J. Plasma Physics (1999), *vol.* 62, *part* 4, *pp.* 389–396. Printed in the United Kingdom

Relativistic self-focusing of a rippled laser beam in a plasma

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(Received 26 June 1998 and in revised form 10 April 1999)

Abstract. This paper presents an analysis of the relativistic self-focusing of a rippled Gaussian laser beam in a plasma. Considering the nonlinearity as arising owing to relativistic variation of mass, and following the WKB and paraxial-ray approximations, the phenomenon of self-focusing of rippled laser beams is studied for arbitrary magnitude of nonlinearity. Pandey et al. [*Phys. Fluids* **82**, 1221 (1990)] have shown that a small ripple on the axis of the main beam grows very rapidly with distance of propagation as compared with the self-focusing of the main beam. Based on this analogy, we have analysed relativistic self-focusing of rippled beams in plasmas. The relativistic intensities with saturation effects of nonlinearity allow the nonlinear refractive index in the paraxial regime to have a slower radial dependence, and thus the ripple extracts relatively less energy from its neighbourhood.

1. Introduction

New short-pulse laser technology (Main and Mourou 1988; Patterson et al. 1991) has recently made possible the production of extremely intense laser sources at multiterawatt level. The focused intensities obtained are very high, of the order of 10^{18} W cm^{-2}, and further developments are aimed at intensities exceeding 10^{20} W cm^{-2}. The development of such high-intensity lasers has lead to the possibility of observing relativistic effects when a laser pulse interacts with a fully ionized plasma. The propagation of a high-intensity laser pulse through a fully ionized plasma is a basic physics problem, and is of great interest with regard to practical applications for compact X-ray lasers (see Sprangle and Esarey 1992, and references therein), laser–plasma-based particle accelerators (Burnett and Corkum 1989) and the fast ignitor scheme for studies of inertial-confinement fusion (ICF) (Tabak et al. 1994). The relativistic filamentation instability can lead to modification of the propagating pulse by spatially modulating the laser intensity transverse to the direction of propagation. The relativistic filamentation and self-focusing instabilities have been studied for a number of years, and are important because one needs to understand laser propagation at high intensities before one can interpret the results from other nonlinear phenomena (Sprangle and Esarey 1991; Monot 1995).

A laser beam propagating in an underdense plasma with a frequency ω_p

smaller than the laser frequency ω undergoes relativistic self-focusing as soon as its total power P exceeds a critical value $P_{\text{cr}} \approx 17(\omega/\omega_p)^2$ GW. This self-focusing is due to the relativistic mass increase of plasma electrons and the ponderomotive expulsion of electrons from the pulse region, creating a density depression. Both effects lead to a local decrease in the plasma frequency and an increase in the refractive index. The medium then acts as a positive lens. However, the self-focusing due to the density depression occurs on a longer time scale than the self-focusing due to the relativistic mass increase. The analysis presented here will be concerned only with relativistic self-focusing on a time scale sufficiently short that the plasma density profile does not evolve significantly under the influence of the laser beam. This implies that the pulse length τ_R of the laser beam must be short compared with $\tau_S = r/C_s$, the time scale for the density depression to occur (here r is the laser beam radius and C_s is the ion sound speed). An analysis in terms of the envelope and paraxial approximations shows that, depending on the laser pulse and plasma parameters, either self-focusing of the whole pulse or pulse filamentation occurs (Asthana et al. 1994; Borisov et al. 1995).

In an experimental situation where intense laser or electromagnetic beams travelling through nonlinear self-focusing media result in multiple filament formation, there is a one-to-one correspondence between filaments and intensity spikes riding with the incident laser beam, as studied earlier by Abbi et al. (1985). The origin of the filamentation instability may be attributed to small-scale density perturbations (resulting from quasineutrality) or small-scale intensity spikes associated with the main beam. The perturbations grows at the cost of the main beam, and this is detrimental to laser-induced fusion. In laser plasma experiments, the filamentary structure created in an underdense plasma undergoes self-focusing. The self-focused filaments spoil the symmetry of the energy deposition as well as triggering parametric instabilities that may lead to back- and side-scattering of the laser beam. Thus direct and indirect experimental evidence reveals that an apparently smooth laser beam has intensity spikes that may lead to distortion of self-focusing in nonlinear media (Kothari and Kobayashi 1983; Pandey et al. 1990).

