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TAX POLICY AND FOOD SECURITY: A DYNAMIC ANALYSIS

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We investigate the sectoral and the distributional effects of a food subsidy program, where food consumption in the economy is subsidized by taxing the manufacturing good producers. In a two-agent model comprising of farmer and industrialist households, agents consume food to accumulate health. Simulations indicate that while the subsidy program increases food output and agents' health both in the short run and the long run, manufacturing output and aggregate real GDP appear to fall in the short run and increase only in the long run. The program does not make both agents better off and exhibits social welfare gains for a limited range of subsidies.

Keywords: Allocative Efficiency, Fiscal Policy, Food Security, Welfare, Economic Growth

1. INTRODUCTION

Developing countries like Egypt and India have rolled out widespread schemes which provide subsidized grains to millions of peoples.[1] These programs aim to meet the urgent need for effective policies to improve the nutritional status of the currently undernourished.[2] While existing studies have primarily investigated the optimal forms of food subsidies—cash or kind (Gentilini (2015)), or have quantified the costs and benefits from different existing schemes (Banerjee et al. (2018)), or have analyzed the international trade, technology improvements, and the agriculture subsidy nexus (Ramaswami (2002) and Tokarick (2008)), this paper analyzes the effects of a food subsidy program on output and welfare by incorporating a role for nutrition in health accumulation over time.

A standard assumption is that expenditure on poverty programs can be treated as a side payment from the rich to the poor to compensate partly for the uneven growth which typically favors the rich (Kotwal et al. (2014)). We build

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a two-agent model to capture how different agents respond differently to a food policy.[3] This allows us to investigate cross-sectoral trade-offs. It brings in a new dimension to the literature, where most theoretical studies on the impact of a food subsidy program have primarily focussed on a one (agriculture) sector economy, as in Besley and Kanbur (1988) and Dutta and Ramaswami (2004).

Our paper also incorporates health in a novel way. We allow food to affect health accumulation of workers. Greater food intake increases investment in health, which in turn increases labor capacity of agents in the economy. This specification draws motivation from Fogel (2004), who postulates two routes through which improvements in nutritional status could contribute to economic growth. First, better nutrition increases labor force participation rates by bringing in extremely undernourished individuals into the labor force, and second, it improves the productivity of workers. We posit that a food subsidy program would have an immediate effect on the productivity of workers, through the second route.[4] There is a vast empirical literature on the positive relationship between per capita income and health indicators, like life expectancy at birth, infant mortality rate, height, to name a few.[5] Across countries and over long time horizons, health has a significant positive impact on output per worker, improves labor productivity, and hence leads to higher economic growth (Arora (2001), Mayer (2001), Bloom et al. (2004), and Weil (2014)). Healthy individuals live longer, work more efficiently, and face lower economic costs of illness. Investments in health interventions may be important for equitable income distributions and also to escape poverty traps (Chakraborty et al. (2016)).

In our model economy, there are two household-producers—a farmer and an industrialist. We allow for different subsidies to these households to be able to track their effects on the economy. The subsidy program imposes an income tax on the industrialist, and thereby the immediate effect is to lower her production of the manufacturing good and accumulation of capital. While the subsidy program unequivocally improves the health status of all beneficiaries, these effects contribute to economic growth after a long lag. Over time as health accumulates, it manifests in higher economic growth and eventually increases manufacturing output and aggregate real GDP.

Our simulations suggest that there may be no subsidy combination in which both agents together are better off. This highlights that the government needs to be prudent in choosing the level of subsidies to agents as different subsidy combinations have differential effects across households. This finding is robust to the distribution of farmer and industrialist households in the economy. For most range of subsidies, social welfare is lower in the subsidy program.

We feel that the paper has a couple of contributions to the literature. First, we model nutrition in a growth framework. While there is extensive empirical documentation on this channel, its implications have not been captured sufficiently in theory. We show that food intake and nutrition improve labor productivity. Second, we emphasize the sectoral and the distributional trade-offs of the subsidy program. The program improves health, which by itself, is an important outcome. It, however, does not guarantee increase in output or welfare.

In the next section, we model the income tax regime and present the simulation results in Section 3. Finally, Section 4 concludes the paper.

2. THE MODEL

The economy comprises of a government and two households: a farm household and an industrialist household. As in Jiny and Zengz (2007), the farmer and the industrialist are household producers. They maximize their own utilities subject to their own budget constraints, where the two households work in different occupations. The government provides a subsidy on food consumption to both households by levying an income tax on the industrialist.

Households consume food for two reasons: to enhance utility and to improve health. We feel that the second purpose is a novelty of the paper. There is a vast existing literature that discusses the effects of food consumption on health outcomes. Better calorie intake is associated with better health outcomes (see Strauss and Thomas 2008). In fact, Deolalikar (1988) stipulates that the stock of health (measured as weight-for-height) is a cumulative outcome of nutrition intake. The effect of nutrition on labor productivity is significant in the medium term to long term. In our framework, we assume a similar role of food—as an input in health investments.

It is hard to calculate how much of food intake is for pleasure and how much goes into health investments. While there are some very clear "bad foods" like sweets or aerated drinks, most food products, like fruits, chocolates, proteins (fish, meat, pulses), to name a few, are hard to categorize by their purpose of consumption. Even when people consciously eat healthy food items, they choose those which give them pleasure. Thus, we assume that the same quantity of food gives agents utility as well contributes to health accumulation—food serves *dual purposes*.

It is important to highlight that in our economy agents do not get utility from health. While this would not have any impact on our analysis in terms of the sectoral effects, the welfare effects may be sensitive to this assumption. If agents cared about being healthy, health benefits from a food subsidy program may outweigh any potential welfare losses due to lower consumption of goods. But do people get utility from health? It seems plausible in developed nations where citizens tend to choose their location of work or stay, their social networks, their habits and cultural activities in accordance with their preference for a healthy life. We believe that such considerations are not widely seen in the emerging or less developed economies. In these economies, consumption of food, durables, and non-durables far outweighs health considerations. Hence, in our model for developing economies, it is not too unrealistic to abstract from incorporating health in the utility function.

Barro (2013) introduces the role of health in economic growth. He assumes that health investments are similar to educational investments or capital investments and do not require different inputs. In our model, health investments require food while capital investments require manufacturing goods.

Differences in occupations stem from differences in asset ownership (see Motiram and Singh (2012), Blanden (2013) and Azam (2013)). We assume that the industrialist is born with capital, while the farmer is not. As the farmer does not own capital, he cannot participate in the production of manufacturing goods. We assume that the industrialist does not want to participate in labor intensive work required for agricultural goods production. In practice, differences in assets between farmers and industrialists are not limited to only ownership of capital goods. There could be differences in access to good schools, or to high-skilled worker networks, or even to work locations with modern facilities like electricity and the Internet. Typically industrialists own better quality assets and the farmers, working in rural settings, find it difficult to acquire them (see Honohan (2006), Honohan (2008), and Bhattacharya and Patnaik (2016)). The question is whether a policy on health improvements is able to achieve economic and welfare gains in the presence of these resource differences between farmers and industrialists. This paper does not find so.

The production structure is as follows. The farmer supplies his own labor to produce two agricultural goods, a food crop and a cash crop.[6] The industrialist produces the manufacturing good using her own labor, cash crop, and capital. As explained earlier, *the same quantity of food gives utility and health benefits*. The manufacturing good is separately used for two purposes: a part of the manufacturing good is consumed by agents for utilitarian purposes and the other part is used to produce capital for the next period. The usage of the manufacturing good is standard in the literature.

2.1. The farm household

The representative farm household has a fixed number of L_f members, each born with H_{ft} units of health at time t. Time is continuous. Each member derives utility from consumption of food (x_{at}), manufacturing goods (x_{mt}), and leisure (x_{lt}). The farm household's instantaneous utility function is

$$U_{ft} = L_f\,[\phi_1 \ln x_{at} + \phi_2 \ln x_{mt} + \phi_3 \ln x_{lt}],$$

where $0 < \phi_1, \phi_2, \phi_3 < 1$ and $\phi_1 + \phi_2 + \phi_3 = 1$. For the sake of simplicity, we use log linear, homothetic preferences.[7]

The same unit of food consumption also adds to farmer's health.[8] Similar to Kelly (2017), an individual's health accumulates as

$$\dot{H}_{ft} = NH_{ft}^{\gamma} x_{at}^{1-\gamma} - \delta_h H_{ft}, \quad 0 < \gamma < 1, \tag{1}$$

where N is the health technology parameter. Instantaneous investment in health is based on a Cobb–Douglas interaction between the existing stock of health and the consumption of food, where γ is the weight on health. In each period, the stock

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of health depreciates at the rate of $\delta_h > 0$, which can be attributed to aging. We look into the aspect of nutrition and health, unlike Kelly (2017) who analyzes the role of medical goods in health investments. Here, more food intake implies faster health accumulation.

The farm household produces a food crop and a cash crop using labor in a CRS technology, that is, $Q_{at} = AL_{at}$, $Q_{ct} = CL_{ct}$, where subscripts a and c denote food and cash crops, respectively, Q is for output, L for labor inputs, A and C are respective food crop and cash crop productivity parameters. The household employs its total health stock for leisure and labor as per the labor allocation rule

$$L_f H_{ft} = L_{at} + L_{ct} + L_f x_{lt}.$$

The revenue from sale of the agricultural goods provides the farmer's income, which he spends on food and manufacturing goods. Therefore, the farm household's budget constraint is given by

$$(1 - s_1)L_f p_{at} x_{at} + L_f x_{mt} = Ap_{at} L_{at} + Cp_{ct}\left(L_f H_{ft} - L_{at} - L_f x_{lt}\right), \qquad (2)$$

where s_1 is the per unit subsidy on food consumption given to the farmer by the government, and p_{at} and p_{ct} are the relative prices of food and cash crops, respectively. Here we have used the farmer's labor allocation rule to substitute out for L_{ct}. The manufacturing good is the numeraire. We term $(1 - s_1)p_{at}$ as the "effective price" of food consumption for the farmer.

