotes real money balances.

The budget constraint faced by the representative household is given by

$$\dot{k}_t + \dot{m}_t = r_t k_t + w_t h_t - \delta k_t - c_t - \pi_t m_t + \tau_t, \quad k_0 > 0 \text{ given}, \tag{7}$$

where π_t denotes the inflation rate, υ_t is real lump-sum transfers from the government, and $\delta \in (0, 1)$ is the capital depreciation rate.

The first-order conditions for the representative agent with respect to the indicated variables and the associated transversality conditions (TVC) are

$$c_t : \quad c_t^{-1} = \lambda_t + \eta_t, \tag{8}$$

$$h_t : \quad \frac{Ah_t^{\gamma}}{\lambda_t} = (1-\alpha)\frac{y_t}{h_t}, \tag{9}$$

$$k_t : \quad \lambda_t\left(\alpha\frac{y_t}{k_t} - \delta\right) = \rho\lambda_t - \dot{\lambda}_t, \tag{10}$$

$$m_t : \quad -\pi_t\lambda_t + \eta_t = \rho\lambda_t - \dot{\lambda}_t, \tag{11}$$

$$\text{TVC} : \quad \lim_{t\to\infty} e^{-\rho t}\lambda_t k_t = \lim_{t\to\infty} e^{-\rho t}\lambda_t m_t = 0, \tag{12}$$

where λ_t is the shadow value of real assets, and η_t represents the Lagrange multiplier for the CIA constraint (6). Equation (8) states that the marginal benefit of consumption equals its marginal cost, which is the marginal utility of having an additional real dollar. Equation (9) equates the marginal disutility of labor to the real wage rate. Equation (10) is the consumption Euler equation. Equations (10) and (11) imply that $\frac{\eta_t}{\lambda_t}$ equals the nominal interest rate, and that (12) is the transversality condition.

2.3. Monetary Authority

The central bank adopts a regime of pure nominal GDP targeting. Let $\bar{n} > 0$ denote the central bank's target level of nominal GDP and p_t represents the price level. It follows that

$$p_t y_t = \bar{n}, \quad \forall t. \tag{13}$$

The central bank adjusts nominal money supply M_t so as to achieve its target level of nominal GDP.[4] Nominal money supply evolves through time according to

$$M_t = M_0 e^{\mu_t t}, \quad M_0 > 0 \text{ given}, \tag{14}$$

where $\mu_t \neq 0$ is the money growth rate, and the resulting seigniorage is transferred in a lump-sum manner to the household; hence, $\tau_t = \mu_t m_t$.

Clearing in the money and goods markets implies that

$$\dot{m}_t = (\mu_t - \pi_t)\, m_t, \tag{15}$$

and

$$\dot{k}_t = y_t - c_t - \delta k_t. \tag{16}$$

3. MACROECONOMIC (IN)STABILITY

To facilitate the analysis of the model's local stability properties, I make the following logarithmic transformation of variables: $\hat{z}_t \equiv \log(z_t)$, where $z_t = \{c_t, k_t, h_t, y_t\}$. I derive that the model exhibits a unique interior steady state, which is given by

$$\hat{k}^* = \frac{\log\left\{\left[\frac{1-\alpha}{A(1+\rho)}\right]^{(1-\alpha)(1+\theta)} x_1^{-\omega} x_2^{-(1-\alpha)(1+\theta)}\right\}}{(1+\gamma)\left[1-\alpha(1+\theta)\right]}, \tag{17}$$

$$\hat{y}^* = \log x_1 + \hat{k}^*, \tag{18}$$

$$\hat{c}^* = \log x_2 + \hat{k}^*, \tag{19}$$

$$\hat{h}^* = \frac{\hat{y}^* - \hat{c}^* + \log\left[\frac{1-\alpha}{A(1+\rho)}\right]}{1+\gamma}, \tag{20}$$

where $\omega \equiv \gamma + 1 - (1-\alpha)(1+\theta)$ denotes the relative slope of the labor supply curve and the equilibrium wage-hour locus, and

$$x_1 \equiv \frac{y^*}{k^*} = \frac{\rho+\delta}{\alpha} > 0 \quad \text{and} \quad x_2 \equiv \frac{c^*}{k^*} = \frac{m^*}{k^*} = \frac{\rho + (1-\alpha)\delta}{\alpha} > 0. \tag{21}$$

In the neighborhood of the steady state, the model's equilibrium conditions can be approximated by the following log-linearized dynamical system:

$$\begin{bmatrix} \dot{\hat{k}}_t \\ \dot{\hat{y}}_t \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} \hat{k}_t - \hat{k}^* \\ \hat{y}_t - \hat{y}^* \end{bmatrix}, \quad k_0 \text{ given}, \tag{22}$$

where

$$\mathbf{J}_{11} = -x_1 - \frac{\Omega_1}{\Delta} x_2,$$

$$\mathbf{J}_{12} = x_1 - \frac{\Omega_2}{\Delta} x_2,$$

$$\mathbf{J}_{21} = -\alpha x_1 + (1+\rho)\left[1 + \frac{\Omega_1}{\Delta} - \frac{\alpha(1+\gamma)}{1-\alpha}\right],$$

$$\mathbf{J}_{22} = \alpha x_1 + (1+\rho)\left[\frac{\Omega_2}{\Delta} + \frac{\omega}{(1-\alpha)(1+\theta)}\right],$$

https://doi.org/10.1017/S1365100518000111 Published online by Cambridge University Press

together with

$$\Omega_1 = \frac{(1+\rho)\left[\alpha(2+\gamma)-1\right]\omega - \alpha x_1(1+\gamma)(1-\alpha)\theta}{(1+\rho)(1-\alpha)}, \tag{23}$$

$$\Omega_2 = \frac{(1-\alpha)(1+\theta)\theta(1+\gamma)\alpha x_1 - (1+\rho)\omega^2}{(1-\alpha)(1+\theta)(1+\rho)}, \tag{24}$$

$$\Delta = \frac{\left[(\alpha+\gamma)(1+\rho)+(1+\gamma)\alpha x_2\right] - \Theta\theta}{1+\rho} = \omega + \frac{(1+\gamma)\alpha(1+\theta)x_2}{1+\rho}, \tag{25}$$

where $\Theta = (1-\alpha)(1+\rho) - (1+\gamma)\alpha x_2$.

It follows that the determinant and trace of the model's Jacobian matrix $\mathbf{J}$ are

$$\text{Det} = \frac{(1+\gamma)\left[\alpha(1+\theta)-1\right]\alpha x_1 x_2}{\Delta} \gtrless 0, \quad \text{if} \ \ \Delta \lesseqgtr 0, \tag{26}$$

$$Tr = \frac{\Phi\theta - \left[\delta(\alpha+\gamma)(1+\rho) + (1+\gamma)(1-\alpha)\alpha x_1 x_2\right]}{(1+\rho)\Delta} \gtrless 0, \tag{27}$$

where $\Phi = (1+\rho)\left[\delta(1-\alpha) + (\alpha+\gamma)\alpha x_1\right] + (1+\gamma)\alpha^2 x_1 x_2 > 0$.

The model's local stability property is determined by comparing the eigenvalues of $\mathbf{J}$ that have negative real parts with the number of initial conditions in the dynamical system (22), which is one, because $\hat{y}_t$ is a nonpredetermined jump variable. As a result, the economy displays saddle-path stability and equilibrium uniqueness if and only if the two eigenvalues of $\mathbf{J}$ exhibit opposite signs (Det < 0). If both eigenvalues have negative real parts (Det > 0 and $Tr < 0$), then the steady state is a locally indeterminate sink that can be exploited to generate endogenous cyclical fluctuations driven by agents' self-fulfilling expectations or sunspots. When both eigenvalues have positive real parts (Det > 0 and $Tr > 0$), the steady state becomes a completely unstable source whereby any trajectory that diverges away from it may settle down to a limit cycle or to some more complicated attracting sets.

Given the analytical results of (26) and (27), Figure 1 depicts the combinations of γ (the inverse of the intertemporal elasticity of substitution in labor supply) and θ (the positive productive externality) that graphically characterize the model's local stability properties.[5] The positively-sloped locus θ^{Det}, given by (26), divides the regions labeled as "sink" and "saddle." The dashed line of θ^{BF} represents the minimum level of productive externalities above which the laissez-faire economy of Benhabib and Farmer (1994) possesses an indeterminate steady state.

Figure 1 clearly depicts that the requisite degree of productive externalities needed for equilibrium indeterminacy under nominal GDP targeting, θ^{Det}, is strictly higher than that under Benhabib and Farmer's (1994) laissez-faire economy, θ^{BF}. In addition, under sufficiently low levels of the intertemporal labor

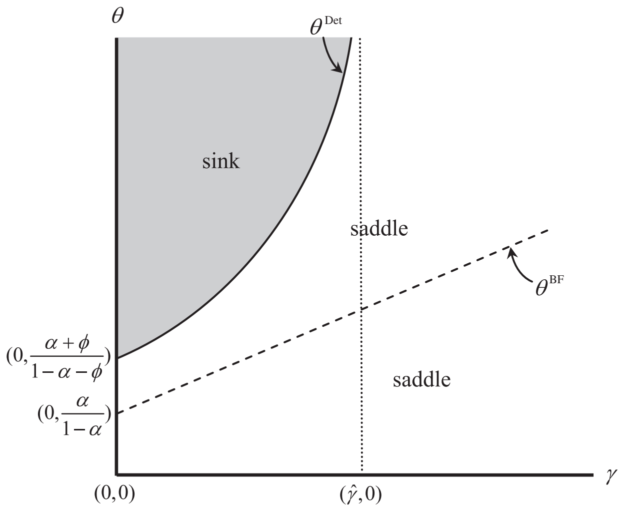

FIGURE 1. Regions of equilibrium (in)determinacy.

supply elasticity such that γ exceeds the threshold value $\hat{\gamma}$, the steady state always exhibits saddle-path stability.

