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Excitation of heater-enhanced plasma and ion lines near the reflection level of a high-frequency pump wave

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Abstract. A theory of enhanced plasma and ion lines near the reflection height of a pump wave is presented. It is argued that the high-frequency pressure of the pump wave and the averaged pressure of plasma waves localized in small-scale cavitons contribute to the rapid creation of a cavity in the main Airy maximum. Langmuir waves excited later by parametric decay instability are trapped in such a nonstationary cavity. Analytical expressions for adiabatic eigenmodes of Langmuir waves in a nonstationary cavity are presented. Collisionless attenuation of the eigenmodes of Langmuir waves trapped in the cavity is investigated. It is shown that parametric decay instability in a cavity may account for the excitation of plasma and ion-acoustic waves measured by EISCAT UHF radar.

1. Introduction

Under the action of a strong high-frequency (HF) electromagnetic wave, various nonlinear processes take place in the F-region of the ionosphere (see e.g. Gurevich 1978; DuBois et al. 1993; Sulzer and Fejer 1994; Stubbe and Hagfors 1997; Mjølhus 1998). In an inhomogeneous ionospheric plasma, there are three particular regions where the strongest excitations of electrostatic waves are expected. One of them is localized near the reflection level of the ordinary electromagnetic pump wave, the second corresponds to the so-called matching height, the third is situated at the upper-hybrid resonance level. Different diagnostic tools are used to investigate interesting phenomena in these regions. One of the most powerful methods that gives important information about Langmuir and ion-acoustic waves excited in the ionosphere due to HF heating is incoherent scatter radar. In Tromsø, Norway, there is a unique opportunity to obtain information about excited waves with considerably different k-numbers using EISCAT radars with the frequencies $f = 224$ MHz (VHF radar) and $f = 933$ MHz (UHF radar) and by making bistatic observations in Kiruna and Sodankylä. With the help of chirped radar technique (Hagfors 1982) unusual behavior of the enhanced ion line was detected (Isham et al. 1990). The ion line was measured by UHF radar near the reflection height of the pump wave (bottomside enhancement) and also above the maximum of the F-layer at the level where the electron concentration is the same as at the reflection point of a pump wave (topside enhancement). The natural plasma line excited by photoelectrons and the usual enhanced plasma line at the matching height were not simultaneously detected in the experiment. Previous experiments with UHF radar occasionally showed the existence of the enchanced ion line without simultaneously excited plasma lines (Stubbe et al. 1985; Kohl et al. 1987). The ion line was detected below the reflection height of the pump wave. A possible explanation for this, based on the decay of the mother Langmuir wave into daughter Langmuir and ion-sound waves, was given by Stubbe et al. (1992). In the experiment performed by Isham et al. (1990), on the other hand, the enhanced ion line was measured within a few milliseconds after a cold start near the reflection height (within 1 km), where the matching conditions in a nondisturbed plasma are not fulfilled. The enhanced plasma line was also measured near the reflection level of the pump wave in the heating experiments. First it was detected in Arecibo with the help of Barker-coded 430 MHz radar pulses (Muldrew and Showen 1977), and later in Tromsø.

The aim of the present paper is to give a theoretical analysis of the processes associated with the almost-instantaneous appearance of enhanced ion and plasma lines near the reflection level of a pump wave. We propose the following mechanism. In the main Airy maximum, an electromagnetic pump wave and small-scale cavitons produce significant HF pressure. The maximum of the pressure is localized in the central part of the Airy maximum. Due to such pressure, a few milliseconds after the heater is switched on, a cavity is formed in the center of the main Airy maximum that is deep enough to retain at least several lowest modes of the Langmuir waves. In such a cavity, the electromagnetic pump wave decays into Langmuir and ion-acoustic waves. The formation of density depletions of different scales near the reflection level of a pump wave has been discussed in several papers (e.g. Morales and Lee 1977; Muldrew 1992; DuBois et al. 1993). Morales and Lee (1977) investigated cavity formation and its dynamics in the case of a laser plasma. Muldrew (1992) argued that magnetospheric ducts can play a significant role in parametric processes near the reflection level of the pump wave. DuBois et al. (1993) investigated the formation of small-scale cavitons in the ionosphere and the interaction of waves in such plasma depletions. We consider a cavity with scales of the order of tens of meters stretched in the horizontal plane and a near-vertical magnetic field line. Such a cavity is formed due to the rapid longitudinal rearrangement of electrons and ions. The idea, based on experimental data, that plasma depletion with the mentioned scales could be quickly formed near the reflection level of the pump wave was previously introduced by Birkmayer et al. (1986) and Isham et al. (1987). Note that such a cavity is a nonstationary one. We discuss the possibility of introducing eigenmodes (adiabatic modes) for Langmuir waves, and obtain analytical expressions for them in a model parabolic cavity changing in time. Langmuir waves are trapped in such a cavity, and grow in time due to the parametric decay instability. Matching conditions for decay instability in a nonstationary cavity are fulfilled only within some restricted time interval for a certain eigenmode of Langmuir waves. Thus the amplification of the excited waves is achieved.

The paper is organized as follows. First, the basic equations describing Langmuir wave trapped in a cavity are derived. Then collisionless attenuation of the eigenmodes of Langmuir waves trapped in a cavity is discussed. The creation of a plasma depletion in the presence of the eletromagnetic pump and Langmuir waves is investigated in Sec. 4. In this section the role of thermal effects in the formation of the depletion is briefly analysed. Adiabatic modes of Langmuir waves trapped in the cavity are considered in Sec. 5. In Sec. 6, the decay instability of the pump electromagnetic wave into Langmuir and ion-acoustic waves is investigated.

2. Basic equations

Let a powerful HF electromagnetic wave with frequency ω_0 be radiated into the ionosphere. The ionosphere is supposed to be homogeneous in the horizontal plane and slowly changing with height. It is convenient to introduce a system of coordinates with the z axis vertical, the y axis horizontal in the meridional plane, and the x axis orthogonal to z and y.

The equation for a Langmuir wave in a homogeneous medium is (Ginzburg 1961)

$$\omega_L^2 = \omega_{Pe}^2 + 3k^2 v_{Te}^2 + \omega_{He}^2 \sin^2\theta. \tag{1}$$

Here $\omega_{Pe} = (e^2 N/\epsilon_0 m)^{1/2}$ is the plasma frequency, $-e$ and m are the electron charge and mass, N is the plasma density, ϵ_0 is the dielectric permeability of the vacuum, $\omega_{He} = eB/m$ is the electron gyrofrequency, B is the Earth's magnetic field, $v_{Te} = (T_e/m)^{1/2}$ is the electron thermal speed, T_e, is the electron temperature (in energy units), k is the wavenumber, θ is the angle between the wave vector and the magnetic field, which is assumed to be small ($\sin\theta^2 \ll 1$, and therefore $\sin^2/\theta \approx \theta^2$). In inhomogeneous media, the equation for Langmuir waves must be discussed in greater detail.

We start with the kinetic equation for the perturbation of the electron distribution function in the form $F_e(\mathbf{r}, \mathbf{v}, t) = F_1(\mathbf{r}, \mathbf{v}, t)\exp(i\omega_1 t)$:

$$i\omega_1 F_1 + (\mathbf{v}\cdot\boldsymbol{\nabla})F_1 - \frac{e}{m}(\mathbf{v}\times\mathbf{B})\cdot\frac{\partial F_1}{\partial\mathbf{v}} = \frac{e}{m}\mathbf{E}\cdot\frac{\partial F_0}{\partial\mathbf{v}}. \tag{2}$$

Here $\mathbf{v}$ is the electron velocity and $\mathbf{E}\exp(i\omega_1 t)$ is the polarization electric field. The function F_1 is assumed to change slowly with t to compare with the frequency ω_1, and hence its derivative with respect to t in the first approximation can be neglected. The nondisturbed electron distribution function is supposed to be Maxwellian.

$$F_0 = \frac{N(z)}{(2\pi v_{Te}^2)^{3/2}}\exp\left(-\frac{v_\parallel^2 + v_\perp^2}{2v_{Te}^2}\right), \tag{3}$$

where $v_\parallel$ is the component of the electron velocity along the magnetic field line and $\mathbf{v}_\perp$ is the projection of the electron velocity on the plane orthogonal to $\mathbf{B}$. The equation for the electric field amplitude $\mathbf{E}$ is

$$\boldsymbol{\nabla}\cdot\mathbf{E} = -\frac{e}{\epsilon_0}\int F_1\, dv_\parallel\, d^2 v_\perp \tag{4}$$

For a Langmuir wave excited along the z axis, (2) takes the form

$$\left(i\omega_1 + v_z\frac{\partial}{\partial z}\right)F_1 + \omega_{Be}\frac{\partial F_1}{\partial\alpha} = -\frac{ev_z}{mv_{Te}^2}\frac{\partial\phi}{\partial z}F_0. \tag{5}$$

The angle α is measured from the x axis in the plane orthogonal to the Earth's magnetic field B. The polarization electric field $\mathbf{E}$ is expressed as $\mathbf{E} = -\boldsymbol{\nabla}\phi$. Note that the vertical component of the velocity, v_z, is a function of the longitudinal ($v_\parallel$) and transverse ($v_\perp$) velocities.