The household maximizes its discounted lifetime utility subject to the law of motion for health accumulation (1) and the per period budget constraint (2). The Hamiltonian is

$$\begin{aligned}\widehat{\mathcal{H}}_f &= L_f e^{-\rho t}\left[\phi_1 \ln x_{at} + \phi_2 \ln x_{mt} + \phi_3 \ln x_{lt}\right] \\ &\quad + \mu_{1t}\left[Ap_{at}L_{at} + Cp_{ct}(L_f H_{ft} - L_{at} - L_f x_{lt}) - (1 - s_1)L_f p_{at} x_{at} - L_f x_{mt}\right] \\ &\quad + \mu_{2t}\left[NH_{ft}^{\gamma} x_{at}^{1-\gamma} - \delta_h H_{ft}\right]\end{aligned}$$

where ρ is the discount rate, and μ_{1t} and μ_{2t} are the co-state variables. The choice variables are x_{at}, x_{mt}, x_{lt} and L_{at}, and the state variable is H_{ft}. The optimization yields:

$$\frac{\phi_1 L_f e^{-\rho t}}{x_{at}} + N(1 - \gamma)\mu_{2t}\left(\frac{H_{ft}}{x_{at}}\right)^{\gamma} = (1 - s_1)L_f \mu_{1t} p_{at}, \qquad (3)$$

$$\frac{\phi_2 e^{-\rho t}}{x_{mt}} = \mu_{1t}, \qquad (4)$$

$$\frac{\phi_3 e^{-\rho t}}{x_{lt}} = C\mu_{1t} p_{ct}, \qquad (5)$$

$$Ap_{at} = Cp_{ct}, \qquad (6)$$

$$\mu_{2t}\left[\gamma N\left(\frac{H_{ft}}{x_{at}}\right)^{\gamma-1} - \delta_h\right] + CL_f \mu_{1t} p_{ct} = -\dot{\mu}_{2t}. \qquad (7)$$

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Equations (3), (4), and (5) are the first-order conditions with respect to food,[9] the manufacturing good and leisure, respectively. The optimal choice of labor gives (6), that is, prices of the two agricultural goods are proportional to each other. The farm household's Euler equation is (7). Substituting (4), (5), and (6) in the farm household's budget (2) gives his demand functions for consumption of the manufacturing good and leisure:

$$x_{mt} = \frac{\phi_2 p_{at} H_{ft}}{\phi_2 + \phi_3}\left[A - (1 - s_1)\frac{x_{at}}{H_{ft}}\right], \tag{8}$$

$$x_{lt} = \frac{\phi_3 H_{ft}}{(\phi_2 + \phi_3)A}\left[A - (1 - s_1)\frac{x_{at}}{H_{ft}}\right]. \tag{9}$$

2.2. The industrialist household

An industrialist (or a manufacturing) household has L_i members, each born with H_{it} units of health in time period t. The industrialist gets utility from consumption of food (y_{at}), the manufacturing good (y_{mt}), and leisure (y_{lt}). Her felicity function has the same form as that of the farmer. Her health accumulation equation is

$$\dot{H}_{it} = NH_{it}^{\gamma} y_{at}^{1-\gamma} - \delta_h H_{it}. \tag{10}$$

The industrialist produces the manufacturing good (Q_{mt}) using labor (L_{mt}), the cash crop (q_{ct}), and capital (K_t) in a CRS technology

$$Q_{mt} = ML_{mt}^{\alpha} q_{ct}^{\beta} K_t^{1-\alpha-\beta}, \quad 0 < \alpha, \beta < 1. \tag{11}$$

Sale of the manufacturing good is the only source of income for the industrialist. She pays a per unit tax, $\tau_t \in (0, 1)$, on her income and spends her after-tax income on consumption of food and the manufacturing good, on purchase of cash crops, and to accumulate capital. Like the farmer, she also gets a per-unit subsidy, s_2, from the government on her food consumption. The industrialist's household budget is

$$(1 - s_2)L_i p_{at} y_{at} + L_i y_{mt} + p_{ct} q_{ct} + \dot{K}_t + \delta_k K_t = (1 - \tau_t)ML_{mt}^{\alpha} q_{ct}^{\beta} K_t^{1-\alpha-\beta}, \tag{12}$$

where δ_k is rate of capital depreciation and her labor allocation rule is $L_i H_{it} = L_{mt} + L_i y_{lt}$. Her "effective price" of food consumption is $(1 - s_2)p_{at}$. The industrialist household's Hamiltonian is given by

$$\begin{aligned}\widehat{\mathcal{H}}_i = {} & L_i e^{-\rho t}\left[\phi_1 \ln y_{at} + \phi_2 \ln y_{mt} + \phi_3 \ln y_{lt}\right] \\ & + \lambda_{1t}\Big[(1 - \tau_t)M(L_i H_{it} - L_i y_{lt})^{\alpha} q_{ct}^{\beta} K_t^{1-\alpha-\beta} \\ & - (1 - s_2)L_i p_{at} y_{at} - L_i y_{mt} - p_{ct} q_{ct} - \delta_k K_t\Big] \\ & + \lambda_{2t}\left[NH_{it}^{\gamma} y_{at}^{1-\gamma} - \delta_h H_{it}\right],\end{aligned}$$

where λ_{1t} and λ_{2t} are the co-state variables. Here, y_{at}, y_{mt}, y_{lt}, and q_{ct} are choice variables, and H_{it} and K_t are state variables. The first-order conditions with respect to consumption of food, manufactures, and leisure are

$$\frac{\phi_1 L_i e^{-\rho t}}{y_{at}} + N(1-\gamma)\lambda_{2t}\left(\frac{H_{it}}{y_{at}}\right)^{\gamma} = (1-s_2)L_i\lambda_{1t}p_{at}, \tag{13}$$

$$\frac{\phi_2 e^{-\rho t}}{y_{mt}} = \lambda_{1t}, \tag{14}$$

$$\frac{\phi_3 L_i e^{-\rho t}}{y_{lt}} = \frac{\alpha\lambda_{1t}(1-\tau_t)Q_{mt}}{(H_{it}-y_{lt})}. \tag{15}$$

The cash crop input is optimally chosen so as to equate its marginal revenue to its marginal cost:

$$\beta(1-\tau_t)\frac{Q_{mt}}{q_{ct}} = p_{ct}. \tag{16}$$

The first-order conditions with respect to the state variables give two dynamic equations:

$$\dot{\lambda}_{1t} = -\lambda_{1t}\left[(1-\alpha-\beta)(1-\tau_t)\frac{Q_{mt}}{K_t} - \delta_k\right], \tag{17}$$

$$-\dot{\lambda}_{2t} = \lambda_{1t}\cdot\frac{\alpha(1-\tau_t)Q_{mt}}{(H_{it}-y_{lt})} + \lambda_{2t}\left[\gamma N\left(\frac{H_{it}}{y_{at}}\right)^{\gamma-1} - \delta_h\right]. \tag{18}$$

2.3. General equilibrium

The food crop market clearing condition states that the demand for food by the two households must equal its supply by the farmer

$$L_f x_{at} + L_i y_{at} = A L_{at}. \tag{19}$$

Similarly, the cash crop and the manufacturing goods market clearing conditions are

$$q_{ct} = C(L_f H_{ft} - L_{at} - L_f x_{lt}), \tag{20}$$

$$\dot{K}_t + \delta_k K_t + L_f x_{mt} + L_i y_{mt} = Q_{mt}. \tag{21}$$

The government spends its tax revenue on providing food subsidies. Its budget constraint is

$$\tau_t Q_{mt} = s_1 L_f p_{at} x_{at} + s_2 L_i p_{at} y_{at}. \tag{22}$$

The farmer's demand functions (3)–(4), (6), (8)–(9), manufacturing production function (11), industrialist's optimization conditions (13)–(16), the goods market clearing conditions (19)–(20), the government budget constraint (22), and industrialist's labor allocation rule $L_i H_{it} = L_{mt} + L_i y_{lt}$ constitute

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the fourteen equations of the static system. The 14 dependent variables $\{p_{ct}, p_{at}, x_{at}, x_{mt}, x_{lt}, y_{at}, y_{mt}, y_{lt}, L_{at}, q_{ct}, L_{mt}, Q_{mt}, \tau_t, \mu_{1t}\}$ can be solved as functions of six "independent" variables $\left(\lambda_{1t}, \lambda_{2t}, \mu_{2t}, K_t, H_{it}, H_{ft}\right)$. The evolution of these independent variables is determined by the dynamic equations (1), (7), (10), (17), (18), and (21).

2.4. The steady state

Here, we first derive the dynamic system in terms of the normalized variables and then derive the steady state of the economy. The normalized variables are

$$\chi_{1t} = \frac{x_{at}}{H_{ft}}, \quad \chi_{2t} = \frac{e^{-\rho t}}{\mu_{2t} H_{ft}}, \quad \psi_{1t} = \frac{e^{-\rho t}}{\lambda_{1t} H_{ft}}, \quad \psi_{2t} = \frac{\lambda_{1t}}{\lambda_{2t}}, \quad \psi_{it} = \frac{H_{it}}{H_{ft}},$$
$$\psi_{ht} = \frac{H_{it} - y_{lt}}{H_{ft}}, \quad \psi_{at} = \frac{y_{at}}{H_{ft}}, \quad \psi_{mt} = \frac{y_{mt}}{H_{ft}}, \quad \psi_{kt} = \frac{K_t}{H_{ft}}, \quad \psi_{qt} = \frac{Q_{mt}}{H_{ft}},$$

In Appendix A, we reduce the static system to six equations in six normalized variables $\left\{\psi_{qt}, \psi_{ht}, \psi_{at}, p_{at}, \tau_t, \chi_{2t}\right\}$. These variables can be written as functions of five "independent" normalized variables $(\psi_{1t}, \psi_{2t}, \psi_{kt}, \psi_{it}, \chi_{1t})$, whose evolution is determined by the dynamic system. We enumerate the steps to derive the dynamic system in normalized variables in Appendix B.

PROPOSITION 1. *There exists a unique steady state for the economy.*

We prove this in Appendix C.

COROLLARY 1. *In long run, the food to health ratio is same for both agents.*

In the steady state, $\dot{\psi}_{it} = 0$ which implies,

$$\frac{\psi_a^*}{\psi_i^*} = \chi_1^*. \tag{23}$$

COROLLARY 2. *The farmer's normalized food consumption at the steady state, χ_1^*, is increasing in his subsidy, s_1, and independent of s_2.*

Proof. It follows from (A.17) that $d\chi_1^*/ds_1 > 0$. ■

It follows,

PROPOSITION 2. *The steady-state growth rate of farmer's health $\left(\dot{H}_{ft}/H_{ft} \equiv g\right)$ depends on the farmer's subsidy, parameters of the preference function, discount rate, and health accumulation function parameters, $g = N\chi_1^{*1-\gamma} - \delta_h$.*

We prove this in Appendix D.

