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A NOTE ON ENVIRONMENTAL POLICY AND PUBLIC DEBT STABILIZATION

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This article analyzes the consequences of environmental tax policy under public debt stabilization constraint. A public sector of pollution abatement is financed by a tax on pollutant emissions and/or by public debt. At the same time, households can also invest in private pollution abatement activities. We show that the economy may be characterized by an environmental-poverty trap if debt is too large or public abatement is not sufficiently efficient with respect to the private one. However, there exists a level of public abatement and debt at which a stable steady state is optimal.

Keywords: Environmental Taxation, Public and Private Abatements, Public Debt, Trap, Optimality

1. INTRODUCTION

Growing environmental concerns have forced several countries to adapt their tax structures by introducing new taxes on pollutants. France, following some Scandinavian countries such as Sweden, has planned to adopt a carbon tax on energy use in the next few years. The revenues of these green taxes are used to limit the economic distortions of the reform by reducing other taxes, or alternatively, are allocated to pollution abatement programs. In France, the main environmental protection agency (ADEME) is entirely financed[1] by revenues of taxes on pollutants, called the General Tax on Polluting Activities. However, whatever the government's decision about distribution of the environmental tax revenues, public engagements in environmental protection are often constrained by long-term fiscal

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objectives that impose the need to control public deficits and public debt evolution. The aim of this paper is to analyze the economic consequences of environmental policy under a debt stabilization constraint.

Alongside the rise of public expenditures for environmental protection, households have also massively increased their environmental spending. Households' environmental actions were initially limited to the purely personal, e.g., protection of homes against noise pollution, lawn and garden maintenance, and individual wastewater purification of homes not connected to a collective sewage system. But these actions are nowadays widened to the entire environmental field: combating biodiversity loss, sorting and recycling waste, and fighting air, water, and ground pollution. This increasing engagement of individuals in environmental protection corresponds to an increasing demand for more and more environmental requirements in the OECD countries. In France, over the period 1990–2003, households' spending for environmental protection has increased on the average by 3.5% per year, while public spending has increased by nearly 7% per year. Thus, the growth of total expenditure on environmental protection is between 3% and 6%, and even approached 10% in the years 1993–1995. At the same time, GDP only grew at an annual average rate of 2%.[2]

Based on these observations, our objective is to study the consequences of environmental tax policy for capital accumulation, environmental quality, and welfare when households also invest in pollution abatement. Does the increase of public environmental engagement lead to a green crowding-out effect, penalizing private involvement in environmental protection? When environmental policy is characterized by environmental taxation and a public pollution-abatement sector, does it still respect debt stabilization constraints? We extend the model developed by John and Pecchenino (1994)[3] to take public abatement into account. This paper has introduced pollution externalities in an overlapping-generations model à la Diamond (1965), where the behavior of selfish individuals generates intergenerational inefficiencies and inequities.[4] Because we intend to study the interactions between voluntary commitments of individuals and government's intervention in environmental protection, we focus on an economy where households are involved in environmental maintenance, through a trade-off between polluting consumption and environmental quality, alongside public actions to protect the environment. We further assume that government spending is financed through a tax on polluting consumption on one hand, and through public debt on the other. This means that a share of public environmental abatement is financed by future generations. Basically, we assume that generations that will benefit from the public environmental protection should pay for it. This assumption corresponds to a beneficiary–payer principle, enhancing willingness to implement environmental policy. Indeed, one of the results of the previous literature is to show that environmental taxation implies such a welfare loss for present generations that its implementation cannot be wished: the generation that would decide it would also bear the heaviest burden. Finally, we also consider a debt stabilization constraint that imposes a constant level of debt (per capita) [5] and allows us to treat debt as a policy parameter.[6]

Using this framework, we show that if public debt is low enough and public abatement sufficiently efficient, the economy monotonically converges to a long-run steady state. In contrast, when public debt is not so low and efficiency of private maintenance is sufficiently high with respect to public, an environmental-poverty trap can emerge. Indeed, the capital stock may decrease because a large share of saving is devoted to maintaining debt constant and households increase their labor income share dedicated to "green" investment through private abatement. This a priori negative effect of public policy is mitigated by the welfare analysis. Indeed, we show that the optimal stationary allocation can be decentralized by choosing appropriately the two policy instruments, public abatement and debt. They both affect the consumption tax rate, which is endogenously determined by the intertemporal budget constraint of the government. However, government spending also has a direct impact on environmental quality, as it corresponds to public abatement, whereas debt is linked to productive capital because it captures a share of saving.