To understand the intuition behind the above indeterminacy result, in what follows I present the log linear forms of the labor supply schedule, the labor demand schedule, and the equilibrium wage-hours locus[6]:

$$\hat{w}_t^s = \log A + \gamma \hat{h}_t - \hat{\lambda}_t, \tag{28}$$

$$\hat{w}_t^d = \log (1-\alpha) + \hat{x}_t + \alpha \hat{k}_t - \alpha \hat{h}_t, \tag{29}$$

$$\hat{w}_t^D = \log (1-\alpha) + \alpha (1+\theta) \hat{k}_t + [(1-\alpha)(1+\theta) - 1] \hat{h}_t, \tag{30}$$

where $\hat{x}_t = \alpha\theta\hat{k}_t + (1-\alpha)\theta\hat{h}_t$ and $\hat{\lambda}_t$ respectively represent productive externalities and the shadow value of wealth. The above equations state that, as Figure 2 illustrates: (i) the labor supply curve, L^s, is upward sloping: $\frac{\partial \hat{w}_t^s}{\partial \hat{h}_t} = \gamma > 0$, and an increase in $\hat{\lambda}_t$ shifts L^s downward: $\frac{\partial \hat{w}_t^s}{\partial \hat{\lambda}_t} = -1 < 0$; (ii) as each individual firm takes productive externalities $\hat{x}_t$ as given, the labor demand curve of each individual firm, L^d, exhibits a negative slope: $\frac{\partial \hat{w}_t^d}{\partial \hat{h}_t} = -\alpha < 0$; in addition, an increase in $\hat{x}_t$ or $\hat{k}_t$ shifts L^d upward: $\frac{\partial \hat{w}_t^d}{\partial \hat{x}_t} = 1 > 0$ and $\frac{\partial \hat{w}_t^d}{\partial \hat{k}_t} = \alpha > 0$; and (iii) after incorporating productive externalities, the equilibrium wage-hours locus, L^D, is positively sloped when θ is sufficiently high: $\frac{\partial \hat{w}_t^D}{\partial \hat{h}_t} = (1-\alpha)(1+\theta) - 1 > 0$, when $\theta > \frac{\alpha}{1-\alpha}$; moreover, an increase in $\hat{k}_t$ shifts L^d upward: $\frac{\partial \hat{w}_t^D}{\partial \hat{k}_t} = \alpha (1+\theta) > 0$. Both the left and the right panels of Figure 2 illustrate equilibrium in the labor market.

https://doi.org/10.1017/S1365100518000111 Published online by Cambridge University Press

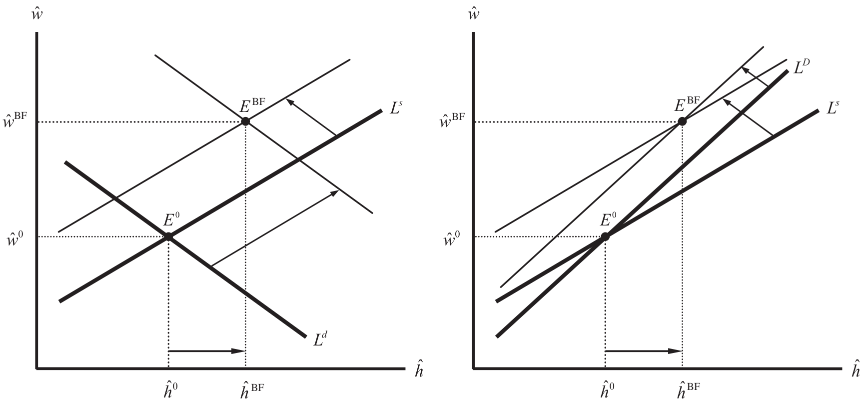

FIGURE 2. Labor market adjustment to a sunspot shock in the Benhabib–Farmer model.

It is helpful to start by understanding the mechanism that productive externalities produce belief-driven business fluctuations in the Benhabib–Farmer real economy, where $\hat{\lambda}_t$ is given by

$$\hat{\lambda}_t = -\hat{c}_t. \tag{31}$$

Start from the model's steady-state equilibrium E^0 illustrated in Figure 2, and suppose that agents anticipate an increase in future economic activity. Acting upon this belief, households will consume less and invest more today. By raising the capital stock, this shifts the labor demand curve L^d (the left panel of Figure 2) up and to the right; labor hours are therefore enhanced. In the presence of productive externalities, the higher economy-wide levels of capital and hours worked by raising $\hat{x}_t$ cause a further shift in L^d to the right; note from the expression $\hat{x}_t = \alpha\theta\hat{k}_t + (1-\alpha)\,\theta\hat{h}_t$ that a higher value of θ will magnify the increase in $\hat{x}_t$ and the consequential shift in L^d. The resulting higher level of real output by enhancing consumption lowers the shadow value of wealth $\hat{\lambda}_t$, thereby reducing labor supply (the income effect). Thus, as the left panel of Figure 2 shows, the economy moves from E^0 to E^{BF}, leading labor hours to increase from $\hat{h}^0$ to $\hat{h}^{\mathrm{BF}}$. In terms of the right panel of Figure 2, the above sunspot shock by raising future capital stock and consumption shifts both the equilibrium wage-hour locus L^D and the labor supply curve L^s upward. The economy thus moves from E^0 to E^{BF}, and hours worked rises from $\hat{h}^0$ to $\hat{h}^{BF}$.

For agents' initial rosy expectations about the economy's future to be validated in equilibrium, the increase in hours worked needs to be sufficient such that the return on investment, $r_t = \alpha\, k_t^{\alpha(1+\theta)-1} h_t^{(1-\alpha)(1+\theta)}$, can be increasing in transition. As it turns out, this requires a degree of productive externality θ that exceeds a threshold level, $\theta^{\mathrm{BF}} = \frac{\alpha+\gamma}{1-\alpha}$. This condition is met when the equilibrium wage-hours locus is positively sloped and steeper than the labor supply curve:

$$(1-\alpha)\,(1+\theta) - 1 > \gamma. \tag{32}$$

If the degree of productive externality θ is not high enough to meet the above condition, then the Benhabib–Farmer model's steady state will be a locally determinate saddle point. Given that the stock of capital is predetermined, any belief-driven deviation from the initial steady state will result in divergent trajectories that will violate the transversality condition. Hence, indeterminacy and sunspots will not arise.

In this paper's monetary economy under the CIA constraint (6), the shadow value of wealth $\hat{\lambda}_t$ is given by

$$\hat{\lambda}_t = -\hat{c}_t - \log\left(1 + \frac{\eta_t}{\lambda_t}\right), \tag{33}$$

where, as noted in Section 2.2, $\frac{\eta_t}{\lambda_t}$ is the model's implied nominal interest rate.

Through the influences on consumption and the nominal interest rate and hence the shadow value of wealth, the conduct of monetary policy under nominal GDP creates a reinforcement effect on labor supply that reduces hours worked and the possibility of belief-driven business fluctuations. Specifically, when the production of real output increases as a result of agents' anticipation of an expansion in future real activities, the price level falls. Regardless of the magnitude of the decline in the price level, the central bank will conduct monetary policy in such a way that the decline in the price level offsets the increase in real output, and thus nominal GDP can be maintained at the target level. This induces the household to have more real money balances. Complementarity between consumption and real balances under the CIA constraint then leads to more consumption purchases, which on one hand suppress labor supply, while on the other hand induce more factor employments by the firm. As a result of a higher stock of capital, both the labor demand curve and the equilibrium wage-hour locus shift upward. In addition, by raising the nominal interest rate, the increased real rate of return on investment under sufficient aggregate increasing returns further reduces labor supply. Therefore, as the dashed lines in Figure 3 illustrate, the central bank's contractionary monetary policy under nominal GDP targeting leads the economy to move from E^{BF} to E^{GDP} and hours worked to decrease from $\hat{h}^{\mathrm{BF}}$ to $\hat{h}^{\mathrm{GDP}}$.

