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Dual role of friction in granular flows: attenuation versus enhancement of instabilities

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Flow instabilities driven by the dissipative nature of particle–particle interactions have been well documented in granular flows. The bulk of previous studies on such instabilities have considered the impact of inelastic dissipation only and shown that instabilities are enhanced with increased dissipation. The impact of frictional dissipation on the stability of grains in a homogeneous cooling system is studied in this work using molecular dynamics (MD) simulations and kinetic-theory-based predictions. Surprisingly, both MD simulations and theory indicate that high levels of friction actually attenuate instabilities relative to the frictionless case, whereas moderate levels enhance instabilities compared to frictionless systems, as expected. The mechanism responsible for this behaviour is identified as the coupling between rotational and translational motion. These results have implications not only for granular materials, but also more generally to flows with dissipative interactions between constituent particles – cohesive systems with agglomeration, multiphase flows with viscous dissipation, etc.

Key words: complex fluids, granular media, instability

1. Introduction

Instabilities in molecular fluids (e.g. turbulence) have been studied extensively, and their impact on numerous applications is without question. Granular materials also display flow instabilities (Goldhirsch, Tan & Zanetti 1993; Goldhirsch & Zanetti 1993; Brey, Ruiz-Montero & Moreno 1998; Brito & Ernst 1998; Luding & Herrmann 1999; Petzschmann *et al.* 1999; Soto, Mareschal & Mansour 2000; Brilliantov *et al.* 2004; Garzó 2005; Mitrano *et al.* 2011, 2012), some of which have similarities to those found in their molecular counterparts (Shinbrot, Alexander & Muzzio 1999; Goldfarb, Glasser & Shinbrot 2002; Ciamarra, Coniglio & Nicodemi 2005) and others that do not. The latter arises due to the dissipative nature of collisions between grains (Hopkins & Louge 1991; Goldhirsch & Zanetti 1993). Previous work has shown that dissipation-driven instabilities can take the form of velocity vortices and particle clusters (Luding & Herrmann 1999; Mitrano *et al.* 2011, 2012), as illustrated in figure 1 for a homogeneous cooling system (HCS). Moreover, previous experiments (Kudrolli, Wolpert & Gollub 1997; Conway, Shinbrot & Glasser 2004),

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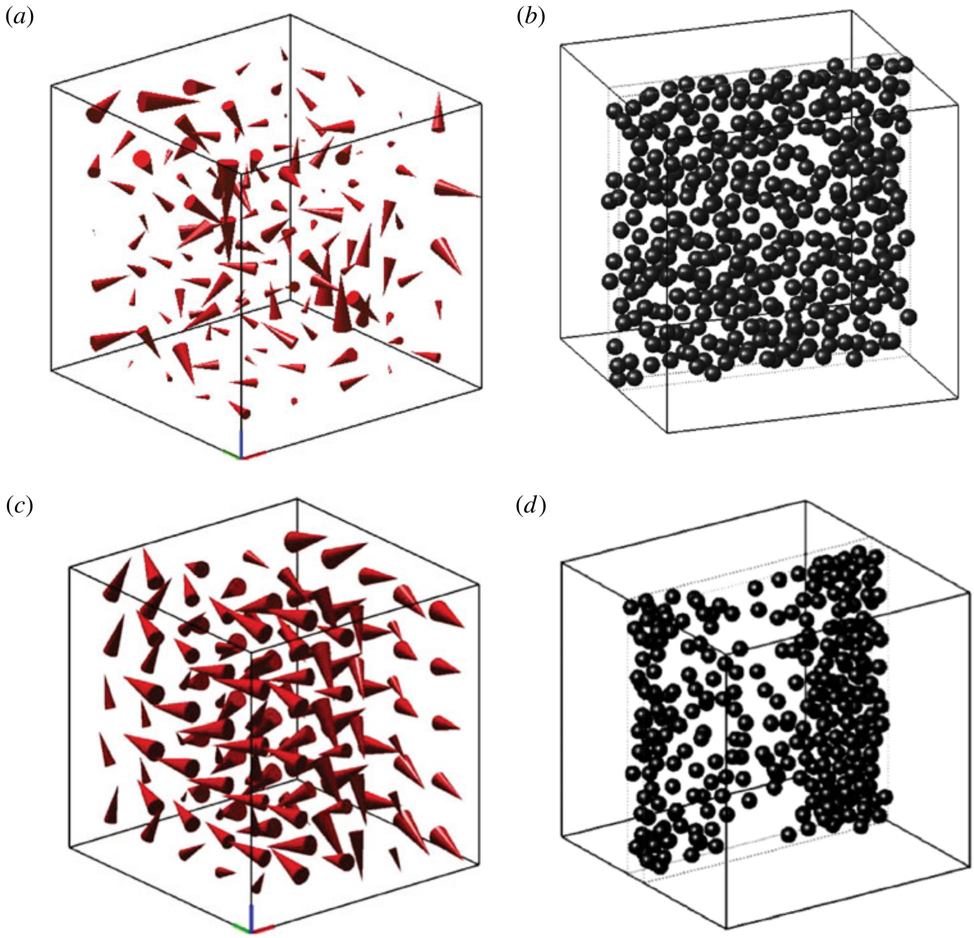

