Industrial and Organizational Psychology, **3** (2010), 224–227.

COMMENTARIES

Using Meta-Analysis to Increase Power in Differential Prediction Analyses

MATTHEW J. BORNEMAN
Southern Illinois University

Meade and Tonidandel (2010) correctly note several of the interpretive and statistical problems associated with the analysis of differential prediction. These include, most notably, problems with low statistical power. However, modern techniques of meta-analysis can alleviate this problem by combining the regression analyses from multiple smaller sample studies, thereby reducing standard errors for the regression coefficients and increasing statistical power. In addition, these meta-analytic techniques can provide more accurate estimates of intercept differences, allowing for more accurate interpretation of differential prediction analyses. This commentary will serve two purposes. First, I will briefly introduce these meta-analytic techniques and discuss some of the analytic care that must be taken in primary studies assessing differential prediction. This directly addresses concerns regarding statistical power discussed in the focal article. Second, I will note additional research questions that can be addressed using these meta-analytic techniques, which can help build theory and recommendations for practice.

Correspondence concerning this article should be addressed to Matthew J. Borneman.
E-mail: mborneman@siu.edu
Address: Department of Psychology, Southern Illinois University, 1125 Lincoln Drive, Rm. 281 Life Science II, Carbondale, IL 62901

Meta-Analysis of Regression Weights

Although techniques to meta-analyze regression weights have been known for decades, Becker and Wu (2007) recently introduced a new method to meta-analytically synthesize regression slopes from multiple independent studies. This method can be applied to any set of regression models, including those with and without an intercept, quadratic terms, and interaction terms; more pertinently, these methods can be used to synthesize regression slopes in differential prediction analyses (i.e., the Cleary model, 1968). In fact, they have also shown that the results from these techniques can be exactly equivalent to the results from a full dataset containing all cases from all of the primary studies. Basically, these techniques weight the regression coefficients by the inverse of the standard error. To conduct these analyses, three supermatrices (i.e., matrices of matrices) need to be constructed; these are described in more detail in the Appendix using the Cleary model of differential prediction as an example.

The primary concern with the meta-analysis of regression coefficients is that each study must calculate the exact same regression equation. Although this is a serious concern with this technique generally, two facts mitigate this concern. First, this meta-analytic technique is flexible

https://doi.org/10.1111/j.1754-9434.2010.01228.x Published online by Cambridge University Press

enough to analyze regression coefficients from multiple sets of regression equations and compare results with and without certain terms through the use of moderator analyses (see Becker and Wu [2007] for additional comments regarding this issue). Second, and more importantly, studies that assess differential prediction with the Cleary model have fairly standard output and nearly always include all four terms, precluding any need for these more complex analyses.

However, studies do not always use the same metric for the predictors. Although this is less of a concern when the intercept is not of interest (because unstandardized coefficients can readily be converted to standardized coefficients given the standard deviations [*SD*s] of the predictors/criteria), it is of grave importance when assessing the Cleary model. To be interpretable when aggregating unstandardized regression coefficients, all the studies must be on the same metric; averaging regression coefficients with different scales is tantamount to mixing apples and bricks. For an example from the standardized test literature, consider the SAT-Quant, which has $SD \approx 100$, and the ACT Math, which has $SD \approx 5.0$; a one unit increase on the ACT is substantially more than a one unit increase on the SAT. To help standardize the metric moving forward, I recommend first standardizing the focal predictor/criterion to have $M = 0.0$ and $SD = 1.0$ for the total combined group and using a 0/1 coding scheme for the dichotomous group indicator variable. This will then force the interaction term and the intercept to a common metric as well, allowing interpretable accumulation across studies.

Third, the variance–covariance matrix for the regression weights is nonstandard output in software packages and typically unreported in articles.[1] Although standard errors are usually reported (i.e., the diagonal elements of the matrix), the covariance elements are not. As such, I also recommend that this matrix be calculated and reported along with the regression weights; this will help facilitate future accumulation of regression coefficients moving forward. Although it is not technically necessary to have the off-diagonal elements (Becker and Wu [2007] present examples where zeroes are assumed for covariances among regression coefficients and note for possibilities for the use of artifact distributions), results are most accurate when these values are available.

Finally, it should be noted that this call is for reporting research moving forward; it is unlikely there are sufficient data in published manuscripts lying around for meta-analysis. Published and unpublished data sets can certainly contribute to future meta-analyses, though some reanalysis will need to take place to contribute to a future meta-analysis using the techniques presented here.

Additional Research Questions

Similar to more traditional uses of meta-analytic techniques, the benefits of this new meta-analytic technique are twofold. First, cumulating across multiple studies will provide more accurate estimates of the regression coefficients, allowing the researcher to be more confident when interpreting the magnitude of these coefficients. Second, these techniques allow for the possibility of increasing statistical power and decreasing confidence intervals around these estimates, decreasing error rates of these statistical analyses. However, these analyses permit additional research questions to be asked.

The Becker and Wu (2007) methods of meta-analysis also permit moderator analyses to be conducted. At minimum, this will allow for differences to be computed between different assessments, test batteries, or constructs; this helps provide aggregated estimates of group differences in slopes and intercepts, allowing additional theory to be developed regarding the causes and correlates of these differences.