COROLLARY 3. *The long-run growth rate of sectoral outputs is g. The relative price of food is constant in the steady state.*

In the steady state, the normalized variables ψ^* are constant. Hence, H_{it}, Q_{mt}, and Q_{at} grow at the same rate as H_{ft}.

PROPOSITION 3. *There exists an upper limit on* s_1.

Proof. Substituting the agricultural goods' relative price expression (6), the demand functions (9) and (16), and the food market clearing condition (19) in the cash crop market clearing condition (20) gives

$$\beta(1-\tau_t)\frac{Q_{mt}}{H_{ft}p_{at}} + \frac{L_i y_{at}}{CH_{ft}} = \frac{\phi_2}{\phi_2+\phi_3}AL_f - \frac{(\phi_2+s_1\phi_3)L_f x_{at}}{(\phi_2+\phi_3)H_{ft}}.$$

In the steady state, this relationship in normalized variables is

$$\beta(1-\tau^*)\frac{\psi_q^*}{p_a^*} + \frac{L_i\psi_a^*}{C} = \frac{\phi_2}{\phi_2+\phi_3}AL_f - \frac{(\phi_2+s_1\phi_3)L_f\chi_1^*}{(\phi_2+\phi_3)}.$$

As χ_1^* is increasing in s_1 (Proposition 2), the RHS of the above equation is negatively related to s_1. The LHS of the above equation must be positive, which limits the value of s_1. Intuitively, an increase in farmer's subsidy increases his food consumption and leisure. This increase may be so high that the farmer may choose to not supply any food or cash crop to the industrialist. Thus, there is an upper limit, denoted by $\bar{s}_1$, at which

$$\frac{\phi_2}{\phi_2+\phi_3}AL_f - \frac{(\phi_2+\bar{s}_1\phi_3)L_f\chi_1^*(\bar{s}_1)}{(\phi_2+\phi_3)} = 0$$

where $\chi_1^*(\cdot)$ derived from (A.17). ■

We resort to numerical simulations to characterize the steady-state properties. Let us first define the conditions for stability in a multi-variable dynamic system:

DEFINITION. *Consider the following linear differential equation system:*

$$\dot{\hat{y}}(t) = A\hat{y}(t); \quad \hat{y} = y - y^*,$$

where y *is a vector of* n *variables, with* $m \leq n$ *is the predetermined number of state variables, initial value* $y(0)$ *is given,* y^* *is the steady state of the system, and* A *is an* $n \times n$ *matrix. Suppose that* $l(\leq n)$ *is the number of the eigenvalues of* A *have negative real parts. Then, there exists an* l*-dimensional subspace* L *of* $\Re^n$ *such that that starting from any* $\hat{y}(0) \in L$*, the differential equation has a unique solution with* $\hat{y}(t) \to 0$*. Further, if* $l = m$*, the system is saddle-path stable, with unique optimal trajectory. If* $l < m$ *(or* $l > m$*) the system is unstable and* $y(t)$ *does not converge to the steady state (or the system is indeterminate and multiple optimal trajectories exist).* ■

We derive the linearized system of dynamic equations around the steady state in Appendix E. For any initial values of K_0, H_{f0}, and H_{i0}, we get two initial values of normalized variables ψ_{k0} and ψ_{i0}. Through numerical simulations, we find that the steady state is saddle point stable. We get two real negative eigenvalues of the transition matrix, which imply that the economy has a unique path to the steady state. Simulations show that very high values of manufacturer's subsidy (s_2) may make one of the eigenvalues imaginary with negative real part. In this case, the system would spiral to the steady state and the dynamics will be asymptotically stable. We do not consider such range of subsidies which yield imaginary eigenvalues.

3. NUMERICAL SIMULATIONS

We base our numerical simulations on India, which is the latest developing country to have implemented a large food subsidy program. The effects of the food subsidy program, however, would be relevant for other developing countries.

The structural parameters for India are fixed in accordance with the existing literature. As in Gabriel et al. (2012), we set the discount rate, ρ, at 0.02 and the annual depreciation rate of physical capital, δ_k, at 0.1. We calculate the preference parameters using data from the RBI Handbook of Statistics and the CSO database, which gives detailed input-use data by sectors for the period 1999–2007.

In the utility function, $\phi_1 + \phi_2$ is the weight on consumption goods hence the remaining weight, ϕ_3, is

$$\phi_3 = 1 - \frac{C}{Y},$$

where C/Y is the ratio of private final consumption expenditure to GDP, averaged over the years 1999–2007. We get $\phi_3 = 0.4$. As the households consume two goods, food and the manufacturing good, their weights are calculated as per their share in value added:

$$\phi_1 = \left(\frac{V_A}{V_M + V_A}\right) \times \frac{C}{Y}, \quad \phi_2 = \left(\frac{V_M}{V_M + V_A}\right) \times \frac{C}{Y},$$

where V_A and V_M are the average agriculture and manufacturing value added for the period 1999–2007. We obtain $\phi_1 = 0.36$ and $\phi_2 = 0.24$.

The value of manufacturing output equals the sum of expenditure on its inputs, that is, wage payments, capital payments, and the spending on cash crop intermediates. Similar to the methodology in Verma (2012), wage payments are estimated by compensation to employees, and the capital payments by the sum of consumption of fixed capital and operating surplus. The estimation of expenditure on cash crops inputs is a more involved process. Dholakia et al. (2009) tabulate the input–output (I–O) tables for the years 1999 and 2003 for India. They report that the cash crop constitutes about 8.7% of the total intermediate input costs. Together we get,

$$\alpha = \frac{\begin{array}{c}\text{Wage}\\ \text{Compensation}\end{array}}{\begin{array}{c}\text{Wage}\\ \text{Compensation}\end{array} + \left(\begin{array}{c}\text{Fixed}\\ \text{Capital Costs}\end{array} + \begin{array}{c}\text{Operating}\\ \text{Surplus}\end{array}\right) + \begin{array}{c}\text{8.7\% of}\\ \text{Intermediates Costs}\end{array}},$$

which gives $\alpha = 0.19$. Similarly, we calculate the share of cash crop and capital: $\beta = 0.25$ and $1 - \alpha - \beta = 0.56$.

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We look into the health and nutrition literature to estimate the parameter γ. Scientists have documented the importance of both diet and exercise in improving health outcomes, such has to increase metabolism or lower obesity (Curioni and Lourenco (2005) and Bo et al. (2008)). In our health investment function, diet could be captured by food intake and exercise by initial health levels. Healthier individuals exercise more. Practitioners put a greater emphasis on healthy diet in comparison to exercise—around the order of 80% on food intake and 20% on exercise (Lawler (2017), Leal (2018), and Edwards (2018)). Based on National Health expenditures accounts of the USA, Kelly (2017) estimates expenditures on medical goods. In her model, healthcare alone contributes to health investment and she estimates 67% (or 33%) of expenditure on health investment is due to her choice variable, healthcare (or non-controllable factors). We assume the initial stock of an individual's health is a proxy for Kelly's non-controllable factors. Based on nutrition studies or on the USA health expenditure accounts, the estimated γ comes out to be in the range of 0.2–0.33. In absence of any similar India-specific studies, we assume the parameter to be somewhere in between, that is, $\gamma = 0.25$. Further, based on Kelly (2017), Mitnitski et al. (2002), and Rockwood and Mitnitski (2007), we assume aging is equivalent to a per period depreciation in health and in accordance with their estimates we set the health depreciation rate, δ_h, at 0.04.

We assume the productivity parameters at $A = C = M = 1$. We set $N = 0.137$ to match the growth rate of GDP in our model with the 8.67% annual growth rate calculated from data. The farmer's household is assumed to be twice as large as the manufacturer household, $L_f = 600$, $L_i = 300$. Since we are interested in analyzing the effect of the subsidies, we conduct our numerical experiments for different values of s_1 and $s_2 \in [0, 1)$.

For our model, we find that the highest feasible value for the farmer's subsidy is 3.8%. The industrialist's subsidy could be as high as 63%. We look at the effect of the subsidies first on the steady state of normalized variables; second on the growth rates and the time trends of sectoral outputs and prices; and finally, on the welfare of agents.

3.1. Effect on taxes

An income tax on the manufacturer's income is imposed to finance the food subsidy program. Hence, the steady-state income tax rate increases with the subsidies, that is, $\tau^* = \tau^*(\underset{+}{s_1}, \underset{+}{s_2})$. We plot the steady-state tax rates for different subsidy combinations in Figure 1. The x-axis denotes the farmer's subsidy (s_1) and the y-axis captures the variable of interest, here tax rates. We plot separate curves for different values of industrialist's subsidies (s_2). As one moves from the black solid line to the purple dotted line, the entrepreneur's subsidy increases. In the absence of a subsidy program, tax rate is zero which is depicted by the blue square. We follow this graph color key for all plots.

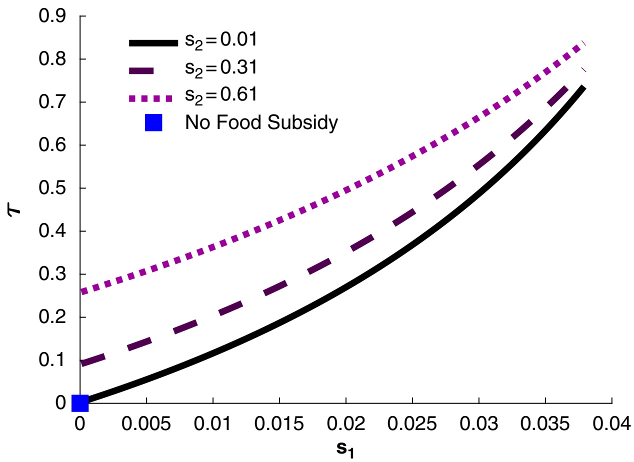

FIGURE 1. Effect of subsidies on tax rate, τ^*.