In view of the ongoing development of ultra-intense short-pulse lasers, we present here an analysis of the relativistic self-focusing of a Gaussian laser beam with a ring ripple superimposed on it for arbitrary large nonlinearity. In Sec. 2, the general equations for the self-focusing are presented. In Sec. 3, an expression for the nonlinearity is given, and an equation governing the variation of the beam width parameter with distance of propagation, the self-trapping condition and the critical power are presented. Results and a discussion are given in Sec. 4, supported by numerical analysis.

2. General equations for self-focusing of the beam

Consider the propagation of a Gaussian laser beam with a ring ripple superimposed on it in a homogeneous collisionless plasma along the z direction. The electric field at the fixed plane $z = 0$ of the main beam may be represented by

$$\mathbf{E}_0|_{z=0} = \mathbf{E}_{00} \exp\left(-\frac{r^2}{2r_0^2}\right) \exp(i\omega t), \tag{1a}$$

where ω is the angular frequency of the laser beam, r is the radial coordinate of the cylindrical coordinate system and r_0 is the initial width of the main beam. The electric field of the ring superimposed on the main beam may be expressed as

$$\mathbf{E}_1|_{z=0} = \mathbf{E}_{10}\frac{r}{r_{10}}\exp\left(-\frac{r^2}{2r_{10}^2}\right)\exp[(i\omega t - \Phi_p)], \tag{1b}$$

where Φ_p is the phase difference between the main beam and the ripple; the maximum field of the ring is at $r = r_{10}$. The total electric vector of the beam can thus be written as $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$. The intensity distribution of the rippled Gaussian laser beam is thus given by

$$\mathbf{E}\cdot\mathbf{E}^*|_{z=0} = E_{00}^2\exp\left(-\frac{r^2}{r_0^2}\right)\left\{1 + 2\frac{E_{10}}{E_{00}}\frac{r}{r_{10}}\cos\Phi_p\exp\left[\frac{r^2}{2}\left(\frac{1}{r_0^2}-\frac{1}{r_{10}^2}\right)\right]\right.$$
$$\left. + \frac{E_{10}^2}{E_{00}^2}\left(\frac{r}{r_{10}}\right)^2\exp\left[r^2\left(\frac{1}{r_0^2}-\frac{1}{r_{10}^2}\right)\right]\right\}. \tag{2}$$

The wave equation governing the electric vector of the beam in a plasma with dielectric constant ε can be written as

$$\nabla^2\mathbf{E} + \frac{\omega^2}{c^2}\varepsilon\mathbf{E} = 0. \tag{3}$$

In writing (3), the term $\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E})$ has been neglected, which is justified when $(c^2/\omega^2)\,|(1/\varepsilon)\nabla^2\ln\varepsilon| \ll 1$. The nonlinear dielectric constant of the medium for arbitrary large nonlinearity is (Asthana et al. 1994)

$$\varepsilon(\langle\mathbf{E}\cdot\mathbf{E}\rangle) = \varepsilon_0 + \Phi(\langle\mathbf{E}\cdot\mathbf{E}\rangle). \tag{4}$$

In the paraxial-ray approximation, one generally expands Φ around $\Phi \approx 0$. However, with such an expansion, one can study only those cases where $\Phi \ll \varepsilon_0$. To study self-focusing for arbitrary large Φ, the nonlinear dielectric constant of the medium at $r = 0$ is needed.