FIGURE 1. (Colour online) Visualizations from a molecular dynamics simulation of an HCS with normal restitution coefficient $e = 0.7$, solids fraction $\phi = 0.3$, tangential restitution coefficient $\beta = -0.7$ and $L/d = 18$, where L is the domain length and d is the particle diameter. (*a*,*c*) The coarse-grained (over cell size $L/5$) velocity field after four and 100 collisions per particle, respectively. The cones in these subplots are sized according to the magnitude of the coarse-grained velocity and point towards its corresponding direction. The stable flow field in panel (*a*) is characterized by random orientation of cones (random motion), whereas the vortex instability in panel (*c*) is characterized by nearby cones pointing in the same direction (aligned motion). (*b*,*d*) The particle positions within a $L/10$ domain slice after four and 100 collisions per particle, respectively. A random distribution of particles is observed in the stable flow field of panel (*b*), whereas a clustering instability is apparent in panel (*d*). Note that only a two-dimensional slice is shown, since a three-dimensional rendering would block from view particles significantly behind the front face, making it difficult to identify particle clusters.

simulations (Hopkins & Louge 1991; Goldhirsch *et al.* 1993; Luding & Herrmann 1999; Petzschmann *et al.* 1999; Brilliantov *et al.* 2004; Mitrano *et al.* 2011) and theories (Goldhirsch & Zanetti 1993; Brey *et al.* 1998; Brito & Ernst 1998; Soto *et al.* 2000; Brilliantov *et al.* 2004; Garzó 2005) have shown a monotonic dependence of instabilities on dissipation. Put simply, instabilities are well known to intensify with increasing dissipation levels.

Dissipation in non-cohesive systems occurs via inelastic or frictional particle–particle contacts. Previous simulations and theoretical analysis (Hopkins & Louge 1991; Goldhirsch *et al.* 1993; Goldhirsch & Zanetti 1993; Brey *et al.* 1998; Brito & Ernst 1998; Luding & Herrmann 1999; Petzschmann *et al.* 1999; Soto *et al.* 2000; Brilliantov *et al.* 2004; Garzó 2005; Mitrano *et al.* 2011, 2012) of instabilities focused on inelastic dissipation, characterized by a normal restitution coefficient, which governs the ratio of the normal, relative velocity on rebound to that on approach. However, these works ignored the influence of friction, which impacts the tangential component of relative velocity. Surprisingly, recent experiments indicate that clusters, forming in streams of particles falling from a nozzle, were less pronounced for rougher copper particles compared to smoother glass particles (Royer *et al.* 2009). At first glance, this attenuation of clustering for increased roughness (increase in total energy dissipation) appears contradictory to the many previous works in which increased inelasticity alone leads to enhanced clustering.

In this work, we confirm and expand on the counterintuitive effect of friction on instabilities, and identify the underlying mechanism. First, our molecular dynamics (MD) simulations of the HCS indicate that highly frictional particles actually serve to attenuate instabilities (i.e. increase the critical dimensionless length scales L_{Vortex}/d and $L_{Cluster}/d$, which demarcate stability in the velocity and concentration fields, respectively) relative to the frictionless case, while particles with moderate friction enhance instabilities compared to frictionless ones, as expected. Second, we find that increasing friction from moderate to higher levels also attenuates instabilities, consistent with recent experiments (Royer *et al.* 2009). Both behaviours are explained here in terms of the coupling of rotational and translational dynamics. Additionally, we obtain simple granular hydrodynamic predictions by inserting a cooling rate (Goldshtein & Shapiro 1995) – also available for polydisperse systems (Santos, Kremer & Garzó 2010) – that includes friction and inelasticity into a linear stability analysis (Garzó 2005) obtained for frictionless particles. These theoretical predictions, which use the same collision rules as MD, support our findings.