On a practical aspect, the model cautions about the magnitudes of the subsidy. There is a range of feasible subsidies as well as a range of implementable ones. For example, in our model a 3.5% subsidy on food consumption to the farmer and a 1% subsidy to the manufacturer is equivalent to an income tax rate of 60%, which is clearly an exorbitant number (Figure 1). We estimate the burden of the program to be about 1–6.5% of GDP. This is clearly more than, say, the Indian government's allocation of 1% of GDP for the food subsidy program for the year 2018 (India Budget 2018). Our estimated fiscal burden of the food subsidy program is consistent with Kozicka et al. (2017), who estimate it to be at least 1.6% of GDP.

To compare food subsidy costs across developing countries, we look at the study Ter-Minassian et al. (2008). They report that in order to address concerns of food security some countries have lowered import taxes on agricultural goods, while others have increased public outlays in food subsidy programs. In 2007–2008, the median fiscal costs of such "tax decreases" was around 0.1% of GDP and median costs of "increasing subsidy" was 0.2%. More than 22 countries incurred fiscal costs by increasing the food subsidies. In particular, countries like Burundi, Egypt, Jordan, Maldives, Morocco, and Timor-Leste approximately spent over 1% of GDP on food subsidies, with Maldives allocating the highest share, 3.6% of GDP, to their food subsidy program (Demeke et al. 2009). These numbers from developing economies' food subsidy experiences are comparable to our estimates based on the Indian data.

3.2. Effect on agricultural outputs

To understand the effect of the subsidy program on agricultural outputs, we first look at their effects on the demand for food, χ_1^* and ψ_a^*.

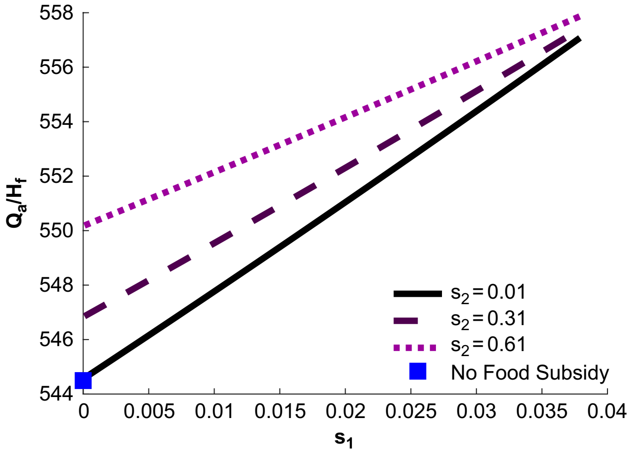

FIGURE 2. Effect of subsidies on normalized food output.

As shown in Proposition 2, $\chi_1^* = \lim_{t\to\infty}\left(x_{at}/H_{ft}\right) = \chi_1^*(\underset{+}{s_1}, \underset{0}{s_2})$. Further, simulations show that the normalized industrialist's food consumption falls in s_1 and rises in s_2, $\psi_a^* = \psi_a^*(\underset{-}{s_1}, \underset{+}{s_2})$. For given prices, an increase in the industrialist's subsidy lowers her effective price on food, $(1 - s_2)p_a^*$. However higher subsidies, s_1 and s_2, also increase the tax rate and hence lower her after-tax income. The former price effect would increase demand for food, while the latter income effect would lower demand for food. The subsidy s_1 affects only through income so it unequivocally lowers ψ_a^*. In contrast, the effect of s_2 is theoretically ambiguous—there is a positive effect through effective prices and a negative effect due to lower after-tax income. Simulations yield that in log linear preferences the effective price effect dominates.

The food subsidies to an agent increase their own demand for food. The cross effects of subsidies on the other agent's food demand is weak. Adding both agents' food demands,

$$\left(\frac{Q_a}{H_f}\right)^* = L_f\chi_1^* + L_i\psi_a^*$$

we get that subsidies increase food output, $(Q_a/H_f)^*(\underset{+}{s_1}, \underset{+}{s_2})$. We show this in Figure 2.

Higher taxes in the subsidy program lower the marginal product of the cash crop and hence its demand. Thus, subsidies have a contractionary effect on the cash crop output, $Q_c/H_f(\underset{-}{s_1}, \underset{-}{s_2})$.

3.3. Effect on capital and manufacturing output

Subsidies affect manufacturing production primarily through income taxes. In the steady state, subsidies unequivocally increase taxes, and lower the marginal

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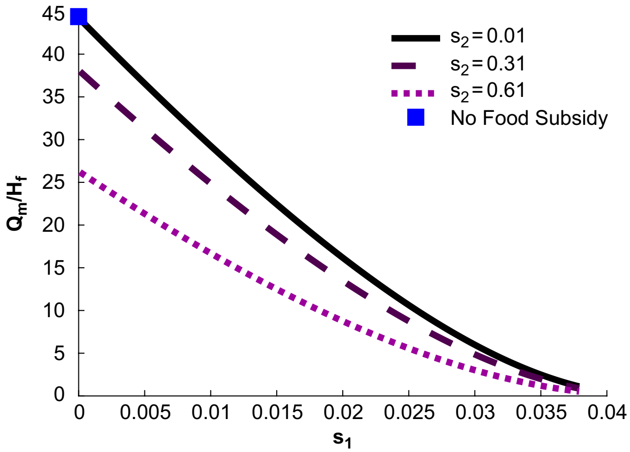

FIGURE 3. Effect of subsidies on normalized manufacturing output, ψ_q^*.

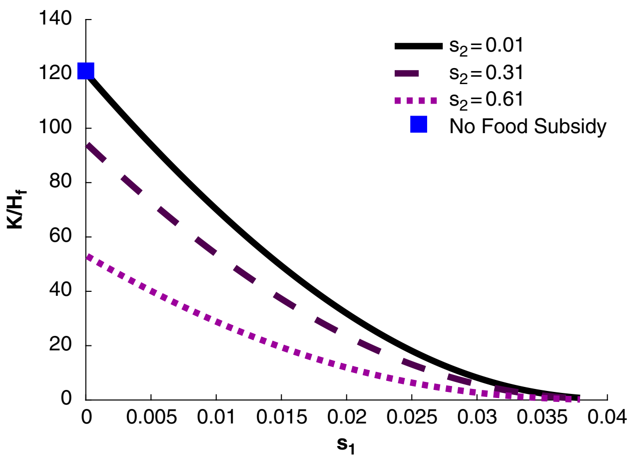

FIGURE 4. Effect of subsidies on normalized capital, ψ_k^*.

products of labor, capital and the cash crop. This disincentivizes production of the manufacturing good. We show in Figure 3 that the higher the subsidy, lower is the normalized manufacturing output, $\psi_q^* = \psi_q^*(\underset{-}{s_1}, \underset{-}{s_2})$. Lower after-tax income also depresses capital accumulation and hence leads to lower levels of steady-state normalized capital, that is, $\psi_k^* = \psi_k^*(\underset{-}{s_1}, \underset{-}{s_2})$. We plot these subsidy effects in Figure 4.

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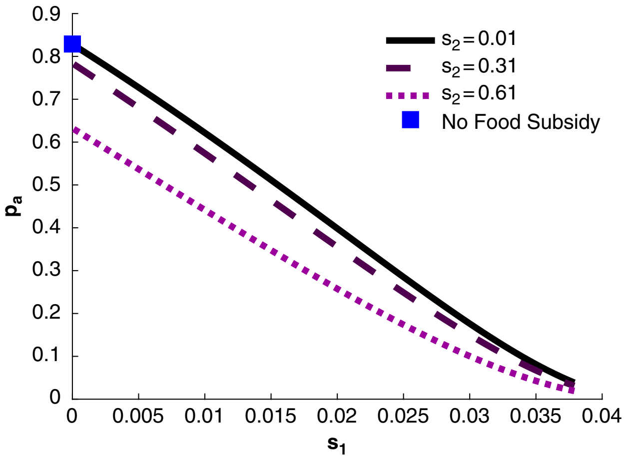

FIGURE 5. Effect of subsidies on relative price of food, p_a^*.

3.4. Effect on relative prices and GDP

As shown in Figure 5, the relative prices of food are negatively related to the two subsidies, that is, $p_a^* = p_a^*(\underset{-}{s_1}, \underset{-}{s_2})$. To understand this, recall that the subsidies increase the demand for the food crop and reduce the supply of the manufacturing good. The nominal price of both the food crop and the manufacturing good increases. Our simulations suggest that the increase in the nominal price of the manufacturing good is higher than that of the food crop, which implies that the price of the food crop relative to the manufacturing good falls with subsidies. Thus, both subsidies lower p_a^*. Further, since p_c^* is proportional to p_a^*, as in equation (6), the steady-state relative price of the cash crop also falls with subsidies.

In the steady state, the subsidy program has an overall negative effect on the normalized aggregate real GDP, shown in Figure 6, $\psi_{\mathcal{Y}} = \psi_{\mathcal{Y}}(\underset{-}{s_1}, \underset{-}{s_2})$. Similar to Ghate et al. (2016), we normalize aggregate real GDP as

$$\psi_{\mathcal{Y}}^* \equiv \lim_{t\to\infty} \frac{Y_{rt}}{H_{ft}} = \lim_{t\to\infty} \frac{Y_t / p_{at}^{\frac{\phi_1}{\phi_1+\phi_2}}}{H_{ft}} = \frac{p_a^*\left(L_f \chi_1^* + L_i \psi_a^*\right) + \psi_q^*}{p_a^{*\frac{\phi_1}{\phi_1+\phi_2}}},$$

where we assume that the price index is a composite of prices of final goods, each weighed by their relative shares in the utility function. Food and manufacturing consumption have a total weight of $(\phi_1 + \phi_2)$ in the utility function of which ϕ_1 is the weight on food. The fall in relative price, p_a^*, and manufacturing output, ψ_q^*, is more than the increase agents' food demand, $L_f \chi_1^* + L_i \psi_a^*$. Subsidies, thus, lower normalized real GDP.

