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A NOTE ON THE SECTORAL COMPOSITION OF GOVERNMENT SPENDING, PROGRESSIVE TAXATION AND AGGREGATE (IN)STABILITY

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This paper analyzes the effect of the sectoral composition of government spending on the stability properties of a two-sector economy with a progressive tax structure. The results suggest that indeterminacy is more likely to occur if the fraction of government spending on consumption goods increases. This study also finds that, under progressive taxation, a sufficiently high public-consumption share is needed to generate indeterminacy. It is shown that, with the benchmark parameterization, a higher fraction of government spending on consumption goods needs to be implemented with a more progressive tax scheme to stabilize the economy. Moreover, it is emphasized that belief-driven economic fluctuations may indeed be a feature of the U.S. economy.

Keywords: Sectoral Composition of Government Spending, Tax Progressivity, Stability Properties

1. INTRODUCTION

Starting with the work of Schmitt-Grohé and Uribe (1997), there has been an extensive body of literature studying the stabilization effects of various fiscal policies in the context of real business cycle models.[1] If a policy rule can eliminate indeterminacy, then it stabilizes the economy in the sense that it prevents sunspot-driven economic fluctuations. However, many previous studies have restricted attention to the impact of different tax rules on the (in)stability properties of economies. The (de)stabilizing role of government spending is relatively underexplored.[2] Motivated by this gap, Chang et al. (2015, 2019) consider a two-sector economy, in which total government spending is decomposed into public-consumption and public-investment goods, to explore the interrelationship between the sectoral distribution of government expenditures and macroeconomic (in)stability. They find that it is the sectoral composition of government spending, rather than total government spending, that matters for stabilization policy design.

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In light of the fact that the income tax system in the United States is considered to be a progressive system wherein tax burden increases with taxable income, this paper seeks to examine the quantitative relationship between the sectoral distribution of government spending, tax progressivity, and equilibrium (in)determinacy. To do so, I embed Guo and Harrison's (2001) progressive tax schedule and Chang et al.'s (2019) government spending decomposition into a discrete time, two-sector model with sector-specific externalities, in which productive externalities in each sector depend on the aggregate output of its own sector.[3]

A novel feature of this work is that I study how the sectoral composition of government spending affects the occurrence of indeterminacy in the model using parameter values that are consistent with U.S. post-war data. In particular, the degree of tax progressivity is set to equal 0.181, a point estimate by Heathcote et al. (2017).

The key results can be summarized as follows. The minimum degree of externalities required to bring about indeterminacy is 0.0877 when the public-consumption share equals 1. In this case, the model is reduced to a modified version of Guo and Harrison (2001, 2015). It can be shown that indeterminacy is harder to obtain if more government expenditures are devoted to the investment sector. The results are robust under different parameterizations and an alternative preference specification.

The role of the public-consumption share on the occurrence of indeterminacy is as follows. Under optimistic beliefs, the agent is willing to substitute investment for consumption and reallocate resources from the consumption sector to the investment sector, thereby increasing the future capital stock. On the one hand, the expected rate of return declines due to the diminishing marginal product of capital (diminishing returns effect). On the other hand, with positive production externalities, the prices of investment goods decline as inputs flow into the investment sector (price effect). Indeterminacy implies that the price effect dominates. Higher public-investment share implies that fewer resources are devoted to the consumption sector in the steady state, which makes relative price movement smaller as inputs flow into the investment sector. As a result, higher production externalities are needed to generate indeterminacy.

Another novel feature of this work is that I study how the model's (in)stability properties are affected by the combination of tax progressivity and the sectoral distribution of government spending. This research provides useful insights on how policymakers can combine their government spending policy and tax policy to stabilize the economy. In their study, Chang et al. (2019) examine the relationship of government spending and distortionary income taxation in generating (in)determinacy. They incorporate two stylized balanced-budget rules: Schmitt-Grohé and Uribe (1997) rule and Guo and Harrison's (2004) rule. This paper considers another type of tax rule that is more realistic.

When the tax rate is constant (no tax progressivity), the model collapses to a modified version of Chang et al. (2019), and the threshold public-consumption share is 0.5626. It is shown that, with the benchmark parameterization, a more

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progressive tax is required to remove indeterminacy arising from a higher public-consumption share. However, the results from the sensitivity analysis indicate that there is mixed evidence about the role of a higher tax progressivity on the stability properties. Generally speaking, under progressive taxation, a sufficiently high public-consumption share is needed to generate indeterminacy. Whether a higher tax progressivity can (de)stabilize the economy depends on its impact on the relative strength of the diminishing returns effect and the price effect.