2. Methods

The event-driven, hard-sphere MD simulations employed here are composed of N monodisperse spheres in a three-dimensional, cubic volume L^3 with solids fraction $\phi = N\pi d^3/(6L^3)$. Each collision is resolved via a constant normal $0 \leqslant e \leqslant 1$ and tangential $-1 \leqslant \beta \leqslant 1$ restitution coefficient:

$$\boldsymbol{n} \cdot \boldsymbol{u}'_{12} = -e(\boldsymbol{n} \cdot \boldsymbol{u}_{12}), \tag{2.1}$$

$$\boldsymbol{n} \times \boldsymbol{u}'_{12} = -\beta(\boldsymbol{n} \times \boldsymbol{u}_{12}), \tag{2.2}$$

where $\boldsymbol{u}_{12} = \boldsymbol{u}_1 - \boldsymbol{u}_2 - (R\boldsymbol{\omega}_1 + R\boldsymbol{\omega}_2)$ is the relative velocity at the point of contact, $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ are the pre-collisional translational velocities of particle 1 and 2, R is the particle radius, $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ are the pre-collisional angular velocities of particle 1 and 2, a prime indicates post-collisional velocities, and $\boldsymbol{n}$ is the unit vector pointing from the centre of particle 2 to that of particle 1. Coefficient β governs the relative tangential velocity *at the point of contact* (i.e. relative surface velocity), where $\beta = -1$ represents frictionless particles (zero tangential impulse, $\boldsymbol{J}_t$), $\beta = 0$ gives maximum energy dissipation (zero relative tangential velocity), and $\beta = 1$ represents elastically rough particles (maximum $\boldsymbol{J}_t$ and fully reversed, relative tangential velocity). Accordingly, the post-collisional velocities (McNamara & Luding 1998; Hoomans *et al.* 2001) are given by

$$\boldsymbol{u}_1' = \boldsymbol{u}_1 - \frac{(1+e)}{2}\boldsymbol{u}_n - \frac{(1+\beta)}{7}\boldsymbol{u}_t, \tag{2.3}$$

$$\boldsymbol{u}_2' = \boldsymbol{u}_2 + \frac{(1+e)}{2}\boldsymbol{u}_n + \frac{(1+\beta)}{7}\boldsymbol{u}_t, \tag{2.4}$$

where $\boldsymbol{u}_n = [(\boldsymbol{u}_1 - \boldsymbol{u}_2)\boldsymbol{\cdot}\boldsymbol{n}]\boldsymbol{n}$ is the normal component of relative velocity, $\boldsymbol{u}_t = [(\boldsymbol{u}_1 - \boldsymbol{u}_2 - (R\boldsymbol{\omega}_1 + R\boldsymbol{\omega}_2)\times\boldsymbol{n})\boldsymbol{\cdot}\boldsymbol{t}]\boldsymbol{t}$is the tangential component of relative velocity, and $\boldsymbol{t}$ is the tangential unit vector. Collisions resolved in this manner are known as 'sticking' (even if the relative tangential velocity is non-zero). After confirming that our MD predictions for the ratio of particle rotational to translational energy coincide with previous theory (McNamara & Luding 1998), we used this expression for initialization.

It is worthwhile to note that other, more complex friction models are available; see, for example, Hoomans *et al.* (2001), who utilize a Coulomb friction coefficient $\mu = 0$ to ∞ for sliding-type collisions along $\beta = 0$ to 1 for sticking-type (non-sliding) collisions. Our motivation for instead using the simple β-only friction model is that this is the only friction model that has been incorporated into kinetic-theory-based continuum descriptions of granular flows. Comparison between such theoretical descriptions and our MD data is a secondary objective of our work. It is worth emphasizing, however, that the main finding emanating from our work (i.e. particles with high levels of friction may serve to attenuate instabilities compared to their frictionless counterparts) is true regardless of which friction model is used, as discussed in § 3. Some differences resulting from the use of different friction models are also discussed for the benefit of readers who may be interested in such details as they relate to other work.

At the centre of this work is the identification of a critical dimensionless length scale, L_{Vortex}/d or $L_{Cluster}/d$, associated with a vortex or clustering instability, respectively. In particular, for relatively small system sizes, no instabilities will develop regardless of the duration of the simulation. For larger system sizes, however, instabilities may be observed. This notion of a critical length scale is analogous to that of a critical Reynolds number for pipe flow – i.e. for a given fluid and flow rate, the critical Reynolds number will determine the pipe diameter that serves as the transition between laminar and turbulent flows. Accordingly, we perform MD simulations over a range of domain sizes L/d in order to identify the critical length scale. To determine if our MD simulations of a given domain size become unstable, a Fourier analysis (Mitrano *et al.* 2012) of the momentum and concentration fields is used to detect vortices and clusters, respectively. For each (integer) value of L/d simulated, 10 simulation replicates with varied initial conditions are run to determine a one-unit L/d range for the critical length scale (i.e. L_{Vortex}/d or $L_{Cluster}/d$). At the bottom of this range, the instability of interest has not been detected in any of the 10 replicates, while the instability has been detected at least once at the upper end of the range. Simulations end when the total number of collisions reaches $800N$ and are insensitive to simulated time.

