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Application of non-linear mixed models for modelling the quail growth curve for meat and laying

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Abstract

The objective of the current paper was to apply mixed models to adjust the growth curve of quail lines for meat and laying hens and present the rates of instantaneous, relative and absolute growth. A database was used with birth weight records up to the 148th day of female quail of the lines for meat and posture. The models evaluated were Brody, Von Bertalanffy, Logistic and Gompertz and the types of residues were constant, combined, proportional and exponential. The Gompertz model with the combined residue presented the best fit. Both strains present a high correlation between the parameters asymptotic weight (A) and average growth rate (k). The two strains presented a different growth profile. However, growth rates allow greater discernment of growth profiles. The meat line presented a higher growth rate (6.95 g/day) than the lineage for laying (3.65 g/day). The relative growth rate was higher for lineage for laying (0.15%) in relation to the lineage for meat (0.13%). The inflection point of both lines is on the first third of the growth curve (up to 15 days). All results suggest that changes in management or nutrition could optimize quail production.

Introduction

The growth and development of animals is a complex phenomenon, influenced by various factors, such as feeding, climate conditions, health and genetics. Species of poultry have widely differing sizes and growth rates as a result of natural selection (Buzala and Janicki, 2016).

The comprehension of growth is necessary to formulate simulation models able to predict the nutritional demands of birds and determine the effects of feeding and environmental conditions on their performance (Gous *et al.*, 1999). These models make it possible to improve management strategies for each life stage or genetic type, with focus on improving important growth traits, and thus enhancing performance and reducing feed costs (Grieser *et al.*, 2018).

Non-linear mathematical models are used to describe the growth of animals during their lifetime, relating weight and age. These models allow datasets consisting of series of weights by age to be condensed into a small number of parameters, to facilitate interpretation and understanding of the phenomenon (Oliveira *et al.*, 2000).

Mixed non-linear modelling permits consideration of the heterogeneity among individuals arising from variables not measured through the inclusion of random effects in the model (Hall and Clutter, 2004). Therefore, by assuming that the live weight measures of each animal follow the same functional form, the method permits variation of individual parameters to consider deviations from the average curve (Lindstrom and Bates, 1990). Mixed non-linear models have been used previously in studies involving quail growth curves (Kizilkaya *et al.*, 2006; Aggrey, 2009; Karaman *et al.*, 2013). However, few studies reporting growth rates have applied mixed models to compare meat and laying quail lines.

Therefore, the objective of the current study was to select the non-linear model with mixed effects that best fits the growth curve of meat and laying quails, employing various types of residuals, and to obtain instantaneous, relative and absolute growth rates of these bird lines.

Materials and methods

The database used came from Mato Grosso Federal University, Rondonópolis Campus, Brazil, containing weights of female quails bred for meat (*Coturnix coturnix coturnix*) and laying (*Coturnix c. japonica*). The weight records referred to 0, 8, 15, 22, 29, 36, 50, 64, 78, 92, 106, 120, 134 and 148 days of age. There was a reduction in the number of records due to the consistency process performed in the database and natural mortality of the birds (Fig. 1). The birds had free access to water and feed and were kept in groups of eight birds

