Quantum Reference Frames in the Context of EPR

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Taking a cue from Bohr's use of the notion of a reference frame in his reply to EPR's argument against the completeness (and consistency) of standard quantum theory, this paper presents an analysis ofthe role of reference frames in the situation considered by EPR, using a quantum-theoretical account of physical reference frames based on the work of Mackey, and Aharonov and Kaufherr. That analysis appears to justify at least some crucial aspects of a Bohrian reply to EPR.

1. Taking a Cue from Bohr. In his 1935 reply to Einstein, Podolsky, and Rosen (1935), henceforth 'EPR', Bohr makes use of the notion of a 'reference frame', echoing his frequent discussions of the notion elsewhere. This aspect of Bohr's language rarely makes it into the official account of his reply, but it is there nonetheless, and its presence prompts me, in this paper, to attempt to understand EPR's argument in terms of a quantum theory of reference frames and see what emerges.

Bohr never provided a formal analysis of the notion of a reference frame in quantum-theoretic terms, but he did discuss it informally. Indeed, careful formal discussions are hard to find. I shall attempt here to explicate a quantum-theoretic account of physically-specified Galilean reference frames as I understand it from combining the work ofAharonov and Kaufherr (1988) and Mackey (1978), and to apply that account to the case considered by EPR. By a 'physically-specified' reference frame I mean one that is given by some physical body (e.g., a 'laboratory with clocks

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‡My thinking about the reply to EPR (and especially possible connections with the historical Bohr, which are not considered here) has benefited enormously from the historical and philosophical work of Tanona (2002). A helpful discussion with Harvey Brown cleared up some points about reference frames. Thanks also to audiences at Indiana, Vienna, Krakow, Utrecht, London, and Oxford, for their patience and helpful comments on half-baked versions of these ideas.

Philosophy of Science, 71 (December 2004) pp. 655–668. 0031-8248/2004/7105-0001$10.00

and rods rigidly attached'). I shall often use the term 'reference body' to highlight this point.

This paper is *not* an attempt to defend Bohr's reply to EPR, but it *is* an attempt to justify what might be certain crucial parts of the reply, by making use of a quantum theory of reference bodies. With that justification in hand, one can then consider how reasonable it might be to endorse Bohr's reply.

In section one, I shall consider this last issue, albeit briefly. After describing the points on which Bohr and EPR agree, I shall make it clear what logical space remains for Bohr's reply. I shall not give much attention to the question whether one *ought* to occupy that space. However, in section three, I shall analyze EPR's argument in terms of a quantum theory of reference frames, described in section two. This analysis implicitly provides some motivation for Bohr's reply.

I emphasize that I do not pretend to be giving an historically accurate account of Bohr's own thoughts. I am only taking a cue from his use of the language of reference frames. Others have pondered—much more carefully and deeply than I—Bohr's own thinking on these and related matters, and I enthusiastically refer the reader to their work.[1] My invocation of the name 'Bohr' should not be taken too seriously.

2. EPR versus Bohr.

2.1. The EPR Argument. Purely as a notational convenience,let M_O^n denote 'the observable O is measured on particle n'; let D_O^n denote 'the observable O is has a definite value for particle n'; and let $p \mathrel{\Box\!\!\rightarrow} q$ denote the counterfactual conditional 'if it had been that p, then it would have been that q. For now, $\boldsymbol{O} = \boldsymbol{Q}$ or $\boldsymbol{P}$ (position or momentum), and $n =$ 1 or 2 (particle 1 or particle 2).