The distribution function F_1 can be represented as a sum of two parts: $F_1 = \bar{F} + \tilde{F}(\alpha)$, where $\bar{F}$ does not depend on α and $\tilde{F}(\alpha)$ has zero average, meaning $\int_0^{2\pi}\tilde{F}(\alpha)\, d\alpha = 0$. To obtain a simple approximate equation for the electron distribution $\bar{F}$, we assume that plasma concentration changes rather slowly along the z axis and introduce the following small parameter: $v_{Te}/l_z\omega_1 \ll 1$, where l_z is the characteristic scale of plasma inhomogeneity along the z axis. Note that v_z for

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small angles θ takes the form $v_z \approx v_\parallel - \theta v_\perp \sin\alpha$, and $\tilde{F}(\alpha)$ can be expressed as a sinusoidal variation in α with in-phase and quadrature components that can be eliminated to obtain an equation for $\bar{F}$ alone. As a result, we come to the following approximate equation for $\bar{F}$:

$$\left(i\omega_1 + v_\parallel \frac{\partial}{\partial z}\right)\bar{F} = -\frac{e}{mv_{Te}^2}\left[\frac{\partial\phi}{\partial z}v_\parallel F_0 + \frac{\theta^2}{2}\frac{i\omega_1 v_\perp^2}{\omega_1^2 - \omega_{He}^2}\frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial z}F_0\right)\right]. \tag{6}$$

In a homogeneous medium, $N_0 = \text{const}$, and taking the dependence of $\bar{F}$ and ϕ on the z coordinate as $\exp(-ikz)$, (6) is reduced to

$$i(\omega_1 - kv_\parallel)\bar{F} = \frac{iek\phi F_0}{mv_{Te}^2}\left(v_\parallel + \frac{\theta^2}{2}\frac{\omega_1 k v_\perp^2}{\omega_1^2 - \omega_{He}^2}\right). \tag{7}$$

Substituting (7) into (4), we obtain the usual dispersion relation (1) for Langmuir waves. In an inhomogeneous plasma, it is convenient to represent the distribution function $\bar{F}$ according to (6) in the form

$$\bar{F} = \frac{ie}{\omega_1 m v_{Te}^2}\left[\frac{\partial\phi}{\partial z}v_\parallel F_0 + \frac{\theta^2}{2}\frac{i\omega_1 v_\perp^2}{\omega_1^2 - \omega_{He}^2}\frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial z}F_0\right)\right] + \frac{iv_\parallel}{\omega_1}\frac{\partial\bar{F}}{\partial z}. \tag{8}$$

The last term on the right-hand side of (8) can be considered as a small perturbation and calculated subsequently with the required accuracy. Again substituting the distribution function (8) into (4), we find the equation for plasma waves in a weakly inhomogeneous medium:

$$\frac{\partial^2\Phi}{\partial z^2} + \frac{\omega_1^2 - \omega_{Pe}^2(z) - \theta^2\omega_{He}^2}{3v_{Te}^2}\Phi = 0. \tag{9}$$

Here $\Phi = \partial\phi/\partial z$ is the eigenfunction describing Langmuir waves in an inhomogeneous plasma. Suppose that Langmuir waves are excited in a plasma cavity where the following model of the plasma distribution applies:

$$N(z) = N_0\left\{1 - A\exp\left[-\left(\frac{z - z_0}{b}\right)^2\right]\right\} \tag{10}$$

Here A is a small amplitude, b is the scale of the inhomogeneity, and z_0 is the center of the cavity. Near the local minimum of plasma concentration $|z| < b$, the distribution of the plasma takes the form

$$N(z) \approx N_0(1 - A)\left[1 + \left(\frac{z - z_0}{a}\right)^2\right], \tag{11}$$

where $a = b/A^{1/2}$. For a cavity (11), well-known solutions describing stationary eigenstates exist (Landau and Lifshitz 1958). Equation (9) with plasma distribution (11) can be represented in the dimensionless form

$$\frac{\partial^2\Phi}{\partial\zeta^2} + (\epsilon - \zeta^2)\Phi = 0, \tag{12}$$

where

$$\zeta = \left(\frac{\omega_{Pe}^{(0)}}{\sqrt{3}v_{Te}a}\right)^{1/2}(z - z_0), \qquad \epsilon = \frac{(\omega_1^2 - \omega_{Pe}^{(0)2} - \theta^2\omega_{He}^2)a}{\sqrt{3}\omega_{Pe}^{(0)}v_{Te}};$$

$\omega_{Pe}^{(0)}$ is the electron plasma frequency in the local minimum of the concentration.

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Equation (12) has a discrete set of eigenfunctions

$$\Phi_n = \frac{1}{(\sqrt{\pi} n! 2^n)^{1/2}} \exp(-\tfrac{1}{2}\zeta^2) H_n(\zeta) \tag{13}$$

corresponding to eigenvalues $\epsilon_n = 2n + 1$. Here H_n are Hermite polynomials, $n = 0, 1, 2, \ldots$. The eigenvalues ϵ_n determine the set of eigenfrequencies $\omega_{1,n} = \omega_{Pe}^{(0)} + \Delta\omega_n$ of the Langmuir waves, where the corrections $\Delta\omega_n$ are

$$\Delta\omega_n = (2n + 1)\frac{\sqrt{3} v_{Te}}{2a}. \tag{14}$$

Near the center of the cavity $|z - z_0| < b$, only a finite number of modes can be retained

$$n < \frac{b\omega_1 A^{1/2}}{2\sqrt{3} v_{Te}}. \tag{15}$$

In typical ionospheric conditions, the restriction (15) is weak and the parabolic model (11) can be used even for high modes $n \gg 1$. For such modes, the approximation of geometric optics is valid. In the general case, eigenfunctions describing stationary states of (12) in the approximation of geometric optics take the form

$$\Phi(z) = \frac{B}{[k(z)]^{1/2}} \cos\left(\int_{z_1}^{z} k(z)\, dz - \frac{\pi}{4} \right). \tag{16}$$

Here B is a constant and $k(z) = k_n(z)$ are discrete wavenumbers, corresponding to different states n,

$$k_n(z) = \frac{\left[\omega_{1,n}^2 - \omega_{Pe}^2(z) - \theta^2 \omega_{He}^2\right]^{1/2}}{\sqrt{3} v_{Te}}. \tag{17}$$

These wavenumbers can be found from the equation

$$\int_{z_1}^{z_2} k_n(z) dz = n + \tfrac{1}{2}, \tag{18}$$

where z_1 and z_2 are the turning points with $k_n(z_{1,2}) = 0$

3. Attenuation of Langmuir waves in a cavity

In the homogeneous case, the usual expression for Landau damping in plasma with Maxwellian electron distribution follows from (6).

$$\nu_1^{(L)} = \sqrt{\frac{\pi}{8}} \frac{\omega_{Lo}^4}{k^3 v_{Te}^3} \exp\left(-\frac{\omega_{Lo}^2}{2k^2 v_{Te}^2} \right), \tag{19}$$

where $\omega_{Lo}^2 = \omega_{Pe}^2 + 3k^2 v_{Te}^2$. The damping of the Langmuir waves in a cavity may differ significantly from the damping in a homogeneous plasma. The eigenmodes in a cavity, Φ_n, are localized functions, and hence wavepackets rather than plane waves with fixed k-numbers. To include attenuation of Langmuir waves in a cavity (11), we start with (6). It is convenient to perform calculations in k space. As a real cavity is rather shallow, the plasma inhomogeneity is a small term (proportional to θ^2) in (6) and can be neglected. As a result, (6) is reduced to

$$i(\omega_1 - kv_\parallel)\bar{F}_k = C(v_\parallel, v_\perp) \left[v_\parallel \left(1 - \frac{1}{a^2} \frac{\partial^2}{\partial k^2} \right) E_k + \frac{\theta^2}{2} \frac{\omega_1 k v_\perp^2}{\omega_1^2 - \omega_{He}^2} E_k \right], \tag{20}$$