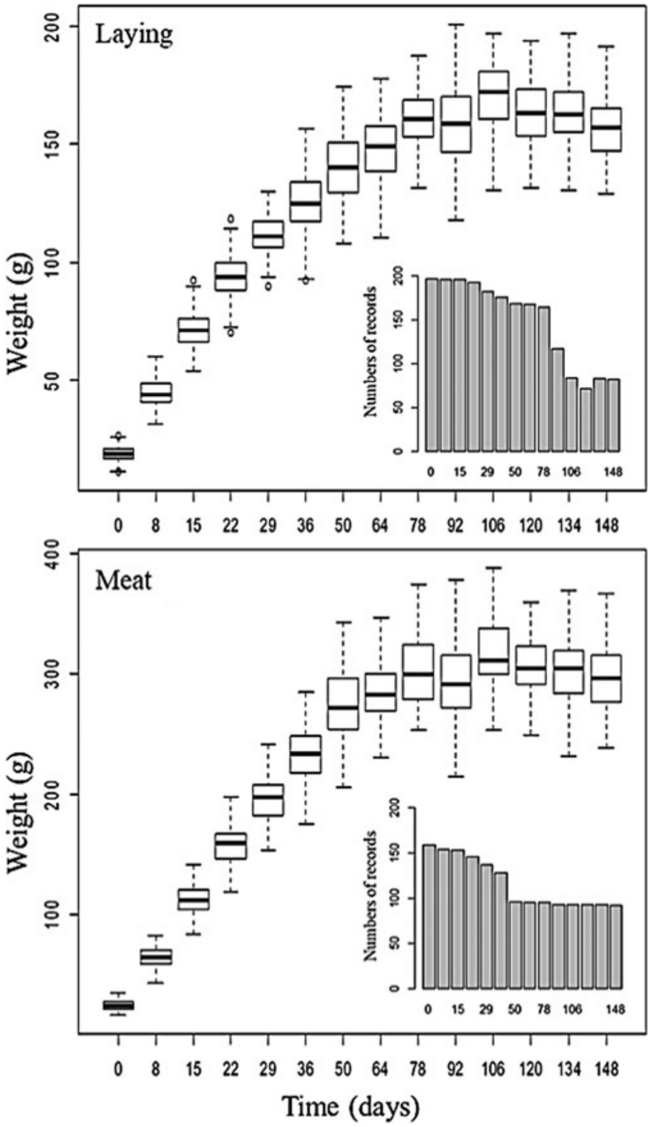

Fig. 1. Description of the database composed of lines of quail for meat and laying.

in cages with dimensions of 100 cm length × 25 cm width × 20 cm height, equipped with automatic water dispensers and feed troughs.

The feed consisted mainly of maize meal and soy meal, containing 250 g/kg crude protein (CP) and 2682 kcal metabolizable energy (ME)/kg, from birth to 21 days of age, 230 g/kg CP and 2774 kcal of ME/kg from 22 to 25 days of age, and 219 g/kg CP and 2591 kcal of ME/kg from 26 to 148 days of age, adjusted considering the chemical composition of the feed and nutritional requirements proposed by Albino and Barreto (2003).

The mathematical models Brody (Brody, 1945), Von Bertalanffy (Von Bertalanffy (1957), Logistic (Nelder, 1961) and Gompertz (Laird, 1965) were considered to adjust the growth curves of both lines using non-linear models with mixed effects according to the following equation:

$$Y_{ij} = f(x_{ij},\ \psi_i) + g(x_{ij},\ \psi_i, \epsilon),\quad 1 \le i \le N,\ 1 \le j \le ni,$$

$$\epsilon_{ij} \sim N(0, \sigma_e^2),$$

$$\psi_i = H(\mu, c_i, \pi_i).$$

where Y_{ij} is the j-th record of the weight of the i-th bird; N is the number of birds; ni is the number of records of the bird i; f is non-linear growth function; x_{ij} is the matrix of independent variables (j-th recording age and i-th heavy bird); ψ_i is the vector of individual parameters; H is a function which describes the covariate model; c_i is a vector of known variables; μ is an unknown fixed vector; π_i is a random unknown vector; g is the residue function of the model; ϵ is a vector of residual variance; ϵ_{ij} is random residuals with mean zero and variance 1; σ_e^2 is the residual variance. Thus, assuming an unknown vector of random normal distribution of size n and where the random residuals are mutually independent, modelling of the residues was performed: constant ($g = a$ and $\epsilon = a$), proportional ($g = b\,f$ and $\epsilon = b$), combined ($g = a + b\,f$ and $\epsilon = a,\ b$) and exponential ($t_{(y)} = \log(y); Y = fe^{g\epsilon}$).