$$\varepsilon(\langle\mathbf{E}\cdot\mathbf{E}\rangle) = \varepsilon_0'(f) + \psi(f), \tag{5}$$

where

$$\varepsilon_0'(f) = \varepsilon_0 + \Phi\left(\left\langle\frac{k(0)E_{00}^2}{k(f2f)^2}\right\rangle\right),$$

$$\psi(f) = \Phi(\mathbf{E}\cdot\mathbf{E}\rangle) - \Phi\left(\left\langle\frac{k(0)E_{00}^2}{k(f)2f^2}\right\rangle\right).$$

Here f is the dimensionless beam-width parameter, defined below in (10), and k is the propagation constant, defined below in (6). Using the WKB approximation and following Akhmanov et al. (1968) and Sodha et al. (1974), one can write

$$E(r,z) = A(r,z)\left[\frac{k(0)}{k(z)}\right]^{1/2}\exp\left[-i\int k(f)\,dz\right], \tag{6}$$

where

$$k(z) = \frac{\omega}{c}[\varepsilon_0'(f)]^{1/2}, \quad k(0) = \frac{\omega}{c}[\varepsilon_0'(f=1)]^{1/2}.$$

Substituting for E and ε in (3), one obtains

$$-2ik(f)\frac{\partial A}{\partial z}+\nabla^2 A+\frac{\omega^2}{c^2}\Psi(f)A=0. \tag{7}$$

Putting

$$A(r,z)=A_0(r,z)\exp\left[-i\int k(f)\,dS\right]$$

in (7) and separating real and imaginary parts, we find

$$2\frac{\partial S}{\partial z'}+\left(\frac{\partial S}{\partial r}\right)^2+\frac{\omega^2}{c^2}\frac{\varepsilon_1(f)}{k^2(f)}=\frac{1}{k^2A_0}\left(\frac{\partial^2 A_0}{\partial r^2}+\frac{1}{r}\frac{\partial A_0}{\partial r}\right), \tag{8}$$

$$\frac{\partial A_0^2}{\partial z}+\frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r}+A_0^2\left(\frac{\partial^2 S}{\partial r^2}+\frac{1}{r}\frac{\partial S}{\partial r}\right)=0. \tag{9}$$

The solutions of (8) and (9) are

$$A_0^2=\frac{E_{00}^2}{f^2}\exp\left(-\frac{r^2}{r_0^2f^2}\right)\left\{1+\frac{2E_{10}}{E_{00}}\frac{r}{r_{10}f}\cos\phi_p\exp\left(\frac{r^2}{2r_0^2f^2}\left(1-\frac{r_0^2}{r_{10}^2}\right)\right]\right.$$
$$\left.+\left(\frac{E_{10}}{E_{00}}\frac{r_0}{r_{10}f}\right)^2\exp\left[\frac{r^2}{2r_0^2f^2}\left(1-\frac{r_0^2}{r_{10}^2}\right)\right]\right\}, \tag{10a}$$

$$S=\tfrac{1}{2}r^2\beta(z)+\eta(z), \tag{10b}$$

$$\beta=\frac{1}{f}\frac{df}{dz}. \tag{10c}$$

It should be noted that β is the inverse radius of curvature of the front and $r_0f(z)$ is the width of the main beam. The intensity profile of the beam inside the plasma can be put in the form

$$\mathbf{E}\cdot\mathbf{E}^*=I_0(r,z)+I_1(r,z), \tag{11}$$

where

$$I_0(r,z)=\frac{k(0)E_{00}^2}{k(z)f^2}\exp\left(-\frac{r}{r_0^2f^2}\right),$$

$$I_1(r,z)=\frac{k(0)E_{00}^2}{k(f)f^2}\left\{\frac{2E_{10}}{E_{00}}\frac{r}{r_{10}f}\cos\phi_p\exp\left[-\frac{r^2}{2r_0^2f^2}\left(1+\frac{r_0^2}{r_{10}^2}\right)\right]\right.$$
$$\left.+\left(\frac{E_{10}}{E_{00}}\frac{r}{r_{10}f}\right)^2\exp\left(-\frac{r^2}{r_{10}^2f^2}\right)\right\}.$$