Previous papers have analyzed the consequences of some environmental policies for environmental quality, growth, and welfare. Nevertheless, in all these studies, public and private choices for abatement are always exclusive.[7] In particular, they have focused on the consequences of environmental taxes whose revenues are redistributed to households. In John et al. (1995), the optimal policy relies on a tax scheme composed of taxes on labor and capital incomes. In the same manner, Ono (1996) shows that the optimal allocation can also be achieved by a combination of taxes on consumption and capital income. In these papers, the authors develop the economic policy tools to reach the optimal capital stock and environmental quality simultaneously. The efficient policy rules have to influence the saving–abatement private arbitrage in this way. Therefore, the economy experiences a sustainable path. As, in our article, we consider both private and public abatement technologies, we allow for interactions between these two environmental involvements, which may even generate an environmental-poverty trap. Nevertheless, public debt is considered as an economic policy tool that could help to reach economic efficiency. Therefore, in contrast to John et al. (1995) and Ono (1996), optimality is obtained with a unique tax rate. Another difference of our paper is the presence of public debt to finance the pollution abatement sector. However, debt has already been introduced in dynamic models with environmental concerns [Bovenberg and Heijdra (1998), Heijdra et al. (2006)], but these contributions focus on a different issue than ours. Instead of using debt to finance a share of public maintenance, debt policy makes it possible to redistribute welfare gains from future to existing generations. In our model, the role of public debt is twofold: as usual, it redistributes welfare among existing and future generations, but first of all, it also finances the public pollution abatement sector. Hence, the redistribution properties of public debt are limited by the environmental engagement of the government.

In the next section, we present the model. The intertemporal equilibrium is defined in Section 3. Section 4 looks at the dynamics, and Section 5 is devoted to long-run welfare analysis. The last section concludes. Several technical details are relegated to the Appendix.

2. A SIMPLE OVERLAPPING-GENERATIONS MODEL

We consider an overlapping-generations model with discrete time, $t = 0, 1, \ldots, +\infty$, and three types of agents: consumers, firms, and a government.

2.1. Consumers

Consumers live for two periods and the population size of each generation is constant and normalized to one. Preferences of a household born at period t are represented by a simple log-linear utility function defined over future consumption c_{t+1} and environmental quality E_{t+1}:

$$\ln(c_{t+1}) + \epsilon \ln(E_{t+1}), \tag{1}$$

where $\epsilon \geq 0$ is a measure of the degree of "green" preferences.

In the first period of life, a household born in period t supplies inelastically one unit of labor, remunerated at the competitive real wage w_t, and shares its labor income between saving e_t, through available assets, and positive environmental abatement $m_t \geq 0$. At the second period of life, saving, remunerated at the real interest factor r_{t+1},[8] is used to consume the final good and pay consumption taxes, at a rate $\tau_{t+1} \geq 0$. Hence, a consumer faces the two following budget constraints:

$$w_t = e_t + m_t, \tag{2}$$

$$(1 + \tau_{t+1})\, c_{t+1} = r_{t+1} e_t. \tag{3}$$

We further assume that private consumption degrades environmental quality, whereas private environmental abatement and public spending, i.e., public environmental abatement, $G_t \geq 0$, can improve it. Assuming linear relationships, environmental quality follows the motion

$$E_{t+1} = \gamma_1 m_t + \gamma_2 G_t - \alpha c_t, \tag{4}$$

where $\alpha > 0$ represents the rate of pollution coming from private activities, whereas $\gamma_1 > 0$ and $\gamma_2 > 0$ are measures of the efficiency of private and public abatement, respectively. Note that, as soon as $\gamma_1 \neq \gamma_2$, private and public abatement have distinct effects on environmental quality. This representation of distinct techniques and/or of distinct productivities for environmental protection[9] may be illustrated by many examples. Indeed, household spending is geared primarily toward curative actions (soundproofing, wastewater treatment, waste separation), whereas public spending is more preventive (air purity protection, protection of biodiversity, species conservation). The distinct consequences of public (γ_2) and private (γ_1) abatement may refer, for example, to the case of urban pollution and

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environmental performance of cities: without specific maintenance, the quality of the urban environment will degrade. Users would then derive less value from public parks, planted pathways, flower gardens, bicycle tracks, etc. Therefore, the municipality supports the preservation of the environment by replacing and maintaining trees, grass, and flowers in public areas, whereas private agents intervene directly themselves (lower quantities of waste by collection and sorting; reduction of gaseous pollutants through investment in power-saving appliances and fuel-efficient cars).

Notice that $-E_{t+1}$ can be interpreted as pollution. Assuming that E_{t+1} does not depend on the current level of environmental quality E_t means that pollution is a flow or a stock with full regeneration after one period. Regarding the main pollutants, such as sulfur dioxide, suspended particulate matter, carbon monoxide, and some sorts of river pollution, although all these pollutants are stock pollutants, they all have short lifetimes and can therefore be considered as flow pollutants from a long-run point of view [IPCC (1996); Lieb (2004); Liu and Liptak (2000)]. In the atmosphere the lifetime of sulfur dioxide is no more than four days and that of carbon monoxide is about three months. Suspended particulate matter is washed out by rain and thus has only a short lifetime. Because rivers are flowing, the concentrations of water pollutants would quickly decline if emissions stopped. So river pollutants are short-lived. Thus they can also be considered as flow pollutants. Because we consider an overlapping-generation model with two-period lived agents, i.e., the length of the period is quite large, it does not seem to be too restrictive to consider that E_{t+1} does not depend on E_t.[10]

A consumer maximizes his or her utility function (1) under the constraints (2)–(4) and $m_t \geq 0$. One obtains

$$\frac{r_{t+1}}{1+\tau_{t+1}} E_{t+1} \geq \gamma_1 \epsilon c_{t+1}, \tag{5}$$

with equality when $m_t > 0$.