Growth curve adjustment was performed using a stochastic approximation version of the Expectation Maximization (EM) algorithm for maximum likelihood estimation, the Stochastic Approximation Expectation Maximization (SAEM) algorithm developed by Kühn and Lavielle (2005) and implemented in the Saemix package (Comets *et al.*, 2017) of R version 3.5.1 (2018) (https://www.r-project.org/).

Selection of the best fitting model is related to the explanation of the observed event in a small number of parameters with biological interpretation, i.e. the best model is one that presents a good fit to the observed data with the lowest number of parameters. The Bayesian Information Criterion (BIC) proposed by Schwarz (1978) was used to evaluate the quality of fit between observed and predicted data, penalizing the model according to the number of parameters. Therefore, the lowest value for BIC characterizes the model with the highest adjustment quality. The BIC was calculated considering the modification proposed by Kass and Raftery (1995) for mixed models, defined as:

$$BIC = -2 \log l_M(y; \hat{\theta}) + p\log(n)$$

where $l_M(y; \hat{\theta})$ represents the likelihood function, considering the approximation method by linearization; n is number of observations; and p is number of parameters adjusted.

After selecting the model with the best fit, the following were estimated: instantaneous growth rate (IGR), the derivative of Y_{ij} as a function of time (t), representing the increase in weight at each unit of t; absolute growth rate (AGR), the ratio of IGR to asymptotic weight (A), representing the rate of weight gain proportional to the estimated final weight; RIGR, the ratio of IGR in t to the estimated weight (Y) at t, which represents the efficiency of the bird in the conversion of food by body mass; and inflection point (IP), the t when the bird's IGR goes from increasing to decreasing (Table 1).

Results

The Gompertz model, independent of residue type, presented the best fit for the growth curve, followed by the Von Bertalanffy, Logistic and Brody models for the meat line, and the Von Bertalanffy, Brody and Logistic models for the laying line (Table 2). Regarding the types of residue as a function of the model, the combined residue provided the best fit for both strains, except for the Brody (Proportional) model for the meat line.

Table 1. Non-linear models with mixed effects, growth rates and inflection point

	Models			
Parameters	Brody	Gompertz	Logistic	Von Bertalanffy
Equation	$Y_t = A(1–Be^{-kt}) + \epsilon$	$Y_t = Ae\text{-}Be^{-kt} + \epsilon$	$Y_t = A(1 + Be^{-kt})^{-1} + \epsilon$	$Y_t = A(1 - Be^{-kt})^3 + \epsilon$
IGR	$ABke^{-kt}$	Bke^{-kt}	$yBk/(1 + Be^{-kt})e^{-kt}$	$3ABke^{-kt}(1\text{-}Be^{-kt})^2$
RIGR	$ABke^{-kt}/y$	Bke^{-kt}	$Bk/(1 + B^{-kt})e^{-kt}$	$3yBke^{-kt}/(1\text{-}Be^{-kt})$
AGR	Bke^{-kt}	$(Bkye^{-kt})/A$	$yBk/(1 + Be^{-kt})e^{-kt}$	$3Bke^{-kt}(1\text{-}Be^{-kt})^2$
IP	–	A/e; $\log(B)/k$	$A/2$; $\log(B)/k$	$8A/27$; $\log(3B)/k$

A, asymptotic weight (g); *B*, integration constant; *k*, average growth rate; *t*, age (days); e, exponential; IGR, instantaneous growth rate; RIGR, relative growth rate; AGR, absolute growth rate; IP, inflection point; *y*, equation of the model used; ϵ, error

Table 2. Values of Bayesian Information Criterion (BIC) of mixed non-linear models with different types of residues for quail lines for meat and laying