Last, I calibrate the public-consumption share to match U.S. data and find that belief-driven economic fluctuations may indeed be a feature of the U.S. economy when other parameter values are empirically plausible.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the local dynamics and provides the main results. The economic interpretations are also discussed in this section. A model with utility-generating government consumption is discussed in Section 4. The last section concludes.

2. THE ECONOMY

This paper incorporates the sectoral distribution of government spending *a la* Chang et al. (2019) and Guo and Harrison's (2001) progressive tax policy in a discrete time two-sector model with sector-specific externalities. The economy is populated by a unit measure of identical, infinitely living households who own their capital and labor and lend them to firms, taking a rate of return on capital and labor as given. There are two sectors of production: the consumption and investment sectors. External effects in each sector depend on the aggregate output of its own sector.

2.1. Households

A representative household maximizes its lifetime utility by choosing a sequence of consumption $\{C_t\}_{t=0}^{\infty}$, labor supply $\{L_t\}_{t=0}^{\infty}$, and capital $\{K_t\}_{t=0}^{\infty}$. The household discounts future utility by the subjective discount factor $\beta \in (0, 1)$. The lifetime utility function is

$$\sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{L_t^{1+\chi}}{1+\chi} \right], \tag{1}$$

where χ denotes the inverse elasticity of labor supply. The representative household faces a budget constraint

$$C_t + p_t X_t = (1 - \tau_t)(r_t K_t + w_t L_t), \tag{2}$$

where X_t is the household's new investment in new capital, and p_t is the relative price of investment goods in units of consumption goods. r_t and w_t are the prices of capital and labor, respectively. τ_t is the income tax rate. Following Guo and Harrison (2001), τ_t takes the form

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$$\tau_t = 1 - \eta \left(\frac{Y}{Y_t}\right)^{\phi}, \tag{3}$$

where $\eta \in (0, 1)$ and $\phi \in [0, 1)$. η and ϕ capture the level and slope of the tax schedule. Y_t is the total taxable per capita income, and Y denotes its steady-state level which is taken as given by the household. When $\phi = 0$, the household faces a constant tax rate $1 - \eta$. $\phi > 0$ implies that the tax rate τ_t is increasing with Y_t. To see the progressiveness feature of the tax schedule, let τ_t^m be the marginal tax rate, which is defined as the change in taxes paid by the household divided by the change in its taxable income, then

$$\tau_t^m = \frac{\partial(\tau_t Y_t)}{\partial Y_t} = 1 - \eta(1-\phi)\left(\frac{Y}{Y_t}\right)^{\phi}. \tag{4}$$

Equations (3) and (4) imply that the marginal rate is higher than the average tax rate when $\phi > 0$. In this case, the tax schedule is said to be "progressive." Guo and Lansing (1998) point out that it is the marginal tax rate that affects the household's decisions, as it considers the way in which the tax scheme affects its earnings.

Let $\delta \in (0, 1)$ denote the rate of capital depreciation. The law of motion for capital accumulation is given by

$$K_{t+1} = (1-\delta)K_t + X_t. \tag{5}$$

Let Λ_t be the costate variable associated with the Lagrangian setup of the household's optimization problem. The Lagrangian setup is

$$\begin{aligned} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \log C_t - \frac{L_t^{1+\chi}}{1+\chi} + \Lambda_t \{(1-\delta)K_t \\ & + \left[\eta Y^{\phi}(r_t K_t + w_t L_t)^{1-\phi} - C_t\right] p_t^{-1} - K_{t+1}\} \Bigg\}. \end{aligned} \tag{6}$$

The first-order conditions are

$$\frac{1}{C_t} = \Lambda_t p_t^{-1}, \tag{7}$$

$$L_t^{\chi} = \Lambda_t (1-\tau_t^m) w_t p_t^{-1}, \tag{8}$$

$$\frac{C_{t+1}}{C_t} = \beta \left[\frac{(1-\delta)p_{t+1} + (1-\tau_{t+1}^m) r_{t+1}}{p_t}\right], \tag{9}$$

$$\lim_{t\to\infty} \beta^t \Lambda_t K_{t+1} = 0, \tag{10}$$

where equations (7) and (8) show the intratemporal trade-off between consumption and leisure. Equation (9) is the intertemporal Euler equation, and equation (10) is the transversality condition.