By 'the EPR argument'[2] I mean the argument that for some quantum-mechanical state (the 'EPR state'), where $\boldsymbol{O} = \boldsymbol{Q}$ or $\boldsymbol{P}$, $n = 1$ or 2, $\boldsymbol{O} \neq \boldsymbol{O}'$, and $\Diamond$ is the modal 'possibility' operator:

Measurability. $\Diamond M_{\boldsymbol{Q}}^1$ & $\Diamond M_{\boldsymbol{P}}^1$

Reality. $M_{\boldsymbol{O}}^1 \Rightarrow (D_{\boldsymbol{O}}^1$ & $D_{\boldsymbol{O}}^2)$

Epistemic Uncertainty. $M_{\boldsymbol{O}}^n \Rightarrow \neg M_{\boldsymbol{O}'}^n$

1. I have been strongly influenced by Tanona (2002), as well as Folse (1985) and Howard (1999), among others.

2. One can find much more careful accounts of the EPR argument in other places, for example, Fine (1986), and the uninitiated reader is encouraged to consult such sources.

and yet,

Strong Non-Disturbance. $D^2_O \Rightarrow (M^1_{O'} \square\!\!\rightarrow D^2_O)$

and therefore, $\Diamond(D^2_Q \;\&\; D^2_P)$.

The existence (not to mention persistence in time) of the EPR state, as well as the measurability of $\boldsymbol{Q}$ and $\boldsymbol{P}$, are problematic at best. However, we can learn something useful by ignoring these problems (as the original interlocutors did), and I shall, for now. However, ultimately we must consider the EPR argument properly, and doing so requires either a satisfactory account of the EPR state and of position and momentum measurements in quantum theory, or a consideration of other observables—I shall briefly consider Bohm's version of the argument in terms of spin later. 'Reality' is essentially EPR's well-known 'criterion for physical reality' as applied to the EPR state. 'Epistemic Uncertainty' is just the recognition that position and momentum cannot be measured simultaneously.

The conclusion is a problem not only for Bohr, but also for standard quantum theory, which apparently has no way to represent this possibility. It is obtained from the preceding premises roughly as follows.[3] Imagine that M^1_Q (for example), and therefore D^2_Q, by Reality. Strong Non-Disturbance implies that if (counterfactually) M^1_P then still D^2_Q, and yet in this case (where M^1_P) Reality implies that D^2_P. In other words, Strong Non-Disturbance allows one to say that the definiteness of particle 2's position inferred from the result of measuring particle 1's position is not 'counterfactually undone' by a counterfactual case in which we measure particle 1's momentum; but in that case, particle 2 must also have a definite momentum. Of course, 'counterfactual measurements' are not possible for us to do, which is why nothing in EPR's argument violates Epistemic Uncertainty. Indeed, their acknowledgement of Epistemic Uncertainty *requires* that they consider such counterfactual situations—the counterfactual nature of their argument is thus inescapable.

2.2. The Disputed Premise. Bohr does not dispute anything in the EPR argument except Strong Non-Disturbance. Instead, he admits only 'Weak Non-Disturbance': $D^2_O \Rightarrow (\neg M^1_O \square\!\!\rightarrow D^2_O)$. This condition is weaker than Strong Non-Disturbance (because its consequent is weaker), which is normally (and not unreasonably) conceived as a condition of locality. Indeed, Weak Non-Disturbance is sufficiently weaker than Strong Non-Disturbance to block the inference to the conclusion.

3. There are, of course, many more detailed accounts of EPR's argument. See, for example, Fine (1986) and a variety of comments about the limitations of the argument in Dickson (2002).

But is it plausible to substitute Weak Non-Disturbance for Strong Non-Disturbance? Apart from whatever justification flows from the discussion later, I shall not consider this question in much detail, but a few comments are in order.

The EPR state is a state of perfect correlation in position and momentum precisely because it is a simultaneous eigenstate[4] of the two (compatible) observables $\boldsymbol{Q}_1 - \boldsymbol{Q}_2$ and $\boldsymbol{P}_1 + \boldsymbol{P}_2$ (where $\boldsymbol{Q}_1$ is the position observable for particle 1, and so on). Adding *either* $\boldsymbol{Q}_1$ *or* $\boldsymbol{P}_1$ to this pair results in a non-commuting set, because $[\boldsymbol{Q}_1, (\boldsymbol{P}_1 + \boldsymbol{P}_2)] \neq 0$ and $[\boldsymbol{P}_1, (\boldsymbol{Q}_1 - \boldsymbol{Q}_2)] \neq 0$ (with the obvious tensor products left implicit). In other words, measuring (for example) $\boldsymbol{P}_1$ actually destroys the perfect correlation in position.