The maximum of I_1 occurs at $r=r_{\max}$, where

$$\frac{r_{\max}}{r_0f(z)}=\left[\frac{\dfrac{r_0}{r_{10}}+2\dfrac{E_{10}}{E_{00}}\cos\phi_p}{\left(\dfrac{r_0^2}{r_{10}^2}-1\right)\dfrac{r_0^2}{r_{10}^2}+\left(3+\dfrac{r_0^2}{r_{10}^2}\right)\dfrac{E_{00}}{E_{10}}\cos\phi_p}\right]^{1/2}. \tag{12}$$

Equations (8) and (9), together with (10)–(12), can now be used to study the self-focusing of the beam for relativistic nonlinearity.

3. Nonlinear dielectric constant: relativistic nonlinearity

3.1. Self-focusing equation

Following Asthana et al. (1994), the effective dielectric constant in a collisionless plasma for relativistic nonlinearity can be written as

$$\varepsilon = \varepsilon_0 + \phi(\langle \mathbf{E}\cdot\mathbf{E}\rangle), \tag{13a}$$

where $\varepsilon_0 = 1-\omega_p^2/\omega^2$ is the linear part of the dielectric constant and $\omega_p = (4\pi N_0 e^2/m)^{1/2}$ is the plasma frequency in the absence of the beam. The electron density and charge are N_0 and e respectively. The nonlinear term due to relativistic variation of mass for a circularly polarized wave is

$$\phi\langle \mathbf{E}\cdot\mathbf{E}\rangle) = \frac{\omega_p^2}{\omega^2}\{1-[(1+\tfrac{1}{2}\alpha EE^*)C_1]^{-1/2}\}, \tag{13b}$$

with

$$C_1 = 1+\frac{e^2EE^*}{16\lambda^2 m_0^2\omega^2c^2}, \quad \alpha = \frac{e^2}{m_0^2\omega^2c^2}.$$

Substitution for E from (6), and S, A_0 and β from (10) into (13), and following the paraxial-ray approximation, ϕ can be written as (correct to terms in r^2)

$$\phi\left(\frac{k(0)E_{00}^2}{k(f)2f^2}\left(1+\frac{E_{10}r_0}{E_{00}r_{10}}\cos\phi_p\right)\right) = \frac{\omega_p^2}{\omega^2}\left\{1-\left[1+\frac{k(0)\alpha E_{00}^2}{k(f)2f^2}\left(1+\frac{E_{10}r_0}{E_{00}r_{10}}\cos\phi_p\right)\right]\right.$$
$$\left.\times\left[1+\frac{k(0)E_{00}^2}{k(f)16\lambda^2 2f^2}\left(1+\frac{E_{10}r_0}{E_{00}r_{10}}\cos\phi_p\right)\right]^{-1/2}\right\}. \tag{14}$$

Substituting for S and ϕ in (8), using the paraxial-ray approximation (i.e. $(r/r_0 f)^4 \ll 1$), equating the coefficients of r^2 on both sides of the resulting equation, and substituting for β, one obtains

$$\frac{d^2f}{dz^2} = \frac{1}{k^2(f)r_0^4f^3} - \left(\frac{\omega_p r_0}{c}\right)^2\frac{k(0)\alpha E_{00}^2}{k(f)4r_0^4f^3k^2}$$
$$\times\frac{1+\dfrac{1}{4\lambda^2}\dfrac{k(0)}{k(f)}\dfrac{\alpha E_0^2}{2f^2}X}{\left[1+\dfrac{k(0)}{k(f)}\dfrac{\alpha E_{00}^2}{2f^2}X\left(1+\dfrac{k(0)}{k(f)}\dfrac{1}{16\lambda^2}\dfrac{\alpha E_{00}^2}{2f^2}X\right)\right]^{3/2}}XY, \tag{15a}$$

with

$$X = 1+\frac{E_{10}r_0}{E_{00}r_{10}}\cos\phi_p, \tag{15b}$$

$$Y = 1-\frac{E_{10}r_0}{E_{00}r_{10}}\cos\phi_p\left[1-\frac{1}{2}\left(\frac{r_0^2}{r_{10}}+1\right)\right]-\left(\frac{E_{10}r_0}{E_{00}r_{10}}\right)^2. \tag{15c}$$