3. Results

From this Fourier analysis, we have determined L_{Vortex}/d and $L_{Cluster}/d$ for $0.7 \leqslant e \leqslant 0.9$, $-1 \leqslant \beta \leqslant 1$ and $0.1 \leqslant \phi \leqslant 0.3$. These MD data are plotted in figure 2 as a

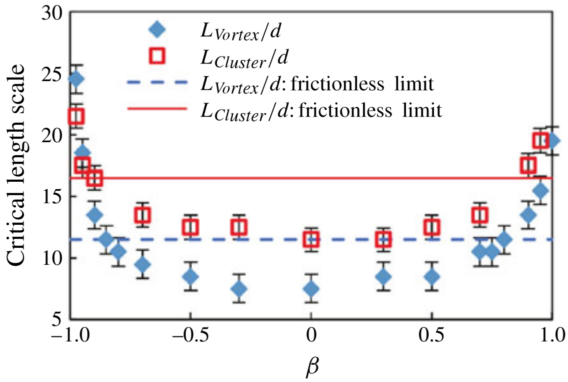

FIGURE 2. (Colour online) MD data for critical dimensionless length scale for vortex, L_{Vortex}/d (filled diamonds, blue online), and cluster, $L_{Cluster}/d$ (open squares, red online), instabilities as a function of tangential restitution coefficient for $e = 0.9$ and $\phi = 0.3$. Lines represent critical dimensionless length scales in the frictionless limit ($\beta = -1$).

function of β for $e = 0.9$ and $\phi = 0.3$. Surprisingly, the inclusion of friction does not always enhance instabilities relative to the frictionless case. First, nearly frictionless ($\beta \approx -1$) and highly rough ($\beta \approx 1$) systems attenuate instabilities compared to their frictionless counterparts ($\beta = -1$); in other words, larger domains are needed at these extremes for the instabilities to occur. Second, increasing the level of friction does not always enhance instabilities. Specifically, for $\beta > 0$, instabilities are attenuated as roughness (β) increases. This latter result is perhaps not as surprising as the former, since, as β increases from zero at constant e, frictional dissipation, and thus total dissipation, decreases (i.e. more energy is put towards reversing tangential relative velocity as opposed to being dissipated). Regarding the first observation, the attenuation of instabilities relative to the frictionless case is physically meaningful in highly rough ($\beta \approx 1$) systems, whereas its nearly frictionless counterpart ($\beta \approx -1$) is an artifact of the friction model, which only allows for sticking collisions. This $\beta \approx -1$ attenuation would be observed in practice only for $\beta \approx -1$ combined with a quite high Coulomb friction coefficient, which is unrealistic for practical materials. As further described below, the resulting accumulation of rotation that occurs in the HCS for $\beta \approx -1$ is also responsible for the discontinuity in the critical length scale observed at $\beta = -1$.

The intriguing attenuation of instabilities noted at the high-friction limit ($\beta \approx 1$) in figure 2 can be understood by considering the particle motion associated with velocity vortices and the collision outcomes that help to induce (or hinder) such instabilities. In particular, vortices (figure 1*a*) are characterized by the alignment of tangential particle translation. Such motion is not *directly* dependent on particle rotation. Instead, collisions that increase the alignment of tangential translation will help induce vortices. Such alignment can be quantified via another restitution coefficient,

$$e_t = -\frac{\boldsymbol{t} \cdot (\boldsymbol{u}_1' - \boldsymbol{u}_2')}{\boldsymbol{t} \cdot (\boldsymbol{u}_1 - \boldsymbol{u}_2)}, \tag{3.1}$$

which relates this pre-collisional, tangential component of the relative translational velocity to the post-collisional value. In contrast to β (2.2), e_t does not directly depend on particle rotation, since only translational motion is considered. Furthermore, e_t is a