So suppose that $D^2_{\boldsymbol{Q}}$, in virtue of $M^1_{\boldsymbol{Q}}$ and Reality. Now consider the (counterfactual) possibility that $M^1_{\boldsymbol{P}}$. In this situation, there is *not* a perfect correlation between $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$. But we originally inferred that $D^2_{\boldsymbol{Q}}$ precisely on the basis of this correlation, a correlation that does not exist in this other (counterfactually) possible situation. So what do we say, then, about the definiteness of $\boldsymbol{Q}_2$, which, in the context of the argument, depended on the existence of this correlation? At the very least, it is not clear. Weak Non-Disturbance does not make a claim about this case. Strong Non-Disturbance does. Resolving this question ultimately turns (unavoidably, because of the counterfactual nature of the EPR argument itself) on subtle issues surrounding the evaluation of counterfactuals, and I shall not consider them further here.

I have no idea whether Bohr's refusal to accept Strong Non-Disturbance implies an endorsement of 'non-locality' in any sense that should cause concern, nor whether a denial of Strong Non-Disturbance is 'anti-realist', whatever that term may mean. It is fairly clear, on the other hand, that a mere denial of Strong Non-Disturbance as presented here does not lead to a solution to the problem of measurement in quantum theory; nor does it constitute an 'interpretation of quantum theory', in the contemporary sense of that phrase. Nonetheless, Bohr's reply to EPR does have value, and its value lies primarily in the explanation that it offers for the non-commutativity of $\boldsymbol{Q}_1$ with $\boldsymbol{P}_1 + \boldsymbol{P}_2$ (and of $\boldsymbol{P}_1$ with $\boldsymbol{Q}_1 - \boldsymbol{Q}_2$). Bohr's account is supposed to give us some physical insight into, perhaps even explanation of, these failures of commutativity, and I believe it does. The next two sections are my attempt to explicate the explanation.

4. I have alluded once to the fact that strictly speaking this claim is false, because the EPR state does not exist in the Hilbert space—as a function on the configuration space of the pair of particles, its support has measure zero. I shall not continue to do so.

3. Quantum Reference Frames.

3.1. Informal Discussion. Bohr argued that one must *stipulate* a physical object as defining a frame of reference. Given this conception of a quantum-theoretic frame of reference, Bohr attempted to explain in a physically natural way the uncertainty relation between position and momentum. For consider a measurement of the position of a particle relative to the reference frame. If the apparatus that makes the measurement is allowed to move with respect to the body that defines the frame of reference, then any exchange of momentum between the particle and the apparatus will set the apparatus in motion relative to the frame, so that even if we knew its position relative to the frame prior to measurement, we do not know it after the measurement.

Assuming (as Bohr did) that the exchange of momentum is necessary and 'uncontrollable' (a term of art for Bohr), one cannot, at the time of measurement, account for this change in the position of the apparatus (relative to the frame), and so we must not allow the apparatus to move with respect to the frame of reference. In order words, it must be rigidly fixed to the body that defines the frame of reference, so that the ('uncontrollable') exchange of momentum in the act of a measurement of the position of the particle is between the particle and the frame of reference itself. But the frame of reference is *by definition* always 'at rest' (for it determines the meaning of 'at rest'). In other words, so long as we take the stipulated body to define our frame of reference, we are compelled to ignore any exchange of momentum between it and the measured particle. But then, while we are now able to determine the particle's position relative to the frame (because the indications on the apparatus always bear a known and fixed relation to the frame itself), we are necessarily uncertain about the momentum of the particle when we measure its position, because any momentum exchanged between the particle and the apparatus is 'absorbed' into the frame, and subsequently 'lost' because of our continued assumption that the body defining the frame has zero momentum.