Because of the reinforcement effect on labor supply that reduces hours worked, nominal GDP targeting makes indeterminacy more difficult to occur, because it requires a higher degree of productive externalities to produce an indeterminate steady state: $\theta > \theta^{\mathrm{Det}}$, where $\theta^{\mathrm{Det}} > \theta^{\mathrm{BF}}$. By rearranging, the condition $\theta > \theta^{\mathrm{Det}}$ can be written as

$$\underbrace{\left[1 - \alpha - \frac{(1+\gamma)\,\alpha x_2}{1+\rho}\right](1+\theta) - 1}_{<(1-\alpha)(1+\theta)-1} > \gamma. \tag{34}$$

A comparison of (32) and (34) reveals that, under nominal GDP targeting, an additional negative term, $-\frac{(1+\gamma)\alpha x_2(1+\theta)}{1+\rho}$, is added to the left-hand side of the condition for equilibrium indeterminacy. Note that $x_2 = \frac{m^*}{k^*}$ is the real

https://doi.org/10.1017/S1365100518000111 Published online by Cambridge University Press

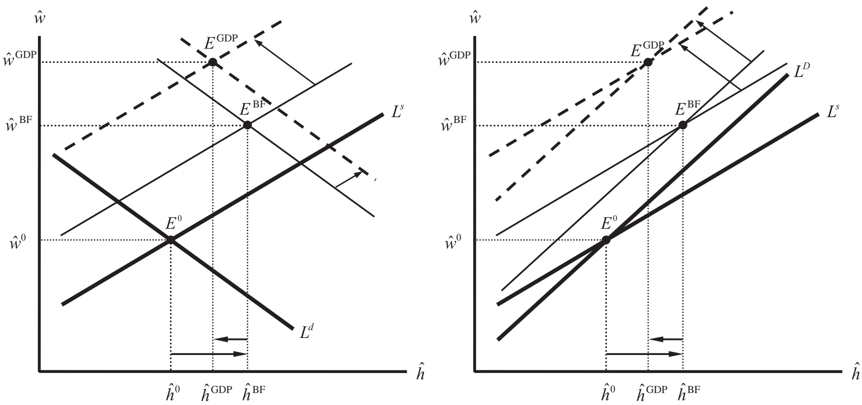

FIGURE 3. Labor market adjustment to a sunspot shock under nominal GDP targeting.

balances-to-capital ratio. This additional term is thus produced by the conduct of monetary policy that affects households' money holdings and hence the labor market. It is clear from (34) that, in the presence of this additional term, it requires a higher θ that makes the left-hand side of (34) larger than the right-hand side of it.[7]

Note from (34) that saddle-path stability is likely to be maintained even when the equilibrium wage-hours locus is positively sloped and steeper than the labor supply curve. Moreover, the term $-\frac{(1+\gamma)\alpha x_2(1+\theta)}{1+\rho}$ has an absolute value that is increasing in the labor supply elasticity parameter γ. Therefore, as in Benhabib and Farmer's (1994) laissez-faire economy, Figure 1 depicts that the requisite degree of productive externalities that fulfills agents' anticipation about the future, θ^{Det}, will *certis paribus* increase as the value of γ rises. Figure 1 also shows that, when γ is sufficiently high such that it exceeds the threshold level $\hat{\gamma}$, the reinforcement effect imposed by the conduct of monetary policy will be so strong that there is no possibility of equilibrium indeterminacy.

Figure 4 presents the impluse-response functions of the economy under indeterminacy to a 0.1% sunspot innovation. When plotting the figure, I follow Benhabib and Farmer (1994) in setting the capital share of national income α at 0.3, the annual rate of time preference ρ at 6.5%, and the annual capital depreciation rate δ at 10%. In addition, the labor supply elasticity parameter γ is set at 0 [Hansen (1985) and Rogerson's (1988) indivisible labor formulation]. Given the baseline calibrations of α, ρ, δ, and γ, the threshold level of productive externalities that satisfies the necessary and sufficient condition for equilibrium indeterminacy under nominal GDP targeting is $\theta^{\text{Det}} = 0.4997$. Figure 4 is plotted under $\theta = 0.52$.

As Figure 4 illustrates, all variables exhibit nonmonotonic adjustment paths that are in line with the previously described mechanism. Real GDP, hours worked, investment, and the capital rental rate rise on impact and also during the period when the economy is converging back to the original steady state. Consumption

https://doi.org/10.1017/S1365100518000111 Published online by Cambridge University Press

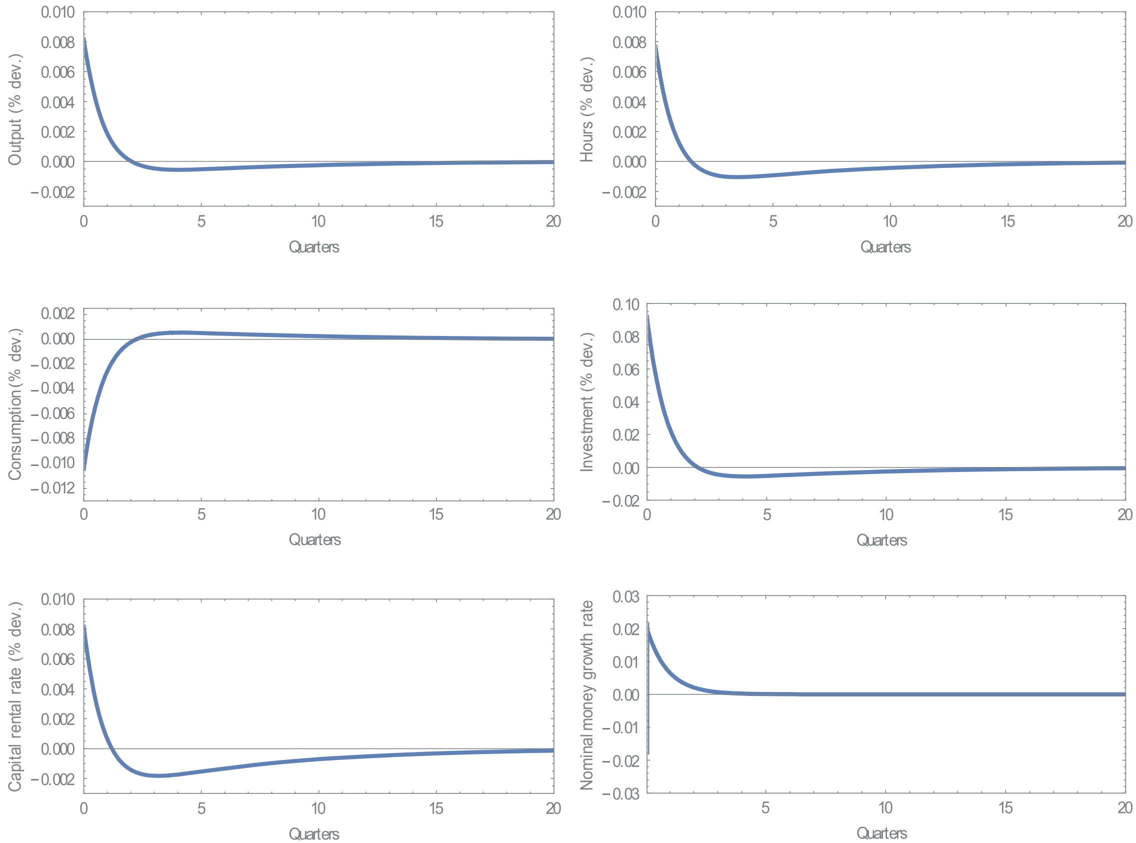

FIGURE 4. Impulse response functions: nominal GDP targeting and indeterminacy ($\theta = 0.52$).

initially drops because of a dominating intertemporal substitution effect (from consumption to investment). Upon the sunspot shock that raises real GDP, the central bank in response conducts a contractionary monetary policy of a negative nominal money growth rate.

A monetary regime that is closely related to nominal GDP targeting is nominal consumption targeting, proposed by Nathan Sheets. In the absence of capital investment, the two monetary regimes are equivalent. However, in this paper's setting where there is physical investment, the two monetary regimes are inequivalent. Since consumption is constrained by the CIA constraint (6), a constant level of nominal consumption, $c_t p_t = \bar{C}$, $\forall t$, is attained by a constant nominal money stock, which implies a zero nominal money growth rate: $\mu_t = 0$, $\forall t$. The central bank thus will not respond to sunspot shocks. In addition, a constant level of nominal consumption also implies that the sum of the growth rates of real consumption and inflation equals zero: $\hat{c}_t + \pi_t = 0$. When the household utility (5) is logarithmic in consumption, this implies that the nominal interest rate is constant over time. As a result, under nominal consumption targeting, the necessary and sufficient condition for the economy to display equilibrium indeterminacy and belief-driven aggregate fluctuations is identical to that in Benhabib and Farmer's (1994) laissez-faire economy, as shown in (32).

Under the same set of parameter values $\{\alpha, \gamma, \rho, \delta\}$, $\theta^{\mathrm{BF}} = 0.4286$. Figure 5 illustrates the impulse response functions of the economy under nominal consumption targeting and under $\theta = 0.47$. As depicted, a positive sunspot shock generates simultaneous expansions in real GDP, consumption, investment, and the return on investment on impact, as well as hump-shaped adjustment paths for all these variables. Consistent with the previously described mechanism, output and labor hours are higher under nominal consumption targeting (Figure 5) than under nominal GDP targeting (Figure 4). The consequential stronger intertemporal income effect under nominal consumption targeting leads consumption to rise on impact, and displays higher levels in the transition period than in the case of nominal GDP targeting.