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where

$$\bar{F}_k = \frac{1}{\sqrt{2\pi}} \int \exp(ikz)\, \bar{F}\, dz, \quad E_k = -\frac{1}{\sqrt{2\pi}} \int \exp(ikz)\, \frac{\partial \phi}{\partial z} dz,$$

$$C(v_\parallel, v_\perp) = \frac{eN_0}{(2\pi)^{3/2} m v_{Te}^5} \exp\left(-\frac{v_\perp^2 + v_\parallel^2}{2v_{Te}^2}\right).$$

Substituting $\bar{F}_k$ into Poisson's equation (4), we find the following equation for Langmuir waves:

$$\frac{\partial^2 \Phi_k}{\partial k^2} + a^2 \frac{\omega_1^2 - (\omega_{Pe}^{(0)2} + \theta^2 \omega_{He}^2 + 3k^2 v_{Te}^2) - i2\omega_1 \hat{\nu}_1^{(L)}}{\omega_{Pe}^{(0)2}} \Phi_k = 0, \tag{21}$$

where $\hat{\nu}_1^{(L)}$ coincides with (19) in k-space. Now we introduce instead of k the dimensionless variable $\kappa = [\sqrt{3} v_{Te} a / \omega_{Pe}(0)]^{1/2} k$. The result is

$$\frac{\partial^2 \Phi(\kappa)}{\partial \kappa^2} + \left(\varepsilon - \kappa^2 - i2 \frac{\hat{\nu}_1^{(L)} a}{\sqrt{3} v_{Te}}\right) \Phi(\kappa) = 0, \tag{22}$$

where

$$\varepsilon = \frac{(\omega_1^2 - \omega_{Pe}^{(0)2} - \theta^2 \omega_{He}^2) a}{\sqrt{3} v_{Te} \omega_{Pe}^{(0)}}.$$

Assuming that Landau damping is small compared with the frequency shift between the adjacent eigenfrequencies $\Omega = \Delta\omega_n - \Delta\omega_{n-1}$ (see (14)), we use the theory of perturbations with discrete spectrum to find the collisionless damping for the nth eigenmode in the cavity (see Landau and Lifshitz, 1958):

$$\nu_{1,n}^{(L)} = \frac{3^{3/4} \pi^{1/2}}{2^{3/2}} \frac{\omega_{1,n}^{5/2} a^{3/2}}{v_{Te}^{3/2}} \int \frac{|\Phi_n(\kappa)|^2}{|\kappa|^3} \exp\left(-\frac{\sqrt{3}\omega_{1,n} a}{2 v_{Te} \kappa^2}\right) d\kappa \tag{23}$$

Here $\Phi_n(\kappa)$ is the nth Fourier component of the eigenfunction $\Phi_n(\zeta)$ in the cavity:

$$\Phi_n(\kappa) = \frac{i^n}{(\sqrt{\pi} n! 2^n)^{1/2}} H_n(\kappa) \exp(-\tfrac{1}{2}\kappa^2);$$

$H_n(\kappa)$ is the Hermite polynomial of argument κ. Taking into account the plasma distribution for the lowest state $n = 0$ in the cavity,

$$\Phi_0(\kappa) = \frac{1}{(\pi)^{1/4}} \exp(-\tfrac{1}{2}\kappa^2), \tag{24}$$

collisionless damping of this state can be easily found with the help of the method of stationary phase:

$$\nu_{1,0}^{(L)} \approx 0.1 \omega_{1,0} q^{3/2} \exp(-2q), \tag{25}$$

where $q = (\sqrt{3} aw / 2v_{Te})^{1/2}$. Note that for typical conditions in the F-region of the ionosphere ($v_{Te} \approx 10^7$ cm s^{-1} $\omega_0 \approx 3 \times 10^7$ s^{-1}, $a \approx 10^3$–10^5 cm, the parameter q is large ($q \gg 1$).

Similarly, for the state Φ_1,

$$\Phi_1(\kappa) = \frac{i\kappa}{(2\sqrt{\pi})^{1/2}} \exp(-\tfrac{1}{2}\kappa^2), \tag{26}$$

the Landau damping is equal to

$$\nu_{1,1}^{(L)} \approx 0.1\omega_{1,1} q^{5/2} \exp(-2q). \tag{27}$$

With the growth of the mode number, n the attenuation increases. For the lowest-order modes this is clear from direct comparison of (25) and (27). If a state with high number $n \gg 1$ is excited, it is difficult to use the general expression (23) for calculation of the damping rate. But, in this case, the approximation of geometric optics is valid, allowing us to find the attenuation. Starting once more with (6), we then arrive at the following expression for collisionless damping:

$$\nu_n^{(L)} = \sqrt{\frac{\pi}{8}} \frac{1}{\int |\Phi_n(z)|^2\, dz} \int |\Phi_n(z)|^2 \frac{\omega_{1,n}^4}{|k_n(z)|^3 v_{Te}^3} \exp\left[-\frac{\omega_{1,n}^2}{2k_n(z)^2 v_{Te}^2}\right] dz, \tag{28}$$

where $k_n(z)$ is the k-number corresponding to the nth eigenmode in the cavity. For a parabolic cavity (11), we obtain

$$k_n^2(z) = \frac{2\omega_{1,n}\Delta\omega_n}{3v_{Te}^2}\left(1 - \frac{\omega_{1,n}}{2\,\Delta\omega_n}\frac{z^2}{a^2}\right). \tag{29}$$

The main contribution to the attenuation in (28) stems from the region near the local minimum of plasma concentration where $k_n^2(z)$ achieves its maximum value. Due to this, the integral in (28) can be evaluated, and the following analytical result is obtained:

$$\nu_{1,n}^{(L)} = \frac{\pi\sqrt{3}a\omega_{1,n}^2}{4(n+\frac{1}{2})v_{Te}} \exp\left(-\frac{\sqrt{3}a\omega_{1,n}}{4(n+\frac{1}{2})v_{Te}}\right). \tag{30}$$

This result is valid if the eigenfunctions $\Phi_n(z)$ oscillate not too fast to compare with the region of order $a(\Delta\omega_n/\omega_1)^{1/2}$ that contributes to the attenuation in (29). The coresponding condition takes the form $(\Delta\omega_n/\omega_1)^{3/2} < r_d/a$, where $r_d = v_{Te}/\omega_{Pe}$ is the Debye radius. Note that the collisionless damping increases with increasing mode number n. It also increases rapidly with the decreasing dimensionless scale of inhomogeneity a/r_d. It should be mentioned that plasma inhomogeneity along the z axis influences the collisionless attenuation. Compared with the Landau damping in a homogeneous plasma, the damping rate in the inhomogeneous case is smaller by a factor of approximately $(\Delta\omega_n/\omega_1)^{1/2}$. In Fig. 1, Landau damping in a parabolic cavity for the eigenmodes $n = 10$ (continuous line) and $n = 1$ (dashed line) as a function of dimensionless scale a/r_d is presented. It is seen that damping for the mode $n = 10$ is very strong, especially in small cavities.