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2.2. Firms

The production technology for the consumption sector and investment sector are given by

$$Y_{ct} = A_t K_{ct}^{\alpha} L_{ct}^{1-\alpha}, \tag{11}$$

$$Y_{xt} = B_t K_{xt}^{\alpha} L_{xt}^{1-\alpha}, \tag{12}$$

$$A_t = (\overline{K}_{ct}^{\alpha} \overline{L}_{ct}^{1-\alpha})^{\theta},\ B_t = (\overline{K}_{xt}^{\alpha} \overline{L}_{xt}^{1-\alpha})^{\theta}, \tag{13}$$

where A_t and B_t are the scaling factors that represent productive externalities. K_{ct} (K_{xt}) and L_{ct} (L_{xt}) are the capital and labor inputs used in the production of consumption (investment) goods. α is the capital share in each sector. A bar over a variable refers to the economy-wide average, which is taken as given by an individual firm. θ captures the degree of sector-specific externalities in the two sectors and $0 < \theta < 1$.[4]

Assume that the factor markets are perfectly competitive. The first-order conditions for firms' profit maximization are

$$r_t = \frac{\alpha Y_{ct}}{K_{ct}} = p_t \frac{\alpha Y_{xt}}{K_{xt}}, \tag{14}$$

$$w_t = \frac{(1-\alpha) Y_{ct}}{L_{ct}} = p_t \frac{(1-\alpha) Y_{xt}}{L_{xt}}. \tag{15}$$

where r_t and w_t are the capital rental price and labor wage, respectively.

2.3. Government

The government chooses the tax policy τ_t and balances its budget at each point in time. Therefore, the government's period budget constraint is given by

$$G_t = \tau_t Y_t = G_{ct} + p_t G_{xt}, \tag{16}$$

where G_t is the total government spending. G_{ct} and G_{xt} are the consumption and investment goods purchased by the government. Following Chang et al. (2019), the government expenditures on consumption and investment goods are permitted to be constant fractions of total government spending:

$$G_{ct} = \psi G_t,\ p_t G_{xt} = (1-\psi) G_t, \tag{17}$$

where $\psi \in [0, 1]$. From (2) and (16), one can get the aggregate resource constraint faced by the economy:

$$Y_t = C_t + p_t X_t + G_t. \tag{18}$$

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2.4. Equilibrium

This research focuses on perfect foresight equilibrium, which is defined as a path $\{C_t, K_t, L_t, \tau_t^m, G_t, Y_t, s_t\}_{t=0}^{\infty}$ and a set of prices $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ satisfying the first-order conditions, (7)–(10) and (14)–(15), and constraints, (11)–(13) and (16)–(18). s_t denotes the fractions of capital and labor inputs used in the consumption sector,

$$s_t = \frac{K_{ct}}{K_t} = \frac{L_{ct}}{L_t}. \tag{19}$$

In equilibrium,

$$K_{ct} = \overline{K}_{ct},\ L_{ct} = \overline{L}_{ct},\ K_{xt} = \overline{K}_{xt},\ L_{xt} = \overline{L}_{xt}. \tag{20}$$

The quantities of consumption and investment goods demanded by households and the government are equal to the quantities supplied by firms. Moreover, all capital and labor inputs are used in the production of consumption and investment goods. Hence,

$$C_t + G_{ct} = Y_{ct},\ X_t + G_{xt} = Y_{xt}, \tag{21}$$

$$K_{ct} + K_{xt} = K_t,\ L_{ct} + L_{xt} = L_t. \tag{22}$$

It is easy to show that the relative price and the factor prices are

$$p_t = \left(\frac{s_t}{1 - s_t}\right)^{\theta}, \tag{23}$$

$$r_t = \frac{\alpha Y_t}{K_t} \tag{24}$$

$$w_t = \frac{(1-\alpha) Y_t}{L_t}. \tag{25}$$

From (16) to (21), one can derive the total production function

$$Y_t = s_t^{\theta} K_t^{\alpha(1+\theta)} L_t^{(1-\alpha)(1+\theta)}. \tag{26}$$

It is easy to show that the steady-state fractions of factor inputs used in consumption sector s are given by the following expression:

$$s = \frac{\eta[1 - \beta(1-\delta) - \delta\alpha\beta(1-\phi)]}{1 - \beta(1-\delta)} + \psi(1-\eta), \tag{27}$$

and then all of other steady-state values could also be easily derived.