So argues Bohr. The next few subsections provide a more formal account (and one that I am more willing to endorse).

Before we turn to that account, however, it is important to realize what it will what it will *not* establish. For Bohr, the essentially new feature of quantum mechanics appears to have been the 'uncontrollability' of the exchange between the measuring apparatus and the measured system. I am far from clear what 'uncontrollable' means in this context[5], but the notion plays a crucial role in the argument, and its presence indicates that

5. Here again I refer the interested reader to the experts.

Bohr's argument cannot be taken to establish uncertainty from the very notion of a physically specified frame of reference. Rather, the argument can at best suggest a physically satisfying way to understand uncertainty *within* a quantum-theoretic context. The trick, of course, is to work in a quantum-theoretic context without assuming the uncertainty relations in a way that makes the argument viciously circular. I shall proceed in a similar manner. While I will certainly be working within a quantum-theoretic context, my aim is to do so in a way that will still shed light on the uncertainty relations, rather than taking them as a starting-point.

3.2. Observables as Invariant and Covariant Quantities. One usually considers observables in quantum theory to be self-adjoint operators on a Hilbert space. One notes that such an operator, $\boldsymbol{F}$, uniquely determines and is determined by a spectral measure, given by a family of projection operators, $\boldsymbol{P}_\Delta$, where Δ is a set of real numbers from the spectrum of $\boldsymbol{F}$. The probability that a measurement of $\boldsymbol{F}$ yields a result in Δ is $\mathrm{Tr}[\boldsymbol{P}_\Delta \boldsymbol{W}]$, where $\boldsymbol{W}$ is the state of the system.

It makes no difference whether we think of observables as self-adjoint operators, or as spectral measures. Indeed, one can generalize the usual scheme by allowing positive-operator-valued (POV) measures without changing much apart from increasing the expressive power of the theory. In general, the approach to observables that I shall describe (albeit briefly) in this subsection requires such generalizations, though the examples that I consider do not.

In some cases, merely by considering the covariances and invariances of the observables, we can determine (up to unitary equivalence) which POV measures correspond to a given observable. By covariance, we mean simply that, given a group (such as the Galilean group), the action of the group on the space of values of the observable, and the action of the group on the Hilbert space, whenever some Δ is changed by the action of some element of the group, $\boldsymbol{P}_\Delta$ changes accordingly. By invariance, we mean just that $\boldsymbol{P}_\Delta$ and $\boldsymbol{P}_{\Delta'}$ are the same if Δ and Δ' are related by the action of some element of the group.

Consider, for example, the position operator and the group of spatial translations. Let α be a specific spatial translation on $\mathbb{R}^3$ and let $\boldsymbol{U}_\alpha$ be the corresponding unitary transformation on the Hilbert space. Then the position operator is *covariant* under spatial translations just in case, for any $\Delta \in \mathbb{R}^3$, $\boldsymbol{U}_\alpha \boldsymbol{P}_\Delta^{\boldsymbol{Q}} \boldsymbol{U}_\alpha^{-1} = \boldsymbol{P}_{\alpha(\Delta)}^{\boldsymbol{Q}}$, where $\boldsymbol{P}_\Delta^{\boldsymbol{Q}}$ is the spectral projection of $\boldsymbol{Q}$, the position operator, corresponding to the set Δ of spectral values. Thus we can characterize position as invariant under boosts and covariant under translations, while momentum should be invariant under translations and covariant under boosts.

It turns out that, as a simple corrollary of the imprimitivity theorem plus a few reasonable assumptions, we need say no more in order to characterize the position and momentum operators uniquely, up to unitary equivalence.[6] In addition, it is an immediate consequence of what we have said thus far that position is the generator of translations in momentum, and vice versa, and from this fact we can almost immediately obtain the Weyl form of the commutation relations.