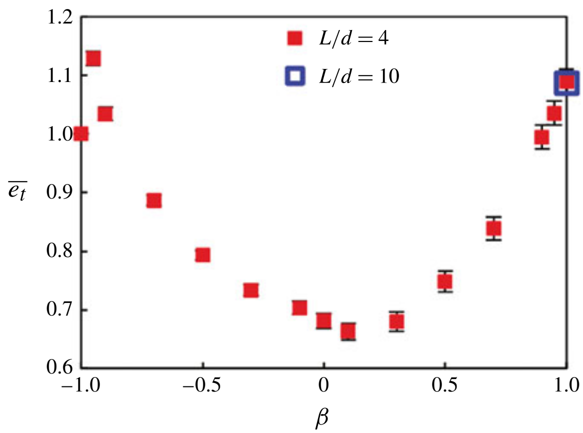

FIGURE 3. (Colour online) Translational tangential restitution coefficient $\overline{e_t}$ extracted from MD simulations as a function of β for $e = 0.9$ and $\phi = 0.3$ in a domain with $L/d = 4$ (filled squares, red online) and $L/d = 10$ (open square, blue online).

measurement characterizing collisional outcomes rather than dictating outcomes, as do the material-property inputs e and β. Collisions that align particle motion (and induce vortices) give rise to $|e_t| < 1$, while collisions that hinder vortex formation lead to $|e_t| > 1$. Frictionless interactions lead to $|e_t| = 1$. Hence, averaging e_t over a simulation provides insight into the critical length scales of figure 2. A $\log_{10}$ average is more appropriate than traditional averaging, which would inappropriately weight high values of $|e_t|$. In figure 3, $\overline{e_t} \equiv 10^{\overline{\log_{10}|e_t|}}$ is plotted as a function of β for (unstable) systems with $L/d \ll L_{Vortex}/d$. As expected, the shape of this curve mimics that of L_{Vortex}/d (figure 2), with values of $\overline{e_t} > 1$ occurring at the $\beta \approx -1$ and $\beta \approx 1$ limits.

An explanation of the β dependence of $\overline{e_t}$ (figure 3) stems from a consideration of the ratio of the rotational to translational kinetic energy (RE/KE), plotted in figure 4 as a function of β for $e = 0.9$ and $\phi = 0.3$. A theoretical prediction (Goldshtein & Shapiro 1995) of this ratio in HCS, based on identical collision rules, is also shown. Note that, although both RE and KE decay in time, their ratio remains constant in time according to both MD and theory. Excellent agreement was found between MD simulations and theory (Goldshtein & Shapiro 1995), supporting a previous assessment (McNamara & Luding 1998). This energy ratio plays an important role in the development of instabilities via the frictional coupling of rotation and translation. First, consider the behaviour when $RE > KE$, which occurs for highly rough ($\beta > 0.75$ for $e = 0.9$) particles (figure 4). (Note that $\beta > 0.75$ is also the region where L_{Vortex}/d and $L_{Cluster}/d$ (figure 2) and $\overline{e_t}$ (figure 3) become greater than their corresponding frictionless values.) For such systems of fast-spinning, highly frictional particles, collisions actually serve to increase the tangential component of relative translational velocity (figure 3), similar to a tennis ball with topspin converting rotation into tangential translation when the tennis ball collides with the surface. Specifically, the relative tangential translation between the ball and ground is increased via collision compared to a non-rotating or frictionless ball. As β increases towards unity, particles transfer greater portions of tangential momentum ($\boldsymbol{J}_t \propto (1+\beta)\boldsymbol{u}_t$; see (2.3) and (2.4)) to collision partners, resulting in increased relative tangential translation. This behaviour causes decreased alignment of particle motion ($|e_t| > 1$) and explains the attenuation of instabilities (figure 2) as particles approach perfect roughness, $\beta = 1$.

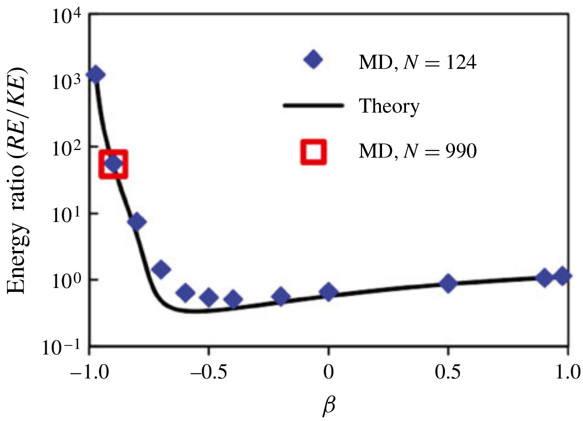

FIGURE 4. (Colour online) The ratio of rotational kinetic energy (*RE*) to translational kinetic energy (*KE*) as a function of β for $e = 0.9$ and $\phi = 0.3$. Data points are averages from MD simulations run for 1600 collision per particle with 124 particles (filled diamonds, blue online) and 990 particles (open square, red online). The line is the theoretical prediction (Goldshtein & Shapiro 1995).