3. DYNAMICS

To analyze the properties of local dynamics of the economy, I take log-linear approximations around the steady state (see Appendix A). Then, the system boils down to

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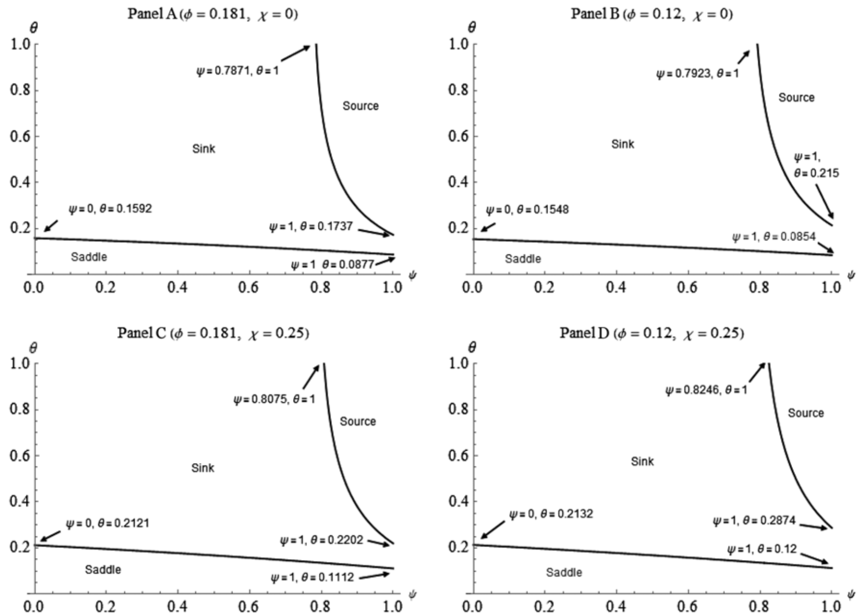

FIGURE 1. Stability properties: $\psi - \theta$.

$$\begin{bmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{bmatrix} = J \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix}, \tag{28}$$

where hat variables refer to their percentage deviations from their respective steady-state values. J is a 2×2 Jacobian matrix.[5] Indeterminacy requires that both eigenvalues of J are inside the unit circle.[6]

For numerical analysis, I calibrate the standard parameters as $\alpha = 0.3$, $\beta = 0.99$, $\delta = 0.025$, and $\chi = 0$, which are commonly used in real business cycle literature. As in Guo and Harrison (2001), the steady-state government spending to output ratio G/Y is set to equal 0.2, implying that the level of tax schedule η is 0.8.

3.1. Sectoral Composition of Government Spending and Returns to Scale

In this subsection, I investigate the relationship between the sectoral composition of government spending and the production externalities in generating indeterminacy. Panel A of Figure 1 depicts the stability properties of the model as a function of the returns to scale parameter θ and the public-consumption share ψ. At each point on the plot, the tax slope parameter ϕ is calibrated to equal Heathcote et al.'s (2017) point estimate of U.S. tax progressivity, 0.181. In this figure, the $\psi - \theta$ constellation is separated into three regions: saddle, sink, and source. The threshold values of θ used to generate indeterminacy can be read off the bottom line on the graph.

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Panel A shows that the minimum degree of externalities $\theta_{\min}$ is inversely related to public-consumption share ψ to produce sunspot equilibria, that is, $\partial\theta_{\min}/\partial\psi < 0$. The policy implication of this result is that the economy is more susceptible to belief-driven fluctuations if the government raises the public-consumption share. At $\psi = 1$, the economy boils down to the modified version of Guo and Harrison's (2001; 2015) model. In this case, multiple equilibria require $0.0877 < \theta < 0.1737$. It can be shown that an empirically realistic value of investment sector-specific externalities $\theta_{US} = 0.108$ (Harrison, 2003) is higher than the threshold value of 0.0877, implying that instability arising from self-fulfilling prophecies may be a feature of the U.S. economy if public-consumption share is one or "relatively large."[7] The steady state turns into a source if θ is greater than 0.1737. Indeterminacy is harder to obtain if more government expenditures are devoted to the investment sector (lower ψ). For example, I find the minimum $\theta = 0.1592$ when $\psi = 0$.

These results can be understood as follows. Starting from a specific equilibrium path, suppose that the agent expects the future rate of return on capital to be high. Accordingly, the agent reallocates factors between sectors, increasing the output of the investment goods sector. As a result, future capital stock will increase. If this is associated with a higher rate of return on capital, the agent's expectation can be self-fulfilled. This can be achieved if sufficiently strong sector-specific externalities are present. This is so because, in this environment, an increase in investment relative to consumption will lead to a decrease in the relative price of investment p_t.[8] A higher public-investment share $(1-\psi)$ implies that fewer resources are devoted to the consumption sector in the steady state (lower s), which makes relative price movement smaller as more factor inputs shift to the investment sector.[9] As a result, higher returns to scale are required to produce sunspot equilibria as ψ declines.