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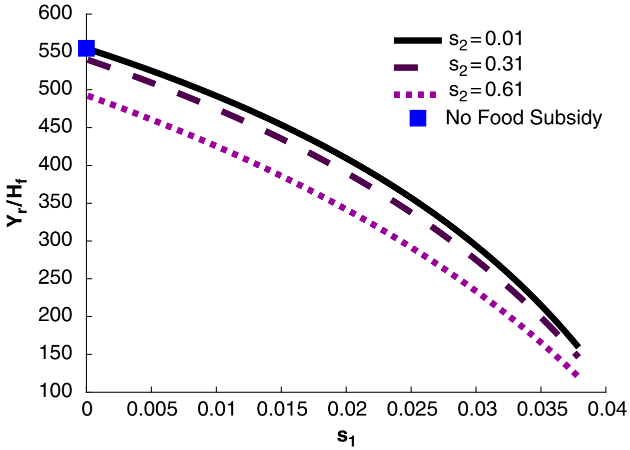

FIGURE 6. Effect of subsidies on normalized real GDP, $\psi^*_{\mathcal{Y}}$.

3.5. Growth effects and time trends

So far, we have focussed only on the effects of subsidies on long-run levels of the normalized variables. We now look at the effect of subsidies on the short-run and the long-run growth rates. The eigenvalues of the transition matrix determine the rate of growth in the short run. Numerically, we get that both subsidies lower these eigenvalues, ζ_1 and ζ_2. The long-run growth of the economy depends positively on normalized food consumption, $(x_a/H_f)^*$, which increases with s_1 and is independent of s_2.

Subsidies may have opposing effects on the levels of normalized variables and their growth rates of the economy. To analyze the net effects of subsidies on selected macroeconomic variables, we plot their time trends. Since all our simulations are based on annualized parameters, each time period corresponds to one year.

We focus on the trajectories of the manufacturing output, food output, the relative price of food and the real GDP. As expected these variables grow over time. Q_{mt}, Q_{at}, p_{at}, and Y_{rt} are positively sloped and convex in time t. The key question is to determine how do these subsidies affect the path of these variables. To answer this, we plot the percentage deviation of the trajectory in a subsidy program from the trajectory in a no-subsidy program in Figure 7. We choose three combination of subsidies. In Figure 7, the solid black line depicts the lowest subsidy combination $s_1 = 0.01, s_2 = 0.15$. The dashed purple line maintains s_1 at 0.01 and increases s_2 to 0.5. The dotted black line increases s_1 such that $s_1 = 0.025, s_2 = 0.15$. The initial values of the state variables in the simulation are set at $H_{f0} = 2, H_{i0} = 0.0357, K_0 = 121.10$.[10]

Consider the black solid line. The subsidy program initially lowers manufacturing output but soon the deviation reduces. Around time period 450, the manufacturing output is higher in presence of the subsidy program. The subsidy

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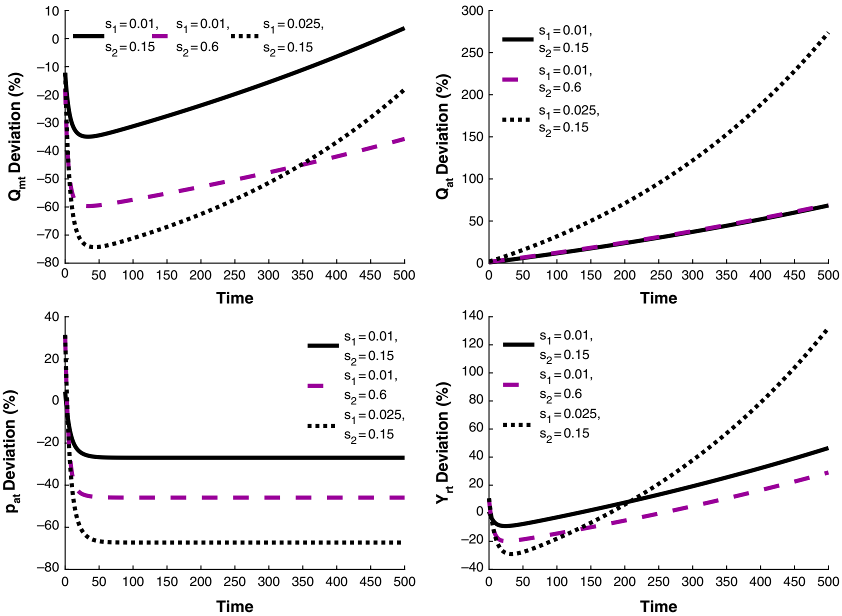

FIGURE 7. Effect of subsidies on trajectories of manufacturing output, food output, relative price of food, and real GDP.

program increases food output for all time periods. With respect to relative food price, it initially increases in the program but falls over time. Finally, the subsidy program has a u-shaped effect on real GDP. Initially when the program increases relative price of food, it also increases real GDP. Then again post time period 150, when increase in food output is more than the fall in manufacturing output, the program again raises the real GDP. However, in the interim for about 140 years, the real GDP is lower in the subsidy program.

Subsidies, therefore, affect output trajectories in three ways—through short-run growth effects, through long-run growth effects and through long-run level effects. Initially, subsidies lower short-run growth rates, captured by ζ_1 and ζ_2, which slows growth of these variables in the presence of the program. The initial effect of the subsidies is to slow down capital accumulation which also lowers GDP. Over time as health accumulation of the agents improve, the subsidy program increases the long-run growth rate of the economy.[11] Along the balanced growth path, real GDP is higher in the presence of the subsidy program. But are the short-term costs worth the gains in the long run? To assess the impact of the subsidies over an agent's lifetime, we look at the welfare effects.

3.6. Effect on welfare

We calculate the discounted life-time welfare as follows:

$$W_f = L_f \int_0^\infty e^{-\rho t} \left[\phi_1 \ln x_{at} + \phi_2 \ln x_{mt} + \phi_3 \ln x_{lt}\right] dt.$$

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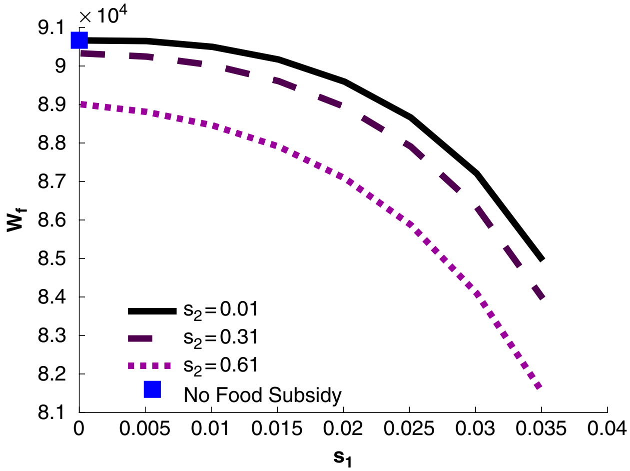

FIGURE 8. Effect of subsidies on farmer's welfare.

The food subsidy program increases the farmer's demand for food, and lowers his leisure and the relative price of food. Lower food prices reduces his income which results in lower consumption of the manufacturing good. As we show in Figure 8, the welfare of the farmer falls with both s_1 and s_2. It appears that the negative effects on manufacturing consumption and leisure affect welfare more than the positive effects of higher food consumption.

We get a similar expression for the welfare of the industrialist

$$W_i = L_i \int_0^{\infty} e^{-\rho t} \left[\phi_1 \ln y_{at} + \phi_2 \ln y_{mt} + \phi_3 \ln y_{lt}\right] dt.$$

As in the farmer's case, a subsidy program lowers the manufacturer's after-tax income. While s_1 has an obvious overall negative effect on W_i through lower after-tax income, the effect of s_2 is theoretically ambiguous. Simulations suggest that s_2 has a positive effect on W_i. For a given level of s_1, a higher s_2 is equivalent to returning some of the manufacturer's taxed income back to her household. In fact, for some subsidy combinations the industrialist may be better off in the subsidy program. If s_1 is small and s_2 is at medium levels, the health benefits outweigh the tax burden. We see this in Figure 9.

Finally, social welfare is calculated as

$$W_{ag} = W_f + W_i.$$

Figure 10 plots the subsidy effects on social welfare. When s_1 is low and s_2 is at medium levels, the program's positive effect on the industrialist's welfare may be more than its negative effect on the farmer's welfare.[12] In such a combination of subsidies, the program may improve social welfare. However, the range of subsidies for such an outcome is very limited. It is unlikely that a fiscal policy may

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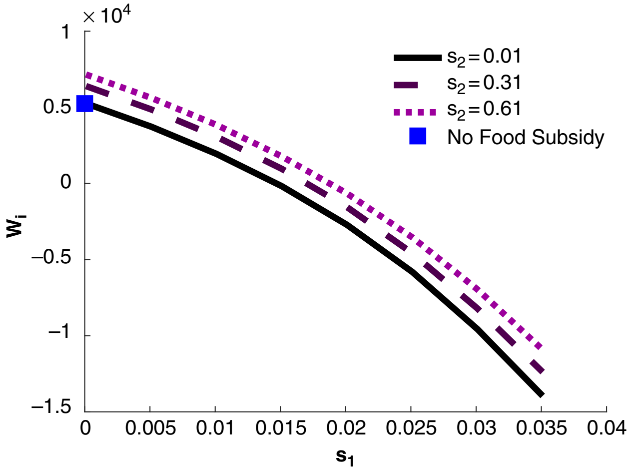

FIGURE 9. Effect of subsidies on entrepreneur's welfare.

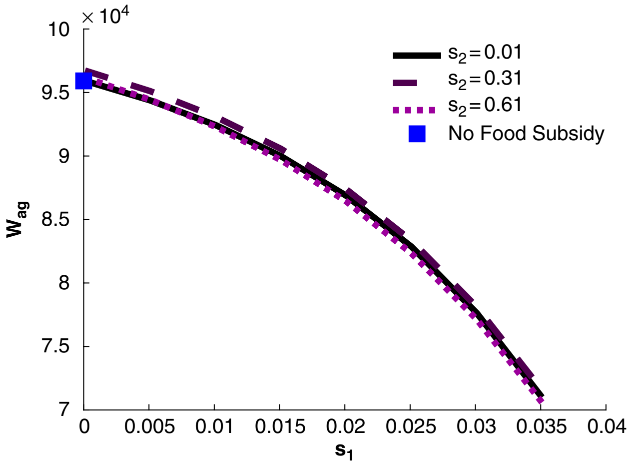

FIGURE 10. Effect of subsidies on social welfare.

be able to correctly target these subsidies. In general, therefore, food subsidies adversely affect the welfare of agents in the economy.

