A Problem in Voting.

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(*Read and Received 14th January 1910.*)

The following is a problem in voting:

A number of n individuals are separately arranged in order of preference by a number of k electors; it is desired to ascertain the order of preference in the opinion of the electors as a whole.

Without discussing what is the best solution of this problem, there are three plausible methods, and it will appear as the result of a curious theorem that two of them lead to identical results.

First Method: Let each individual have the number assigned to him in each order of preference. Add the totals and arrange in ascending order. This gives the order of preference.

Second Method (Francis Galton's): Let a_1 be the number of times A is placed first, a_2 the number of times he is placed second, and so on. Then we have a frequency distribution of the numbers a_1, a_2.... Consider these numbers broken up into units so that we have

$$a_1 \text{ units}, \; a_2 \text{ units}, \ldots.$$

Then single out the middle unit of this series. Suppose it is one of the a_r. Then if this unit were the middle unit of the a_r we should have half the electors considering r too high, and half too low, a position, so that r would be the fair position according to the judgment of the electors as a whole. If a_r is the p^{th} unit of the a_r, we may assign to A provisionally the number $r+p-\frac{1}{2}(a_r-1)$. Then arrange these numbers in ascending order and we get the order of preference.

Third Method: Let a_{rs} be the number of times A_r is placed above A_s, so that $a_{sr} = k - a_{rs}$. Then sum the numbers $\sum_{s=1}^{n} a_{rs}$ and arrange in descending order.

https://doi.org/10.1017/S0013091500034751 Published online by Cambridge University Press

In general the second method will give a different result from the others, but the first and third methods are exactly equivalent. In fact if b_{rp} is the number assigned to A_r by the p^{th} elector according to the first method

$$\sum_{p=1}^{k} b_{rp} + \sum_{s=1}^{n} a_{rs} = nk.$$

To prove this we notice that A_r is placed by the p^{th} elector above $n - b_{rp}$ of the others.

Hence

$$\sum_{s=1}^{n} a_{rs} = \sum_{p=1}^{k} (n - b_{rp}) = nk - \sum_{p=1}^{k} b_{rp}.$$