Towards sub-microarsecond rigid Earth nutation series in the Hamiltonian theory

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Abstract. The nonrigid Earth nutation series adopted by the IAU (International Astronomical Union) are based on the works of Kinoshita (1977) and Wahr (1979). In the first one, the rigid Earth nutation series were calculated by the application of the Hamiltonian canonical equations to the rotation of the rigid and elliptical Earth. In the second one, the transfer function for the nutations of an elliptical, elastic and oceanless Earth with fluid core and a solid inner core was obtained. The nonrigid Earth nutation coefficients were derived from the convolution between Wahr's transfer function and Kinoshita's rigid Earth nutation series.

The improvement in the accuracy of the techniques as a Very Long Baseline (VLBI), Lunar Laser Ranging (LLR) and Global Positioning System (GPS) has led in this decade to the extension of Kinoshita's theory and more precise determination of Wahr's transfer function. In the present paper and starting from Kinoshita's work (1977), we present the different steps carried out, during this last decade, to obtain the submicroarcsecond rigid Earth nutation series REN 2000 from the Hamiltonian study of the rotation of a rigid Earth (Souchay *et al.*, 1999).

1. Introduction

In 1980, the IAU adopted a new series of nutation for a nonrigid Earth model based on Kinoshita's series on the Hamiltonian study of the rotation of a rigid Earth. The nonrigid Earth nutation coefficients were derived from Kinoshita's series (Kinoshita, 1977) and Wahr's transfer function (Wahr, 1979). 0.1 *mas* (milliarcsecond) was the level of accuracy considered for these coefficients of nutation. The effects considered at this level of precision were influence of zonal harmonics: J_2, J_3 and J_4, and the influence of the triaxiality of the Earth.

Kinoshita's work was the first paper to present a rigorous study of the rotation of a rigid body, starting from Hamiltonian theory. This work has been the basis of a large number of papers, and it was the improvement in the accuracy of VLBI, LLR and GPS which has led to extension of Kinoshita's theory. In the following, the different steps carried out are presented.

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2. The first second-order theory of the rotation of a rigid Earth

Kinoshita and Souchay (1990) published a new series of nutation for a rigid Earth model, considering a new level of truncation for the coefficients of nutation: 0.005 *mas*. This new level made necessary the study of new effects and the extension of some developments. The principal influences can be briefly summed up as (Kinoshita & Souchay, 1990): influence of the J_2 harmonic and of the second-order parts of the potential of the Earth (J_3, triaxiality and J_4), direct and indirect planetary effects, and extension of the theory to the second order: coupling effect between the rotational motion of the Earth and the orbital motion of the Moon.

3. New developments in rigid Earth nutation theory

The updated value of the general precession in longitude led to a change of the value of the dynamical ellipticity of the Earth as well as the scaling factors for the nutation coefficients relative to the potential of the Moon and the Sun. For this reason, Souchay and Kinoshita (1996, 1997) recalculated the coefficients of nutation - due to the lunisolar influence on the geopotential (J_2, J_3, C_{22}, S_{22} and J_4) and those coming from the direct torque exerted by the planets - influenced by these changes. Moreover, they presented new contributions to the nutation not contained in (Kinoshita & Souchay, 1990) including the influence of the periodic oscillations of the ecliptic, and the planetary tilt-effect. The agreement of the new coefficients of Souchay & Kinoshita (1996, 1997) with those of Hartmann & Soffel (1994) and Williams (1994, 1995) was remarkable.

4. The effects on nutation of the non-zonal harmonics of third and fourth degree

Taking into account the new accuracy of the precession-nutation observations (a few microarseconds), Folgueira *et al.* (1998a,b) calculated the coefficients of nutation for a rigid Earth model due to C_{3m}, S_{3m}, C_{4m} and S_{4m} $(m \neq 0)$ harmonics of the geopotential, which were not considered in previous studies of nutation involving Hamiltonian theory, with a level of truncation of 0.1 μas (microarcseconds). These non-zonal harmonics, together with C_{22} and S_{22}, give birth to short period nutations, the periods being related to the order m of the corresponding harmonic. That is to say:

- The diurnal terms come from C_{31}, S_{31}, C_{41} and S_{41}.
- The semidiurnal terms are due to the harmonics with $m = 2$.
- The terdiunal terms have their origin in C_{33}, S_{33}, C_{43} and S_{43} harmonics.