Suprathermal electrons produced in the process of cavity collapse cause additional increase of the damping rate. To investigate this effect, the electron distribution function can be taken in the form $F_e = F_0 + \Delta F_e$, where F_0 is given by (3) and ΔF_e is a small part represented by a Maxwellian distribution with high temperature along the magnetic field line

$$\Delta F_e = \xi \frac{N(z)}{(2\pi)^{3/2} v_{Te} v_h^{1/2}} \exp\left[-\left(\frac{v_\perp^2}{v_{Te}^2} + \frac{v_\parallel^2}{v_h^2}\right)\right]. \tag{31}$$

Here $\xi \ll 1$ is the reduced concentration of the suprathermal electrons and v_h is the thermal speed of hot electrons along the magnetic field line. We suppose that small amounts of hot electrons do not influence the dispersion of the trapped Langmuir waves. The corresponding condition takes the form $\xi v_h^2/v_{Te}^2 \ll 1$. At the same time,

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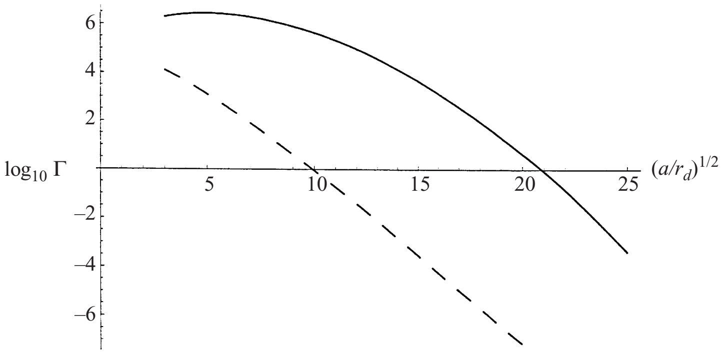

Figure 1. Dependence of collisionless attenuation on the cavity scale a for different modes: $n = 10$ (continuous line) and $n = 1$ (dashed line).

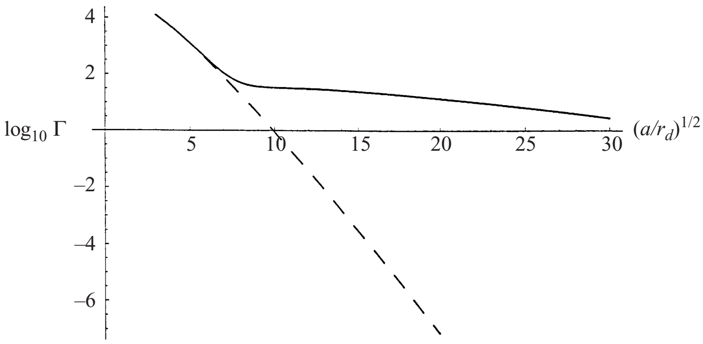

Figure 2. Influence of suprathermal electrons on collisionless attenuation in a cavity for the mode $n = 1$. The dashed line represents collisionless attenuation without suprathermal electrons, while the continuous line shows attenuation in the presence of a small fraction of suprathermal electrons $\xi = 0.01$, $v_h/v_{Te} = 7$.

the existence of suprathermal electrons causes significant additional collisionless attenuation:

$$\Delta\hat{\nu}^L = \sqrt{\frac{\pi}{8}}\xi\frac{\omega_1^4}{k^3v_h^3}\exp\left(-\frac{\omega_1^2}{2k^2v_h^2}\right). \tag{32}$$

Let us discuss the corresponding effect for the first eigenmode $\Phi_1(\kappa)$ and calculate the total collisionless attenuation. Taking (32) into account, we arrive at the result (compare with (27))

$$\nu_{1,1}^{(L)} \approx 0.1\omega_1 q^3\left[\frac{\exp(-2q)}{q^{1/2}} + \xi\frac{v_{Te}^3}{v_h^3}\frac{\exp(-2q_h)}{q_h^{1/2}}\right], \tag{33}$$

where $q_h = qv_{Te}/v_h$. In Fig. 2, the influence of a small fraction of suprathermal electrons $\xi = 0.01$ on collisionless attenuation for different scales of plasma inhomogeneity is presented. The dashed line describes the damping rate without suprathermal electrons, while the continuous line shows the attenuation in the presence of fast electrons. It is clearly seen that even a small amount of fast electrons causes an increase of attenuation by several orders of magnitude. The same effect takes place for modes with higher numbers. In Fig. 3, this is demonstrated for the mode $n = 10$.

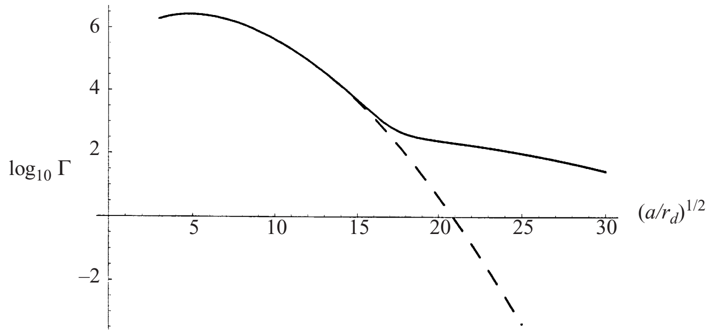

Figure 3. Influence of suprathermal electrons on collisionless attenuation in a cavity for the mode $n = 10$. The dashed line represents collisionless attenuation without suprathermal electrons, while the continuous line shows attenuation in the presence of small fraction of suprathermal electrons $\xi = 0.01$, $v_h/v_{Te} = 7$.

In the above discussion, a very small reduced concentration of suprathermal electrons $\xi = 0.01$ was included. With increasing ξ, the damping rate grows very rapidly. Note that the parameter δ that describes the ratio of inelastic (change of energy) to elastic (change of impulse) collisions is small ($\delta \ll 1$). Due to this, fast electrons return several times ($\sim \delta^{-1}$) to the resonance region before they lose a sufficient fraction of their energy. As a result, the concentration of suprathermal electrons grow in time. Hence the collisionless damping in cavities with scales small enough becomes so strong that it supresses any instability. This means that it is not necessary to artificially introduce the type of increase of attenuation for small scales as was done in numerical simulations of strong turbulence (see DuBois et al. 1993).

4. Formation of a seed plasma cavity

Due to the action of a strong HF pump wave on the ionosphere, a weak plasma depletion appears in the region of the main Airy maximum. To describe its formation, we start with the equation for the motion of electrons in which the ponderomotive force is retained. It is well known that the electric field $\mathbf{E}_0$ of the electromagnetic pump wave at the reflection level is polarized mainly along the Earth's magnetic field B (see Ginzburg 1961). Besides the pressure of the pump wave, we also retain the HF pressure of Langmuir waves. According to the results of numerical computations (DuBois et al. 1993), localized packets of strong Langmuir waves (cavitons) appear in the Airy maximum. Cavitons have very short lifetime and small scales. Nevertheless, their averaged HF pressure contributes to the formation of a plasma cavity with larger scales. The idea that the averaged pressure of cavitons influences the formation of plasma depletion in the Airy maximum was introduced by Cheung et al. (1992). As a result of the HF pressure of the small-scale Langmuir waves the equation of electron motion takes the form

$$\frac{\partial v_{ez}^{(2)}}{\partial t} = -\frac{e}{m}E_z^{(2)} - \gamma_e \frac{v_{Te}^2}{N_0}\frac{\partial \Delta N}{\partial z} - \frac{e^2}{4m^2}\frac{1}{\omega_0^2}\frac{\partial}{\partial z}[|E_0(z)|^2 + |E_L(z)|^2]. \tag{34}$$

Here N_0 is the undisturbed plasma concentration, $E_z^{(2)}$ is a low-frequency electric field, $v_{ez}^{(2)}$ is the speed of low-frequency oscillations of electrons in the z direction, ΔN is the low-frequency plasma perturbation, γ_e is the adiabatic coefficient for electrons, E_0 is the amplitude of the pump wave with frequency ω_0, and $E_L(z,t)$ is the electric field of Langmuir cavitons randomly distributed in the vicinity of the Airy maximum. Below, it is shown that a spatial averaging associated with the action of small-scale Langmuir waves $|E_L|^2$ appears in (34). The pump electric field is represented as

$$E_p(z,t) = \tfrac{1}{2}[E_0(z)\,\exp(i\omega_0 t) + \text{c.c.}]$$

The corresponding equation for the motion of ions takes the form

$$\frac{\partial v_{iz}^{(2)}}{\partial t} = \frac{e}{M}E_z^{(2)} - \gamma_i \frac{v_{Ti}^2}{N_0}\frac{\partial \Delta N}{\partial z} - \nu_2 v_{iz}^{(2)} \tag{35}$$

Here M is the mass of an ion, γ_i is the adiabatic coefficient for ions, $v_{iz}^{(2)}$ is the speed of ions in the low-frequency field $E_z^{(2)}$, and ν_2 is the effective collision frequency of ions, which includes collisions and Landau damping. The continuity equations for electrons and ions should be added to the system (34), (35):

$$\frac{\partial \Delta N_l}{\partial t} + \frac{\partial}{\partial z}N_0 v_{l,z}^{(2)} = 0 \quad (l = e, i) \tag{36}$$

Eliminating the low-frequency polarization electric field $E_z^{(2)}$, which serves only to maintain plasma neutrality, one obtains the following equation describing the formation of the cavity as the heater is turned on at $t = 0$:

$$\frac{\partial}{\partial t}\left[\left(\frac{\partial}{\partial t} + \nu_2\right)\Delta N\right] - \left(\gamma_i + \gamma_e \frac{T_e}{T_i}\right) v_{Ti}^2 \frac{\partial^2}{\partial z^2}\Delta N = \frac{\varepsilon_0}{4M}\frac{\omega_{Pe}^2}{\omega_0^2}\frac{\partial^2}{\partial z^2}[|E_0|^2\Theta(t) + |E_L|^2], \tag{37}$$

where v_{Ti} is the ion thermal velocity, T_i is the ion temperature, and $\Theta(t)$ is the unit step function. Note that γ_i is in general a complex function, for low-frequency oscillations $\gamma_e \approx 1$ (see Stubbe and Hagfors 1997).