2.2. Firms

Taking into account that one unit of labor is inelastically supplied in each period, the production is given by $y_t = k_t^s$, where k_t indifferently denotes the capital stock or the capital–labor ratio, and $s \in (0, 1)$ the capital share in total income. From profit maximization, we get

$$r_t = s k_t^{s-1} \equiv r(k_t), \tag{6}$$

$$w_t = (1-s) k_t^s \equiv w(k_t). \tag{7}$$

2.3. Public Sector

The aim of the government is to improve environmental quality, using public spending G_t to provide public environmental abatement. To finance these

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expenditures, as seen above, the government levies a tax on private consumption, at the rate $\tau_t \geq 0$, or can use debt B_t. The intertemporal budget constraint of the government can be written

$$B_t = r_t B_{t-1} + G_t - \tau_t c_t \tag{8}$$

with $B_{-1} \geq 0$ given.

In this paper, we focus on equilibria with constant debt or constant debt per capita, i.e., $B_t = B > 0$ for all $t \geq 0$.[11] This avoids explosive debt paths. This condition also ensures the long-term credibility of the environmental policy and makes it possible to use debt as an economic policy instrument. We further consider that public spending, which corresponds to the level of public abatement, is an exogenous instrument for environmental policy; i.e., $G_t = G \geq 0$ for all $t \geq 0$. Therefore, the budget constraint of the government (8) can be rewritten as

$$\tau_t \frac{r(k_t)e_{t-1}}{1+\tau_t} = [r(k_t) - 1]B + G. \tag{9}$$

3. INTERTEMPORAL EQUILIBRIUM

In this paper, we are interested in the effect of public debt on the dynamics, but also on the respective roles of private versus public abatement. This explains why we focus on equilibria with strictly positive abatement $m_t > 0$. Besides, in contrast to many papers,[12] an explicit condition is derived below such that the inequality $m_t > 0$ holds along the whole dynamic path. We note also that, as emphasized in the Introduction, positive private and public environmental abatements are supported by empirical evidence.

Equilibrium on the asset market is ensured by

$$e_t = k_{t+1} + B. \tag{10}$$

Therefore, the budget constraint of the government (9) can be rewritten as

$$\tau_t \frac{r(k_t)(k_t + B)}{1+\tau_t} = [r(k_t) - 1]B + G. \tag{11}$$

It defines the consumption tax rate as a function of capital:

$$\tau_t = \frac{B[r(k_t) - 1] + G}{r(k_t)k_t + B - G} \equiv \tau(k_t). \tag{12}$$

We are interested in an economy where public debt is larger than public abatement:

Assumption 1. $B \geq G \geq 0$.

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Hence, $\tau_t \geq 0$ is satisfied if $r(k_t) \geq 1 - G/B$. This requires $k_t \leq \overline{k}$, with

$$\overline{k} \equiv r^{-1}(1 - G/B) = \left(\frac{sB}{B-G}\right)^{1/(1-s)}. \quad \mathbf{(13)}$$

Using (6) and (12), we notice that $\tau(k)$ is strictly decreasing ($\tau'(k) < 0$); i.e., the tax rate is countercyclical.

Assuming $m_t > 0$, the consumer trade-off (5) is written:

$$E_{t+1} = \gamma_1 \epsilon (B + k_{t+1}). \quad \mathbf{(14)}$$

Substituting this expression into (4) and using (2), (3), (6), (7), and (12), we obtain

$$k_{t+1} = \frac{1}{\gamma_1(1+\epsilon)} \left\{[\gamma_1(1-s) - \alpha s]\, k_t^s + X\right\} \equiv H(k_t) \quad \mathbf{(15)}$$

with

$$X \equiv (\alpha + \gamma_2)G - [\alpha + \gamma_1(1+\epsilon)]\, B. \quad \mathbf{(16)}$$

We notice now that, using (2) and (10), $m_t > 0$ is equivalent to $w(k_t) - B - k_{t+1} > 0$. This is satisfied if $(1-s)k_t^s - B > H(k_t)$ holds; i.e., $k_t > \underline{k}$, with

$$\underline{k} \equiv \left[\frac{(\alpha + \gamma_2)G - \alpha B}{\alpha s + \gamma_1 \epsilon (1-s)}\right]^{1/s}. \quad \mathbf{(17)}$$

Assumption 2.

$$\gamma_2 < \frac{\alpha s + \gamma_1 \epsilon (1-s)}{G} \left(\frac{sB}{B-G}\right)^{s/(1-s)} + \alpha \frac{B-G}{G} \equiv \overline{\gamma}_2.$$

Under this assumption, $\underline{k}$ is strictly lower than $\overline{k}$. This assumption simply indicates that for a sufficiently high level of public abatement productivity ($\gamma_2 \geqslant \overline{\gamma}_2$), private abatement falls to zero. In that case, there is obviously no need for private protection, the public protection being highly efficient.