The next monetary regime I shall examine is inflation targeting, under which the central bank targets a specific level of the inflation rate $\bar{\pi}$, and adjusts nominal money supply so as to maintain a constant inflation rate: $\pi_t = \bar{\pi}$, $\forall t$.[8] Under this monetary regime, the fall in the inflation rate caused by agents' animal spirits induces the central bank to engage in monetary expansion so as to raise the inflation rate back to the target level. Complementarity between consumption and real money balances under the CIA constraint subsequently implies a reduction in consumption, which in turn raises labor supply and reduces the derived labor demand. In addition, the increased nominal interest rate by reducing the shadow value of wealth lowers labor supply. The net effect of the expansionary monetary policy on the equilibrium hours worked turns out to be nil. Thus, as is analytically derived, the minimum degree of productive externality above which the model economy exhibits an indeterminate steady state is the same as that in the Benhabib–Farmer model, i.e., θ^{BF}.

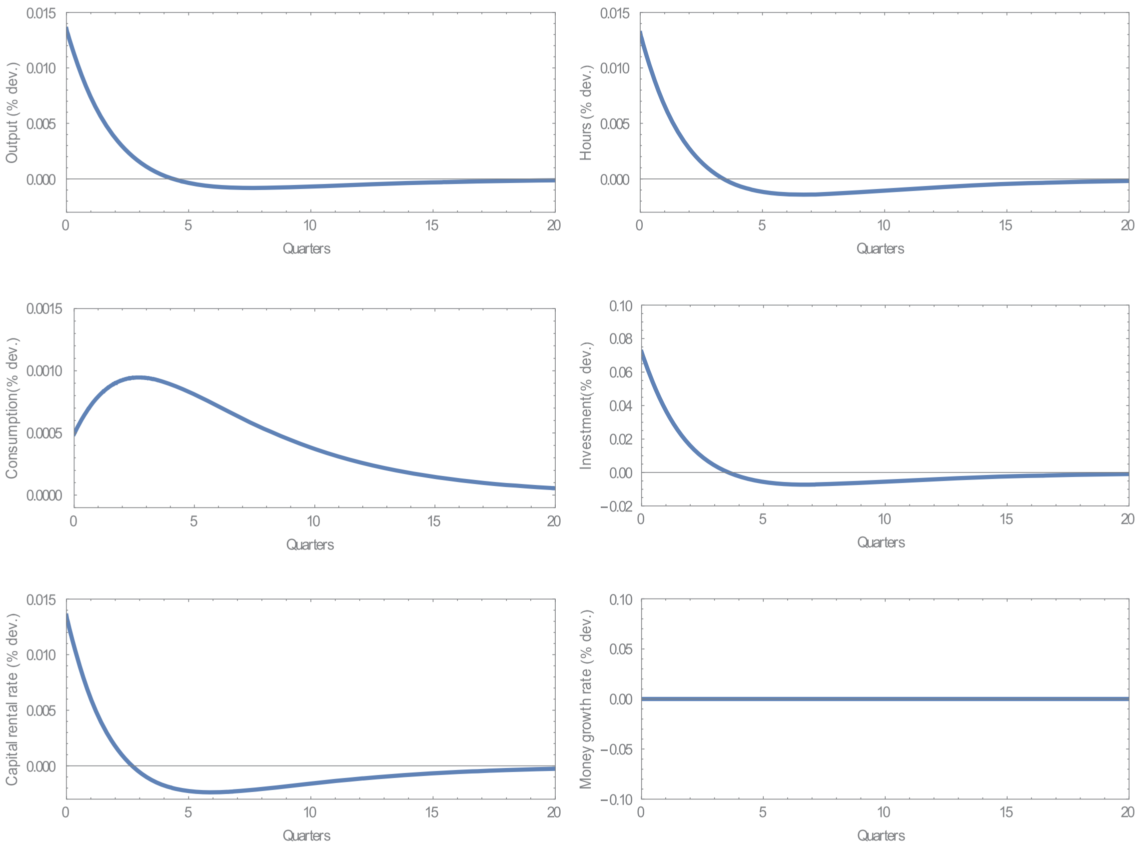

FIGURE 5. Impulse response functions: nominal consumption targeting and indeterminacy ($\theta = 0.47$).

When plotting the economy's impulse response functions in Figure 5, the inflation target is set at 0 for the purpose of comparison between different monetary regimes, since under both nominal GDP targeting and nominal consumption targeting the steady-state rate of inflation is 0. A comparison of Figures 5 and 6 reveals that the only difference of the economy's response between nominal consumption targeting and inflation targeting is that consumption is lower in the latter case, because, as explained, an expansionary monetary policy under inflation targeting induces lowers consumption.

I so far have focused the analysis on a monetary version of the Benhabib–Farmer model, for the purpose of exploring the mechanism that nominal GDP targeting narrows the region of equilibrium indeterminacy. It is well known that the required degree of increasing returns-to-scale that satisfies the Benhabib–Farmer condition for local indeterminacy is too high to be empirically plausible when judged by most recent empirical estimates [e.g., Burnside (1996), Basu and Fernald (1997)]. Therefore, to quantitatively assess how effectively nominal GDP targeting narrows the region of equilibrium indeterminacy, in what follows I incorporate into the model variable capital utilization, which has been demonstrated to be a mechanism that reduces the requisite degree of productive externalities to an empirically plausible level [Wen (1998)]. Of course, models with multiple production sectors with sector-specific externalities [Benhabib and Farmer (1996), Weder (2000), and Harrison (2001), among others] are another direction that further research can pursue.

Under variable capital utilization, the production function becomes: $y_t = \left(\bar{\chi}_t \bar{k}_t\right)^{\alpha\theta} \ \bar{h}_t^{(1-\alpha)\theta} \left(\chi_t k_t\right)^{\alpha} \ h_t^{1-\alpha}$, where χ_t denotes the rate of capital utilization determined by the household, and $\bar{\chi}_t$ is the economy-wide level of utilized capital. A more intensive utilization of capital accelerates capital depreciation, and hence the capital depreciation rate is postulated to take the form: $\delta_t = \frac{1}{\varphi} u_t^{\varphi}$, $\varphi > 1$. By substituting the optimal rate of capital utilization, $\chi_t = (\alpha \frac{y_t}{k_t})^{\frac{1}{\varphi}}$, into the production function and rearranging, I derive the reduced-form social technology as

$$y_t = \alpha^{\frac{\alpha(1+\theta)}{\varphi-\alpha(1+\theta)}} k_t^{\frac{\alpha(1+\theta)(\varphi-1)}{\varphi-\alpha(1+\theta)}} h_t^{\frac{(1-\alpha)(1+\theta)\varphi}{\varphi-\alpha(1+\theta)}}, \tag{35}$$

where $\frac{\alpha(1+\theta)(\varphi-1)}{\varphi-\alpha(1+\theta)} < 1$, such that there exists an interior steady state.

It can be derived that the necessary and sufficient conditions for an indeterminate steady state under inflation targeting and under nominal GDP targeting are

$$\frac{\varphi\,(1-\alpha)\,(1+\theta)}{\varphi-\alpha\,(1+\theta)} - 1 > \gamma, \tag{36}$$

and

$$\underbrace{\left[\varphi\,(1-\alpha) - \frac{(\varphi-1)\,(1+\gamma)\,\alpha x_2}{1+\rho}\right]\frac{(1+\theta)}{\varphi-\alpha\,(1+\theta)} - 1}_{<\frac{\varphi(1-\alpha)(1+\theta)}{\varphi-\alpha(1+\theta)}-1} > \gamma, \tag{37}$$

respectively.