3.2. Sectoral Composition of Government Spending and Tax Progressivity

In this subsection, I investigate the relationship between tax progressivity ϕ and the public-consumption share ψ in producing equilibrium (in)determinacy. The degree of externalities is set to equal the empirically plausible value, $\theta = 0.108$. Panel A of Figure 2 (the solid line) illustrates the local stability properties of the model for different combinations of ψ and ϕ. The region above (below) the solid line is the indeterminate (determinate) zone.[10]

At $\phi = 0$, that is, the tax rate τ is constant at 0.2 $(= 1 - \eta)$, the model collapses to a modified version of Chang et al. (2019) and the threshold value $\psi_{\min} = 0.5626$.[11] As indicated in the graph, the inclusion of a progressive tax raises the critical value of ψ to obtain indeterminacy. For example, $\psi_{\min}$ raises to 0.7735 if the degree of tax progressivity raises to 0.181. The results in Subsection 3.1 indicate that a higher public-consumption share tends to destabilize the economy. Then, Panel A of Figure 2 depicts that, with the benchmark parameterization, a more progressive tax is needed to remove indeterminacy arising from higher ψ.

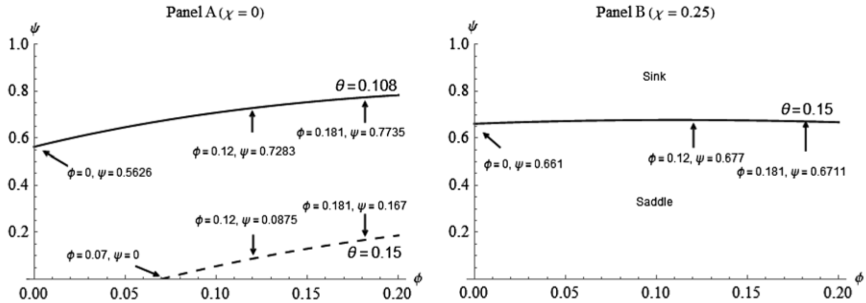

FIGURE 2. Stability properties: $\phi - \psi$.

Intuitively, the higher anticipated rate of return induces an increase in future capital stock. On the one hand, the expected rate of return on capital declines due to the diminishing marginal product of capital (diminishing returns effect). On the other hand, with positive production externalities, the relative price p_t declines as the agent shifts inputs toward the investment sector (price effect). Indeterminacy implies that the price effect dominates the diminishing returns effect. In this environment, whether a higher tax progressivity can remove indeterminacy depends on its impact on the relative strength of the two effects. The expected marginal tax rate increases as the tax structure becomes more progressive.[12] First, a higher marginal tax rate weakens the diminishing returns effect, as the after-tax marginal product of capital declines less. Second, there is a downward shift of the social production frontier as the tax rate increases, weakening the price effect (Chang et al., 2019). The upward-sloping solid line in Panel A of Figure 2 indicates that, *with the benchmark parameterization*, a more progressive tax tends to suppress indeterminacy (at given value of ψ).

Of particular interest, here is whether empirically plausible tax progressivity and public-consumption share are putting the U.S. economy into the sink region. To calibrate the public-consumption share parameter ψ, I compute the average ratio of government consumption expenditure to total government expenditures for the 1979–2018 period (annual data).[13] The average public-consumption share is then found to be 0.7915, which is higher than the critical value of 0.7735, implying that instability arising from agents' self-fulfilling prophecies may indeed be a feature of the U.S. economy.

3.3. Robustness

For sensitivity analysis, I consider three other parameterizations of ϕ and χ that are also considered as empirically plausible to study how public-spending share ψ affects the stability properties of equilibria. As indicated in Panels B, C, and D of Figure 1, it turns out that the results from Subsection 3.1 are robust to these alternative specifications.[14] In addition, Panels A and C, as well as Panels B and D, show that the smaller the elasticity of labor supply (larger χ), the greater the

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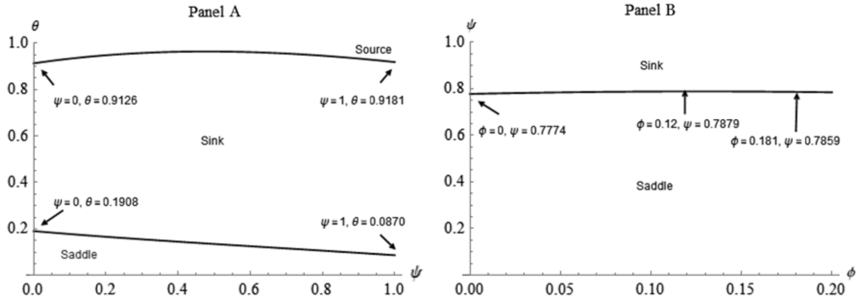