As (37) is linear with respect to ΔN, we can represent ΔN as the sum of two terms: $\Delta N = \Delta N^{(P)} + \Delta N^{(L)}$, where $\Delta N^{(P)}$ and $\Delta N^{(L)}$ describe plasma depletions produced by the pump wave and Langmuir waves.

First we discuss the formation of the depletion caused by the action of Langmuir waves. The corresponding equation takes the form

$$\frac{\partial}{\partial t}\left[\left(\frac{\partial}{\partial t} + \nu_2\right)\Delta N^{(L)}\right] - v_s^2 \frac{\partial^2}{\partial z^2}\Delta N^{(L)} = \frac{\varepsilon_0}{4M}\frac{\omega_{Pe}^2}{\omega_0^2}\frac{\partial^2}{\partial z^2}|E_L|^2 \tag{38}$$

Here $v_s = (\gamma_i + \gamma_e T_\varepsilon/T_i)^{1/2} v_{Ti}$ is the ion-sound velocity. Cavitons have very small lifetime Δt_c and spatial scale Δz_c compared with the time intervals and scales in which we are interested. For this reason, we are able to represent the HF pressure of the small-scale Langmuir waves in the form

$$|E_L|^2(z,t) = |E_L^{(0)}|^2 \sum_j \delta(z - z_0 - z_j)\delta(t - t_j)\,\Delta z_c\,\Delta t_c, \tag{39}$$

where the coordinates z_j and t_j describe the position and the moment of the appearence of the jth caviton and z_0 is the center of the Airy maximum. Now we perform a Fourier transform with respect to the coordinate z in (38):

$$\frac{\partial}{\partial t}\left[\left(\frac{\partial}{\partial t} + \nu_2\right)\Delta N_k^{(L)}\right] + k^2 v_s^2\,\Delta N_k^{(L)} = -k^2 \frac{\varepsilon_0}{4M}\frac{\omega_{Pe}^2}{\omega_0^2}\sum_j |E_L^{(0)}|^2 \times \exp(-ikz_j)\,\delta(t - t_j)\,\Delta z_c\,\Delta t_c, \tag{40}$$

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where $\Delta N_k^{(L)}$ is the kth Fourier component of $\Delta N^{(L)}$. Note that cavitons are distributed stochastically within the Airy maximum. Hence, the statistical averaging in (40) should be with respect to z_j. If the distribution of the positions of the cavitons is Gaussian, then

$$\langle \exp(-ikz_j)\rangle = \exp(-\tfrac{1}{4}k^2L^2), \tag{41}$$

where $L^2 = 2\langle z_j^2\rangle$ and the brackets $\langle\ldots\rangle$ mean statistical averaging. We suppose that the scale L is smaller than the scale of the Airy maximum b, because cavitons are expected to appear more frequently in the region where the electric field of the pump wave is the largest. Indeed, cavitons are excited due to the oscillating two-stream instability, the growth rate of which is sensitive to the distribution of the pump wave in the Airy maximum. We average (40) with respect to time and compute the inverse Fourier transform. We obtain the following equation:

$$\frac{\partial}{\partial t}\left[\left(\frac{\partial}{\partial t}+\nu_2\right)\Delta N^{(L)}\right] - v_s^2\frac{\partial^2}{\partial z^2}\Delta N^{(L)} = \frac{\varepsilon_0}{2ML^2}|\bar{E}_L^{(0)}|^2\left[\frac{2(z-z_0)^2}{L^2}-1\right] \times \exp\left[-\frac{(z-z_0)^2}{L^2}\right]\Theta(t), \tag{42}$$

where

$$|\bar{E}_L^{(0)}|^2 = 2\sqrt{\pi}\frac{\Delta z_c}{L}\frac{\Delta t_c}{\tau}|E_L^{(0)2}|;$$

τ characterizes the frequency of caviton appearence in the Airy maximum,

$$\tau = \lim_{T\to\infty}\frac{1}{T}\int_0^T\sum_j \delta(t-t_j).$$

With increasing time, the solution of (42) tends to

$$\Delta N^{(L)} \approx -\frac{N_0}{4v_s^2}\frac{e^2|E_L^{(0)}|^2}{Mm\omega_0^2}\exp\left[-\frac{(z-z_0)^2}{L^2}\right]. \tag{43}$$

The characteristic time interval within which such a solution is achieved is $\Delta t \sim L/v_s$. For example, if we take $L = 5$ m, Δt is of the order of $\Delta t \approx 5$ ms. Further, we concentrate on the processes that take place at $t > \Delta t$, and hence we assume that the cavity (43) exists.

Turning now to the slower plasma modification due to the action of the ponderomotive force of the electromagnetic pump wave, we represent the distribution of the pump wave near the Airy maximum in the form

$$|E_0|^2(z) = |E_e|^2\left\{1+\eta\exp\left[-\frac{(z-z_0)^2}{b^2}\right]\right\},$$

where E_e is the electromagnetic field amplitude outside of the Airy maximum and η is the swelling of the field in the center $z = z_0$. As the scale of the Airy maximum is large enough, $b > v_s/\nu_2$, we use the diffusion equation to describe the slow modification of the cavity:

$$\frac{\partial}{\partial t}\Delta N^{(P)} - D_a\frac{\partial^2}{\partial z^2}\Delta N^{(P)} = \frac{\varepsilon_0}{4M\nu_2}\frac{\omega_{Pe}^2}{\omega_0^2}\frac{\partial^2}{\partial z^2}|E_0|^2\Theta(t), \tag{44}$$

where $D_a = v_s^2/\nu_s$ is the coefficient of ambipolar diffusion. The solution of the

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diffusion equation (44) with HF pressure produced by the pump wave is as follows:

$$\Delta N^{(P)}(z,t) = -Q_P \int_0^t \frac{1}{\sqrt{2\pi D_a(t-t')}}\,dt' \times \int_{-\infty}^{\infty} \exp\left\{-\left[\frac{(z-z')^2}{4D_a(t-t')} + \frac{(z'-z_0)^2}{b^2}\right]\right\}\left[1 - \frac{2(z'-z_0)^2}{b^2}\right]dz'. \tag{45}$$

Here

$$Q_P = \frac{e^2\eta|E_0|^2 N_0}{2Mm\omega_0^2 b^2 \nu_2}.$$

In explicit form, the decrease in plasma concentration $\Delta N^{(P)}$ at the center of the cavity according to (45) is given by

$$\Delta N^{(P)} = -\frac{Q_P b^3}{2D_a}\left[\frac{1}{b} - \frac{1}{(b^2 + 4D_a t)^{1/2}}\right]. \tag{46}$$

The characteristic time $t_D = b^2/4D_a$ describing cavity change in the lower part of the F-region is about $t_D \approx 50$ ms. In the initial stage, the diffusion can be neglected not only at the center $z = z_0$ but also in the region $|z - z_0| \ll b$. In this case, the small parameter $\beta = D_a t/b^2$ allows one to obtain an approximate solution of (45) near the local minimum as a series of powers of β. In the lowest-order approximation, the corresponding solution is described by

$$\Delta N^{(P)}(z,t) \approx -Q_P t\left[1 - \frac{3(z-z_0)^2}{b^2}\right]. \tag{47}$$