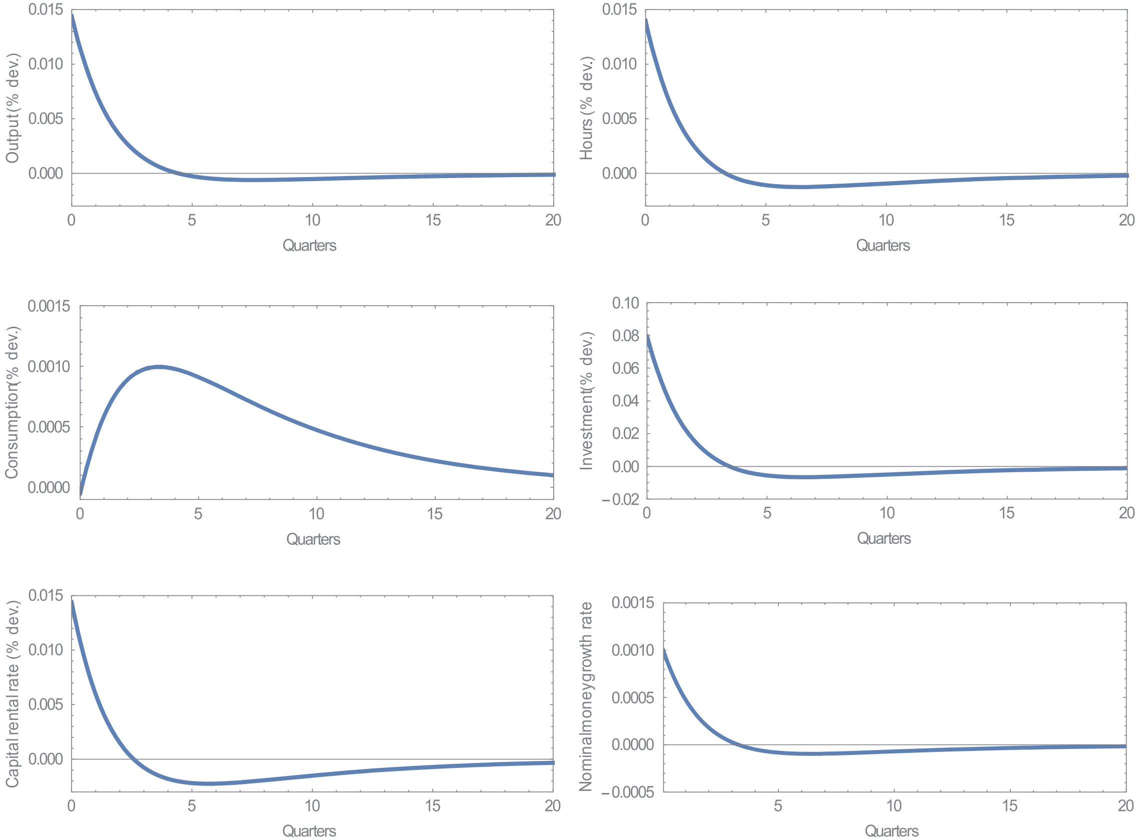

FIGURE 6. Impulse response functions: inflation targeting and indeterminacy ($\theta = 0.47$).

As in (32), (36) is equivalent to the statement that the equilibrium wage-hours locus is positively sloped and steeper than the labor supply curve. As indicated in Wen (1998), by raising the equilibrium labor elasticity of output: $\frac{\varphi(1-\alpha)(1+\theta)}{\varphi-\alpha(1+\theta)} > (1-\alpha)(1+\theta)$, the variable capital utilization reduces the degree of productive externalities needed for equilibrium indeterminacy. On the other hand, as in (34), an additional negative term, $-\frac{(\varphi-1)(1+\gamma)\alpha x_2(1+\theta)}{(1+\rho)[\varphi-\alpha(1+\theta)]}$, which is produced by the conduct of monetary policy, is added to the left-hand side of the condition for equilibrium indeterminacy. As a consequence, it requires a higher θ that makes for equilibrium indeterminacy under nominal GDP targeting.

Under the same calibrated values of α, ρ, δ, and γ as those in the baseline parameterization, the degree of aggregate returns-to-scale needed for equilibrium indeterminacy is 1.134 under inflation targeting, and is 1.202 under nominal GDP targeting. Recent empirical estimates by Burnside (1996) and Basu and Fernald (1997) indicate that a degree of aggregate returns-to-scale within the range of 1.05–1.15 is empirically plausible. Based on their empirical finding, the above numerical result suggests that nominal GDP targeting is stabilizing when compared to inflation targeting.

4. THE DYNAMICS OF ENDOGENOUS GROWTH

This section shows that the preceding section's result that nominal GDP targeting produces a stabilizing effect that narrows the region of indeterminacy holds in an endogenous growth setting. As in Benhabib and Farmer (1994, Sect. 5), when productive externalities are sufficiently strong such that $\alpha(1+\theta) = 1$, the economy exhibits sustained economic growth. Under this circumstance, targeting a level of nominal GDP as given by (13) implies long-run deflation. Thus, the central bank instead targets a rate of nominal GDP growth, and adjusts nominal money supply such that

$$\dot{\hat{y}}_t + \pi_t = \bar{\kappa}, \quad \forall t, \tag{38}$$

where $\bar{\kappa} > 0$ is the target rate of nominal GDP growth.

I focus on the economy's balanced-growth path (BGP) along which output, consumption, physical capital, and real money balances exhibit a common, positive constant growth rate g. Under the policy rule (38), resolving the model in Section 2 with $\alpha(1+\theta) = 1$ leads to the following single differential equation in $z_t \equiv \frac{y_t}{k_t}$ that describes the equilibrium dynamics:

$$\frac{\dot{z}_t}{z_t} = \frac{(\alpha - 1)(1+\theta)[(\alpha-1)z_t + q_t - \rho]}{\omega}, \tag{39}$$

where $q_t \equiv \frac{c_t}{k_t} = q(z_t)$, with $q' = \frac{(1+\gamma)(1-\alpha)Bq_t^2 z_t^{(1+\gamma)/[(1-\alpha)(1+\theta)]} - \omega^2 q_t/(1+\theta)}{(1+\gamma)Bq_t^2 z_t^{(1+\gamma)/[(1-\alpha)(1+\theta)]} + \omega(1-\alpha)z_t}$.

A balanced-growth equilibrium is characterized by a positive real number z^* that satisfies $\dot{z}_t = 0$. It can be derived that z^* is the solution to the following equation:

$$z^* = \frac{(z^*)^{(-\omega)/[(1-\alpha)(1+\theta)]}}{B\,(1+\rho)} - \frac{\rho}{1-\alpha}. \tag{40}$$

With (40), the expression of the consumption-to-capital ratio is $q^* = (1-\alpha)\,z^* + \rho$, and the common (positive) rate of economic growth g is:

$$g = \alpha z^* - \delta - \rho \quad \text{or} \quad g = z^* - q^* - \delta. \tag{41}$$

To examine the existence and number of the economy's balanced-growth paths in a transparent manner, I let $f(z^*) \equiv \frac{(z^*)^{(-\omega)/[(1-\alpha)(1+\theta)]}}{B(1+\rho)} - \frac{\rho}{1-\alpha}$ from (40) and obtain:

$$f' = -\frac{\omega\,(z^*)^{-(1+\gamma)/[(1-\alpha)(1+\theta)]}}{(1-\alpha)\,(1+\theta)\,B\,(1+\rho)} \gtreqless 0 \quad \text{when} \quad \omega \lesseqgtr 0, \tag{42}$$

and

$$f'' = \frac{-(1+\gamma)\,f'}{(1-\alpha)\,(1+\theta)\,z^*} \gtreqless 0 \quad \text{when} \quad f' \lesseqgtr 0. \tag{43}$$

Therefore, the equilibrium z^* can be located from the (possibly more than one) intersection(s) of $f(z^*)$ and the 45-degree line.

In terms of the BGP's local stability properties, I linearize (39) around the BGP and derive that its local stability property is governed by the eigenvalue:

$$e = \frac{(1+\rho)\,(1-\alpha)^2\,(1+\theta)\left(1-f'\right)z^*}{(1+\gamma)\,q^* + \omega\,(1+\rho)}. \tag{44}$$

Since there is no initial condition associated with (39), the BGP displays local determinacy and equilibrium uniqueness when $e > 0$. If $e < 0$, then the BGP is locally indeterminate that can be exploited to generate endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots.

I then analyze the existence and number of the model's BGPs, together with the associated local dynamics, in three parametric specifications. First, when $\omega = 0$ (hence $\theta = \theta^{\mathrm{BF}}$), it is immediately clear from (40) that the economy possesses a unique balanced-growth path along which $z^* = \frac{1}{B(1+\rho)} - \frac{\rho}{1-\alpha}$. Moreover, it can be shown that the eigenvalue of this specification equals $e = \frac{(1+\rho)(1-\alpha)^2(1+\theta)z^*}{(1+\gamma)q^*} > 0$, and thus the unique BGP equilibrium is a saddle.

Second, when $\omega > 0$ (hence $\theta < \theta^{\mathrm{BF}}$), I note that $f(z^*) \to \infty$ as $z^* \to 0$. Figure 7(a) shows that this feature, together with (42) and (43), implies that $f(z^*)$ is a downward-sloping and convex curve that intersects the 45-degree line once in the positive quadrant. Therefore, the economy exhibits a unique BGP. Regarding local dynamics, in this formulation with $f' < 0$, the model exhibits a positive eigenvalue: $e > 0$, indicating the BGP exhibits saddle-path stability.