The effect of subsidies on social welfare may vary with the size of the household. To assess the effect of the household size, we plot social welfare as a function of the relative size of the industrialist's household in Figure 11. We find that social welfare is lower in economies with more industrialist households. A smaller proportion of farmers would produce less food and hence agents' health and growth rate would be lower. As noted earlier, there may be some combinations (low s_1, medium s_2) which may increase social welfare. But for a larger

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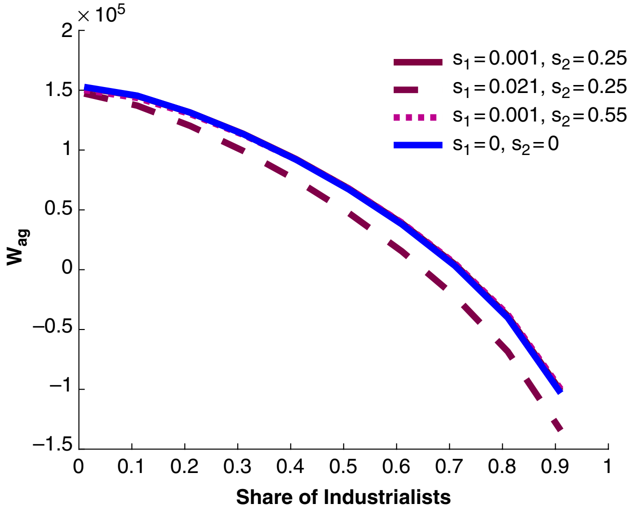

FIGURE 11. Effect of subsidies on social welfare for different shares of industrialists.

range of subsidies, it is not so. The effect on welfare is more sensitive to changes in s_1 than in s_2.

4. CONCLUSION

Food subsidies are good politics, but bad economics. These are typically populist moves which signal benevolence of the government (Kotwal et al. (2014) and The Economist (10 February 2018)). In this regard, the literature has extensively discussed two issues: what should be the form of these policies, and what are the *health effects* from these policies. In contrast, this paper analyzes the effects of a food subsidy program on output and welfare.

It is a well-established fact that improvement in nutritional intake improves health and productivity of workers. The channel of health improvement to economic growth crucially depends on participation in the labor market. This mechanism is weak in the presence of a fragmented labor market. In our model economy, labor is immobile across occupations and asset ownership favors industrialist in terms of access to savings technology. In such a setting, while a food subsidy program improves nutritional intake and hence health, it does so at the expense of other goods. Manufacturing goods production and capital accumulation gets adversely affected for a very long time period. It affects consumption of manufacturing, and hence reduces welfare. In fact, the policy leads to a fall in relative price of agricultural commodities which itself may hurt the farmer. In terms of aggregate GDP, the subsidy effects are mostly negative in the short run and positive in the very long run.

The analysis highlights that quantitatively, the level of subsidies to different agents as well as the tax burden, could be substantial.

We feel that the model can be extended to address more questions. The framework could look at good nutrition and bad nutrition; where the former creates healthy workers and the latter creates obese or malnourished workers. In such an environment, the food subsidy program would incentivize good nutrition and would exhibit larger health benefits. The paper highlights the need for policy sequencing. The benefits of a food subsidy policy on health are unquestionable. However, we need to improve the non-industrialist household's (in our case, the farmer's) access to investment technologies, be it financial markets, improvements in human capital or access to high skill networks, to witness a positive impact on the economy. These potential gains may be higher if skill improvement programs or financial inclusion schemes were introduced before or with the introduction of food subsidy programs. Finally, there are questions on the effects of the program in an open economy. The model predicts that the subsidy program will lower prices of agricultural goods, which in a large open economy, would have global effects on trade patterns. We leave these for future work.

NOTES

1. In 2013, the nine countries that recognized the right to food as a separate and stand-alone right were Bolivia, Brazil, Ecuador, Guyana, Haiti, Kenya, South Africa, Nepal, and Nicaragua (Knuth and Vidar 2011).
2. FAO et al. (2017) document that there are about 815 million undernourished people in the world. See Barrett (2002) for an overview on food insecurity and food assistance programs around the world.
3. Yaniv et al. (2009) conducted a similar empirical study to assess the effects of fat tax and thin subsidy across weight-conscious and non-weight conscious individuals.
4. Increasing labor force participation rates would not only require better nutrition but also ability to meet minimum job requirements, in terms of educational ability or access to job market networks. Thus, the effect of better nutrition on labor force participation, as implied by the first route, may be weak.
5. See Weil (2014) for an overview.
6. Instead of cash crop, this second output may also be considered as low-skill services. This output forms an additional link between the food and the manufacturing sectors. It provides the farmer an alternate form of employment.
7. Alternate utility functions, like Stone–Geary preferences with subsistence consumption of food, or CRRA or CES preferences could also be used for this problem. Stone–Geary preferences (such as $U = L[\phi_1 \ln(c_a - \bar{a}) + \phi_2 \ln c_m + (1 - \phi_1 - \phi_2) \ln c_l]$, where c_j is the consumption bundle of good j and $\bar{a}$ is minimum level of food consumption required for survival) would affect transition dynamics but the system would asymptote to the steady state, exactly the same as in our main model. CRRA or CES preferences, on the other hand, bring in cross-price effects. For one thing, that adds another layer of interaction between sectors through demand functions, and for another yields non-closed form expressions for manufacturing and leisure demand functions. However, even in these preferences the key predictions would not alter. As long as food and manufacturing goods are imperfect substitutes, the subsidy program would adversely hit the manufacturing sector more than it would benefit the agriculture sector—thus the sequence of effects of the subsidy program would follow.
8. This dual benefit of food contributes to the complexity of the model. We do not get closed form solutions for most of our variables.

9. In the presence of non-homothetic preferences, the farmer's optimal choice of food would be such that:

$$\frac{\phi_1 L_f e^{-\rho t}}{x_{at}(1-\bar{a}/x_{at})} + N(1-\gamma)\mu_{2t}\left(\frac{H_{ft}}{x_{at}}\right)^{\gamma} = (1-s_1)L_f\mu_{1t}p_{at},$$

where as the economy grows, the health of agents as well as their food consumption grows. Hence, as $t \to \infty$, $x_{at} \to \infty$ or $\bar{a}/x_{at} \to 0$, so the system asymptotes to the model with homothetic preferences.

10. The initial point of the economy is assumed to be close to the steady state, so that the linear approximation closely matches the actual path. We choose $\psi_{i0} = 0.4\psi_i^*$ and $\psi_{k0} = 0.6\psi_k^*$, where ψ_i^* and ψ_k^* are the steady-state values of manufacturer's normalized health and normalized capital in the no-subsidy program.

11. The long-run benefits of health improvements, through better nutrition and reduction in parasitic and infectious disease, are well documented in Madsen (2018).

12. For $s_1 < 0.0025$ and $s_2 = 0.01$, the social welfare is higher in the subsidy program. Increase in s_1 lowers social welfare. However as the curves cut each other, the effect of s_2 does not appear to be uniform for different values of s_1.

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TECHNICAL APPENDIX

APPENDIX A: THE STATIC SYSTEM

We use (4) and (8) to substitute for $\mu_{1t}p_{at}$ in (3) to get:

$$\frac{\phi_1 L_f e^{-\rho t}}{x_{at}} + N(1-\gamma)\mu_{2t}\left(\frac{H_{ft}}{x_{at}}\right)^{\gamma} = \frac{(1-s_1)L_f(\phi_2+\phi_3)}{A(H_{ft}/x_{at})-(1-s_1)}\frac{e^{-\rho t}}{x_{at}}.$$

In terms of normalized variables, this can be rewritten as

$$\chi_{2t} = \frac{N(1-\gamma)\chi_{1t}^{1-\gamma}}{L_f}\cdot\left[\frac{(\phi_2+\phi_3)(1-s_1)}{A/\chi_{1t}-(1-s_1)} - \phi_1\right]^{-1} \equiv \Theta(\chi_{1t}). \tag{A.1}$$

We rewrite the manufacturing production function (11), the industrialist's optimization conditions (13)–(16), the food and cash crop market clearing conditions (19)–(20) combined into one equation, and the government budget (22) in terms of normalized variables:

$$\psi_{qt}^{1-\beta} = c_1\psi_{ht}^{\alpha}\left(\frac{1-\tau_t}{p_{at}}\right)^{\beta}\psi_{kt}^{1-\alpha-\beta} \quad \text{where} \quad c_1 \equiv ML_i^{\alpha}\left(\frac{\beta C}{A}\right)^{\beta} \tag{A.2}$$

$$\phi_1 L_i\frac{\psi_{1t}}{\psi_{at}} + \frac{N(1-\gamma)}{\psi_{2t}}\left(\frac{\psi_{it}}{\psi_{at}}\right)^{\gamma} = (1-s_2)L_i p_{at} \tag{A.3}$$

$$\psi_{mt} = \phi_2\psi_{1t} \tag{A.4}$$

$$\psi_{ht} = \frac{\alpha(1-\tau_t)\psi_{qt}}{\alpha(1-\tau_t)\psi_{qt}+\phi_3 L_i\psi_{1t}}\psi_{it} \tag{A.5}$$

$$\psi_{ct} = \frac{\beta C(1-\tau_t)\psi_{qt}}{Ap_{at}} \tag{A.6}$$

$$\beta\frac{(1-\tau_t)\psi_{qt}}{p_{at}} = \frac{L_f}{\phi_2+\phi_3}\left[\phi_2 A+\phi_3(1-s_1)\chi_{1t}\right] - L_f\chi_{1t} - L_i\psi_{at} \tag{A.7}$$

$$\tau_t\frac{\psi_{qt}}{p_{at}} = s_1 L_f\chi_{1t} + s_2 L_i\psi_{at} \tag{A.8}$$

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The eight equations (A.1)–(A.8) can be solved to get $\{\chi_{2t}, \psi_{qt}, \psi_{ct}, \psi_{ht}, \psi_{at}, \psi_{mt}, p_{at}, \tau_t, \}$ as functions of the vector $(\psi_{1t}, \psi_{2t}, \psi_{kt}, \psi_{it}, \chi_{1t})$. This is the static system of the normalized variables.