We are now able to define an equilibrium:

DEFINITION 1. *Under Assumptions 1 and 2, an intertemporal equilibrium with strictly positive private abatement ($m_t > 0$) is a sequence $k_t \in (\underline{k}, \overline{k}]$, $t = 0, 1, \ldots, +\infty$, such that equation (15) is satisfied, given $k_0 \in (\underline{k}, \overline{k}]$.*

Therefore, the dynamics is driven by a one-dimensional dynamic equation, where k_t is a predetermined variable. Because $E_t = \gamma_1 \epsilon (B + k_t)$, this also determines the evolution of environmental quality.

4. DYNAMICS AND ENVIRONMENTAL TRAP

We are now able to analyze the roles of debt, public spending, and the effectiveness of private and public environmental abatements in dynamics. By direct inspection

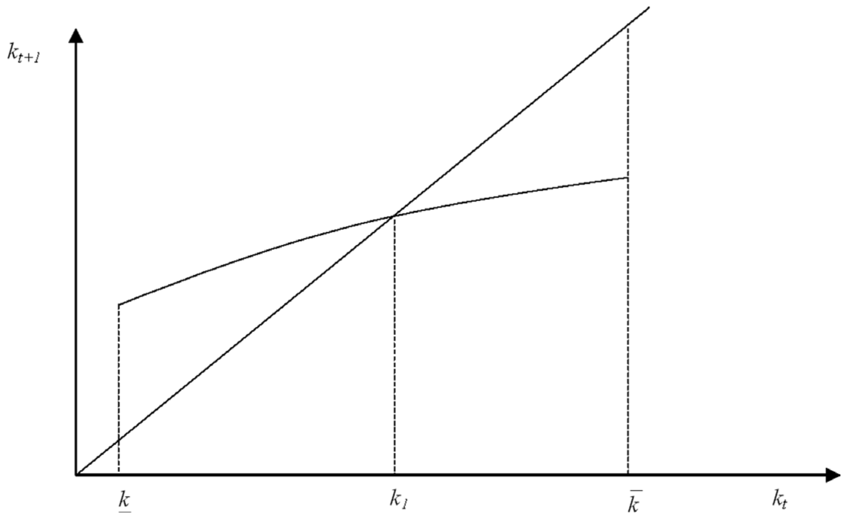

FIGURE 1. Illustration of Proposition 1.

of (15), we immediately see that two main cases may emerge, depending on the sign of $\alpha s - \gamma_1(1-s)$. When $\alpha s > \gamma_1(1-s)$, it is possible to show that, under Assumptions 1 and 2, there is no steady state belonging to $(\underline{k}, \overline{k}]$ and no persistent trajectory staying in this interval. Therefore, we exclude this case and we will focus on the configuration where the following holds.

Assumption 3. $\alpha s < \gamma_1(1-s)$.

For further reference, because $H(0) = X/[\gamma_1(1+\epsilon)]$, it is useful to note that $X \geq 0$ if and only if $\gamma_2 \geq \gamma_{20}$, with

$$\gamma_{20} \equiv (\alpha + \gamma_1(1+\epsilon))B/G - \alpha \tag{18}$$

and $X < 0$ otherwise, where $\gamma_{20} < \overline{\gamma}_2$ if B and G are not too far.[13] Because $\gamma_{20} > \gamma_1$, positive values of X requires $\gamma_2 > \gamma_1$. This means that if public and private abatements have identical efficiency ($\gamma_2 = \gamma_1$) or public abatement is the less efficient ($\gamma_2 < \gamma_1$), we have $X < 0$.

The following proposition examines cases where the economy converges to a unique long-run steady state (see Figure 1).

PROPOSITION 1. *Let*

$$\widetilde{\gamma}_2 \equiv \frac{\alpha s + \gamma_1 \epsilon (1-s)}{G}\left[\frac{\gamma_1(1-s) - \alpha s}{\gamma_1(1+\epsilon)}\right]^{s/(1-s)} + \alpha \frac{B-G}{G}.$$

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Under Assumptions 1–3,

1. *When* $\gamma_2 = \gamma_{20} < \widetilde{\gamma}_2$*, there is one stable steady state given by*

$$k_1 = \left[\frac{\gamma_1(1-s)-\alpha s}{\gamma_1(1+\epsilon)}\right]^{1/(1-s)} \quad \text{if } \frac{B-G}{B} < s\frac{\gamma_1(1+\epsilon)}{\gamma_1(1-s)-\alpha s}.$$

2. *When* $\widetilde{\gamma}_2 > \gamma_2 > \gamma_{20}$*, there is one stable steady state*

$$k_1 > \left[\frac{\gamma_1(1-s)-\alpha s}{\gamma_1(1+\epsilon)}\right]^{1/(1-s)}$$

if B and G are not too far.

Note that $\widetilde{\gamma}_2 > \gamma_{20}$ *requires a not-too-large* B*. Otherwise,* k_1 *becomes smaller than* $\underline{k}$*. In this case,* k_t *decreases until it reaches its lower bound* $\underline{k}$*. Finally, if* B *is too far from* G*,* k_1 *may be larger than* $\overline{k}$*. Then* k_t *grows until it reaches its upper bound* $\overline{k}$*.*

Proof. See the Appendix. ■

This proposition shows that, when $X \geq 0$, there is at most one stable steady state. Obviously, this occurs without government intervention ($B = G = 0$).[14] This proposition shows that this is still relevant if public debt is not too large with respect to public abatement or, as underlined above, public environmental abatement is sufficiently efficient compared to private abatement ($\gamma_2 > \gamma_1$).