Third, when $\omega < 0$ (hence $\theta > \theta^{\mathrm{BF}}$), Figure 7(b) shows that $f(z^*) = -\frac{\rho}{1-\alpha} < 0$ as $z^* = 0$, and that $f(z^*)$ is an upward-sloping concave curve. Therefore, depending on parameter values, the number of intersections between $f(z^*)$ and

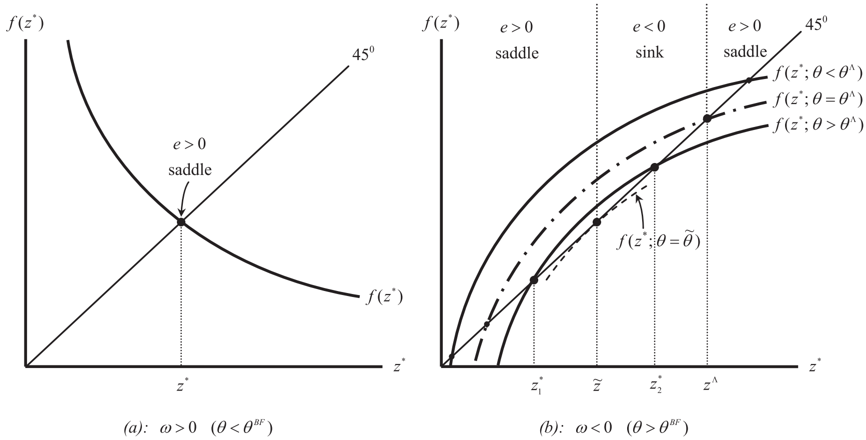

FIGURE 7. Endogenous growth model. (a) $\omega > 0$ ($\theta < \theta^{\mathrm{BF}}$). (b) $\omega < 0$ ($\theta > \theta^{\mathrm{BF}}$).

the 45-degree line in the positive quadrant can be zero, one, or two. In terms of local stability properties, I prove in Appendix C that when the degree of productive externalities is below or equal to a threshold level θ^{Λ} that is strictly higher than θ^{BF}, around either of the BGPs associated with z_1^* and z_2^*, the model's eigenvalue is positive, indicating that both BGPs display saddle path stability. When the degree of productive externalities is higher than θ^{Λ}, around the BGP that is associated with a low output-to-capital ratio z_1^*, the model's eigenvalue is positive. Thus, this low-growth equilibrium path is a saddle. On the other hand, in the neighborhood of the BGP equilibrium associated with a high output-to-capital ratio z_2^*, the model's eigenvalue is negative, and hence the high-growth equilibrium exhibits indeterminacy and sunspots.

It is thus clear that Section 3's result that nominal GDP targeting raises the minimum degree of productive externality above which equilibrium indeterminacy occurs holds in an endogenous growth setting. It can also be derived that the necessary conditions for equilibrium indeterminacy under both nominal consumption growth targeting and inflation targeting are the same as that in Benhabib and Farmer's (1994) endogenous-growth economy under laissez-faire. Specifically, when $\omega > 0$ (hence, $\theta < \theta^{\mathrm{BF}}$), the economy possesses a unique BGP that exhibits saddle-path stability; when $\omega < 0$ (hence $\theta > \theta^{\mathrm{BF}}$), two BGPs emerge, with the low-growth one exhibiting equilibrium uniqueness and the high-growth one displaying equilibrium indeterminacy.

5. MONEY IN THE UTILITY FUNCTION

Up to now I have focused the analysis on a CIA monetary model. Under this monetary approach, more money holdings allow for more consumption purchases, and thus consumption and real money balances exhibit perfect complementarity.

Instead, in Brock's (1974) formulation, households derive utility from holding real money balances. This monetary approach of money-in-the-utility-function (MIUF) is more general than that of CIA, since c_t and m_t can now exhibit complementarity or substutability, or be separated from each other in the utility function.

Following Obstfeld (1985), I consider the following formulation of the household's instantaneous utility function:

$$u_t = \frac{\left(c_t^a m_t^{1-a}\right)^{1-\sigma} - 1}{1-\sigma} - A\frac{h_t^{1+\gamma}}{1+\gamma}, \quad 0 < a < 1, \tag{45}$$

where $\sigma > 0$ is the inverse of relative risk aversion. The instantaneous utility function u_t is increasing and strictly concave with respect to consumption, labor hours, and real money balances. Under (45), the inverse of the intertemporal elasticity of substitution in consumption is $\Sigma = -\frac{u_{cc}c_t}{u_c} = 1 - a(1-\sigma) > 0$. It is clear that $\Sigma \gtreqless 1$, when $\sigma \gtreqless 1$.

When $\sigma = 1$, the household's preference in (45) exhibits additive separability between consumption and real money balances. Since the marginal utility of c_t is independent of m_t ($u_{cm} = 0$), the conduct of monetary policy under nominal GDP targeting in response to a sunspot shock will not change labor demand nor labor supply. Hence, the necessary and sufficient condition for indeterminacy is identical to that in Benhabib and Farmer's (1994) laissez-faire economy.

When $\sigma < 1$ (hence, $\Sigma < 1$), the marginal utility of consumption increases with respect to real money balances ($u_{\text{cm}} > 0$). In this case, c_t and m_t are Edgeworth complements, and the model becomes a transactions service model [Wang and Yip (1992)]. Although complementarity between c_t and m_t leads the household to raise consumption in response to a decline in the inflation rate caused by a contractionary monetary policy, $\Sigma < 1$ also implies that consumers are more willing to sacrifice consumption and invest more today in exchange for higher future consumption (the intertemporal substitution effect). The consequential higher labor supply makes indeterminacy easier to occur, even though in this case c_t and m_t exhibit complementarity. Thus, when α, ρ, δ, and γ are set at the baseline values, even when productive externalities are absent ($\theta = 0$), the model exhibits one degree of indeterminacy for all $\sigma < 1$ and all $0 < a < 1$, as there are two stable roots and only one predetermined initial condition on capital.

When $\sigma > 1$ (hence, $\Sigma > 1$), the marginal utility of consumption decreases with respect to real money balances ($u_{\text{cm}} < 0$). In this case, c_t and m_t are Edgeworth substitutes, and the model becomes an asset substitution model [Wang and Yip (1992)]. Substitutability between c_t and m_t leads the household to shrink consumption and increase labor supply in response to a decline in the inflation rate. At the same time, with $\Sigma > 1$, consumers dislike more the fluctuations in intertemporal consumption. Under the baseline parameterizations of α, ρ, δ, and γ, and setting a at 0.5, in the absence of productive externalities ($\theta = 0$), the model exhibits 2 degrees of indeterminacy when $\sigma < 3$ and saddle-path stability when $\sigma > 3$, whereby in the latter case agents are sufficiently risk averse.[9]

Note that the result that a MIUF economy is prone to self-fulfilling expectations and belief-driven business fluctuations has been demonstrated in Farmer (1997), which shows that, in the presence of nonseparability between real balances and labor supply in the utility function, the indeterminacy result is robust not only to a wide range of parameters that are consistent with the data, but also to changing the monetary policy rule from fixing the nominal interest rate to fixing the money growth rate. This paper obtains the indeterminacy result in the MIUF model under alternative considerations of nonseparability between real balances and consumption in the utility function and the regime of nominal GDP targeting.

Another point worth mentioning is that the interrelation of complementarity/substutability between c_t and m_t and the intertemporal elasticity of substitution in consumption in the MIUF models hold true in a more general formulation of the instantaneous utility function as follows:

$$u_t = \frac{(v_t)^{1-\sigma} - 1}{1-\sigma} - A\frac{h_t^{1+\gamma}}{1+\gamma}, \tag{46}$$

where $v_t = \left[ac_t^{\varepsilon} + (1-a)\,m_t^{\varepsilon}\right]^{\frac{1}{\varepsilon}}$, $0 < \varepsilon < 1$, and the elasticity of substitution between c_t and m_t is $\frac{1}{1-\varepsilon}$. When $\varepsilon \to -\infty$, v_t approaches the Leontief form: $v_t = \min(c_t, m_t)$, and c_t and m_t are perfect complements. When $\varepsilon \to 0$, v_t approaches the Cobb–Douglas form as given by (45): $v_t = c_t^a m_t^{1-a}$, and c_t and m_t are imperfect substitutes. When $\varepsilon \to 1$, v_t is linear: $v_t = ac_t + (1-a)\,m_t$, and c_t and m_t are perfect substitutes.

Let $\Sigma^{\text{CES}} = \frac{\sigma ac_t^{\varepsilon} + (1-\varepsilon)(1-a)m_t^{\varepsilon}}{ac_t^{\varepsilon} + (1-a)m_t^{\varepsilon}} > 0$ denote the inverse of the intertemporal elasticity of substitution in consumption under (46). It is clear that Σ^{CES} is inversely related with the elasticity of substitution between c_t and m_t, i.e., $\frac{1}{1-\varepsilon}$. This indicates that under parameter configurations where c_t and m_t exhibit high elasticity of substitution, agents dislike more the fluctuations in intertemporal consumption, and vice versa. Hence, in the MIUF models, the effect of the degree of substutability/complementarity between c_t and m_t and that of consumers' willingness to shift consumption from the present to the future are tied together and cannot be isolated from each other.

6. CONCLUSION

This paper has explored the theoretical interrelation between nominal GDP targeting and equilibrium (in)determinacy in a monetary version of Benhabib and Farmer's (1994) one-sector RBC model with aggregate increasing returns-to-scale. I analytically show that the implementation of the nominal GDP targeting rule produces a reinforcement effect on labor supply that raises the threshold value of productive externalities above which belief-driven business fluctuations arise. Hence, equilibrium indeterminacy is more difficult to occur.

In this paper I have explored the stabilizing effect of a pure nominal GDP targeting rule where there is no feedback mechanism nor sticky prices. It would be worthwhile to incorporate new Keynesian features such as price stickiness, wage rigidity, and investment adjustment costs, and/or to consider monetary policy rules that allow the central bank's policy rate to respond to the nominal GDP gap. It would also be worthwhile to conduct an open-economy analysis. These possible extensions will allow me to study the robustness of this paper's theoretical results and policy implications, as well as further enhance the understanding of the relationship between nominal GDP targeting and macroeconomic (in)stability. I plan to pursue these research projects in the future.