FIGURE 3. Stability properties.

stabilizing effect, in that higher returns to scale are required to generate indeterminacy at a given value of ψ. This finding is in line with Guo and Harrison (2001).[15]

As shown in Figure 2, I also investigate the $\phi - \psi$ nexus for different values of θ and χ. On the one hand, the indeterminacy zone expands when θ is calibrated to equal 0.15, which is Harrison's (2003) point estimate on the degree of externalities in investment sector from the sample of US two-digit manufacturing data.[16] On the other hand, the zone shrinks as χ increases. Moreover, in the case of $\chi = 0.25$, the relationship between ϕ and ψ becomes nonmonotonic.[17] It seems that the elasticity of labor supply can influence the relationship between ϕ and ψ.[18] Generally, indeterminacy requires a sufficiently high public-consumption share. Notably, in all cases, the economy always lies in the indeterminacy region if the public-consumption share and tax progressivity are calibrated to match that in the United States.

4. ALTERNATIVE SPECIFICATION ON PREFERENCE

In this section, I reexamine the two-sector model with a different preference formulation in which government consumption can generate external effects on households' utility. The specification of the lifetime utility is given by:

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^{\theta_1} G_{ct}^{\theta_2})^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\chi}}{1+\chi} \right], \tag{29}$$

where σ is the inverse of the intertemporal elasticity of substitution in effective consumption $C_t^{\theta_1} G_{ct}^{\theta_2}$.[19] The parameter θ_1 satisfies $\theta_1 > 0$ and $\theta_1(1-\sigma) < 1$ to ensure that the utility function is increasing and strictly concave with respect to private consumption. $\theta_2 > 0$ implies a positive preference externality of government spending. Moreover, $\theta_1 + \theta_2 = 1$ so that the utility exhibits linear homogeneity in effective consumption.

Panels A and B in Figure 3 plot the parameter constellations $\psi - \theta$ and $\phi - \psi$, respectively. At each point on the plots, I calibrate $\theta_1 = 0.7178$ and $\sigma = 0.3308$ based on Ni's (1995) estimates of a constant-relative-risk-aversion Cobb-Douglas preference formulation of effective consumption, as in (29). Other parameters are the same as their benchmark values. The result from Panel A parallels the case of wasteful government spending: the economy becomes more susceptible to indeterminacy as ψ increases. Notably, the externality required to produce indeterminacy is 0.1075 when $\psi = 0.7915$, which is lower than its empirically plausible values. This again indicates that the U.S. economy may be unstable. Panel B indicates that, with these parameterizations, the economy always falls within the indeterminacy region with the observed value of ψ (0.7915). In this case, the role of tax progressivity in the stability properties of the economy is rather limited.

5. CONCLUSION

Evidence suggests that the sectoral composition of government spending plays an important role in a two-sector model economy's (in)stability properties. Motivated by the fact that the income taxation system in the United States is considered to be a progressive system, I study how government spending affects the occurrence of indeterminacy when the model incorporates a progressive tax schedule and is calibrated to match the U.S. estimates. The quantitative analysis shows that a higher public-consumption share requires lower returns to scale to generate sunspot equilibria. I also investigate quantitative interrelations between government spending and tax progressivity in generating (in)determinacy. It is emphasized that fiscal policies aiming to stabilize the economy need to coordinate effectively between public spending and tax policy rules.

NOTES

1. For example, Guo and Harrison (2004), Fernàndez et al. (2004), Gokan (2008), Guo and Harrison (2008), Kamiguchi and Tamai (2011), Ghilardi and Rossi (2014), Abad et al. (2017), among others.
2. Futagami et al. (2008), Minea and Villieu (2012), and Hori and Maebayashi (2019) discuss the effects of government spending on the occurrence of indeterminacy in the context of endogenous growth model.
3. Guo and Lansing (1998), Guo and Harrison (2001, 2015), and Chen and Guo (2014) consider a progressive income taxation in their models to investigate the relationship between tax progressivity and the (in)stability properties.
4. Harrison (2011) shows that the indeterminacy properties are independent of the degree of externalities in the consumption sector if the utility function is logarithmic in consumption. Therefore, in this paper, I assume that the size of externalities in the consumption sector is the same as that in the investment sector.
5. The elements of J are shown in the Appendix A.
6. That is, the trace and determinant of the Jacobian matrix $Tr(J)$ and $Det(J)$ must satisfy: $-1 < Det(J) < 1$ and $-Det(J) - 1 < Tr(J) < Det(J) + 1$. As the expressions of the trace and determinant are too incommodious (see Appendix A), the necessary and sufficient conditions for indeterminacy cannot be derived analytically for all feasible parameter values. Therefore, I perform numerical exercises.
7. When $\theta = 0.108$, $\psi = 0.774$ on the graph.