3.2. Self-trapping and critical power

For an initial plane wave front of the beam, the initial conditions are $f(z=0)=1$ and $(df/dz)|_{z=0}=0$. When the two terms on the right-hand side of (15) cancel each other at $z=0, d^2f/dz^2=0$. If one further considers a parallel beam at $z=0$ then $df/dz = 0$ at $z=0$; thus, if $f=1$ at $z=0$, it remains so for all values of z (in other words, the beam propagates without convergence or divergence). The critical power for self-trapping of a rippled Gaussian laser beam is therefore

$$\left(\frac{\omega_p r_0}{c}\right)^2 = \frac{2\left[1+\frac{\alpha E_{00\mathrm{cr}}^2}{2}X\left(1+\frac{1}{16\lambda^2}\frac{\alpha E_{00\mathrm{cr}}^2}{2}X\right)\right]^{3/2}}{\left(1+\frac{1}{4\lambda^2}\frac{\alpha E_{00\mathrm{cr}}^2}{2}X\right)\frac{\alpha E_{00\mathrm{cr}}^2}{2}XY}. \tag{16}$$

The corresponding critical beam power is

$$\begin{aligned} P_{\mathrm{cr}} &= \frac{c}{8\pi}\int \varepsilon^{1/2} E_{00\mathrm{cr}}^2\, 2\pi r\, dr \\ &= \frac{c r_0^2 E_{00\mathrm{cr}}^2}{8}\left\{1-\frac{\omega_p^2}{\omega^2}\left[1+\frac{\alpha E_{00\mathrm{cr}}^2}{2}X\left(1+\frac{1}{16\lambda^2}\frac{\alpha E_{00\mathrm{cr}}^2}{2}X\right)\right]^{-1/2}\right\}^{1/2} W. \end{aligned} \tag{17}$$

4. Results and discussion

Equation (15a) is the fundamental equation for determining the focusing/defocusing of a rippled Gaussian laser beam in a plasma. The first term on the right-hand side represents the diffraction phenomenon of the ripple. The rather complicated second term, which arises from the relativistic nonlinearity, describes nonlinear refraction. The relative magnitude of these terms determine the focusing/defocusing behaviour of the rippled beam. Equation (16) yields the self-trapping condition, with the corresponding critical power of the beam given by (17). Furthermore, (16) has two roots: $E_{00\mathrm{cr}1}$ (corresponding to a critical power $P_{\mathrm{cr}1}$) and $E_{00\mathrm{cr}2}$ (corresponding to a critical power $P_{\mathrm{cr}2}$) ($E_{00\mathrm{cr}1} < E_{00\mathrm{cr}2}$ and $P_{\mathrm{cr}1} < P_{\mathrm{cr}2}$). Hence the beam becomes self-trapped in this region, and the medium acts as an oscillatory waveguide. To obtain a numerical assessment of self-focusing, (15) is solved numerically for typical sets of parameters for a neodymium glass laser with irradiance exceeding 10^{18} W cm^{-1}, $\omega_{p0} = 0.5\omega$, $r =$ 1–3 μm, $E_{10}/E_{00} = 0.2$, $\phi_p = 0$ and $r_0/r_{10} = 1$. Figure 1 illustrates the variation of critical power with dimensionless beamwidth for a rippled Gaussian laser beam. The variations of the beamwidth parameter f of the ripple and the dimensionless axial intensity with propagation distance are illustrated in Figures 2 and 3 respectively. The periodic structure is a signature of beam propagation in a nonlinear medium with a power in excess of the critical power for self-focussing.