NOTES

1. Recently, Billi (2017) compares nominal GDP level targeting with strict price level targeting in a small New Keynesian model under a zero lower bound on nominal interest rates.

2. Nominal consumption targeting is proposed by Nathan Sheets, a former top official at the US Federal Reserve and currently Chief Economist and Head of Global Macroeconomic Research at PGIM Fixed Income. Robin Harding, the Financial Times' Tokyo Bureau Chief, summarized in the Financial Times on November 30, 2011 the main points of Mr Sheets on the advantages of nominal consumption over nominal GDP as a target: (i) it keeps the focus on consumer prices as the measure of inflation rather than the GDP deflator; (ii) a stable path for consumption, rather than a stable path for GDP, is what consumers want and thus is closer to maximizing their welfare; (iii) consumption is less volatile than GDP, making it easier to target; among others. I appreciate an anonymous referee for suggesting the analysis of this monetary regime.

3. Garnier et al. (2013) explore the role of returns to scale on the local (in)determinacy properties of the steady state in a two-sector economy with endogenous labor supply and sector-specific externalities. Harrison and Weder (2013) show that in a model with collateral constraints and increasing returns to scale in production and in the absence of income effect on the consumer's choice of leisure, indeterminacy of equilibria arises for more realistic parameterization.

4. The policy function of money supply is provided in Appendix B.

5. See Appendix A for the derivation of Figure 1.

6. The term "equilibrium wage-hours locus" is dubbed as "equilibrium labor demand schedule" in Schmitt-Grohé and Uribe (1997) and "aggregate labor demand curve" in Wen (1998).

7. Under empirically plausible parameter values, the bracket $[1 - \alpha - \frac{(1+\gamma)\alpha x_2}{1+\rho}]$ in (34) is positive in sign.

8. When the inflation target is set at 0, the monetary regime is referred to as price level targeting.

9. When a takes on a higher (lower) value of 0.6 (0.4), the minimum value of σ such that saddle-path stability can be maintained rises (falls) to 3.3 (2.8). In particular, a higher value of a reduces Σ and makes consumers more willing to shift consumption from the present to the future. Hence, to rule out the possibility of indeterminacy, a higher degree of risk aversion is needed.

10. See, for example, Farmer and Guo (1994) and Farmer (1999).

REFERENCES

Basu, Susanto and John G. Fernald (1997) Returns to scale in U.S. production: Estimates and implications. *Journal of Political Economy* 105, 249–283.

Bean, Charles R. (1983) Targeting nominal income: An appraisal. *Economic Journal* 93, 806–819.

Benhabib, Jess and Roger E. A. Farmer (1994) Indeterminacy and increasing returns. *Journal of Economic Theory* 63, 19–41.

Benhabib, Jess and Roger E. A. Farmer (1996) Indeterminacy and sector-specific externalities. *Journal of Monetary Economics* 37, 421–444.

Billi, Roberto M. (2017) A note on nominal GDP targeting and the zero lower bound. *Macroeconomic Dynamics* 21, 2138–2157.

Brock, William A. (1974) Money and growth: The case of long-run perfect foresight. *International Economic Review* 15, 750–77.

Burnside, Craig (1996) Production function regressions, returns to scale, and externalities. *Journal of Monetary Economics* 37, 177–201.

Carney, Mark (2012) Guidance. Speech Before the CFA Society Toronto. Toronto, Ontario, 11 December.

Clower, Robert (1967) A reconsideration of the microfoundations of monetary theory. *Western Economic Journal* 6, 1–8.

Farmer, Roger E. A. (1997) Money in a real business cycle model. *Journal of Money, Credit and Banking* 29, 568–611.

Farmer, Roger E. A. (1999) *The Macroeconomics of Self-fulfilling Prophecies*, 2nd ed. Cambridge, MA: MIT Press.

Farmer, Roger E. A. and Jang-Ting Guo (1994) Real business cycles and the animal spirits hypothesis. *Journal of Economic Theory* 63, 42–72.

Frankel, Jeffrey and Menzie Chinn (1995) The stabilizing properties of a nominal GNP rule. *Journal of Money, Credit and Banking* 27, 318–334.

Frankel, Jeffrey (2012) The Death of Inflation Targeting. VoxEU.org, 19 June.

Frankel, Jeffrey (2013) Nominal-GDP targets, without losing the inflation anchor. In Lucrezia Reichlin and Richard Baldwin (eds.), *Is Inflation Targeting Dead? Central Banking After the Crisis*, pp. 90–94. London: Centre for Economic Policy Research.

Garnier, Jean-Philippe, Kazuo Nishimura and Alain Venditti (2013) Local indeterminacy in continuous-time models: The role of returns to scale. *Macroeconomic Dynamics* 17, 326–355.

Hansen, Gary D. (1985) Indivisible labor and the business cycle. *Journal of Monetary Economics* 16, 309–327.

Harrison, Sharon G. (2001) Indeterminacy in a model with sector-specific externalities. *Journal of Economic Dynamics and Control* 25, 747–764.

Harrison, Sharon G. and Mark Weder (2013) Sunspots and credit frictions. *Macroeconomic Dynamics* 17, 1055–1069.

Lucas, Robert E. Jr. (1980) Equilibrium in a pure currency economy. *Economic Inquiry* 18, 203–220.

McCallum, Bennett T. (1985) On consequences and criticisms of monetary targeting. *Journal of Money, Credit and Banking* 17, 570–597.

McCallum, Bennett T. and Edward Nelson (1999) Nominal income targeting in an open-economy optimizing model. *Journal of Monetary Economics* 43, 553–578.

Meade, James (1978) The meaning of "internal balance." *Economic Journal* 88, 423–435.

Obstfeld, Maurice (1985) The capital inflows problem revisited: A stylized model of southern cone disinflation. *Review of Economic Studies* 52, 605–625.

Rogerson, Richard (1988) Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21, 3–16.

Schmitt-Grohé, Stephanie and Martín Uribe (1997) Balanced-budget rules, distortionary taxes, and aggregate instability. *Journal of Political Economy* 105, 976–1000.

Taylor, John B. (1985) What would nominal GNP targeting do to the business cycle? *Carnegie-Rochester Conference Series on Public Policy* 22, 61–84.

Tobin, James (1983) Monetary policy: Rules, targets and shocks. *Journal of Money Credit and Banking* 15, 506–518.

Wang, Ping and Chong K. Yip (1992) Alternative approaches to money and growth. *Journal of Money Credit and Banking* 24, 553–562.

Weder, Mark (2000) Animal spirits, technology shocks and the business cycle. *Journal of Economic Dynamics and Control* 24, 273–295.

Wen, Yi (1998) Capacity utilization under increasing returns to scale. *Journal of Economic Theory* 81, 7–36.

Woodford, Michael (2012) Methods of policy accommodation at the interest-rate lower bound. Presented at the Federal Reserve Bank of Kansas City Symposium on the Changing Policy Landscape, September 2012.

Woodford, Michael (2013) Inflation targeting: Fix it, don't scrap it. In Lucrezia Reichlin and Richard Baldwin (eds.), *Is Inflation Targeting Dead? Central Banking After the Crisis*, pp. 74–89. London: Centre for Economic Policy Research.

APPENDIX A: DERIVATION OF FIGURE 1

It can be derived that $\Theta \gtreqless 0$, if $\gamma \lesseqgtr \hat{\gamma} \equiv \frac{(1-\alpha)(1+\rho)}{\alpha x_2} - 1$. As a result, (25) and (26) indicate that, when $\gamma > \hat{\gamma}$ and hence $\Theta < 0$, I have $\Delta > 0$ and hence Det < 0. In this case, the steady state is always a saddle.

When $\gamma < \hat{\gamma}$, I have $\Theta > 0$ and hence $\Delta \gtrless 0$, if $\theta \lessgtr \theta^{\text{Det}} \equiv \frac{(\alpha+\gamma)(1+\rho)+(1+\gamma)\alpha x_2}{\Theta}$. It can be derived that (i) $\frac{\partial \theta^{\text{Det}}}{\partial \gamma} > 0$; (ii) $\frac{\partial^2 \theta^{\text{Det}}}{\partial \gamma^2} > 0$; (iii) $\theta^{\text{Det}}(\gamma = 0) = \frac{\alpha+\phi}{1-\alpha-\phi} > \frac{\alpha}{1-\alpha}$, where $\phi = \frac{\alpha x_2}{1+\rho}$; and (iv) $\theta^{\text{Det}}(\gamma \to \hat{\gamma}) \to \infty$. Therefore, the locus of θ^{Det} is a positively-sloped convex curve as depicted in Figure 1. The positively-sloped straight line of $\theta^{\text{BF}} = \frac{\alpha+\gamma}{1-\alpha}$ in Figure 1 is such that (32) holds in equality. Since $\theta^{\text{Det}} = \frac{\alpha+\gamma+\frac{(1+\gamma)\alpha x_2}{1+\rho}}{1-\alpha-\frac{(1+\gamma)\alpha x_2}{1+\rho}} > \frac{\alpha+\gamma}{1-\alpha} = \theta^{\text{BF}}$, the locus of θ^{Det} lies entirely above that of θ^{BF}.