With the help of (44) and (47), one finds the shape of the nonstationary plasma cavity near its center for $\Delta t < t < b^2/4D_a$:

$$\Delta N(z,t) \approx -\Delta N_0^{(L)} \exp\left[-\frac{(z-z_0)^2}{L^2}\right] - Q_P t\left[1 - \frac{3(z-z_0)^2}{b^2}\right], \tag{48}$$

where $\Delta N_0^{(L)}$ is the depletion at the center of the cavity, (43). The total plasma concentration in the vicinity of the cavity can be represented approximately in the form

$$N(z,t) = (N_0 - \Delta N^{(ef)})\left[1 + \frac{(z-z_0)^2}{a^2(t)}\right]. \tag{49}$$

Here $\Delta N^{(ef)} = \Delta N_0^{(L)} + Q_p t$ is the depletion at the center and the characteristic scale $a(t)$ is determined by

$$a^{-2}(t) = \frac{\Delta N_0^{(L)}}{N_0} L^{-2} + \frac{Q_P t}{N_0}\frac{3}{b^2}.$$

Note that the main parameters of the cavity, a and $\Delta N^{(ef)}$, are slowly changing with time. With the development of the cavity, trapped Langmuir waves with higher numbers n can be retained. The estimate of the reduced density at the center of the parabolic depletion (49) that is required in order to retain an eigenmode of order n is obtained with the help of (15) as $\Delta N^{(ef)}/N_0 \approx q\sqrt{n}v_{Te}/a\omega_0$, where q is a numerical factor of order unity. This depletion is much broader than the small-scale cavitons discussed by DuBois et al. (1993). As will be shown in Sec. 6, the usual resonant three-wave interaction takes place in such a cavity.

Let us briefly discuss the influence of thermal effects on cavity formation. The heating of electrons is determined by the equation

$$\frac{\partial}{\partial t}\Delta T_e + \delta\nu_e \Delta T_e - D_{Te}\frac{\partial^2}{\partial z^2}\Delta T_e = Q_T(z)\theta(t). \tag{50}$$

Here ΔT_e is the increase in electron temperature, ν_e is the collision frequency of electrons, δ is the average fraction of energy lost by an electron in one collision, D_{Te} is the thermal conductivity of electrons along the magnetic field line, $Q_T = 2e^2|E(z)|^2\nu_e/3m\omega_0^2$ is the heating source, and $|E(z)|^2 = |E_0|^2 + |\bar{E}_L|^2$ is the square of the electric field amplitude. Thermal nonlinearity is significant in quasistationary conditions at $t > (\delta\nu_e)^{-1}$ (see Gurevich 1978). Due to the large thermal conductivity along magnetic field lines, plasma depletion becomes more and more elongated. In the opposite limiting case ($t \ll (\delta\nu_e)^{-1}$), the influence of heating on the formation of the depletion is less pronounced. To analyze the results of Isham et al. (1990), we need concentrate only on the latter case. Let us choose the spacial distribution of the source $Q(z)$ in the form

$$Q_T(z) = Q_L \exp\left[-\frac{(z-z_0)^2}{L^2}\right] + Q_0 \exp\left[-\frac{(z-z_0)^2}{b^2}\right].$$

After substitution of $Q_T(z)$ into (50), we arrive at the following result:

$$\begin{aligned}\Delta T_e(z,t) = Q_L\Bigg\{ L\left(\frac{t}{D_{Te}}\right)^{1/2} - \sqrt{\frac{\pi}{2}}\frac{L}{D_{Te}}\int_{z_0}^{z}\Bigg[\Phi\left(\frac{z_1-z_0}{L}\right) \\ -\Phi\left(\frac{z_1-z_0}{(L^2+4D_{Te}t)^{1/2}}\right)\Bigg]\,dz_1\Bigg\} \\ +Q_0\Bigg\{ b\left(\frac{t}{D_{Te}}\right)^{1/2} - \sqrt{\frac{\pi}{2}}\frac{b}{D_{Te}}\int_{z_0}^{z}\Bigg[\Phi\left(\frac{z_1-z_0}{b}\right) \\ -\Phi\left(\frac{z_1-z_0}{(b^2+4D_{Te}t)^{1/2}}\right)\Bigg]\,dz_1\Bigg\},\end{aligned} \tag{51}$$

where

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-\chi^2)\,d\chi$$

is the error function. According to (51), for moderate heating ($|E(z)| \approx 1$–$3\,\mathrm{V\,m^{-1}}$), the increase in the electron temperature within the first several milliseconds from the cold start is very small, $\Delta T_e/T_e < 10^{-3}$. This means that the influence of heating on cavity formation at such times is weaker than the action of the ponderomotive force, and can be neglected.

5. Adiabatic modes of Langmuir waves

Our aim is to find eigenmodes of Langmuir waves in the cavity (49) that change in time. According to (49), plasma depletion in the Airy maximum is described by two parameters: $\Delta N^{(ef)}(t)$ and $a(t)$. Assuming that the time variation of the scale

$a(t)$ is not too fast, $\omega_1|d\ln a/dt| \ll 1$, the following equation is obtained instead of (9):

$$\frac{\partial^2\Phi}{\partial z^2} + \left[\frac{\omega_1^2 - \omega_{Pe}^2(0,t) - \theta^2\omega_{He}^2}{3v_{Te}^2} - \frac{\omega_{Pe}^2(0,t)}{3v_{Te}^2}\frac{(z-z_0)^2}{a^2}\right]\Phi - i\frac{2\omega_1}{3v_{Te}^2}\frac{\partial\Phi}{\partial t} = 0. \qquad (52)$$

We seek the solution of (52) describing eigenmodes of Langmuir waves in the form

$$\Phi = \Phi_n(\alpha z)\exp\left(i\int^t \Omega_n\,dt + i\mu\alpha^2 z^2\right), \qquad (53)$$

where $\alpha(t)$, $\Omega_n(t)$, and $\mu(t)$ are parameters to be determined. Note that a similar solution is used in the theory of a quantum oscillator with slowly changing frequency. After substitution of (53) into (52), three relations for the unknown parameters α, Ω_n, and μ are obtained:

$$\omega_1^2 - \omega_{Pe}^2(0,t) - \theta^2\omega_{He}^2 + 2\omega_1\Omega_n(t) = (6n+3)v_{Te}^2\alpha^2, \qquad (54a)$$

$$\frac{\omega_1}{v_{Te}^2}\frac{d\alpha}{dt} = 6\alpha^3\mu, \qquad (54b)$$

$$\frac{\omega_{Pe}^2(0,t)}{a^2} - 2\omega_1\alpha\left(\alpha\frac{d\mu}{dt} + \mu\frac{d\alpha}{dt}\right) = 3\alpha^2 v_{Te}^4. \qquad (54c)$$

In a steady state when ω_{Pe} and a have no dependence on time, it follows from (54) that

$$\alpha = \left(\frac{\omega_{Pe}(0)}{\sqrt{3}av_{Te}}\right)^{1/2}, \qquad \mu = 0, \qquad (55a, b)$$

$$\Omega_n = \sqrt{3}(n+\tfrac{1}{2})\frac{v_{Te}}{a} - \frac{\omega_1^2 - \omega_{Pe}^2(0) - \theta^2\omega_{He}^2}{2\omega_1}. \qquad (55c)$$

Now let us assume that the parameters $\alpha(t)$ and $\mu(t)$ vary rather slowly in time and that the terms with derivatives in (54c) can be neglected. The corresponding condition will be given below. In this case we obtain the following approximate solution of (54):

$$\alpha(t) \approx \left(\frac{\omega_{Pe}(0,t)}{\sqrt{3}v_{Te}a(t)}\right)^{1/2}, \qquad \mu(t) \approx -\frac{\omega_1}{12v_{Te}^2\alpha^2}\frac{d\ln a}{dt}. \qquad (56)$$

The expression for the eigenfrequencies Ω_n remains the same as in (55c), with $a = a(t)$ and $\omega_{Pe}(0) = \omega_{Pe}(0,t)$. According to (56), the difference of frequencies between two adjacent eigenmodes is

$$\Delta\Omega \approx \frac{\sqrt{3}v_{Te}}{a(t)}. \qquad (57)$$

The above approximate solution is valid if $|\mu| \ll 1$. Combining (56) and (57), we find the necessary condition for the existence of adiabatic modes.

$$\Delta\Omega \gg \left|\frac{d\ln a}{dt}\right|. \qquad (58)$$

This means that the cavity modification should be sufficiently slow compared with the characteristic frequency $\Delta\Omega$. If the condition (58) is fulfilled, the general solution

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of (52) describing Langmuir waves localized in a parabolic cavity takes the form

$$\Phi = \sum_n c_n \Phi_n(\alpha z) \exp\left(i \int^t \Omega_n \, dt + i\mu\alpha^2 z^2\right), \tag{59}$$

where c_n are arbitrary coefficients. Note that the eigenfunctions Φ_n with different numbers n are orthogonal.