Indeed, when the level of debt is not too high and private environmental abatement not too efficient, a large share of labor income is devoted to capital accumulation, which fosters convergence.

Proposition 1 is also useful to deduce the impact of a (slight) increase of debt or public abatement on the long-run stable steady state. Because X is increasing in G, but decreasing in B, public abatement and debt have opposite effects on the stationary capital stock. The first promotes capital accumulation, reducing private abatement in favor of productive saving. In contrast, the second lowers capital at the steady state, because of a crowding-out effect reducing the share of saving through capital.

When $X < 0$ ($\gamma_2 < \gamma_{20}$), two steady states, a stable one $k_1 > 0$ and an unstable one k_2 ($< k_1$), may coexist. However, when X (γ_2) is sufficiently close to 0 (γ_{20}), k_2 is lower than $\underline{k}$. In this case, Proposition 1 still applies. In contrast, when X is negative enough or γ_2 sufficiently lower than γ_{20}, we show that an environmental trap may emerge. To examine this possibility, we assume that B and G satisfy the following assumption.

Assumption 4. $[\gamma_1(1-s)-\alpha s]^{s/(1-s)}[\alpha s + \gamma_1\epsilon(1-s)] > [\gamma_1(1+\epsilon)]^{1/(1-s)} s^{-s/(1-s)} B - (1-s)[\gamma_1(1-s)-\alpha s]^{1/(1-s)} G$.

This allows us to show the following proposition (see Figure 2).

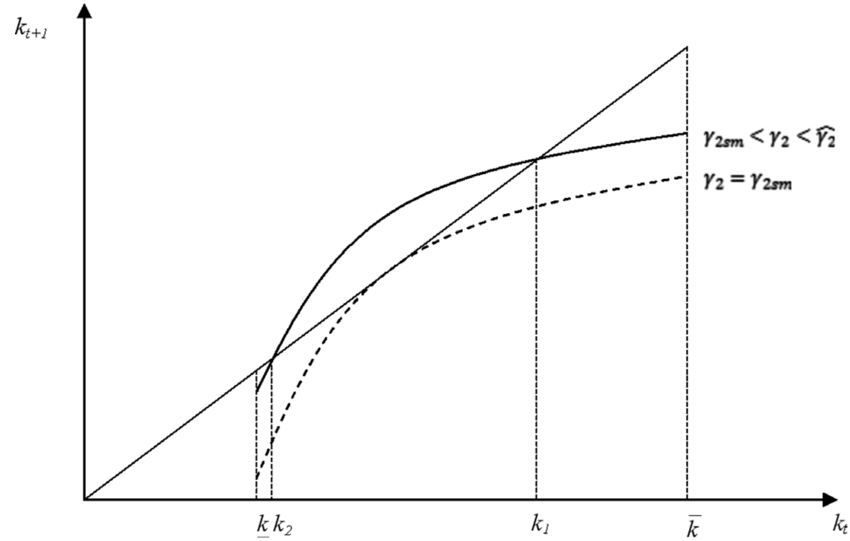

FIGURE 2. Illustration of Proposition 2.

PROPOSITION 2. *Let*

$$\gamma_{2sn} \equiv [\alpha + \gamma_1(1+\epsilon)]\frac{B}{G} - \alpha$$
$$- (1-s)[\gamma_1(1-s) - \alpha s]^{1/(1-s)}\left[\frac{s}{\gamma_1(1+\epsilon)}\right]^{s/(1-s)}$$

$$\widehat{\gamma_2} \equiv \alpha\frac{B-G}{G} + \frac{\alpha s + \gamma_1\epsilon(1-s)}{G}\left[s\frac{\gamma_1(1-s) - \alpha s}{\gamma_1(1+\epsilon)}\right]^{s/(1-s)}.$$

Suppose that Assumptions 1–4 are satisfied. When $\gamma_{2sn} < \gamma_2 < \widehat{\gamma_2}$ with γ_2 close enough to γ_{2sn}, there exist two steady states. One is unstable ($k_2 > \underline{k}$) and the other is stable ($\overline{k} \geq k_1 > k_2$). A saddle-node bifurcation occurs for $\gamma_2 = \gamma_{2sn}$ ($k_2 = k_1$) and there is no steady state for $\gamma_2 < \gamma_{2sn}$. In this last case, $k_{t+1} = H(k_t) < k_t$ for all k_t, and k_t reaches the lower bound $\underline{k}$ after a finite number of periods.

Proof. See the Appendix. ■

This proposition establishes that, for $\gamma_{2sn} < \gamma_2 < \widehat{\gamma_2}$, there is a poverty trap for all $\underline{k} < k_t < k_2$. Because $E_t = \gamma_1\epsilon(B + k_t)$, this also corresponds to an environmental trap, where environmental quality degrades. Furthermore, it is interesting to note that because $X < 0$, Proposition 2 applies for $\gamma_2 \leq \gamma_1$; i.e., public and private abatements have the same effectiveness or public abatement is less efficient. Note also that the inequality $X < 0$ is strengthened by a high level of public debt.