As detailed in the previous paragraph, the attenuation of instabilities at high friction levels can be traced to the coupling of rotational and translation motion via a consideration of particle-level interactions. A similar conclusion is also reached via a consideration of continuum theory. In particular, the ratio of rotational to kinetic (translational) energy depends on both e and β (see e.g. Huthmann & Zippelius 1997). At high friction levels, this ratio can be less than unity, which goes hand in hand with a translational portion of the cooling rate less than its frictionless counterpart. In other words, part of the translational energy is transferred from the rotational to the translational degrees of freedom, thereby lessening the net cooling of translational energy. This reduction in the dissipation of translational energy, while still resulting in an increase in the total dissipation, reduces the likelihood of particle alignment (vortex instability).

Now consider the tangential momentum exchange $\boldsymbol{J}_t \propto (1+\beta)\boldsymbol{u}_t$ as $\beta \rightarrow -1$. The extremely high rates of rotation in this region (figure 4), despite small $(1+\beta)$ values, give rise to tangential impulses that, again, increase relative tangential translation (at the cost of rotation), as shown in figure 3. With such high rotation, particles with aligned tangential translation will tend to be less aligned after colliding. In this way, high rotational velocities, which give rise to high tangential impulses, hinder vortex formation. This result corroborates a previous finding in which rotational driving hindered instabilities (Cafiero, Luding & Hermann 2002).

This consideration of e_t in conjunction with RE/KE provides an explanation of the attenuation of vortices (increased L_{Vortex}/d) observed in the low- and high-friction limits (figure 2), and a similar argument applies to cluster attenuation. Namely, the increased tangential component of relative translation ($|e_t| > 1$) for $\beta \approx 1$ and $\beta \approx -1$ leads to increased tangential separation after particles collide. This increased separation hinders cluster formation relative to the frictionless case (where $|e_t| = 1$). Another mechanism for cluster attenuation, since vortices precede clusters, is the attenuation of vortices themselves, because viscous heating, which leads to cluster formation

(Goldhirsch *et al.* 1993; Goldhirsch & Zanetti 1993; Brey, Ruiz-Montero & Cubero 1999; Soto *et al.* 2000), is smaller for a more homogeneous velocity field.

Recall that we previously described the attenuation of instabilities (figure 2) at the high-friction limit ($\beta \approx 1$) as physically meaningful, while referring to analogous behaviour at low friction levels ($\beta \approx -1$) as an artifact of the friction model used. We hypothesized that this artifact stems from the sticking collisions (2.2) allowed by the model and would not be present in more robust friction models that also allow for (sliding) Coulomb friction contacts. Specifically, the extremely high RE/KE at $\beta \approx -1$ (figure 4) would have a severely diminished effect in a more robust model, since such high levels of rotation would result in sliding collisions. Sliding collisions would lead to smaller tangential impulses (than sticking collisions) near $\beta = -1$, since such impulses are proportional to the relative normal velocity at contact rather than its tangential counterpart. To test our hypothesis, we performed additional simulations with a more robust friction model (Hoomans *et al.* 2001), which allows for both sticking and sliding. The remainder of this paragraph will focus on the results from this more robust model, in which two $\boldsymbol{J}_t$ values are calculated for each collision, the smaller of which is used to resolve the collision: (i) $\boldsymbol{J}_t$ based on sticking interactions (with constant $\beta = 0$–1); and (ii) $\boldsymbol{J}_t$ based on sliding interactions (with constant Coulomb friction coefficient $\mu = 0$–∞). We found that many similarities exist between this more robust model and the simpler β-only model. Most importantly, instabilities are attenuated with highly frictional particles (β and μ of the order of 1), while moderate friction levels (moderate values of μ for all values of β) enhance instabilities compared to the frictionless case. However, the more robust friction model indicates that small levels of friction (small values of μ for all values of β) give to critical length scales that coincide with those of the frictionless case, which is contrary to the β-only model and consistent with our interpretation that the behaviour of the β-only model near the frictionless limit is a model artifact.

One final aspect worth discussing is that the increase in relative tangential translation imposed by the extremely high temperature ratio at $\beta \rightarrow -1$ does not affect vortex and cluster formation equally. As outlined above, vortices depend on the relative tangential translation between colliding particles, while cluster formation depends on particle separation. Thus, collisions that selectively increase relative tangential translation (rather than normal translation) hinder vortices to a greater extent than clusters. This selective attenuation of vortices is exemplified in figure 2, where, for approximately $\beta \leqslant -0.95$, clusters manifest more readily (at smaller L/d) than vortices.

