

THE SHALLOW WATER EQUATIONS ON THE UNIT SPHERE WITH SCATTERED DATA

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This dissertation deals with different aspects of the shallow water equations (SWEs) on the unit sphere in the three-dimensional Euclidean space. Such equations arise by vertically integrating the Navier–Stokes equations and the mass conservation equation. They are a simplification of the equations describing various meteorological phenomena such as tides in coastal areas. The SWEs are used as a primary test problem for new efficient methods for real-life hydrodynamical problems.

In the dissertation, firstly, we prove the existence of weak solutions for the SWEs by considering the Galerkin method. We prove that the Galerkin approximations converge to the solution of the original system.

When solving global meteorological problems, data are often collected by satellites. Therefore, it becomes inconvenient to use methods that require structured grids. Consequently, the development of finite-element methods that have fewer geometric requirements for mesh generation is becoming more and more popular.

A second contribution of the dissertation is the design of a finite-element method using spherical splines in the case of scattered data, which are involved in the modelling of the SWEs. We implement our proposed method to the standard Williamson test set for the inviscid SWEs with satellite data.

To generate more realistic flows it is important to include viscosity in the model. As a third contribution, we derive *a priori* error estimates for this method using spherical splines, when viscosity is included in both the momentum and the continuity equations. We also calculate estimated orders of convergence that support our theoretical results.

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As is well known, the matrices arising when using finite-element methods are often ill conditioned. To overcome the ill conditioning, as a fourth contribution of the dissertation, we study two different types of preconditioners, namely the additive Schwarz and the alternative triangular preconditioners. We calculate numerical bounds for the condition numbers to verify the effectiveness of both preconditioners for the spherical SWE problem.

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