		Lines	
		BIC	
Models	Type of residuals	Meat	Laying
Gompertz	Constant	14 120	15 584
	Proportional	13 523	15 054
	Combined	13 454	14 953
	Exponential	13 552	15 112
Logistic	Constant	14 237	15 844
	Proportional	13 764	15 382
	Combined	13 723	15 289
	Exponential	13 793	15 443
Brody	Constant	14 334	15 736
	Proportional	13 775	15 174
	Combined	13 794	15 059
	Exponential	13 954	15 226
Bertalanffy	Constant	14 326	15 765
	Proportional	13 892	15 203
	Combined	13 706	15 055
	Exponential	13 899	15 220

Applying the selected model (Gompertz with combined residue), it was possible to verify differences in the magnitude of asymptotic weight (*A*), but it was also noticed that both lines presented values of close to the integration constant (*B*), average growth rate (*k*) and IP (Fig. 2). The model showed a good fit to observed data in the initial phase of growth, but less so in the asymptotic phase of the curve of the two lines, over-estimating values for the laying line, but under-estimating and over-estimating for the meat line.

Regarding the estimated parameters, there was a greater amplitude for the A value of the meat line in relation to the laying line (Table 3). For parameter *B*, the highest amplitude was seen for the laying line, but the same amplitude was verified for *k* between the lines. There was greater variability for the *k* parameter in both lines, followed by *B* and *A*. Among the lines, there is greater variability in *B* and *k* for the meat lines, and A for laying lines. However, estimated variability of the parameters for the studied populations was low (variation coefficient <3.0). The correlation between *A* and *k* was significant ($P < 0.001$) with a value of −0.99 for both lines.

The meat line presented higher initial IGR than the laying line (Fig. 3), with a maximum growth rate of 6.95 g/day at 15 days and 3.65 g/day at 11 days of age for the meat and laying lines, respectively, after which both lines began to grow more slowly.

The RIGR presented initial values of 0.13% for the meat line and 0.15% for the laying line, reaching minimum values after 50 days of age (Fig. 3). The AGR estimated at birth were 0.004 and 0.007% for the European and Japanese lines, respectively. Thereafter, the AGR steadily increased until the IP (0.008 and 0.01%) and then declined until reaching minimum values near zero at 100 days.

Discussion

Corroborating the results obtained in the current study for the two lines, the Gompertz model has been applied in other studies to fit growth curves for quail (Narinç *et al.*, 2017; Rossi *et al.*, 2017). The longitudinal nature of the data, where variance with age is not constant, led to the selection of a heterogeneous residual structure (Craig and Schinckel, 2001; Schinckel and Craig, 2002).

The estimates of *A* obtained in the analyses were lower than those described in other studies, which have presented values between 357 and 410 g for meat quail and between 166 and 222 g for laying quail (Mota *et al.*, 2015; Firat *et al.*, 2016; Grieser *et al.*, 2018). However, AGR in the current study was similar to the values reported by the aforementioned authors, near 0.07%. Besides the model used, a possible explanation for the smaller values of *A* is related to the time period utilized in modelling the curves, since Koncagul and Cadirci (2009) found a reduction in the estimate of *A* with increasing age of birds.

The antagonistic relation found between the parameters *A* and *k* also has been described in cattle (Lopes *et al.*, 2016), buffaloes (Malhado *et al.*, 2017), chickens (Manjula *et al.*, 2018) and pigs (Coyne *et al.*, 2015), where the estimates vary between −0.33 and −0.70. Mota *et al.* (2015) reported correlations of −0.94 and −0.95 for meat and laying quail. This antagonistic association indicates that animals having higher growth rates have lower asymptotic weight or reach their final weight at a younger age (Knižetova *et al.*, 1991; Lopes *et al.*, 2016).