The depletion in the center of the Airy maximum is formed quickly. According to the theory, the growth rate of the oscillating two-stream instability responsible for caviton creation is proportional to the electric field amplitude of the pump (DuBois et al. 1993). Hence, cavitons appear mainly in the center of the Airy maximum, where the pump field reaches its maximum value. We therefore assume that the corresponding scale L (see (41)) is less than the scale of the Airy maximum b. The initial cavity with scale L appears in the center of the Airy maximum at $\Delta t \approx L/v_s$. The shape of the depletion at $t \sim \Delta t$ is determined mainly by the distribution of the averaged HF pressure of the small-scale plasma waves (see (43)). It is possible under real ionospheric conditions to expect a depletion $\Delta N_L^{(0)}/N_0 \approx 0.01$ to be formed within several milliseconds after the cold start. Furthermore, the cavity will be modified due to the action of the ponderomotive force of the pump field E_0 and growing Langmuir waves trapped in the cavity.

The adiabatic modes of Langmuir waves trapped in the cavity can be used if the cavity modification is slow enough (see (58)). Substituting into the results obtained above the parameters related to the cavity $L \approx 10^3$ cm, $b \approx 6 \times 10^3$ cm, $\Delta N_L^{(0)}/N_0 \approx 10^{-2}$, and for the ionospheric plasma $\nu_2 \approx 6\,\mathrm{s}^{-1}$ and $E_0 \approx 1\,\mathrm{m}^{-1}$, it is easy to verify that adiabatic modes can be used as soon as the initial cavity is formed that is, a few milliseconds after the cold start.

6. Parametric decay instability in the nonstationary cavity

To investigate the parametric decay instability in the time-varying cavity, we start with the coupled equations for the electric field of the Langmuir wave and the low-frequency plasma oscillations. The electric field can be represented in the form

$$E_z = \frac{1}{2}\left[E_z^{(0)}(z)\exp(i\omega_0 t) + E_z^{(1)}(z,t)\exp\left(i\int^t \omega_1\, dt_1\right)\right] + \text{c.c.} + E_z^{(2)}(z,t), \tag{60}$$

where $E_z^{(0)}$ and $E_z^{(1)}$ are the slowly varying amplitudes of the electromagnetic pump wave and the Langmuir wave, $E_z^{(2)}$ is the low-frequency polarization electric field. We assume that the difference between the frequencies ω_0 and ω_1 is much smaller than the frequency of the pump wave, $\delta\omega = (\omega_0 - \omega_1) \ll \omega_0$. At the same time, it is assumed that the frequency shift between two adjacent modes of the trapped Langmuir waves (see (57)) is large enough compared with the growth rate of the parametric decay instability. To obtain the correct equation for $E_z^{(1)*}$, we must take into account nonlinear terms in the electron equation of motion:

$$\left[\frac{\partial^2}{\partial z^2} + i\frac{2\omega_1}{3v_{Te}^2}\frac{\partial}{\partial t} + \frac{\omega_1^2 - \omega_{Pe}^2(z,t) - \theta^2\omega_{He}^2 + i2\hat{\nu}_1^{(ef)}\omega_1}{3v_{Te}^2}\right]E_z^{(1)*}$$
$$= \frac{\omega_{Pe}^2(N_0)}{3v_{Te}^2 N_0}\left[\exp\left(-i\int^t \delta\omega\, dt_1\right)E_z^{(0)*} + E_z^{(1)*}\right]n_2. \tag{61}$$

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Here $\hat{\nu}_1^{(ef)}$ describes the total damping of the Langmuir wave: $\hat{\nu}_1^{(ef)} = \nu_e + \hat{\nu}_1^{(L)}$, ν_e is the electron collision frequency, $\hat{\nu}_1^{(L)}$ is the damping caused by the interaction with the resonance electrons in the cavity, and n_2 is the plasma density of the low-frequency perturbations. Note that the plasma frequency $\omega_{Pe}(N)$ in (61) is determined by the sum of the undisturbed concentration N_0 and the slowly varying part $\Delta N(z,t)$ from the previous section. Hence the eigenfrequency ω_1 depends on time: $\omega_1 = \omega_1(t)$.

The equation for the low-frequency plasma oscillations takes the form

$$\left[\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t} + \nu_2\right) - v_s^2 \frac{\partial^2}{\partial z^2}\right] n_2 = \frac{\varepsilon_0}{4M}\frac{\partial^2}{\partial z^2}\left[E_z^{(0)} E_z^{(1)*} \exp\left(i \int^t \delta\omega \, dt_1\right)\right]. \tag{62}$$

We assume that the pump frequency ω_0 coincides with the plasma frequency in the undisturbed plasma, $\omega_{Pe}(N_0)$. If $E_z^{(1)*}$ corresponds to the eigenmode of the Langmuir wave in the cavity, the difference $\delta\omega$ is equal to

$$\delta\omega_{0,n} = \frac{\Delta N^{(ef)}(t)}{2N_0}\omega_0 - \Delta\omega_n(t), \tag{63}$$

where $\Delta\omega_n$ and $\Delta N^{(ef)}$ are determined by (14) and (49). It follows from (63) that the frequency $\delta\omega_{0,n}$ decreases with increasing eigenmode number n of the trapped Langmuir wave. Equations (61) and (62) describe the interaction of high-frequency and low-frequency waves in a nonstationary cavity.

To investigate the parametric decay instability in the cavity, we single out the time dependence $\exp(i\omega_2 t)$ in the low-frequency plasma perturbation n_2 and represent it in the form $n_2 = n_s \exp(i\omega_2 t)$.

It is more convenient to analyze the decay instability in k-space. So we apply a Fourier transform with respect to k to (61) and (62). As a result, the system of coupled equations takes the form

$$\left[\frac{\partial^2}{\partial k^2} + a^2 \frac{\omega_1^2 - (\omega_{Pe}^{(0)2} - \theta^2\omega_{He}^2 + 3k^2 v_{Te}^2) + i\nu_1^{(L)}\omega_1}{\omega_{Pe}^{(0)2}} + i2a^2 \frac{\omega_1}{\omega_{Pe}^{(0)2}}\frac{\partial}{\partial t}\right]\Phi_n^{(1)*}(k)$$
$$= \frac{a^2}{N_0} E_z^{*(0)} n_s(k) \exp\left(-i\int^t \delta\Omega \, dt_1\right), \tag{64}$$

$$\left[\omega_2(\omega_2 - i\nu_2) - k^2 v_s^2 - 2i\omega_2 \frac{\partial}{\partial t}\right] n_2 = k^2 \frac{\varepsilon_0}{4M} E_z^{(0)} \Phi_n^{(1)*} \exp\left(i\int^t \delta\Omega \, dt_1\right). \tag{65}$$

Here $\Phi_n^{(1)*}(k)$ and $n_s(k)$ are the Fourier transforms of $E_z^{(1)*}$ and n_2, and $\delta\Omega = \omega_0 - \omega_1(t) - \omega_2$. The frequency difference $\delta\Omega$ of the three interacting waves, and hence the phase $F(t) \int^t \delta\Omega(t_1)\, dt_1$, vary in time due to modification of the cavity.