So the covariances and invariances of position and momentum are apparently closely connected—at least mathematically—to the uncertainty relations. What do we make of this connection? If we wished only to derive the mathematical expression of the uncertainty relations, then the discussion would be over. The point here, however, is not merely to derive the uncertainty relations mathematically, but to understand why the derivation works from a physical point of view. The initial physical demand of invariance and covariance with respect to (certain parts of) the Galilean group strongly suggests that we try to understand this final piece of mathematics in the same terms. Towards that end, I turn now to an account of Galilean reference frames in quantum theory. The connection between the Galilean group and reference frames will emerge in that discussion.

3.3. The 'Absolute' in Quantum Theory. In order to keep the situation clear, notice that in general, in non-relativistic quantum theory, we work with absolute coordinates of some background reference frame, presumed to be inertial. This fact is implicit, for example, in the assumption that the coordinates appearing in a wavefunction, $\psi(x)$, refer to 'physical space'. I shall refer to this 'absolute' point of view as the 'external viewpoint'. My aim, here, is to describe a physically-specified reference frame, and the measurements that take place 'internally' to it, from such an external point of view. In this strategy (though not in all details), I follow Aharonov and Kaufherr (1988).

An example will make this point clearer. In the usual approach to the hydrogen atom in quantum theory, one begins with the observation that the Coulomb potential depends only on the distance between the proton and the electron, and using this symmetry, the problem is reduced to that of a single particle in a central potential. Working in spherical coordinates, one performs a separation of variables into the radial and angular parts, then solves the resulting equations. It is less often observed (but true nonetheless) that this procedure involves working in the center-of-mass coordinates, rather than the 'absolute' coordinates with which one implicitly began (prior to the identification of the relevant symmetry of the problem). These absolute coordinates appear in the (usually unmentioned)

6. See Mackey (1978) and Varadarajan (1985). As Halvorson has shown, the 'reasonable assumptions' are not unimpeachable.

initial formulation of the problem in terms of a two-particle wavefunction with six arguments (for the configuration space of the two particles). The relational coordinates in which we solve the problem are defined in terms of the coordinates of this background space.

One might object, asking why we could not take the relational space parametrized by the center-of-mass and relative distance coordinates as fundamental? The issue here—though I do not have the space to consider it in detail[7]—is that the equations that we solve in the relational space are derived from the Schrödinger equation on the configuration space, and its validity as an equation of motion for wavefunctions on this space is based on the assumption that the configuration space for the two particles describes an inertial system. Indeed, the separation of variables required to solve the problem of the hydrogen atom relies on the assumption that the compound system is inertial.

So to be more precise, now, about the 'absolute viewpoint' presumed by quantum theory, we can say that it is not quite the assumption that the background configuration space for any quantum-mechanical system is 'absolute space', but rather that it describes an inertial system. (Of course, the presumption that it is absolute space would suffice.) But that assumption is not something we know, either empirically or a priori. We do not *know* which systems are inertial. Instead, we *presume* certain systems to be interial (or 'close enough'), and this presumption allows us to write down the Schrödinger equation in the coordinates describing those systems and get to work.

A quantum theory of reference bodies must, therefore, be a theory with this background assumption in place. For to start, we must know that, in the coordinate system given by a body that is to serve the role of fixing a reference frame, the Schrödinger equation is valid. In order to know that the Schrödinger equation is valid in these coordinates, we must know that they describe an inertial frame. In order to know that they describe an inertial frame, we must know that they are related in the right way (in our case, by a Galilean transformation) to some given inertial frame. This given inertial frame is just that, *given*. I shall call it the 'background space'.