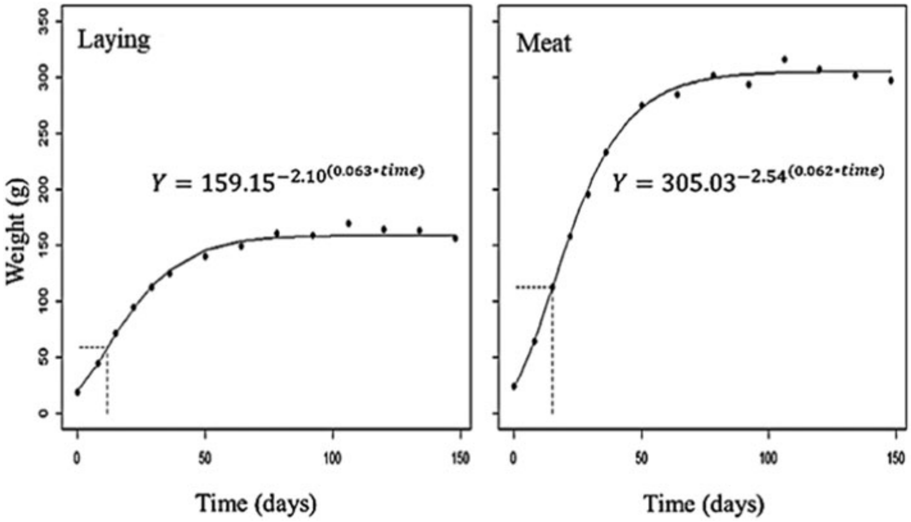

Fig. 2. Observed growth curves (●●●), predicted (−−−−−) and inflection point (-----) adjusted by the Gompertz model with combined residue for meat and laying quails lines.

Table 3. Statistics of the parameters estimated by the Gompertz model with combined residue for quail lines for cutting and laying

	Meat			Laying		
Parameters	*A*	*B*	*k*	*A*	*B*	*k*
Minimum	288	2.3	0.057	148	1.9	0.058
Median	304	2.5	0.061	159	2.0	0.061
Mean	305	2.5	0.062	159	2.0	0.063
Maximum	319	2.5	0.066	168	2.2	0.067
VC (%)	2.1	2.7	2.8	2.3	2.7	2.7

A, asymptotic weight (g); *B*, integration constant; *k*, average growth rate; VC, variation coefficient

A topic for discussion is the importance of employing growth rates, because only by observing the value of *k* is it possible to differentiate the growth profile between the lines. The IGR is very important for genetic selection and/or nutritional management, as the IP would be the ideal time to change the diet, due to the changes in the animals' nutritional requirements (Grieser *et al.*, 2015). Differences were expected between the quail lines for growth rates. The faster growing line, i.e. the line that reaches final weight earlier, has greater nutritional demand than the slower growing line (Mignon-Grasteau *et al.*, 1999; Narinç *et al.*, 2017).

The RIGR represents the efficiency of the animal in converting feed into body mass (Aggrey, 2003). Therefore, the higher values observed for this rate is a result of genetic improvement of the European line for production of meat, i.e. greater accumulation of body mass. Mota *et al.* (2015) reported higher RIGR values than observed in the current study, ranging from 0.23 to 0.28% for meat quail and 0.22% for laying quail.

The IP values for the meat and laying lines are located in the first third of the growth curve (Mota *et al.*, 2015; Firat *et al.*, 2016). The selection of individuals with late IP, near the slaughter age, would possibly result in greater efficiency of production systems, as is the case of meat chickens, where IP values vary from 32 to 41 days of age (Mohammed, 2015; Demuner *et al.*, 2017). However, despite being a trait with high heritability, (0.36), the IP has presented low variability in the populations studied, making it unfeasible to select individuals due to lower genetic weight gain obtained in each generation (Narinç *et al.*, 2010). The low variability is reflected in the small difference between the maximum and minimum values estimated.

In this respect, manipulation of the diet or feeding phases might be an alternative to optimize the productive efficiency of quail breeding. A plausible strategy is to reduce the first feeding phase considering the age of reaching the IP as reference (15 days for meat and 11 days for laying quails), since the IP coincides with the point of maximum deposition of water, minerals and proteins (Grieser *et al.*, 2018). Another possibility is to improve the feed conversion in the initial growth period (up to 14 days) for laying quail (Škrobánek *et al.*, 2004).