If public debt is sufficiently large, a low share of income is devoted to productive saving. In this case, capital accumulation may decrease. This effect is reinforced by great efficiency of private environmental abatement, because in this case, households invest a large part of income in private abatement. However, environmental quality degrades because of the consumer trade-off between consumption and environmental quality, which stipulates that this last evolves in the same direction as capital accumulation.

Therefore, being a source of the environmental-poverty trap, it could seem that the policy introduced in this paper should not be recommended. Analyzing welfare in the steady state, we will see, in the next section, that this conclusion may be mitigated.

5. WELFARE ANALYSIS

As is well known, an overlapping-generations economy may be characterized by over- or underaccumulation of capital. As is emphasized by John and Pecchenino (1994), the same happens for environmental quality: one may have over- or undermaintenance. We reexamine this issue, determining first the optimal stationary allocation. Then we will see that this allocation can be decentralized by an appropriate choice of our two policy parameters, B and G.

We start by solving the planner problem. Using the two resource constraints,

$$c + k + m = k^s - G, \tag{19}$$

$$E = \gamma_1 m - \alpha c + \gamma_2 G, \tag{20}$$

we get[15]

$$c = \frac{\gamma_1(k^s - k) - E + (\gamma_2 - \gamma_1)G}{\alpha + \gamma_1}. \tag{21}$$

Substituting this expression into the utility function $\ln c + \epsilon \ln E$, the planner solves

$$\max_{k,E} \ln\left[\frac{\gamma_1(k^s - k) - E + (\gamma_2 - \gamma_1)G}{\alpha + \gamma_1}\right] + \epsilon \ln E, \tag{22}$$

taking $G \geq 0$ as given. Using the first-order conditions

$$\frac{1}{c}\frac{\gamma_1}{\alpha + \gamma_1}(sk^{s-1} - 1) = 0, \tag{23}$$

$$\frac{\epsilon}{E} - \frac{1}{(\alpha + \gamma_1)c} = 0, \tag{24}$$

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we deduce that the optimal stationary allocation $(\widetilde{k}, \widetilde{E})$ is given by

$$s\widetilde{k}^{s-1} = 1 \Leftrightarrow \widetilde{k} = s^{\frac{1}{1-s}}, \tag{25}$$

$$\begin{aligned}\widetilde{E} &= \frac{\epsilon}{1+\epsilon}[\gamma_1(\widetilde{k}^s - \widetilde{k}) + (\gamma_2 - \gamma_1)G] \\ &= \frac{\epsilon}{1+\epsilon}[\gamma_1 s^{\frac{s}{1-s}}(1-s) + (\gamma_2 - \gamma_1)G],\end{aligned} \tag{26}$$

where $\widetilde{k} < \overline{k}$ corresponds to the standard golden rule. We also notice that $\widetilde{k} > \underline{k}$ for $\gamma_2 < \widetilde{\gamma}_2$, with

$$\widetilde{\gamma}_2 \equiv \frac{\alpha s + \gamma_1\epsilon(1-s)}{G} s^{s/(1-s)} + \alpha\frac{B-G}{G} < \overline{\gamma}_2. \tag{27}$$

Thus, the optimal allocation corresponds to a stationary solution with strictly positive tax rate ($\tau > 0$) and private abatement ($m > 0$).

We are now able to evaluate whether a steady state can be optimal. A stationary solution is defined by $k = H(k)$. Hence, the level of capital is optimal if $\widetilde{k} = H(\widetilde{k})$, which is equivalent to $X = \widetilde{X}$, with

$$\widetilde{X} = s^{\frac{s}{1-s}}[\alpha s - \gamma_1(1 - s(2+\epsilon))]. \tag{28}$$

There is a unique level of X, determined by a combination of the policy parameters G and B, such that the stationary level of capital is optimal. However, because the inequality $H'(\widetilde{k}) < 1$ always holds under Assumption 3, a monotonically unstable steady state, k_2, can never be optimal. In contrast, a monotonically stable steady state, k_1, can be optimal for an appropriate choice of B and G.

Differentiating the equation $k = H(k)$ with respect to k and X, we get

$$\frac{dk}{dX} = \frac{1}{\gamma_1(1+\epsilon)\,[1 - H'(k)]} > 0 \quad \text{iff } H'(k) < 1.$$

Therefore, a stable steady state (k_1) is characterized by overaccumulation if $X > \widetilde{X}$ and by underaccumulation if $X < \widetilde{X}$. In the first case, the optimal allocation can be reached by increasing debt and/or decreasing public abatement, whereas the opposite recommendation is relevant when there is underaccumulation. Indeed, as already emphasized, G and B have opposite effects on the stationary level of capital. Increasing public abatement, by reducing the incentive to provide private abatement, raises productive saving, whereas increasing debt lowers capital, because a smaller share of saving is devoted to capital holding.

In the steady state, the level of environmental quality is given by $E = \gamma_1\epsilon(B+k)$, which is generically different from its optimal level $\widetilde{E}$. Hence, the question we address now is the following: by an appropriate choice of the two policy parameters B and G, can a decentralized steady state be optimal?