In the Appendices, we denote the steady-state variables without subscript t. Thus, the variables x^* in the main paper are described as x in the Appendices. Differentiating this static system *around the steady state*, we get the matrices:

$$M_1 V_D = M_2 V_I, \tag{A.9}$$

where V_D and V_I are the matrices of dependent and independent variables

$$V_D = \begin{bmatrix} d\psi_{qt} \\ d\psi_{ht} \\ d\psi_{at} \\ dp_{at} \\ d\tau_t \end{bmatrix}, \qquad V_I = \begin{bmatrix} d\psi_{1t} \\ d\psi_{2t} \\ d\psi_{kt} \\ d\psi_{it} \\ d\chi_{1t} \end{bmatrix}.$$

The coefficient matrices M_1 and M_2 are

$$M_1 = \begin{bmatrix} 0 & 0 & m_{13}^{(1)} & -(1-s_2)L_i & 0 \\ m_{21}^{(1)} & 1 & 0 & 0 & m_{25}^{(1)} \\ \dfrac{\beta(1-\tau)}{p_a} & 0 & L_i & -\dfrac{\beta(1-\tau)\psi_q}{p_a^2} & -\dfrac{\beta\psi_q}{p_a} \\ \dfrac{\tau}{p_a} & 0 & -s_2L_i & -\dfrac{\tau\psi_q}{p_a^2} & \dfrac{\psi_q}{p_a} \\ \dfrac{1-\beta}{\psi_q} & -\dfrac{\alpha}{\psi_h} & 0 & \dfrac{\beta}{p_a} & \dfrac{\beta}{(1-\tau)} \end{bmatrix}, \quad M_2 = \begin{bmatrix} -\dfrac{\phi_1 L_i}{\psi_a} & m_{12}^{(2)} & 0 & m_{14}^{(2)} & 0 \\ m_{21}^{(2)} & 0 & 0 & m_{24}^{(2)} & 0 \\ 0 & 0 & 0 & 0 & m_{35}^{(2)} \\ 0 & 0 & 0 & 0 & s_1L_f \\ 0 & 0 & \dfrac{1-\alpha-\beta}{\psi_k} & 0 & 0 \end{bmatrix},$$

where the terms $m_{ab}^{(c)}$ are entries for matrix M_c in row a and column b:

$$m_{13}^{(1)} = -\frac{\phi_1 L_i \psi_1}{\psi_a^2} - \frac{N(1-\gamma)\gamma}{\psi_2\psi_a}\left(\frac{\psi_i}{\psi_a}\right)^{\gamma}, \qquad m_{21}^{(1)} = -\frac{\phi_3 L_i \alpha(1-\tau)\psi_1\psi_i}{\left(\alpha(1-\tau)\psi_q + \phi_3 L_i \psi_1\right)^2},$$

$$m_{25}^{(1)} = \frac{\phi_3 L_i \alpha \psi_q \psi_1 \psi_i}{\left(\alpha(1-\tau)\psi_q + \phi_3 L_i \psi_1\right)^2}, \qquad m_{12}^{(2)} = \frac{N(1-\gamma)}{\psi_2^2}\left(\frac{\psi_i}{\psi_a}\right)^{\gamma},$$

$$m_{14}^{(2)} = -\frac{N(1-\gamma)\gamma}{\psi_2\psi_i}\left(\frac{\psi_i}{\psi_a}\right)^{\gamma}, \qquad m_{21}^{(2)} = -\frac{\phi_3\alpha(1-\tau)\psi_q\psi_i L_i}{\left(\alpha(1-\tau)\psi_q + \phi_3 L_i \psi_1\right)^2},$$

$$m_{24}^{(2)} = \frac{\alpha(1-\tau)\psi_q}{\alpha(1-\tau)\psi_q + \phi_3 L_i \psi_1}, \qquad m_{35}^{(2)} = \left(\frac{\phi_3(1-s_1)}{\phi_2+\phi_3} - 1\right)L_f.$$

APPENDIX B: DYNAMIC SYSTEM IN NORMALIZED VARIABLES

We use (4), (6), and (8) to substitute for $\mu_{1t}p_{ct}$ in (7) to get:

$$\mu_{2t}\left[\gamma N\left(\frac{H_{ft}}{x_{at}}\right)^{\gamma-1} - \delta_h\right] + \frac{AL_f(\phi_2+\phi_3)}{A(H_{ft}/x_{at}) - (1-s_1)}\frac{e^{-\rho t}}{x_{at}} = -\dot{\mu}_{2t}.$$

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This expression and the farmer's health accumulation equation (1) in terms of normalized variables yield

$$\frac{\dot{\chi}_{2t}}{\chi_{2t}} = -\rho - \frac{\dot{\mu}_{2t}}{\mu_{2t}} - \frac{\dot{H}_{ft}}{H_{ft}}$$
$$= -\rho - (1-\gamma)N\chi_{1t}^{1-\gamma} + \frac{(\phi_2+\phi_3)AL_f}{A/\chi_{1t} - (1-s_1)}\frac{\chi_{2t}}{\chi_{1t}} \equiv \Upsilon(\chi_{1t}, \chi_{2t}). \quad \textbf{(A.10)}$$

The above equation together with (A.1) gives

$$\dot{\chi}_{1t} = \frac{\Upsilon(\chi_{1t}, \Theta(\chi_{1t}))\Theta(\chi_{1t})}{\Theta'(\chi_{1t})}. \quad \textbf{(A.11)}$$

Now, the difference in growth rates of health accumulation of the two agents gives $\dot{\psi}_{it}/\psi_{it} = \dot{H}_{it}/H_{it} - \dot{H}_{ft}/H_{ft}$ and hence,

$$\frac{\dot{\psi}_{it}}{\psi_{it}} = N\left(\frac{\psi_{it}}{\psi_{at}}\right)^{-(1-\gamma)} - N\chi_{1t}^{1-\gamma}. \quad \textbf{(A.12)}$$

Similarly, from the difference of growth rates of the co-state variables, $\dot{\psi}_{2t}/\psi_{2t} = \dot{\lambda}_{1t}/\lambda_{1t} - \dot{\lambda}_{2t}/\lambda_{2t}$:

$$\frac{\dot{\psi}_{2t}}{\psi_{2t}} = \alpha(1-\tau_t)\frac{\psi_{qt}\psi_{2t}}{\psi_{ht}} + \gamma N\left(\frac{\psi_{it}}{\psi_{at}}\right)^{-(1-\gamma)} - (1-\alpha-\beta)(1-\tau_t)\frac{\psi_{qt}}{\psi_{kt}} - \delta_h + \delta_k. \quad \textbf{(A.13)}$$

Next, we rewrite the dynamic equations (17) and (21) in normalized variables,

$$\frac{\dot{\psi}_{1t}}{\psi_{1t}} = -\rho + (1-\alpha-\beta)(1-\tau_t)\frac{\psi_{qt}}{\psi_{kt}} - N\chi_{1t}^{1-\gamma} + \delta_h - \delta_k, \quad \textbf{(A.14)}$$

$$\frac{\dot{\psi}_{kt}}{\psi_{kt}} = \frac{\psi_{qt}}{\psi_{kt}} - \frac{\phi_2 L_f p_{at}}{(\phi_2+\phi_3)\psi_{kt}}(A - (1-s_1)\chi_{1t}) - \phi_2 L_i\frac{\psi_{1t}}{\psi_{kt}} - N\chi_{1t}^{1-\gamma} + \delta_h - \delta_k. \quad \textbf{(A.15)}$$

Equations (A.11)–(A.15) constitute the dynamic system in normalized variables.

APPENDIX C: EXISTENCE OF UNIQUE STEADY STATE

At the steady state, $\dot{\psi}_1 = \dot{\psi}_2 = \dot{\psi}_i = \dot{\psi}_k = \dot{\chi}_1 = 0$. We suppress the time subscript to denote the steady-state values. The system of 13 equations (A.1)–(A.8) and (A.11)–(A.15) can be solved to get the steady-state values of 13 variables: ψ_1, ψ_2, ψ_i, ψ_k, χ_1, χ_2, ψ_q, ψ_c, ψ_m, ψ_h, ψ_a, p_a, τ. We get a unique value of χ_1 from equation (A.11) at $\dot{\chi}_1 = 0$. It follows from (A.1) and (A.11) that at the steady state both χ_1 and χ_2 are constants. Further,

$$\chi_2 = \frac{\rho}{L_f}, \quad \textbf{(A.16)}$$

$$\rho = N(1-\gamma)\chi_1{}^{1-\gamma}\left[\frac{(\phi_2+\phi_3)(1-s_1)}{A/\chi_1^* - (1-s_1)} - \phi_1\right]^{-1}. \quad \textbf{(A.17)}$$

Note, a feasible solution such that $\chi_1, x_l > 0$ exists for $A/\chi_1(s_1) + s_1 > 1 > \phi_1 A/\chi_1(s_1) + s_1$.

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At $\dot{\psi}_i = 0$, $\psi_a/\psi_i = \chi_1$. At $\dot{\psi}_1 = 0$, we get that marginal product of capital is constant,

$$\frac{(1-\tau)\psi_q}{\psi_k} = \frac{N\chi_1^{1-\gamma} + \rho - \delta_h + \delta_k}{1-\alpha-\beta} \equiv c_2 \tag{A.18}$$

We follow a series of algebraic manipulations to get the steady-state expression for ψ_a. First, equations (A.7) and (A.8) together yield

$$\frac{\psi_q}{p_a} = \frac{c_3}{\beta} + s_1 L_f \chi_1 + \left(s_2 L_i - \frac{L_i}{\beta}\right)\psi_a, \tag{A.19}$$

$$\text{where} \quad c_3 \equiv \frac{L_f}{\phi_2 + \phi_3}\left(\phi_2 A + \phi_3(1-s_1)\chi_1\right) - L_f\chi_1.$$

Second, substituting (A.18) and (A.19) into (A.15) with $\dot{\psi}_k = 0$ gives

$$\frac{\psi_1}{p_a} = c_4 + c_5\psi_a \tag{A.20}$$

$$\text{where} \quad c_4 \equiv \frac{1}{\phi_2 L_i}\left[\frac{c_3}{\beta} + s_1 L_f \chi_1 + \frac{c_3}{\beta c_2}\left(\delta_h - \delta_k - N\chi_1^{1-\gamma}\right) - \frac{\phi_2 L_f}{\phi_2+\phi_3}\left(A - (1-s_1)\chi_1\right)\right],$$

$$c_5 \equiv \frac{1}{\phi_2}\left[s_2 - \frac{1}{\beta} - \frac{1}{\beta c_2}\left(\delta_h - \delta_k - N\chi_1^{1-\gamma}\right)\right].$$