Let us suppose first that the frequency difference of the interacting waves is equal to zero: $\delta\Omega = 0$. By setting $\Phi_{1,n} = \tilde{\Phi}_{1,n} \exp(\Gamma t)$ and $n_s = \tilde{n}_s \exp(\Gamma t)$ and eliminating the ion-sound perturbation with the help of (65), we arrive at the following equation for the growth rate of the parametric decay instability in the cavity.

$$\left[\frac{\partial^2}{\partial k^2} + a^2 \frac{\omega_1^2 - (\omega_{Pe}^{(0)2}\theta^2\omega_{He}^2 + 3k^2 v_{Te}^2) + i\nu_1^{(L)}\omega_1}{\omega_{Pe}^{(0)2}} + i2a^2 \frac{\omega_1}{\omega_{Pe}^{(0)2}}\Gamma\right]\tilde{\Phi}_n^{(1)*}$$
$$= \frac{m}{4M} v_E^2 \frac{k^2 a^2}{\omega_2(\omega_2 - i(2\Gamma + \nu_i)) - k^2 v_s^2}\tilde{\Phi}_n^{(1)*}. \tag{66}$$

Here v_E is the quiver velocity of electrons in the pump field, $v_E = eE_0/m\omega_0$. The parameter Γ determines the growth rate of instability (the real part of Γ) and corrections to the eigenfrequencies of Langmuir waves in the cavity (the imaginary part of Γ). If the imaginary part in the denominator of (66) is small enough, the resonance region $\omega_2^2 \approx k^2 v_s^2$ provides the main contribution to the growth rate. In this case, the simple approximate value of the growth rate follows from (66):

$$\mathrm{Re}\,\Gamma_n \approx 0.3\frac{m}{M}\omega_2\frac{v_E^2}{v_s^3}(\omega_{Pe}^{(0)}v_{Te}a)^{1/2}\frac{|\tilde{\Phi}_n^{(1)}|^2(k_r)}{\int|\tilde{\Phi}_n^{(1)}|^2\,dk} - \tfrac{1}{2}\nu_{1,n}^{(L)} \tag{67}$$

With increasing mode number n, the resonant frequency $\omega_2 = \omega_0 - \omega_1$ and the corresponding wavenumber $k_r = \omega_2/v_s$ decrease. At the same time the Fourier component $\tilde{\Phi}_{1,n}$ contains wavenumbers $k \leqslant (2n+1)^{1/2}(\omega_1/av_{Te})^{1/2}$ that increase with n. This means that resonance conditions are achieved at some intermediate values of n. The results obtained above are valid if the frequency shift between two adjacent eigenmodes, $\Delta\Omega$, is larger than the growth rate: $\Delta\Omega \gg \Gamma$. Numerical estimates show that the characteristic growth rate (67) is $\Gamma \approx 10^3\,\mathrm{s}^{-1}$. The difference of two adjacent eigenfrequencies of Langmuir waves depends on the cavity scale, and can be of the same order of magnitude. This causes significant difficulties in the investigation of instability. But in the limiting case $\Delta\Omega \ll \Gamma \ll \omega_2$, it is also possible to find approximately the growth rate from (67).

$$\Gamma \approx 0.3\left(\frac{m}{M}\right)^{1/2}(\omega_{Pe}^{(0)}\omega_2)^{1/2}\left(\int_{\kappa_r-\Delta\kappa}^{\kappa_r+\Delta\kappa}|\Phi_1|^2\,dk\right)^{1/2}. \tag{68}$$

Here

$$\kappa_r = \left(\frac{\sqrt{3}av_{Te}}{\omega_{Pe}^{(0)}}\right)^{1/2}\frac{\omega_2}{V_s}, \quad \Delta\kappa = \frac{\sqrt{3}av_{Te}}{\omega_{Pe}^{(0)}}\frac{\omega_2\Gamma}{v_s^2\kappa_r}.$$

It is assumed that the eigenfunctions Φ_1 are normalized: $\int|\Phi_1|^2\,d\kappa = 1$. The estimate (68) gives a value $\Gamma \approx 10^4\,\mathrm{s}^{-1}$ that is approximately one order of magnitude larger than in the previous case. It is possible to expect that, in real ionospheric conditions, the typical magnitude of the growth rate is between these two limiting values.

Let us discuss now the influence of cavity modification on the amplification of waves. The time interval Δt within which resonance conditions are fulfilled can be estimated as

$$\Delta t \approx \left|\frac{d}{dt}\delta\Omega\right|^{-1/2} \tag{69}$$

Substituting into (69) the dependence $\delta\Omega(t)$ from (63), we arrive at the relation

$$\Delta t \approx 5\left(\frac{M\nu_2}{m\omega_{Pe}^{(0)}}\right)^{1/2}\frac{b}{v_E}. \tag{70}$$

It follows that the amplification of Langmuir and ion-sound waves $A = \Gamma\,\Delta t$ is significant: $A \approx 5$–7.

The characteristic vertical scale of the cavity should not be too small. Otherwise, collisionless damping becomes very strong and suppresses the parametric decay instability. According to Figs 1 and 2, the damping rate for the eigenstate with $n = 10$ becomes of the order of the growth rate Γ obtained above at $(a/r_d)^{1/2} \geqslant 17$.

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Hence, the scale of the cavity a should be $a \geqslant 300/r_d$. At the same time, the maximum k-number of the excited eigenmode with number n is determined by the scale a. The corresponding value $k_{n,\max}$ is easily obtained with the help of (29):

$$k_{n,\max} \approx \frac{0.6}{r_d}\left[(n+\tfrac{1}{2})\frac{r_d}{a}\right]^{1/2}. \tag{71}$$

Hence the smaller the scale a, the larger is the maximum excited k-number of the trapped Langmuir waves.

The generation of ion-acoustic waves with large k is possible only if such values of k exist in the spectrum of the corresponding Langmuir mode. For example, UHF radar can detect ion-acoustic waves near the reflection height of the pump wave in a small-scale cavity $a \approx 5 \times 10^2$ cm if Langmuir waves with mode numbers $n \geqslant 60$ are strongly amplified. However, high-order modes of the trapped Langmuir waves cannot be excited due to strong damping by resonant electrons. This means that it is difficult to imagine how UHF radar could measure low frequency small-scale plasma perturbations excited by the decay of the HF pump wave into Langmuir and ion-acoustic waves near its reflection height. Indeed, for $k = 0.4\,\mathrm{cm}^{-1}$ and $r_d = 0.5$ cm, it follows from (68) that $a/r_d \approx 9(n + 1/2)$. Substituting this estimate into (30), we find that the collisionless damping rate is higher than the expected growth rate of the parametric decay instability. At the same time, damping by the resonance electrons decreases exponentially with decreasing k. So Langmuir waves with smaller characteristic k-numbers can be efficiently generated near the reflection height of the HF pump radio wave. Hence, the decay of trapped large-amplitude Langmuir waves into other Langmuir waves and ion-acoustic waves may produce small-scale low-frequency plasma perturbations measured by UHF radar. This also explains why only an ion line was measured by UHF radar in the experiment by Isham et al. (1990), while the enhanced plasma line was not detected.

It follows from the analysis presented above that high collisionless damping is probably the main reason why it is difficult to measure simultaneously enhanced plasma and ion lines by UHF radar at the reflection height of the pump wave. One may expect that it is easier to perform such measurements for higher pump frequencies. The higher the pump frequency, the smaller is the Debye radius r_d at the reflection height. Hence, the smaller is the attenuation. Also, it should be easier to measure the enhanced plasma and ion perturbations near the reflection height by VHF rader.

7. Discussion

It has been shown that the plasma cavity created in the main Airy maximum of an HF ordinary electromagnetic wave may play a significant role in the excitation of enhanced plasma and ion lines nearthe reflection level of the pump wave. After the heater is switched on, such a cavity is able to retain Langmuir waves that grow due to the parametric decay instability in a few milliseconds. The instability for the trapped Langmuir waves is an absolute one, which is why the waves grow quickly in time. It is argued that the direct decay process T → L+S when the natural electron temperature is high enough is not able to provide the excitation of very small-scale ion-acoustic perturbations due to strong resonance damping of the Langmuir wave. However, in this case, the decay of the trapped Langmuir wave with large amplitude into another Langmuir wave and an ion-acoustic wave is thought to be the main process for the excitation of enhanced ion line measured by UHF radar near the reflection height of the pump wave.

The electron temperature in the F-region rises considerably in a few hundreds of milliseconds under the action of a strong HF radio wave. Besides that, the concentration of suprathermal electrons in the resonance region grow in time due to elastic scattering from molecules. Both of these effects cause a substantial increase in collisionless damping. Due to this, one may expect UHF radar to be able to measure the enhanced ion line only within a restricted period from the cold start. If the natural electron temperature is not enhanced enough and a rather high pump frequency is used, the plasma line can also be detected for such time intervals by UHF radar. It is possible to expect the appearance even of several plasma lines with different frequencies coming from the same height. Such lines correspond to different eigenmodes in a cavity. According to our estimates, the maximum value of k excited in the direct process $T \rightarrow L + S$ is approximately half the value of k detected by the EISCAT UHF radar. So VHF radar should be able to measure the enhanced plasma line even when it is not seen by UHF radar.

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