Overall, our MD findings of the attenuation of instabilities in highly rough systems compared to their smooth counterparts are surprising given the conventional wisdom that instabilities are enhanced with increased dissipation. As a first attempt to corroborate these findings theoretically, we use a simple theoretical construct to estimate the critical length scales for instability. The basis of this construct is the granular continuum theory developed by Garzó & Dufty (1999), in which a Chapman–Enskog expansion was performed using the Enskog equation as a starting point. They considered inelastic, but frictionless, particles. The resulting balances for mass, momentum and granular energy take the form

$$D_t n + n\nabla \cdot \boldsymbol{u} = 0, \tag{3.2}$$

$$\rho D_t \boldsymbol{u} + \nabla \boldsymbol{P} = \boldsymbol{0}, \tag{3.3}$$

$$D_t T + \frac{2}{3n}(\nabla \cdot \boldsymbol{q} + \boldsymbol{P} : \nabla \boldsymbol{u}) = -\zeta T, \tag{3.4}$$

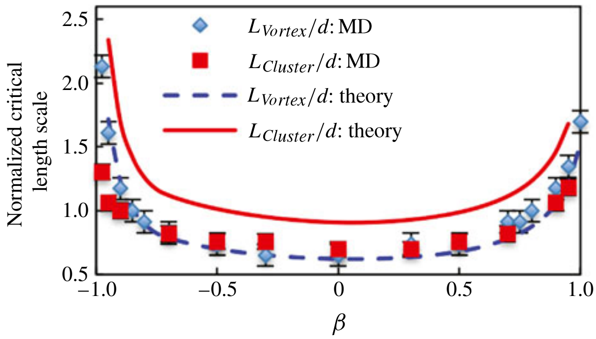

FIGURE 5. (Colour online) Plot of L_{Vortex}/d (filled diamonds, dashed curve; blue online) and $L_{Cluster}/d$ (filled squares, full curve; red online), normalized to the respective frictionless-particle MD prediction (see figure 2), as a function of the tangential restitution coefficient, as predicted by MD (data points) and theory (curves) for $e = 0.9$ and $\phi = 0.3$. Values greater than 1 refer to attenuation of instabilities relative to the frictionless case.

where $D_t = \partial_t + \boldsymbol{u} \cdot \nabla$ is the material derivative, $\rho = mn$ is the mass density, m is the particle mass, n is the number density of particles, $\boldsymbol{u}$ is the local (mean) velocity, T is the granular temperature, $\boldsymbol{P}$ is the stress tensor, $\boldsymbol{q}$ is the heat flux, and ζ is the cooling rate of granular energy arising from particle collisions. Explicit forms of the transport coefficients (shear viscosity, bulk viscosity, conductivity and Dufour coefficient) appearing in the fluxes $\boldsymbol{P}$ and $\boldsymbol{q}$ were also obtained, but are not repeated here for the sake of brevity. Garzó (2005) later performed a linear stability analysis of the above theory (Garzó & Dufty 1999) for inelastic, frictionless particles. The critical domain sizes found from his stability analysis are

$$L_{Cluster} = \frac{5\pi}{4}\sqrt{\frac{5\pi}{2}}\chi\lambda_0\sqrt{\frac{\kappa^* C_\rho - \mu^*}{\zeta_0^*\left(2g - C_\rho\right)}}, \tag{3.5}$$

$$L_{Vortex} = \frac{5\pi}{4}\sqrt{\pi}\chi\lambda_0\sqrt{\frac{\eta^*}{\zeta_0^*}}, \tag{3.6}$$

where η^* is the shear viscosity, κ^* is the thermal conductivity, μ^* is the Dufour coefficient, which relates concentration gradients to the heat flux, χ is the pair correlation function at contact, λ_0 is the mean free path, $g = 1 + \phi(\partial/\partial\phi)\ln\chi$, and $C_p = 1 + g - g/(1 + 2\phi(1 + e)\chi)$. Asterisks indicate the same non-dimensionality as used by Garzó (2005). In this study, we insert a cooling rate (ζ_0^*) that incorporates both inelasticity and friction (Goldshtein & Shapiro 1995) into the stability-analysis expressions shown above. All other quantities are for frictionless particles (Garzó 2005). Consequently, the theoretical predictions shown here are based on an *ad hoc* combination of descriptions that incorporate friction (where available) and others that do not.