We find that self-focusing of a rippled Gaussian laser beam can be analysed like the self-focusing of a Gaussian beam in a plasma. The power of the beam and the phase difference between the electric vector of the main beam and the ripple are found to change the nature of the self-focusing of the ripple significantly. Comparing our results with earlier work by Pandey et al. (1990) shows that a small ripple on the axis of the main beam grows very rapidly with distance of propagation as compared with self-focusing of the main beam, which helps the relativistic self-focusing of a rippled laser beam in a plasma. Using (15), one can write equations similar to (15)–(17) of Pandey et al. (1990) to find the growth of a ripple in a plasma. It is further evident from (13) that the

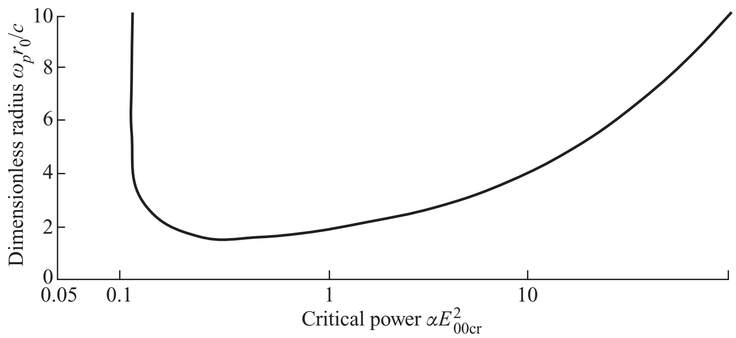

Figure 1. Variation of critical power for self-focusing (relativistic nonlinearity) with dimensionless rippled beamwidth radius $\omega_{p0} r_0/c$.

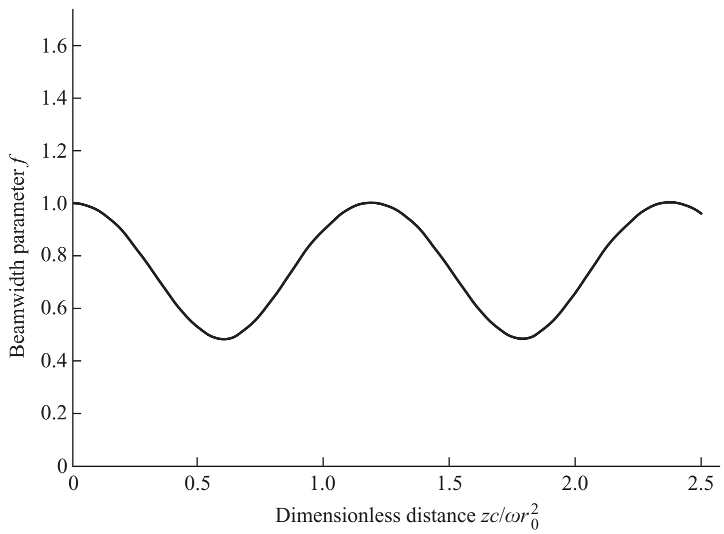

Figure 2. Variation of beamwidth parameter f with propagation distance z.

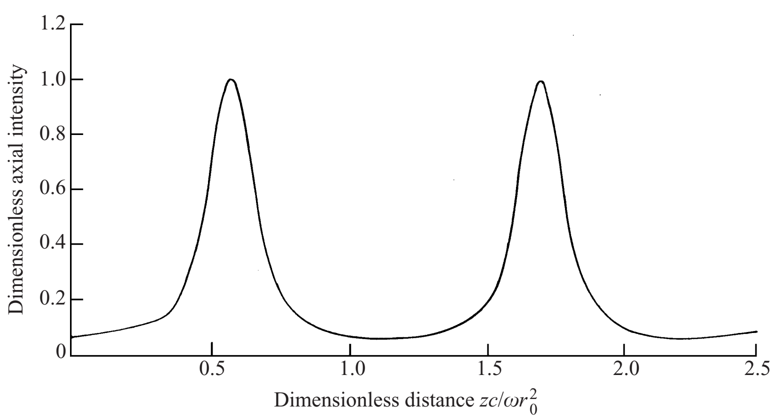

Figure 3. Variation of dimensionless axial intensity with propagation distance z.