1. It is possible to calculate this matrix from a correlation matrix and the R^2 value when there are only linear terms in the regression equation and the intercept is not of interest; however, neither is true when meta-analyzing results from Cleary analyses.

https://doi.org/10.1111/j.1754-9434.2010.01228.x Published online by Cambridge University Press

Another application of these moderator analyses is for advice on the practice of employee selection. Meade and Tonidandel suggest that practitioners and consultants work together to discover the causes of slope and intercept differences, should they be found. This work can be extremely beneficial to others as it can allow other researchers, consultants, and practitioners to avoid similar pitfalls in their selection work. And as before, this will also allow for theory to be developed as to what HR practices, test development efforts, and so on can lead to slope and intercept differences.

Conclusion

Meta-analytic techniques for regression coefficients are a new and useful tool to enhance differential prediction analyses. These techniques can increase the statistical power of Cleary model analyses and provide more accurate estimates of the regression coefficients. In addition, through the use of moderator analyses, theory can be built to help both selection research and practice.

References

Becker, B. J., & Wu, M.-J. (2007). The synthesis of regression slopes in meta-analysis. *Statistical Science*, *22*, 414–429.
Cleary, T. A. (1968). Test bias: Prediction of grades of Negro and White students in integrated colleges. *Journal of Educational Measurement*, *5*, 115–124.
Meade, A. W., & Tonidandel, S. (2010). Not seeing clearly with Cleary: What test bias analyses do and do not tell us. *Industrial and Organizational Psychology*, *3*, 192–205.

Appendix

First, consider a column vector of the regression weights from the Cleary model, $\mathbf{b_i}' = [b_{i0}, b_{iX}, b_{iY}, b_{i(XY)}]$, where i indexes the study, b_0 is the intercept, b_X is the regression weight for the predictor, b_Y is the regression weight for the group indicator, and b_{XY} is the regression weight for the interaction term. The first supermatrix, $\mathbf{b}$, is created by vertically stacking all of the $\mathbf{b_i}'$ column vectors yielding a $[(4k) \times 1]$ vector[2] where k is the number of studies and

$$\mathbf{b} = \begin{bmatrix} b_{10} \\ b_{1X} \\ b_{1Y} \\ b_{1(XY)} \\ \vdots \\ b_{k0} \\ b_{kX} \\ b_{kY} \\ b_{k(XY)} \end{bmatrix}. \tag{A1}$$

The second supermatrix is created by diagonally binding the variance–covariance matrices of the regression weights for each study. This turns out to be

$$\boldsymbol{\Sigma} = \begin{bmatrix} \mathrm{Cov}(b_1) & 0 & 0 & 0 \\ 0 & \mathrm{Cov}(b_2) & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \mathrm{Cov}(b_k) \end{bmatrix}, \tag{A2}$$

where $\boldsymbol{\Sigma}$ is a $(4k \times 4k)$ supermatrix, and $\mathrm{Cov}(b_k)$ is the variance–covariance matrix of the regression weights [i.e., $(\mathbf{X}_i'\mathbf{X}_i)^{-1}S_i^2$], where $\mathbf{X}$ is a matrix of predictor scores[3] and S is the mean squared error for the regression model in study i. The final supermatrix is a weight matrix $\mathbf{W}$ of ones and zeros denoting which study includes which regression weights; in the simplest case, where each study includes all four regression coefficients of the Cleary model, the resulting $\mathbf{W}$ matrix would be k stacked

2. The number of rows is $4k$ when meta-analyzing the Cleary (1968) model only; it would be $k*p$ more generally, where p is the number of regression slopes.
3. For analyses including a regression intercept (including those with the Cleary model), a column vector of ones is appended to the predictor score matrix such that the ones appear as the first column.

https://doi.org/10.1111/j.1754-9434.2010.01228.x Published online by Cambridge University Press

4 × 4 identity matrices, yielding a $(4k \times 4)$ supermatrix.[4]

$$\mathbf{W} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad \text{(A3)}$$

With the three supermatrices from Equations (A1)–(A3) having been constructed, two additional steps need to be conducted to calculate the weighted mean (which is, in effect, weighted by the inverse of the standard error) and aggregated variance–covariance matrix of the regression coefficients (which will yield the standard errors). The mean effect is calculated by

$$\boldsymbol{\beta} = (\mathbf{W}'\boldsymbol{\Sigma}^{-1}\mathbf{W})^{-1}\mathbf{W}'\boldsymbol{\Sigma}^{-1}\mathbf{b}. \qquad \text{(A4)}$$

The aggregated variance–covariance matrix can then be calculated by

$$\text{Cov}(\boldsymbol{\beta}) = (\mathbf{W}'\boldsymbol{\Sigma}^{-1}\mathbf{W})^{-1}. \qquad \text{(A5)}$$

In short, though considerably more complex, this method of meta-analysis provides accurate estimates of aggregated regression coefficients in a manner similar to more traditional methods of meta-analysis.

4. It is also important to note that the **W** matrix is where additional weights would be included for moderator analyses; see Becker and Wu (2007) for additional details on how to conduct moderator analyses.

https://doi.org/10.1111/j.1754-9434.2010.01228.x Published online by Cambridge University Press