The results from this simplified theory are provided in figure 5, along with MD predictions of L_{Vortex}/d and $L_{Cluster}/d$, all normalized to their respective frictionless-particle MD predictions. Excellent agreement occurs for the case of velocity vortices. This (somewhat surprising) result suggests that the influence of friction on the cooling rate is critical to accurately describing granular flows via hydrodynamics. Such agreement is encouraging, since it implies that complexities associated with incorporating friction into other transport coefficients could potentially be avoided without a significant loss of accuracy. The discrepancy between the linear stability analysis of theory and MD simulations for the case of clusters is also telling. In particular, previous studies (Goldhirsch *et al.* 1993; Goldhirsch & Zanetti 1993; Brey *et al.* 1999; Soto *et al.* 2000) suggest that nonlinear mechanisms (e.g. viscous heating) are important in cluster formation. The results contained in figure 5 are consistent with this notion, since the linear theory overpredicts $L_{Cluster}/d$ compared to MD (owing to the importance of nonlinear mechanisms). Finally, although not shown in figure 5, the theory also predicts the same discontinuity exhibited by MD at $\beta=-1$. As discussed above, this discontinuity has its origin in the friction model, which is the same for MD and theory. Nonetheless, these interpretations of MD–theory comparisons should be taken with a grain of salt given the *ad hoc* nature of the theory used (i.e. the effect of friction is incorporated in ζ_0^*, but similar expressions are not yet available for κ^*, μ^* or η^*). Also note the striking similarity between the dependence of instabilities (figure 5) and $\overline{e_t}$ (figure 3) on β.

4. Summary

Our work shows, via MD simulations and theory, that high levels of friction can actually attenuate instabilities relative to the frictionless case. This result is surprising, since previous work shows that increased (inelastic) dissipation leads to enhanced instabilities, whereas here we see that adding another source of dissipation (friction) on top of inelastic dissipation can result in the opposite behaviour. We also find that, when sticking collisions dominate, increasing roughness (β) from moderate levels ($\beta>0$) attenuates instabilities. The physical origin of both findings is traced to the coupling of rotational and translational motion. The results presented here are representative of findings for lower restitution ($e=0.7$) and solids fraction ($\phi=0.1$). The typical cross-overs between enhancing and attenuating instabilities (for this parameter space) occur when $|\beta/e|\approx 0.8$–1 (i.e. when normal and tangential dissipation are of the same order) and are insensitive to ϕ.

Follow-on work is needed to confirm the robustness of these findings beyond the HCS, though some previous work appears encouraging in this regard. Specifically, Alam & Nott (1997) showed through a linear stability analysis of an *ad hoc* frictional, hydrodynamic model that specific levels of friction can actually stabilize an inelastic granular shear flow, though they also reported that particles that only dissipate energy through friction are stable regardless of the level of friction, which is in conflict with our current findings for $e=1$ (not shown here for the sake of brevity, but they have a qualitatively similar form to figure 2 for $-1<\beta<1$). Our observation that friction may attenuate or enhance instabilities is expected to impact not only granular flows that display such instabilities (e.g. planetary rings, asteroid belts and ejection of lunar soil upon spacecraft landing (Immer *et al.* 2011)), but also their gas–solid counterparts. Examples of the latter include the well-documented clustering phenomenon in circulating fluidized bed reactors (Gidaspow 1994; Jackson 2000; Fan & Zhu 2005) and roping observed in pneumatic transport lines (Yilmaz 1997; Schallert & Levy 2000; Yilmaz & Levy 2001). It is also worth noting that the high roughness levels investigated here occur in a range of materials, including biomass feedstock for energy production (Phani, Lope & Scoenau 2010) and materials made with customized roughness (Majidi *et al.* 2006). Finally, our results may have implications for other forms of energy dissipation experienced by the particles, including cohesion.

On a final note, it is worth remarking that our work bears some resemblance to previous work by Louge & Adams (2002), in that a counterintuitive observation can be traced to the rotational–translational coupling in both works. The two observations are quite distinct, nonetheless, since Louge & Adams (2002) observed experimentally that a restitution coefficient greater than unity is realistic for dissipative particles colliding at very oblique angles (whereas $e > 1$ would typically be associated with unphysical, energy-producing collisions). On the other hand, we observed that increasing energy dissipation via the addition of friction could serve to attenuate instabilities (whereas normally an increase in dissipation is associated with the enhancement of instabilities) in some parts of the parameter space. Because all of our simulations were performed with $e < 1$, the two sets of observations are not interrelated. With regard to the bigger picture, however, both works highlight the importance of frictional effects and more specifically how the frictional–translational coupling can lead to a variety of surprising behaviours. Such findings underscore the importance of including frictional effects in future theoretical contributions, as until now the bulk of work has focused on inelastic effects alone.

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