https://doi.org/10.1017/S0022377899008016 Published online by Cambridge University Press

relativistic-intensity with saturation effects of nonlinearity allows the nonlinear refractive index in the paraxial region to have a slower radial dependence, and thus the ripple extracts relatively less energy from its neighbourhood. The present study has been performed under the paraxial-ray approximation, which assumes that $k_{\perp} \ll k$, where $k_{\perp}$ is the transverse wave vector. For still higher values of the rippled laser intensity, $k_{\perp} > k$, and hence the paraxial approximation does not hold.

At this stage, it is worth comparing our results when a sizeable region of underdense plasma is placed in front of an overdense plasma. As the laser light enters the system from the left and begins to penetrate through the critical density, the surface of the plasma becomes corrugated (Wilks et al. 1992). This behaviour is related to the 'bubble' formation studied by Valeo and Estabrook (1975) and Estabrook (1976). Our case differs in that the perturbation on the critical surface is seeded by the electron motion in the underdense plasma well before the ions have a chance to move, in contrast to the previous work, which assumes the ions to be responsible for the perturbation. This Rayleigh–Taylor-like instability is due to the fact that the photons effectively accelerate the plasma interface. As the pulse continues to propagate, the absorption increases as a function of time, such that, after a certain propagation distance, the divergence of diffraction overcomes the self-contraction, and therefore the ripple intensity decays.

Acknowledgements

One of the authors (M. V. Asthana) undertook this work with the support of the Abdus Salam International Centre for Theoretical Physics, Programme for Training and Research in Italian Laboratories, Trieste, Italy.

References

Abbi, S. C., Kothari, N. C. and Subramanyam, V. R. 1985 *Phys. Rev. Lett.* **55**, 71.

Akhamanov, S. A., Sukhorukov, A. P. and Khukhlov, R. V. 1968 *Soviet Phys. Usp.* **10**, 609.

Asthana, M., Sodha, M. S. and Maheswari, K. P. 1994 *J. Plasma Phys.* **51**, 155.

Borisov, A. B., Shiryaev, O. B., McPherson, A., Bover, K. and Rhodes, C. K. 1995 *Plasma Phys. Contr. Fusion* **37**, 569.

Burnett, N. H. and Corkum, P. B. 1989 *J. Opt. Soc. Am.* **6**, 1195.

Estabrook, K. 1976 *Phys. Fluids* **19**, 1733.

Kothari, N. C. and Kobayashi, T. 1983 *Phys. Rev. Lett.* **50**, 160.

Main, P. and Mourou, G. 1988 *Opt. Lett.* **13**, 467.

Monot, P. 1995 *Phys. Rev. Lett.* **74**, 2953.

Pandey, H. D., Tripathi, V. K. and Sodha, M. S. 1990 *Phys. Fluids* **B2**, 1221.

Patterson, F. G., Gonzales, R. and Perry, M. D. 1991 *Opt. Lett.* **16**, 1107.

Sodha, M. S., Ghatak, A. K. and Tripathi, V. K. 1974 *Self Focusing of Laser Beams in Plasmas, Semiconductors and Dielectrics.* Tata McGraw Hill, New Delhi.

Sprangle, P. and Esarey, E. 1991 *Phys. Rev. Lett.* **67**, 2021.

Sprangle, P. and Esarey, E. 1992 *Phys. Fluids.* **B4**, 2241.

Tabak, M., Hammer, J., Glinsky, M. E., Kruer, W. L., Wilks, S. C., Woodworth, J., Campbell, E. M., Perry, M. D. and Mason, R. J. 1994 *Phys. Plasmas* **1**, 1626.

Wilks, S. C., Kruer, W. L., Tabak, M. and Langdon, A. B. 1992 *Phys. Rev. Lett.* **69**, 1383.