PROPOSITION 3. *Let*

$$\gamma_2^a \equiv [\alpha(2+\epsilon) + \gamma_1(1+\epsilon)]\gamma_1/\alpha$$
$$\gamma_2^b \equiv \{\alpha s(2+\epsilon) - \gamma_1[1 - s(2+\epsilon)]\}\gamma_1/(\alpha s).$$

Under Assumptions 1 and 2, $\alpha s < \gamma_1(1-2s)$ *and* $\gamma_2^b \leq \gamma_2 < \min\{\gamma_2^a, \widetilde{\gamma}_2\}$, *where* $\widetilde{\gamma}_2$ *is given by (27), there is a unique value of the policy parameters B and G, with* $B \geq G > 0$, *such that the stationary levels of capital and environmental quality are optimal.*

Proof. See the Appendix. ■

This proposition shows that there exists a unique choice of public environmental abatement and debt that allow a steady state to be characterized by optimal levels of capital and environmental quality. This provides an argument in favor of the policy considered in this paper. We notice that this result arises even if, in contrast to John et al. (1995) and Ono (1996), we introduce a unique tax rate. However, there are still two policy parameters, B and G. In fact, G has an impact through two channels, the level of public abatement of environmental quality and the tax rate. The level of debt B affects the tax rate as well, but also capital accumulation and private abatement, because it captures a share of saving.

We finally notice that $\widetilde{X} \geq 0$ if and only if $\epsilon \geq \widetilde{\epsilon}$, with

$$\widetilde{\epsilon} \equiv \frac{\gamma_1(1-2s) - \alpha s}{\gamma_1 s}.$$

In this case, the optimal policy corresponds to a configuration where there is a unique stable steady state. However, when $\epsilon < \widetilde{\epsilon}$, multiplicity of steady states and a trap may not be excluded a priori.

6. CONCLUSION

In several countries, nonexplosive public debt is a major constraint. Nevertheless, growing concerns about environmental degradation (biodiversity losses, climate change, etc.) lead many governments to fight against pollution and hence to increase environmental spending. In many countries, pollution mitigation induces adoption of environmental taxes bearing on households, alongside with an increase of individual environmental engagements.

In this paper, we show that, when the environmental tax is allocated to environmental protection, and in the same time, it aims to stabilize public debt, environmental public policy may lead the economy to a poverty-environmental trap. Indeed, if the public debt is high enough, its stabilization reduces households' share of income devoted to productive saving. This effect is reinforced by private abatement that is sufficiently productive with respect to public. Indeed, households are encouraged to protect the environment instead of saving for future consumption. Nevertheless, welfare analysis shows that, whatever the initial economic conditions are, there exists a unique value of the policy parameters, namely the debt level and the public spending, such that the decentralized steady state is optimal. This allows us to recommend that policy-makers should carefully evaluate the efficiency of private versus public environmental abatement and the level of public debt before increasing their environmental engagement and introducing new taxes.

NOTES

1. Its budget is about 1% of the French GDP.
2. See Ifen (2006).
3. Ono and Maeda (2001) introduce uncertain lifetime in such a framework to study the consequences of distribution of ages among agents for environmental quality.
4. See Solow (1974, 1986) for seminal contributions.
5. There is no population growth.
6. See Diamond (1965) for a seminal model without any environmental externalities.
7. The distinction between households and (short-lived) governments is not precisely defined in this literature.
8. We assume complete depreciation of capital after one period of use. Therefore, r_{t+1} also denotes the real interest rate.
9. In the following, we do not distinguish between pollution abatement and protection or maintenance of the environment.
10. Seegmuller and Verchère (2007) use a similar assumption.
11. See the seminal paper by Diamond (1965), which, however, ignores environmental issues.
12. See, for instance, Ono (1996) or Zhang (1999).
13. Indeed, this requires that

$$B\left(\frac{B-G}{B}\right)^{s/(1-s)} < \frac{\alpha s + \gamma_1(1-s)\epsilon}{\gamma_1(1+\epsilon)} s^{s/(1-s)}.$$

14. In this case, one converges to the steady state for all $k > 0$.
15. We can also deduce the value of private abatement, given by $m = [\alpha(k^s - k) - (\alpha + \gamma_2)G + E]/(\alpha + \gamma_1)$.

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APPENDIX

PROOF OF PROPOSITION 1

Under Assumption 3, the function $H(k)$ is strictly increasing ($H'(k) > 0$) and concave ($H''(k) < 0$). When $\gamma_2 = \gamma_{20}$, we further have $H(0) = 0$. In this case, there is a unique strictly positive steady state

$$k_1 = \left[\frac{\gamma_1(1-s) - \alpha s}{\gamma_1(1+\epsilon)}\right]^{1/(1-s)}$$

solving the equation $k = H(k)$. Moreover, using (13) and (17), we deduce that k_1 is smaller than $\overline{k}$ if B and G are not too far, such that

$$\frac{B-G}{B} < s\frac{\gamma_1(1+\epsilon)}{\gamma_1(1-s)-\alpha s},$$

and is larger than $\underline{k}$ if $\gamma_2 < \widetilde{\gamma}_2$, where $\widetilde{\gamma}_2 > \gamma_{20}$ is equivalent to

$$B < \frac{\alpha s + \gamma_1 \epsilon(1-s)}{[\gamma_1(1+\epsilon)]^{1/(1-s)}}[\gamma_1(1-s) - \alpha s]^{s/(1-s)}.$$

We immediately see that this last inequality is satisfied if B is not too large. We may also easily conclude that, because $H(k)$ is strictly increasing and concave, the steady state k_1 is stable.