Third, at $\dot{\psi}_2 = 0$, we get from equation (A.13)

$$\frac{(1-\tau)\psi_q\psi_2}{\psi_h} = \frac{1}{\alpha}\left[(1-\alpha-\beta)c_2 + \delta_h - \delta_k - \gamma N\chi_1^{1-\gamma}\right] \equiv c_6 \tag{A.21}$$

This expression together with equations (A.5), (A.7), and $\psi_a/\psi_i = \chi_1$ gives

$$\frac{\psi_a}{\psi_2 p_a} = \frac{(1-\tau)\psi_q}{p_a}\cdot\frac{\psi_a}{\psi_h}\cdot\frac{\psi_h}{(1-\tau)\psi_q\psi_2} = c_7 + c_8\psi_a, \tag{A.22}$$

where

$$c_7 \equiv \frac{\chi_1}{c_6}\left[\frac{c_3}{\beta} + \frac{\phi_3 L_i c_4}{\alpha}\right], \quad c_8 \equiv \frac{L_i\chi_1}{c_6}\left[-\frac{1}{\beta} + \frac{\phi_3 c_5}{\alpha}\right].$$

Note, each constant, c, is a function of subsidies. Finally, using equations (A.20) and (A.22) in (A.3), we get a unique value of ψ_a,

$$\psi_a = \frac{\phi_1 L_i c_4 + (1-\gamma)Nc_7\chi_1^{-\gamma}}{(1-s_2)L_i - \phi_1 L_i c_5 - (1-\gamma)Nc_8\chi_1^{-\gamma}}.$$

We can retrace our steps to derive the steady-state values of the remaining normalized variables.

APPENDIX D: LONG-TERM GROWTH

The variable χ_{1t} is food to health ratio for the farmer. In the steady state, (A.17) gives constant χ_1 and hence from (1) we get that $\dot{H}_{ft}/H_{ft}$ is constant. Equations (1) and (A.17) together imply that the long-run growth rate of H_{ft} is determined by s_1, ρ, preference parameters (ϕ_1, ϕ_2, ϕ_3) and health accumulation function parameters (N, γ, δ_h). The steady-state growth rate of farmer's health is

$$g \equiv N\chi_1^{1-\gamma} - \delta_h. \tag{A.23}$$

APPENDIX E: TRANSITION DYNAMICS

Differentiating the dynamic system (A.11)–(A.15) around the steady-state yields

$$\dot{V}_I = M_3 V_D + M_4 V_I = \left[M_3 M_1^{-1} M_2 + M_4\right] V_I \qquad \textbf{(A.24)}$$

where we have also used the matrix relation (A.9). The matrices are

$$M_3 = \begin{bmatrix} m_{11}^{(3)} & 0 & 0 & 0 & m_{15}^{(3)} \\ m_{21}^{(3)} & m_{22}^{(3)} & m_{23}^{(3)} & 0 & m_{25}^{(3)} \\ 1 & 0 & 0 & m_{34}^{(3)} & 0 \\ 0 & 0 & N(1-\gamma)\left(\frac{\psi_i}{\psi_a}\right)^{\gamma} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad M_4 = \begin{bmatrix} 0 & 0 & m_{13}^{(4)} & 0 & -N(1-\gamma)\chi_1^{-\gamma}\psi_1 \\ 0 & m_{22}^{(4)} & m_{23}^{(4)} & m_{24}^{(4)} & 0 \\ -\phi_2 L_i & 0 & m_{33}^{(4)} & 0 & m_{35}^{(4)} \\ 0 & 0 & 0 & m_{44}^{(4)} & m_{45}^{(4)} \\ 0 & 0 & 0 & 0 & m_{55}^{(4)} \end{bmatrix}$$

$$m_{11}^{(3)} = \frac{(1-\alpha-\beta)(1-\tau)\psi_1}{\psi_k}, \qquad m_{15}^{(3)} = -\frac{(1-\alpha-\beta)\psi_q\psi_1}{\psi_k},$$

$$m_{21}^{(3)} = (1-\tau)\psi_2\left[\frac{\alpha\psi_2}{\psi_h} - \frac{1-\alpha-\beta}{\psi_k}\right], \qquad m_{22}^{(3)} = -\frac{\alpha(1-\tau)\psi_2^2\psi_q}{\psi_h^2},$$

$$m_{23}^{(3)} = \frac{N\gamma(1-\gamma)\psi_2}{\psi_a}\left(\frac{\psi_i}{\psi_a}\right)^{-(1-\gamma)}, \qquad m_{25}^{(3)} = \psi_2\left[-\frac{\alpha\psi_q\psi_2}{\psi_h} + \frac{(1-\alpha-\beta)\psi_q}{\psi_k}\right],$$

$$m_{34}^{(3)} = -\frac{\phi_2 L_f(A-(1-s_1)\chi_1)}{(\phi_2+\phi_3)}, \qquad m_{13}^{(4)} = -\frac{(1-\alpha-\beta)(1-\tau)\psi_q\psi_1}{\psi_k^2},$$

$$m_{22}^{(4)} = \frac{\alpha(1-\tau)\psi_q\psi_2}{\psi_h}, \qquad m_{23}^{(4)} = \frac{(1-\alpha-\beta)(1-\tau)\psi_q\psi_2}{\psi_k^2},$$

$$m_{24}^{(4)} = -\frac{N\gamma(1-\gamma)\psi_2}{\psi_i}\left(\frac{\psi_i}{\psi_a}\right)^{-(1-\gamma)},$$

$$m_{33}^{(4)} = \frac{1}{\psi_k}\left[-\psi_q + \frac{\phi_2 L_f p_a(A-(1-s_1)\chi_1)}{\phi_2+\phi_3} + \phi_2 L_i\psi_1\right],$$

$$m_{35}^{(4)} = \frac{\phi_2 L_f(1-s_1)p_a}{(\phi_2+\phi_3)} - N(1-\gamma)\psi_k\chi_1^{-\gamma}, \qquad m_{44}^{(4)} = -N(1-\gamma)\left(\frac{\psi_i}{\psi_a}\right)^{-(1-\gamma)},$$

$$m_{45}^{(4)} = -N(1-\gamma)\psi_i\chi_1^{-\gamma}, \qquad m_{55}^{(4)} = \frac{\Theta(\chi_1)\Upsilon_1^{'}(\chi_1,\chi_2)}{\Theta'(\chi_1)} + \Theta(\chi_1)\Upsilon_2^{'}(\chi_1,\chi_2)$$

where the last term $m_{55}^{(4)}$ is expanded as

$$\Theta'(\chi_1) = \frac{\chi_2}{\chi_1}\left[1-\gamma - \frac{AL_f\chi_2}{N(1-\gamma)\chi_1^{2-\gamma}} \cdot \frac{(\phi_2+\phi_3)(1-s_1)}{(A/\chi_1-(1-s_1))^2}\right],$$

$$\Upsilon_1^{'}(\chi_1,\chi_2) = -(1-\gamma)^2 N\chi_1^{-\gamma} + \frac{(\phi_2+\phi_3)(1-s_1)AL_f\chi_2}{(A-(1-s_1)\chi_1)^2},$$

$$\Upsilon_2^{'}(\chi_1,\chi_2) = \frac{(\phi_2+\phi_3)AL_f}{A-(1-s_1)\chi_1}.$$

The $[\cdot]$ term in (A.24) is the transition matrix. We computationally solve for the eigenvalues of the transition matrix.

We find two negative real eigenvalues (ζ_1, ζ_2). As the system has two state variables, two stable roots imply that the system is saddle path stable. The dynamics of the system around the steady state is

$$V_I = n_1 e^{\zeta_1 t} \cdot v_1 + n_2 e^{\zeta_2 t} \cdot v_2,$$

where v_1 and v_2 are eigenvectors associated with ζ_1 and ζ_2, respectively. We know the initial values ψ_{k0} and ψ_{i0}. Hence, we can solve for the values of n_1 and n_2 from the equations

$$\psi_{k0} - \psi_k = n_1 v_{1k} + n_2 v_{2k}$$
$$\psi_{i0} - \psi_i = n_1 v_{1i} + n_2 v_{2i}.$$

Once we calculate n_1 and n_2, we get initial values of the remaining variables in the dynamic system:

$$\psi_{10} = \psi_1 + n_1 v_{11} + n_2 v_{21}, \quad \psi_{20} = \psi_2 + n_1 v_{12} + n_2 v_{22}, \quad \chi_{10} = \chi_1 + n_1 v_{1\chi} + n_2 v_{2\chi}.$$

We approximate the linear trajectories of the remaining variables around the steady state. In (A.9), assuming $dv_t = v_t - v$ for a variable v_t to get

$$\begin{bmatrix} \psi_{qt} \\ \psi_{ht} \\ \psi_{at} \\ p_{at} \\ \tau_t \end{bmatrix} = M_1^{-1} M_2 \begin{bmatrix} \psi_{1t} - \psi_1 \\ \psi_{2t} - \psi_2 \\ \psi_{kt} - \psi_k \\ \psi_{it} - \psi_i \\ \chi_{1t} - \chi_1 \end{bmatrix} + \begin{bmatrix} \psi_q \\ \psi_h \\ \psi_a \\ p_a \\ \tau \end{bmatrix}$$

It follows

$$x_{at} = \chi_{1t} H_{ft}, \quad H_{it} = \psi_{it} H_{ft}, \quad K_t = \psi_{kt} H_{ft}, \quad Q_{mt} = \psi_{qt} H_{ft}, \quad y_{at} = \psi_{at} H_{ft},$$
$$y_{mt} = \phi_2 \psi_{1t} H_{ft}, \quad y_{lt} = (\psi_{it} - \psi_{ht}) H_{ft}, \quad x_{mt} = \frac{\phi_2 p_{at}}{\phi_2 + \phi_3} \left[A - (1 - s_1)\chi_{1t}\right] H_{ft},$$
$$x_{lt} = \frac{\phi_3}{A(\phi_2 + \phi_3)} \left[A - (1 - s_1)\chi_{1t}\right] H_{ft}, \quad Q_{at} = AL_{at} = L_f x_{at} + L_i y_{at},$$
$$Q_{ct} = CL_{ct} = C(L_f H_{ft} + L_{at} - L_f x_{lt}).$$

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