Convection in Astrophysics
Proceedings IAU Symposium No. 239, 2006
F. Kupka, I. W. Roxburgh & K. L. Chan, eds.

doi:10.1017/S1743921307000178

Probing Reynolds stress models of convection with numerical simulations: I. Overall properties: fluxes, mean profiles

F. Kupka[1] **and H. J. Muthsam**[2]

[1]Max-Planck-Institute for Astrophysics, Karl-Schwarzschild-Str. 1, 85741 Garching, Germany
email: fk@mpa-garching.mpg.de

[2]Faculty of Mathematics, Univ. of Vienna, Nordbergstraße 15, A-1090 Vienna, Austria
email: Herbert.Muthsam@univie.ac.at

Abstract. We introduce an extended 3D numerical simulation study of Reynolds stress models of stellar convection and probe fluxes as well as mean temperature gradient profiles.

Keywords. Convection, turbulence, stars: interiors

1. Introduction and motivation

To account for the non-local nature of turbulent convection numerous models have been proposed. The Reynolds stress approach is the most systematic one among them (cf. Canuto 1993) and predicts lower order moments of the ensemble averaged fluctuations of velocity, temperature, and density around their mean values as functions of location and time. As the hydrodynamical equations are non-linear, moment expansions yield an unclosed hierarchy of equations. Closure approximations truncate the hierarchy at third or fourth order. It has been subject of a long-standing debate, whether such approximations can be made for a physical parameter space large enough to avoid tuning of closure 'constants' for each case and whether they provide predictive capability beyond integral quantities (stellar radius, depth of a convective zone, etc.). Systematic studies have remained small in number. The most extended one has been published by Chan & Sofia (1996), but focused on consistency tests of individual approximations. Grossman (1996) used particle simulations which contained their own approximations to the hydrodynamical equations. In Kupka (1999) solutions of a closed set of moment equations (Canuto & Dubovikov 1998 combined with results from Canuto 1992, 1993) were compared to 3D numerical simulations published in Muthsam *et al.* (1995, 1999). While implications were promising for thin convection zones dominated by radiative transport (as in A-stars and hot DA/DB white dwarfs, see Kupka & Montgomery 2002 and Montgomery & Kupka 2004), their relevance for deep, quasi-adiabatic convection zones as in our Sun remained unclear. Here, we present results from an extended study of Reynolds stress convection models using numerical simulations for a much wider range of physical parameters aimed at probing the models and their approximations for different types of convection zones: shallow and deep, coupled and isolated. We first focus on fluxes and mean structure.

2. 3D numerical simulations as a test case

We have performed direct numerical simulations of turbulent convection assuming idealised microphysics. This includes a perfect gas equation of state ($\gamma = 5/3$), prescribed radiative conductivities (time independent piecewise quadratic functions of depth), and a cartesian geometry with a constant, downwards pointing gravitational acceleration. Horizontal boundary conditions are periodic while vertical ones are closed and stress-free with a constant energy flux imposed at the bottom and a constant temperature at the top. Radiative transfer is treated in the diffusion approximation. We explicitly prescribe viscous dissipation through a fixed Prandtl number Pr. Thus, small-scale shear instabilities with length scales <10% of the width of a downdraft are suppressed. In most 'large-eddy simulations' (e.g., solar convection simulations with realistic microphysics) these remain unresolved, too (damped by numerical viscosity or a subgrid-scale model). Configurations include a convection zone embedded between two stable layers as well as two convection zones separated by a stable layer inbetween and another one at the bottom of the simulation box. Unstable zones of various sizes ($1-3$ pressure scale heights) and convective efficiencies (peak enthalpy fluxes from 25% to ∼100%) are simulated. Early simulations with $\mathrm{Pr}=1$ have used a box with $72\times 50\times 50$ grid points, while more recent runs for $\mathrm{Pr}=1$ and 0.25 have used a box with $125\times 100\times 100$ points. Companion runs with $\mathrm{Pr}=0.1$ are based on grids with $160\times 140\times 140$ points (first coordinate denotes the vertical component). This way Rayleigh numbers Ra between 10^5 and 10^7 are achieved.

After thermal relaxation of the simulations horizontal averages were computed for the lower order moments of velocity, temperature, and pressure and some of their gradients. The horizontal averages were averaged for 10 to 100 turn-over times of the flow to obtain convergence to a statistically steady state. This was equivalent to $\sim 10^3$ to $\sim 10^4$ sound crossing times or ∼0.1 to ∼1 thermal time scales. Such long runs were necessary due to the stably stratified layer at the bottom of the domain. For solar granulation relaxation can be achieved in a small fraction of the thermal time scale, as it becomes irrelevant because of quasi-adiabatic stratification down to the bottom of the simulation box.