However, we can still be relationalists of a sort, and abjure the true existence of such a background space. We can study quantum reference bodies from an imagined 'absolute' frame, relative to which the bodies are defined. Such is the strategy used here. While this study will not reveal, for example, how to find those frames (as given by bodies) in which the Schrödinger equation is valid, it will reveal certain qualitative features of

7. I am indebted to the Oxford Philosophy of Physics Seminar, and especially to Harvey Brown, for forcing me to consider this issue more carefully.

frames in which the equation is valid. One of those, I claim, is the uncertainty relations. That is, a consideration of how (inertial) bodies determine frames of reference suggests how one might come to understand the uncertainty relations in a physically satisfying way.

3.4. Quantum Reference Bodies. So suppose that we are *given* some absolute frame. From the point of view of this frame, we wish to describe, quantum-mechanically, the physics of measurements made by an observer 'inside a lab', who takes the lab to define his or her frame of reference. The absolute frame should be considered a sort of 'crutch', which we shall throw away once we have studied the physics of measurements in the lab frame. The presumption of this discussion, then, is that real-life measurements of position are best modeled as measurements made by an observer 'in a lab frame'.

We further presume, as observers in a lab frame always do (and must, as we have already mentioned) that the lab is inertial in the appropriate sense; for us, this presumption means that the coordinate system determined by the rods and clocks of the lab is related to the given coordinate system of the background space by a Galilean transformation. Given the discussion above, we know how to implement these transformations as unitary transformations on the Hilbert space, and most important, we know how those transformations are related to the position and momentum operators. The lesson of the brief discussion of the imprimitivity theorem is that there is little room for choice, here.

This situation has been considered by Aharonov and Kaufherr (1988). While not couching their discussion in these terms, they consider a lab ('system 0') together with a collection of systems (1, 2, 3, . . .) and ask how the description of systems 1, 2, 3, . . . given by an observer who takes the lab to define a spatial frame of reference can be transformed into a description in terms of the given background space normally.

To address this issue, let the position and momentum observables of an observer *inside* the lab be denoted $\boldsymbol{Q}$ and $\boldsymbol{P}$. Let the position and momentum observables of an 'external' observer (who has been granted an inertial reference frame) be $\overline{\boldsymbol{Q}}$ and $\overline{\boldsymbol{P}}$. How are these observables related? Restricting our attention to the spatial translation between the lab and the absolute coordinates (that is, restricting our attention to contexts in which the internal observer is using just the rods of the lab frame to define 'position'), the answer, as they note, is given by the unitary transformation:

$$\boldsymbol{U}_{\text{A-K}} = \exp\Big(-i\sum_{n>0}\overline{P}_n\overline{Q}_0\Big). \tag{1}$$

This answer gives the intuitively demanded result, namely, $\boldsymbol{U}_{\text{A-K}}\boldsymbol{Q}_n\boldsymbol{U}_{\text{A-K}}^{-1} = \overline{\boldsymbol{Q}}_n - \overline{\boldsymbol{Q}}_0$ $(n > 0)$. It also, upon reflection, does the right

https://doi.org/10.1086/425939 Published online by Cambridge University Press

thing to the free Hamiltonian for the compound system. To see that it does, consider the case where the total momentum of the lab (and its content) is zero with respect to the external frame. Then

$$\boldsymbol{U}_{\text{A-K}}\boldsymbol{H}_{\text{free}}\boldsymbol{U}_{\text{A-K}}^{-1} = \left(\sum_{n>0}\frac{\overline{\boldsymbol{P}}_n^2}{2m_n}\right) + \frac{\Pi^2}{2m_0}, \tag{2}$$

where $\Pi = \sum_{n>0}\overline{\boldsymbol{P}}_n$. The first term in (2) is just the expected term for the free particles inside the lab. The second term represents, as Aharaonov and Kaufherr term it, the 'drift' of the lab with respect to the absolute coordinates. (The assumption that the total momentum is zero is purely for convenience, to make it easier to see that $\boldsymbol{U}_{\text{A-K}}$ does the right thing in a case where it is clear what the right thing is. Note also that the assumption that the total momentum of the lab and its contents is zero in no way precludes the internal observer from assigning definite values to the positions of the particles in the lab, for from the external viewpoint, these positions are relative positions, which can be definite simultaneously with the definiteness of the total momentum. Indeed, this point is at the heart of Aharonov and Kaufherr's motivation.)