I next reexpress (27) as $Tr = \frac{\Phi(\theta-\theta^{Tr})}{(1+\rho)\Delta}$, where $\theta^{Tr} \equiv \frac{\delta(\alpha+\gamma)(1+\rho)+(1+\gamma)(1-\alpha)\alpha x_1 x_2}{\Phi}$. Under empirically plausible parameter values, $\theta^{Tr} < \theta^{\text{BF}}$, the locus of θ^{Tr} therefore lies entirely below that of θ^{BF}. Since the region above the locus of θ^{Det} exhibits $\theta > \theta^{Tr}$ and $\Delta < 0$, the steady state exhibits a negative trace and a positive determinant, and hence is a (locally indeterminate) sink. On the other hand, the region below the locus of θ^{Det} exhibits a negative determinant and hence equilibrium determinacy.

APPENDIX B: DYNAMICAL SYSTEMS AND POLICY FUNCTIONS OF MONEY SUPPLY

This appendix provides the dynamical systems and the policy functions of money supply under different monetary regimes. The impulse response functions under equilibrium indeterminacy are then provided.

The pair of differential equations that governs the dynamics of the model is given by

$$\dot{\hat{k}}_t^j = \exp\left(\hat{y}_t^j - \hat{k}_t^j\right) - \exp\left(\hat{c}_t^j - \hat{k}_t^j\right) - \delta, \tag{B.1}$$

$$\dot{\hat{y}}_t^j = \frac{(1+\gamma)\,\alpha\,(1+\theta)\,\dot{\hat{k}}_t^j - (1-\alpha)\,(1+\theta)\left[\alpha \exp\left(\hat{y}_t^j - \hat{k}_t^j\right) - \delta - \rho + \epsilon_t\right]}{\omega}, \tag{B.2}$$

where $\hat{c}_t^j = c^j(\hat{k}_t^j, \hat{y}_t^j)$, $j =$ GT, CT, and IT, respectively, represent nominal GDP targeting, nominal consumption targeting, and inflation targeting; and ϵ_t is the sunspot innovation that has zero unconditional mean.[10] The partial derivatives of $c^j(\cdot)$ are $\hat{c}_k^{\mathrm{GT}} = \frac{\Omega_1}{\Delta} + 1$, $\hat{c}_y^{\mathrm{GT}} = \frac{\Omega_2}{\Delta}$, $\hat{c}_k^{\mathrm{CT}} = \frac{(1+\gamma)\alpha}{1-\alpha}$, $\hat{c}_y^{\mathrm{CT}} = \frac{-\omega}{(1-\alpha)(1+\theta)}$, $\hat{c}_k^{\mathrm{IT}} = \frac{(1+\gamma)\alpha}{1-\alpha} + \frac{\alpha x_1}{1+\bar{\pi}+\rho}$, and $\hat{c}_y^{\mathrm{IT}} = \frac{-\omega}{(1-\alpha)(1+\theta)} - \frac{\alpha x_1}{1+\bar{\pi}+\rho}$.

The policy functions of money supply under monetary regimes $j = \mathrm{NT}$, CT, and IT are given by

$$\mu_t^j = \dot{\hat{c}}_t^j + \pi_t^j, \tag{B.3}$$

where $\pi_t^{\mathrm{GT}} = -\dot{\hat{y}}_t^{\mathrm{GT}}$, $\pi_t^{\mathrm{CT}} = -\dot{\hat{c}}_t^{\mathrm{CT}}$, and $\pi_t^{\mathrm{IT}} = \bar{\pi}$. With (B.1), (B.2), and $\hat{c}_t^j = c^j\left(\hat{k}_t^j, \hat{y}_t^j\right)$, it is thus clear how the central bank reacts when the economy is hit by a sunspot shock.

Under the condition that $\hat{k}_t$ is predetermined in the impact period, I derive the solution paths for $\hat{k}_t$ and $\hat{y}_t$ as follows:

$$\hat{k}_t = \hat{k}^* + B\left(e^{s_1 t} - e^{s_2 t}\right), \tag{B.4}$$

$$\hat{y}_t = \hat{y}^* + B\left(\frac{s_1 - \mathbf{J}_{11}}{\mathbf{J}_{12}} e^{s_1 t} - \frac{s_2 - \mathbf{J}_{11}}{\mathbf{J}_{12}} e^{s_2 t}\right), \tag{B.5}$$

where $s_1 < s_2 < 0$ are eigenvalues of the Jacobian matrix. With (B.4) and (B.5) and under a specific value of an unanticipated sunspot shock occurring at time $t = 0$, denoted as ϵ_0, the value of B in (B.4) and (B.5) is selected such that (B.2) is met.

APPENDIX C. DERIVATION OF FIGURE 7(B)

I first derive that

$$\frac{\partial f(z^*)}{\partial \theta} = \frac{q^*(1+\gamma)\overbrace{\log(h^*)}^{(-)}}{(1-\alpha)(1+\theta)} < 0, \tag{C.1}$$

which means that a higher θ shifts the locus of $f(z^*)$ downward. Let $\tilde{z}$ denote the output-to-capital ratio such that $f(z^*)$ is tangent to the 45-degree line. Using (42) with $f'(\tilde{z}) = 1$ and (40) evaluated at $\tilde{z}$, it is straightforward to obtain that $\tilde{z} = \frac{-\omega\rho}{(1+\gamma)(1-\alpha)} > 0$. Thus, if θ is higher than $\tilde{\theta}$, which is the solution to $\tilde{z} = \frac{(\tilde{z})^{\frac{-\omega}{(1-\alpha)\left(1+\tilde{\theta}\right)}}}{B(1+\rho)} - \frac{\rho}{1-\alpha}$, then no BGP exists.

I then reexpress the model's eigenvalue given by (44) as $e(z_i^*) = \frac{(1+\rho)(1-\alpha)^2[1-f'(z_i^*)]z_i^*}{\Lambda(z_i^*)}$, $i = 1, 2$, where

$$\Lambda\left(z_i^*\right) = \frac{(1+\gamma)(1-\alpha)z^* + \omega}{1+\theta} - (1-\alpha)\rho \gtreqless 0 \text{ when } z^* \gtreqless z^{\Lambda} \equiv \frac{(1-\alpha)(1+\theta)\rho - \omega}{(1+\gamma)(1-\alpha)}. \tag{C.2}$$

It can be derived that $z^{\Lambda} - \tilde{z} = \frac{(1-\alpha)(1+\theta)\rho - \omega(1+\rho)}{(1+\gamma)(1-\alpha)} > 0$; hence, as Figure 7(b) illustrates, z^{Λ} is higher than $\tilde{z}$. It can then be inferred that (i) if $z^* > z^{\Lambda}$, then $\Lambda > 0$ and $1 - f' > 0$, and therefore $e > 0$; (ii) if $\tilde{z} < z^* < z^{\Lambda}$, then $\Lambda < 0$ and $1 - f' > 0$, and therefore $e < 0$; and (iii) if $z^* < \tilde{z}$, then $\Lambda < 0$ and $1 - f' < 0$, and hence $e > 0$.

Let θ^{Λ} denote the value of θ such that, as Figure 7(b) illustrates, $f(z^*; \theta = \theta^{\Lambda})$ intersects the 45-degree line from above when the high-z^* BGP is z^{Λ}. It then follows that when $\theta \leq \theta^{\Lambda}$, both BGPs exhibit positive eigenvalues and equilibrium determinacy; when

https://doi.org/10.1017/S1365100518000111 Published online by Cambridge University Press

$\theta > \theta^{\Lambda}$, the BGP associated with z_1^* (z_2^*) displays a positive (negative) eigenvalue and equilibrium (in)determinacy.

I next show that θ^{Λ} is strictly higher than θ^{BF}; hence, under nominal GDP targeting, a higher degree of productive externalities is needed to generate equilibrium indeterminacy. To demonstrate this, I express $\Lambda\left(z_i^*\right)$ as

$$\Lambda\left(z_i^*\right) = (1-\alpha)^2\left[1 - f'\left(z_i^*\right)\right] z_i^* + \Pi, \tag{C.3}$$

where

$$\Pi \equiv \frac{(1+\gamma)(1+\rho)}{1+\theta} - (1-\alpha) \gtreqless 0 \quad \text{when} \quad \theta \lesseqgtr \theta^{\Pi} \equiv \frac{(1+\gamma)(1+\rho)}{1-\alpha} - 1. \tag{C.4}$$

It is straightforward to derive that $\theta^{\Pi} - \theta^{\mathrm{BF}} = \frac{(1+\gamma)\rho}{1-\alpha} > 0$, and hence $\theta^{\Pi} > \theta^{\mathrm{BF}}$.

Note that at the BGP associated with z^{Λ}, the first term on the right hand-side of (C.3) is positive: $(1-\alpha)^2\,[1 - f'(z^{\Lambda})]z^{\Lambda} > 0$. Since Π is decreasing in θ, it follows that a θ (which is θ^{Λ}) that is higher than θ^{Π} is needed for Π to be small (and negative) enough that $\Lambda\left(z_2^*\right)$ and $e\left(z_2^*\right)$ become negative. Hence, $\theta^{\Lambda} > \theta^{\Pi}$.