3. Configurations and models studied

In Fig. 1 we show results for 'model 3J' from Muthsam *et al.* (1995) (cf. Kupka 1999). Its convective zone is inefficient (enthalpy flux <25% of the total flux) and just 1 pressure scale height (H_p) deep, but the temperature gradient is close to the adiabatic one (its most important difference to envelopes of A-stars and hot DA/DB white dwarfs). The second case shown in Fig. 1, 'model 155X', has a $3H_p$ deep, efficient convection zone (in terms of fluxes and temperature gradients) and a large super-adiabatic peak (as in the Sun). Two more cases were shown at IAU S239. Here, the direct averages from the simulations are compared to self-consistent solutions of Reynolds stress models computed with a modified version of the code of Kupka (1999). The model equations are those suggested by Canuto & Dubovikov (1998) (CD98) with extensions taken from (Canuto 1992, 1993, 1997) as in Kupka (1999) and Kupka & Montgomery (2002) (KM2002). The cases shown in Fig. 1 assume different approximations for third order moments (TOMs) and pressure fluxes ($\langle p'w\rangle$). They include the down-gradient approximation (DGA; cf. also Xiong 1978) and the full dynamical equations for the TOMs (CD98, as in KM2002) with and without pressure fluxes (taken from Canuto 1993, 1997), and the intermediate model from (Kupka 1999). The local limit model shown is also taken from CD98.

4. Discussion of results and conclusions

Compared to the local limit solution the Reynolds stress models provide major improvements when predicting convective fluxes. As the latter are negative in the overshooting layers, the temperature gradient in adjacent stable layers becomes closer to the simulation averages. The accuracy of the match depends also on how the pressure fluxes

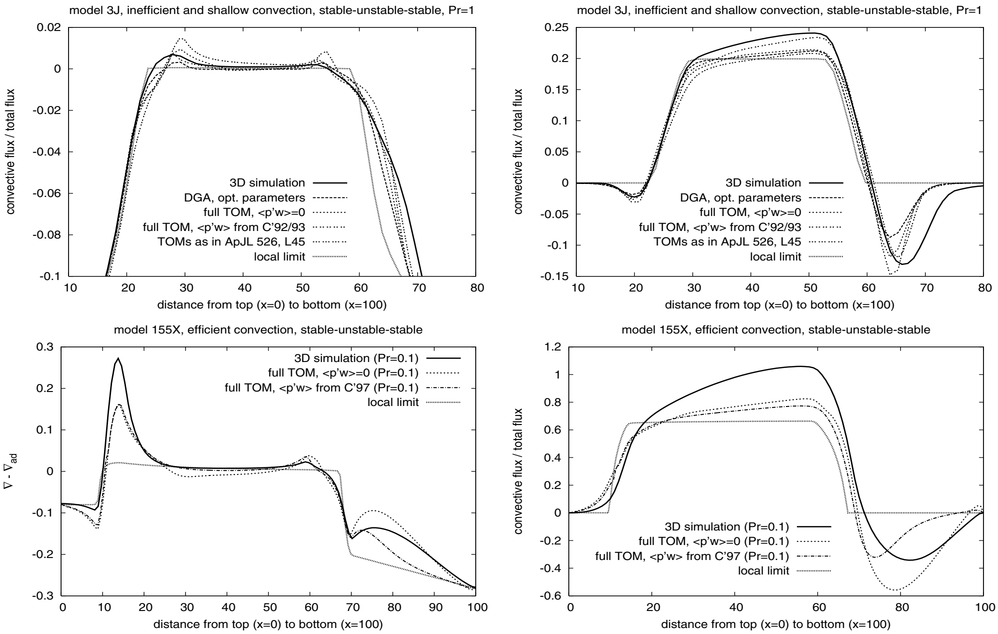

Figure 1. Super-adiabatic temperature gradient (lefts panels) and relative convective flux (right panels) for case '3J' (top row, only central region shown) and '155X' (bottom row).

and third order moments (TOMs) are modelled. It is found that the temperature gradient inside the convection zones can become sub-adiabatic, even for the DGA, although not for the TOM model used in Kupka (1999). Since this is resolved, if contributions from $\langle p'w \rangle$ are accounted for, pressure fluctuations and other compressibility effects cannot be neglected. The cases in Kupka & Montgomery (2002) and Montgomery & Kupka (2004) had temperature gradients far from adiabatic, hence this problem did not appear, although it should be expected for the solar case (see also Kupka & Muthsam 2007a,b).

Acknowledgements

H.J. Muthsam acknowledges support from FwF projects P17024 and P18224.

References

Canuto, V. M. 1992, *ApJ* 392, 218 (C'92)
Canuto, V. M. 1993, *ApJ* 416, 331 (C'93)
Canuto, V. M. 1997, *ApJ* 482, 827 (C'97)
Canuto, V. M. & Dubovikov, M. S. 1998, *ApJ* 493, 834 (CD98)
Chan, K. L. & Sofia, S. 1996, *ApJ* 466, 372
Grossman, S. A. 1996, *MNRAS* 279, 305
Kupka, F. 1999, *ApJL* 526, L45
Kupka, F. & Montgomery, M. H. 2002, *MNRAS* 330, L6 (KM2002)
Kupka, F. & Muthsam, H. J. 2007a, this volume p. 83
Kupka, F. & Muthsam, H. J. 2007b, this volume p. 86
Montgomery, M. H. & Kupka, F. 2004, *MNRAS* 350, 267
Muthsam, H. J., Göb, W., Kupka, F., Liebich, W. & Zöchling, J. 1995, *Astron. & Astrophys.* 293, 127
Muthsam, H. J., Göb, W., Kupka, F. & Liebich, W. 1999, *New Astronomy* 4, 405
Xiong, D. R. 1978, *Chinese Astronomy* 2, 118