8. With positive production externalities, the social production frontier is convex to the origin. Firms are able to produce more investment goods for a given reduction in the production of consumption goods, because the sector-specific externalities increase the productivity of the investment sector as more inputs flow into it (Benhabib and Farmer (1996)). Notably, indeterminacy is not possible if $\theta < 0.0877$, regardless of how government spending is distributed between consumption and investment goods.

9. See (27), $\partial s/\partial \psi > 0$.

10. In this graph, the value of ϕ is restricted to being no greater than 0.2, as it is the largest estimated value for progressivity in Heathcote et al. (2017).

11. In Chang et al. (2019), the critical level of the public-consumption share is 0.5665 when $\tau = 0.2$.

12. That is, $\partial \tau_t^m/\partial \phi > 0$. Tax rate τ_t also increases as ϕ increases: $\partial \tau_t/\partial \phi > 0$.

13. Data source: U.S. Bureau of Economic Analysis, "Table 3.9.5. Government Consumption Expenditures and Gross Investment."

14. These parameter values are chosen because: (1) $\phi = 0.12$ is the average point estimate of Chen and Guo (2013) and is used as benchmark parameterization in Guo and Harrison (2015); (2) $\chi = 0.25$, has been adopted in many related studies such as Guo and Harrison (2001).

15. Intuitively, the agent is less willing to move out of leisure into labor as the labor supply becomes less elastic.

16. When $\theta = 0.108$ and $\chi = 0.25$, the whole $\phi - \psi$ space is a saddle, implying that indeterminacy is not possible for all feasible combinations of ϕ and ψ.

17. In fact, from Figure 1, one can also observe that the effect of ϕ on the threshold value of θ is ambiguous. As shown earlier in Subsection 3.2, the role of ϕ in relation to the stability properties depends on its impact on the relative strength of the two opposite effects.

18. This is an issue that is worth exploring in future research.

19. When $\sigma < 1$ ($\sigma > 1$), C_t, and G_{ct} are Edgeworth complements (Edgeworth substitutes). C_t and G_{ct} are separable if $\sigma = 1$.

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APPENDIX A

The appendix sets out the log-linearized system, the elements that make up the Jacobian matrix of J in (28), and the expressions for the trace and determinant of J.

A.1. THE LOG-LINEARIZED MODEL

$$\begin{aligned}
\hat{C}_t + \chi \hat{L}_t &= -\frac{\tau^m}{1-\tau^m}\hat{\tau_t^m} + \hat{w}_t,\\
\hat{r}_t &= \hat{Y}_t - \hat{K}_t,\\
\hat{w}_t &= \hat{Y}_t - \hat{L}_t,\\
\hat{Y}_t &= \theta\hat{s}_t + \alpha(1+\theta)\hat{K}_t + (1-\alpha)(1+\theta)\hat{L}_t,\\
\frac{G}{Y}\hat{G}_t &= [1-(1-\phi)\eta]\hat{Y}_t,\\
\hat{p}_t &= \frac{\theta}{1-s}\hat{s}_t,\\
\frac{C}{Y}\hat{C}_t &= s\hat{s}_t + s\hat{Y}_t - \psi\frac{G}{Y}\hat{G}_t,\\
\tau^m\hat{\tau_t^m} &= \eta(1-\phi)\phi\hat{Y}_t,\\
\widehat{C_{t+1}} + \hat{p}_t &= \hat{C}_t + \beta(1-\delta)\widehat{p_{t+1}} - \beta\frac{r}{p}\tau^m\widehat{\tau_{t+1}^m} + \beta(1-\tau^m)\frac{r}{p}\widehat{r_{t+1}},\\
\widehat{K_{t+1}} &= (1-\delta)\hat{K}_t + \eta(1-\phi)\frac{Y}{Kp}\hat{Y}_t - \frac{C}{Kp}\hat{C}_t - \delta\hat{p}_t,
\end{aligned}$$

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where

$$
\begin{aligned}
1-\tau^m &= \eta(1-\phi),\\
\frac{G}{Y} &= 1-\eta,\\
\frac{r}{p} &= \frac{1-\beta(1-\delta)}{\beta\eta(1-\phi)},\\
\frac{Y}{Kp} &= \frac{1-\beta(1-\delta)}{\alpha\beta\eta(1-\phi)},\\
\frac{C}{Kp} &= \frac{1-\beta(1-\delta)}{\alpha\beta(1-\phi)}-\delta,\\
\frac{C}{Y} &= \frac{\eta[1-\beta(1-\delta)-\delta\alpha\beta(1-\phi)]}{1-\beta(1-\delta)},\\
s &= \frac{\eta[1-\beta(1-\delta)-\delta\alpha\beta(1-\phi)]}{1-\beta(1-\delta)}+\psi(1-\eta).
\end{aligned}
$$