Finally, when $\gamma_2 > \gamma_{20}$, $H(0)$ becomes strictly positive. Because the steady state k_1 is stable, it becomes larger than

$$\left[\frac{\gamma_1(1-s) - \alpha s}{\gamma_1(1+\epsilon)}\right]^{1/(1-s)},$$

but retains the same properties as under $\gamma_2 = \gamma_{20}$. In particular, we still have $\underline{k} < k_1 < \overline{k}$ if $\gamma_2 < \widetilde{\gamma}_2$ and B and G are not too far. ■

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PROOF OF PROPOSITION 2

Using (15), we find that $H'(k) = 1$ is equivalent to $k = k_{sn}$, with

$$k_{sn} \equiv \left[s \frac{\gamma_1(1-s) - \alpha s}{\gamma_1(1+\epsilon)} \right]^{1/(1-s)}.$$

Moreover, we have $H(k_{sn}) \leq k_{sn}$ when $\gamma_2 \leq \gamma_{2sn}$. Because $H(k)$ is strictly increasing and concave, we deduce that $H(k_t) < k_t$ for $\gamma_2 < \gamma_{2sn}$. In this case, there is no steady state and $k_{t+1} < k_t$ for all $k_t \in (\underline{k}, \overline{k}]$.

Notice now that $k_{sn} < \overline{k}$ is always satisfied and $k_{sn} > \underline{k}$ if $\gamma_2 < \widehat{\gamma}_2$, with $\widehat{\gamma}_2 > \gamma_{2sn}$ under Assumption 4.

Therefore, when $\widehat{\gamma}_2 > \gamma_2 > \gamma_{2sn}$ with γ_2 sufficiently close to γ_{2sn}, there are a stable steady state k_1 and an unstable one k_2, such that $\underline{k} < k_2 < k_{sn} < k_1 < \overline{k}$.

When $\gamma_2 = \gamma_{2sn}$, we have $k_2 = k_{sn} = k_1$, which corresponds to the critical point where saddle-node bifurcation occurs. ■

PROOF OF PROPOSITION 3

A steady state (k_i, E_i) is optimal if $k_i = \widetilde{k}$ and $E_i = \widetilde{E}$, i.e., $X = \widetilde{X}$ and $\widetilde{E} = \gamma_1\epsilon(B + \widetilde{k})$. Using (16), (25), (26), and (28), this is equivalent to

$$\begin{pmatrix} 1+\epsilon & 1-\gamma_2/\gamma_1 \\ -(\alpha+\gamma_1(1+\epsilon)) & \alpha+\gamma_2 \end{pmatrix} \begin{pmatrix} B \\ G \end{pmatrix} = s^{\frac{s}{1-s}} \begin{pmatrix} 1-s(2+\epsilon) \\ \alpha s - \gamma_1(1-s(2+\epsilon)) \end{pmatrix}.$$

Let $\Delta \equiv (\gamma_2^a - \gamma_2)\alpha/\gamma_1$. Because $\Delta > 0$, B and G are uniquely determined by

$$\begin{pmatrix} B \\ G \end{pmatrix} = s^{\frac{s}{1-s}} \Delta^{-1} \begin{pmatrix} \alpha+\gamma_2 & \gamma_2/\gamma_1 - 1 \\ \alpha+\gamma_1(1+\epsilon) & 1+\epsilon \end{pmatrix} \begin{pmatrix} 1-s(2+\epsilon) \\ \alpha s - \gamma_1(1-s(2+\epsilon)) \end{pmatrix},$$

which is equivalent to

$$\begin{pmatrix} B \\ G \end{pmatrix} = s^{\frac{s}{1-s}} \Delta^{-1} \begin{pmatrix} \alpha[1-s(3+\epsilon)] + \gamma_1[1-s(2+\epsilon)] + \gamma_2 s\alpha/\gamma_1 \\ (1-s)\alpha \end{pmatrix}.$$

Note that $G > 0$ for $\gamma_2 < \gamma_2^a$. Hence, Assumption 1 is satisfied if $B \geq G$. This is ensured by $\gamma_2 \geq \gamma_2^b$, where $\gamma_2^b < \gamma_2^a$.

Finally, we have seen that $\widetilde{k} < \overline{k}$, but $\widetilde{k} > \underline{k}$ requires $\gamma_2 < \widetilde{\gamma}_2$. Therefore, to complete this proof, we need to show that the interval $[\gamma_2^b, \min\{\gamma_2^a, \widetilde{\gamma}_2\})$ is nonempty. We have $\gamma_2^b < \gamma_2^a$. Taking $\gamma_2 = \gamma_2^b$, we obtain $\Delta = \gamma_1(1-s)/s$ and $B = G$. In this case, the inequality $\gamma_2^b < \widetilde{\gamma}_2$ is satisfied when $\alpha s < \gamma_1(1-2s)$. ■