3.5. Position Measurements. Position measurements are much easier to consider than momentum measurements, and so I shall focus exclusively on them. So let us consider, first, how the lab observer thinks about these measurements. That observer measures position relative to the rods given by the lab (instead of relative to some presumed absolute background space), but otherwise proceeds just as one normally does in standard quantum theory, representing the interaction by some interaction Hamiltonian, and solving the Schrödinger equation with that Hamiltonian. This procedure should be familiar. As an extremely simple case, let the interaction Hamiltonian be:

$$H_{\text{int}} = g(t)\boldsymbol{Q}_1\boldsymbol{P}_2, \tag{3}$$

letting the measured system be labeled by '1' and the apparatus (or its pointer degree of freedom) by '2'. The function $g(t)$ determines the strength of the interaction, and if it is sharply peaked about some time, then standard techniques of approximation yield the familiar result, namely, that after the interaction is (essentially) over, there is a (near) perfect correlation between the position of system 1 and the value of the pointer observable for system 2 (which is an observable conjugate to $\boldsymbol{P}_2$).

How do we translate this story into the observables of the 'absolute' frame? We can study the physics of this measurement from the absolute point of view by transforming the total Hamiltonian (the free part plus the interaction part) as written down by the internal observer. Doing so, we note the following commutation relations:

$$[\overline{\boldsymbol{H}}, \overline{\boldsymbol{P}}_{\text{total}}] = 0, \tag{4}$$

$$[\overline{\boldsymbol{H}}, \overline{\boldsymbol{P}}_0] \neq 0, \tag{5}$$

$$[\overline{\boldsymbol{H}}, \overline{\boldsymbol{P}}_1] \neq 0. \tag{6}$$

The relation (4) is comforting, because it assures us that the measurement (which, after all, involves particles solely inside the lab) is momentum-conserving. The relation (6) is perhaps unsurprising as well, for it merely indicates that the measurement of particle 1's position involves a change in its momentum (because the commutator of $\overline{\boldsymbol{H}}$ and $\overline{\boldsymbol{P}}_0$ is the time-derivative of $\overline{\boldsymbol{P}}_0$). However, in conjunction with (5), and in light of how we arrived at these commutation relations, one might be led (as I am) to consider the following *interpretation* of these relations: the measurement of particle 1's position disturbs not only the particle's momentum, but also the momentum of the lab itself. (By 'disturb', I mean just that the time-derivative of the momentum during the interaction is non-zero.)

This story brings together, in a somewhat more formal way, two themes that Bohr himself seemed to have linked quite often, namely, the necessity to refer position measurements to a physical reference body, and the inevitability of an exchange of momentum between the measured system and the reference body itself during a measurement of position.

In what sense, though, is the exchange 'uncontrollable'? Bohr did not, so far as I can tell, offer an explicit answer to this question, but here is a suggestion.[8] Remember that we have been studying the physics of a position measurement as made by the internal observer, but from the external point of view. In particular, the external observer has access to two observables that are simply not available to the internal observer, namely, the position and momentum of the lab itself. The external observer can say that the lab's momentum is disturbed during the measurement, and can even, if inclined, measure the disturbance. The internal observer can say, at *best*, that *if* an external viewpoint were available, then from that viewpoint, the momentum of the lab would be disturbed during measurement.