A.2. ELEMENTS OF THE BENCHMARK MODEL'S JACOBIAN MATRIX

$$
\begin{aligned}
J_{1,1} &= (1-\delta)+\left[\frac{1-\beta(1-\delta)}{\alpha\beta}+\delta\Gamma_1\right]\Lambda_1,\\
J_{1,2} &= \left[\frac{1-\beta(1-\delta)}{\alpha\beta}+\delta\Gamma_1\right]\Lambda_2-\delta\Gamma_2-x_1,\\
J_{2,1} &= \frac{\Gamma_1\Lambda_1+\{\{(1-\phi)[1-\beta(1-\delta)]-\beta(1-\delta)\Gamma_1\}\Lambda_1-[1-\beta(1-\delta)]\}\,J_{1,1}}{1-\beta(1-\delta)\Gamma_2-\{(1-\phi)[1-\beta(1-\delta)]-\beta(1-\delta)\Gamma_1\}\Lambda_2},\\
J_{2,2} &= \frac{1-\Gamma_2+\Gamma_1\Lambda_2+\{\{(1-\phi)[1-\beta(1-\delta)]-\beta(1-\delta)\Gamma_1\}\Lambda_1-[1-\beta(1-\delta)]\}\,J_{1,2}}{1-\beta(1-\delta)\Gamma_2-\{(1-\phi)[1-\beta(1-\delta)]-\beta(1-\delta)\Gamma_1\}\Lambda_2},
\end{aligned}
$$

where the $J_{i,j}$ are the elements of the Jacobian matrix J and

$$
\begin{aligned}
\Gamma_1 &= \frac{\Omega_2}{x_2 s(1-s)}>0,\ \Gamma_2=\frac{\theta x_1}{x_2 s(1-s)}>0,\\
\Lambda_1 &= \frac{\alpha(1+\theta)x_2 s}{\Omega_1+\Omega_2}>0,\ \Lambda_2=\frac{\theta x_1-x_2 s(1-\alpha)(1+\theta)}{\Omega_1+\Omega_2}\gtrless 0,\\
\Omega_1 &= x_2 s[1-(1-\phi)(1-\alpha)(1+\theta)]>0,\ \Omega_2=\theta x_2\{s-\psi[1-(1-\phi)\eta]\}>0,\\
x_1 &= \frac{C}{Kp}=\frac{1-\beta(1-\delta)}{\alpha\beta(1-\phi)}-\delta>0,\ x_2=\frac{Y}{Kp}=\frac{1-\beta(1-\delta)}{\alpha\beta\eta(1-\phi)}>0.
\end{aligned}
$$

A.3. TRACE AND DETERMINANT

The trace and determinant of J in (28) are given by:

$$
\begin{aligned}
Tr(J) &= J_{1,1}+J_{1,2}\\
&= 1-\delta+\frac{1-\beta[1-\delta(1+\alpha\Gamma_1)]}{\alpha\beta}\Lambda_1
\end{aligned}
$$

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$$+\frac{\alpha\beta(1-\Gamma_2+\Gamma_1\Lambda_2)-\{\alpha\beta(x_1+\Gamma_2\delta)-\{1-\beta[1-\delta(1+\alpha\Gamma_1)]\}\Lambda_2\}\{(\Lambda_1-1)-\beta(1-\delta)[\Lambda_1(1-\phi+\Gamma_1)-1]-\Lambda_1\phi\}}{\alpha\beta\,\{1-\Lambda_2(1-\phi)-\beta(1-\delta)[\Gamma_2-\Lambda_2(1-\phi+\Gamma_1)]\}},$$

$$\begin{aligned} Det(J) &= J_{1,1}J_{2,2}-J_{1,2}J_{2,1} \\ &= \frac{(1-\Gamma_2)[1-\beta(1-\delta)]\Lambda_1+\alpha\beta\{1-\Gamma_2(1-\delta)+\Gamma_1\Lambda_1x_1+\Gamma_1\Lambda_2+\delta[\Gamma_1(\Lambda_1-\Lambda_2)-1]\}}{\alpha\beta\,\{1-\Lambda_2(1-\phi)-\beta(1-\delta)[\Gamma_2-\Lambda_2(1-\phi+\Gamma_1)]\}}. \end{aligned}$$

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