The principal diurnal and subdiurnal terms for the nutation in longitude and obliquity are shown in Tables 1 and 2.

Table 1. Principal terms for quasidiurnal nutations in longitude and obliquity for the figure axis. The unit is μas

Argument						Period	Longitude ($\Delta\psi$)		Obliquity ($\Delta\varepsilon$)	
Φ	l_M	l_S	F	D	Ω		sin	cos	sin	cos
1	0	0	1	0	1	0.96215	-38.2313	-4.6980	-1.8567	15.1063
1	0	0	-1	0	-1	1.03505	-35.4036	-4.3513	-1.5868	12.9109
1	-1	0	1	0	1	0.99696	24.1443	2.9675	1.1636	-9.4678
1	1	0	-1	0	-1	0.99758	-19.9400	-2.4507	-0.9716	7.9055
1	1	0	1	-2	1	0.99216	-7.0629	-0.8681	-0.3452	2.8089

Table 2. Principal terms for subdiurnal nutations in longitude and obliquity for the figure axis. The unit is μas

Argument						Period	Longitude ($\Delta\psi$)		Obliquity ($\Delta\varepsilon$)	
Φ	l_M	l_S	F	D	Ω		sin	cos	sin	cos
2	0	0	0	0	0	0.49863	31.2522	-17.9397	-7.1157	-12.3961
2	0	0	-2	0	-2	0.51753	-25.5271	14.6533	5.6775	9.8906
2	0	0	-2	2	-2	0.50000	-10.4455	5.9960	2.3682	4.1255
2	-1	0	-2	0	-2	0.52743	-5.0979	2.9264	1.1818	2.0588
2	0	0	-2	0	-1	0.51756	-4.7105	2.7040	1.0816	1.8842

5. The sub-microarcsecond rigid Earth nutation series REN 2000

Souchay *et al.* (1999) presented the new tables REN 2000 of the nutation of a rigid Earth, starting from Hamiltonian theory, with sub-microarcsecond level of truncation. All the previous studied influences, together with the second-order effects due to crossed-nutations and spin-orbit coupling, are included in this paper.

6. The diurnal and sub-diurnal nutations of REN 2000

Recently other rigid Earth nutation series, calculated using different analytical methods, have come forth. These are SMART97 and RDAN97 series (Bretagnon *et al.*, 1997, 1998; Roosbeek & Dehant, 1998). Folgueira & Souchay (1999) and Folgueira *et al.* (1999) have compared the nutations given by REN 2000 series with the corresponding ones in SMART97 and RDAN97. Table 3 gives the comparison for the short period nutations in the time domain between these three theories.

Moreover, they have computed the corresponding values for a nonrigid Earth of REN 2000 prograde nutations, using a new transfer function developed by Mathews (1999) and showed that some coefficients changed at the level of a few μas. Finally, they have also explained how short period nutations can perturb the analysis of the polar motion (Bizouard *et al.*, 1999a;b).

Table 3. Comparison in time domain between SMART97, RDAN97 and REN 2000 short period nutations. The unit is μas

	$\Delta\psi \sin \varepsilon_0$	$\Delta\varepsilon$
SMART97 - REN 2000	1.74	3.52
RDAN97 - REN 2000	2.40	6.80
RDAN97 - SMART97	1.40	6.80

7. Conclusions

In the present paper, we have analyzed the different steps carried out, in this last decade, to reach at the final sub-microarcsecond tables of nutation for a rigid Earth model REN 2000 (Souchay *et al.*, 1999). As summary, we can conclude that:

1. The tables REN 2000 of nutation for a rigid Earth model, starting from the Hamiltonian theory, catch all the coefficients at a *sub-microarcsecond* level.
2. There is a good agreement between REN 2000 and other recent works about rigid Earth nutations (Bretagnon *et al.*, 1997, 1998; Roosbeek & Dehant, 1998).
3. The short period nutations are the last most important contribution to reach at the 0.1 μas level in REN 2000 tables.
4. The validity of the analytical series REN 2000 has been checked by a numerical integration of the nutation developed by Souchay & Kinoshita (1991) and Souchay (1998) and by the study of the residuals between the results given by these two different methods.

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