This interpretation explains two things, one which Bohr would surely endorse, the other perhaps not. First, it explains the indefiniteness of

8. I am unclear whether Bohr would endorse this view, and even less so the consequences that seem to follow, but again, I refer the reader to the work of those more qualified than me to assess Bohr's attitude.

momentum during a measurement of position. When the internal observer asks 'is momentum well-defined' the question must be addressed, it seems, by asking whether the lab is the appropriate sort of object to serve the purpose of defining momenta. While the internal observer has no direct access to the observables describing the lab, he can still go through the exercise that we have gone through here, considering what an imagined external observer would say about the lab, and on that basis decide whether the lab can be taken to define momenta. But we know what the external observer says—she says that lab accelerates during the measurement. An accelerating lab is *not* appropriately taken to define momenta (because it does not have a constant momentum itself, relative to which we could define the momenta of other particles), and so the internal observer must conclude that momentum (relative to the lab!) is undefined during the measurement.

Second, the interpretation explains the well-known fact that while one can never know, at a time t, the position and momentum of a particle at t, one *can* retrodict them. Here we have a quick explanation why: the internal observer can always step outside of the lab and measure the affect of the measurement on the lab, using this information to retrodict.

In any case, while I have had to move rather quickly past some quite subtle points, I hope to have indicated that a quantum theory of reference bodies is suggestive of an interpretation of the uncertainty relations (at least in their qualitative, if not quantitative form) as the result of the (supposed) necessity to refer measurements of position and momentum to a reference body. Let us turn, finally, to consider the application of this view to our original issue, the EPR argument.

3.6. A Bohrian Reply to the EPR Argument. Given the above, we can say fairly quickly how one might reply to EPR, at least in the sense of explaining, from a quantum-theoretic point of view (but in manner that is not *too* viciously circular), the incompatibility of, for example, $\boldsymbol{Q}_1$ with $\boldsymbol{P}_1 + \boldsymbol{P}_2$ (where the indices 1 and 2 now refer to the two entangled particles, and we label the measuring apparatus system '3'.) So we grant, to begin, that these observables are necessarily referred to some reference body, 'the lab'. The discussion of position measurements, above, can be repeated, *mutatis mutandis* for the measurement of $\boldsymbol{Q}_1$ in this situation, and the result is (unsurprisingly) the same. We can interpret the result the same as well: the conditions required for the definiteness of the total momentum, and indeed of $\boldsymbol{P}_2$, are not satisfied during the measurement, because the time-derivatives of the momenta of both the compound system and the lab itself are non-zero. Again, I leave it to the reader to decide whether these observations form the basis of a satisfying reply (independently of the fact, as I see it, that Bohr does not supply us with an interpretation of quantum theory itself).

But what about entanglement in observables other than position and momentum? Can they be handled in the same way? Here I can only indicate the nature of an answer by sketching what one would say in the case of two directions of spin. The point is that reference frames play an essential role here as well. Indeed, in order to know what 'spin-x' and 'spin-y' mean at the two measurement-stations in the EPR-Bohm experiment, experimenters at the two sides must share a direction in space, which they arbitrarily call 'x', and one of the experimenters must be able to determine an angle of 90 degrees (in this example) from this direction. The corresponding 'direction' and 'angle' observables are incompatible. One can arrive at their incompatibility in a way that is more or less analogous to the route, discussed above, to the Weyl relations via the imprimitivity theorem. One begins by requiring that the angle and direction observables have the obvious invariances and covariances with respect to various rotations, then derives, via the imprimitivity theorem and the presumption of a continuous unitary representation of these rotations on the appropriate Hilbert space, a Weyl (uncertainty) relation for these observables.[9]

What is missing, however, is any indication of a physically satisfying account of these relations analogous to the account for position and momentum, above. It was part of Bohr's genius to couch the experiments involving position and momentum in terms that are more or less immediately amenable to understanding the essential role that reference frames play. Though it may be clear how, in broad outline, one would attempt to do so for the case of spin (for example, by allowing a large number of spin-$\frac{1}{2}$ particles prepared identically to determine a direction in space), I am unaware of any detailed account, for which, in any case, there is no room here.

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9. A sketch of the procedure is in Busch et al. (1995). The discussion is necessarily brief and informal, here, because several complications of a more or less purely mathematical character arise.

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