In support of the hypothesis of changes in the IP and IGR through diet manipulation, Santos (2008) reported an increase of 2 days in the IP for the Hy-Line Brown line when fed diets formulated to meet 95% of the nutritional requirements, compared with birds that consumed feed containing 105% of the requirements.

The non-linear mixed Gompertz model with combined residuals produced the best fit for the growth curves of meat and laying quail lines. The growth rates allowed differentiation of the birds' growth profiles. Future studies should investigate the effect of manipulating the diet on the shape of the growth curve and evaluate the effect of possible changes in the feeding phases on the performance and financial return.

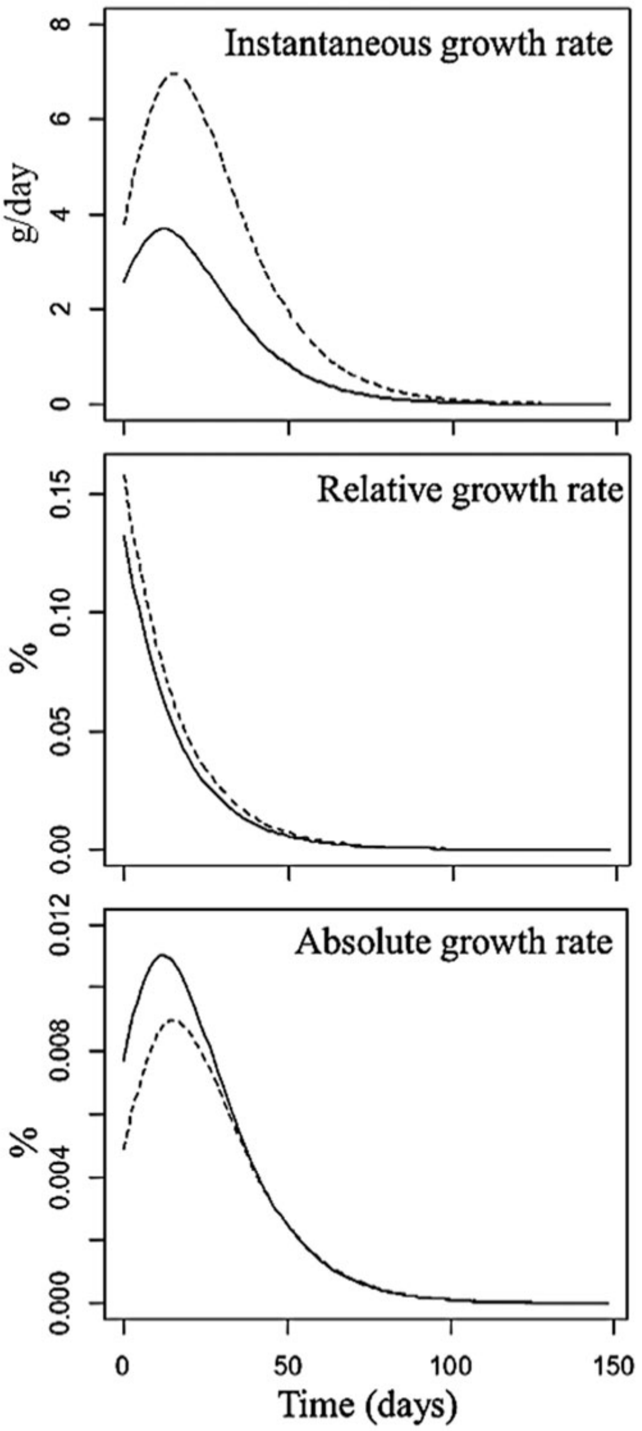

Fig. 3. Instantaneous growth rate (IGR), relative growth rate (RIGR) and absolute growth rate (AGR) for meat (-----) and laying (——————) quail lines.

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