Macroeconomic Dynamics, **25**, 2021, 651–668. Printed in the United States of America.
doi:10.1017/S136510051900035X

CHILDREN'S HEALTH, HUMAN CAPITAL ACCUMULATION, AND R&D-BASED ECONOMIC GROWTH

Annarita Baldanzi
University of Milan and University of Pavia

Alberto Bucci
University of Milan
RCEA (Rimini Center for Economic Analysis)
CEIS (University of Rome-'Tor Vergata')

Klaus Prettner
University of Hohenheim

We analyze the effects of children's health on human capital accumulation and on long-run economic growth. For this purpose, we design an R&D-based growth model in which the stock of human capital of the next generation is determined by parental education and health investments. We show that (i) there is a complementarity between education and health: if parents want to have better educated children, they also raise health investments and vice versa; (ii) parental health investments exert an unambiguously positive effect on long-run economic growth, (iii) faster population growth reduces long-run economic growth. These results are consistent with the empirical evidence for modern economies in the twentieth century.

Keywords: Children's Health, Education, Fertility, Long-run Economic Development

1. INTRODUCTION

There has been a substantial improvement in childhood health within all industrialized countries over the last decades. According to the World Bank (2016)'s Health Nutrition and Population Statistics, the mortality rate of children under the age of 5 has decreased in the OECD from 63 deaths per 1000 children in 1960 to 7 deaths in 2015. This corresponds to a reduction of the child mortality rate of almost 90% within two generations. The substantial improvements in children's

We would like to thank the editor of the special issue Luca Gori, two anonymous referees, Fabio Mariani, and Paul Tscheuschner for valuable comments and suggestions. Klaus Prettner gratefully acknowledges the funding provided by the Faculty of Economics and Social Sciences at the University of Hohenheim within its research focus "Inequality and Economic Policy Analysis" (INEPA). This research did not receive any other grant from funding agencies in the public, commercial, or not-for-profit sectors. Address correspondence to: Klaus Prettner, Institute of Economics, University of Hohenheim, Schloss, Osthof-West, 70593 Stuttgart, Germany. e-mail: klaus.prettner@uni-hohenheim.de. Phone: +49 711 459 23453

 1469-8056/19

health are therefore an important driver of the rise in the survival probability to the age of 65, which has increased between 1960 and 2015 from 64% to 83% for men and from 75% to 90% for women.

As far as the relation between health and economic prosperity is concerned, there is a strong positive association between these two variables, as reflected in the famous "Preston Curve" (Preston, 1975) that was originally meant to illustrate the positive effect of income on health. Later, however, Bloom and Canning (2000) emphasized that there is a reverse causality in the sense that better health leads to higher per capita income. While the causal positive effects of health on income are also emphasized by, for example, Cervellati and Sunde (2005) and Lorentzen et al. (2008),[1] some economists claim the opposite: lower mortality—as induced by a better health condition of the population—might trigger faster population growth and therefore a *reduction* in medium-run growth of income per capita due to the well-known capital dilution effect (cf. Solow, 1956). In their influential work, Acemoglu and Johnson (2007) show that a 1% increase in life expectancy leads to a 1.7–2% increase in the population size but it raises aggregate GDP to a lesser extent. Consequently, according to their findings, a better health condition of the population reduces income per capita.

Aghion et al. (2011) and Bloom et al. (2014) in turn criticize the findings of Acemoglu and Johnson (2007). Their argument is that the negative effect of higher life expectancy on economic growth might come from the omission of a measure for the initial health condition from the regression specifications. Countries with a lower initial health condition of the population have a greater potential to improve health, but, at the same time, they have a lower economic growth potential. Including initial life expectancy as a proxy for initial health in the regressions, Bloom et al. (2014) show that the negative effect of better health on economic growth vanishes. Furthermore, using the same panel data for the period 1940–2000 as Acemoglu and Johnson (2007), Cervellati and Sunde (2011) find that the effect of life expectancy on economic growth might have been negative after the onset of the mortality transition but prior to the fertility transition, that is, when fertility rates stayed constant in the face of decreasing mortality such that population growth gained momentum. However, they show that the effect of life expectancy on economic growth is unambiguously positive after the onset of the fertility transition when higher life expectancy is associated with reductions in the fertility rate to such an extent that population growth slows down. This implies a positive effect of health on income per capita in a neoclassical-type of growth model because the capital dilution effect is reduced.

We contribute to this debate by showing another pathway by which health has the potential to impact on long-run economic growth, especially in modern knowledge-based economies such as the countries of the OECD that have already experienced the demographic transition in the past and that have sizeable R&D sectors. Our argument is based on an endogenous growth mechanism where new ideas are created in a research sector by the human capital that a society devotes to R&D.[2] The aggregate human capital stock of a country is in turn determined by the education level and the health condition of the population and there are feedback effects between these two variables (Schultz, 1961; Becker, 2007; Madsen, 2016). On the household side, health enters the utility function of parents who choose how much to invest in children's health and in children's education. We show that, if parents want to have better educated children, they also increase health investments in their children. This result is consistent with the empirical findings of Perri (1984), Behrman and Rosenzweig (2004), and Currie (2009), who document a positive effect of childhood health on educational achievements. In addition, healthier children perform better in school and will themselves have a higher health-related knowledge (Behrman, 2009). Overall, in our framework, human capital is used as an input in the production functions of the final goods sector, the R&D sector, the education sector, and the health sector. Given the positive role of health in the creation of human capital, there are more productive resources available for R&D in a healthier economy and this has the potential to lead to faster long-run economic growth (cf. Prettner et al., 2013; Kuhn and Prettner, 2016). Our model therefore characterizes an additional channel by which health exerts a positive effect on economic growth besides the neoclassical capital dilution effect (Cervellati and Sunde, 2011) and the Ben–Porath mechanism (Ben-Porath, 1967; Cervellati and Sunde, 2005, 2013).

The paper is organized as follows. We set up the model in Section 2 and describe the consumption side, the production side, and the market clearing conditions. Section 3 contains the long-run solution of the economy, the main analytical results, and a numerical illustration of the transitional dynamics and the long-run solution. In Section 4, we draw our conclusions.

2. THE MODEL

In this section, we develop a general equilibrium growth model with endogenous technological progress that builds on the partial equilibrium household model of Prettner et al. (2013). They use their framework to derive the interrelations between fertility decisions of households and investments in education and health of children under constant wages to derive the elasticity of aggregate human capital with respect to fertility. The main aim of their paper is to estimate this elasticity empirically and the theoretical framework is merely the motivation for the empirical analysis. While their partial equilibrium model abstracts from the production side of the economy and, thus, from technological progress altogether, the aim of our paper is to integrate the household decision framework into a general equilibrium endogenous growth model with five production sectors. In contrast to Prettner et al. (2013), this allows us to analyze the repercussions of the choices of households with respect to the number of children and the education and health investments in their children on R&D and long-run economic growth.

Consider a knowledge-based economy a la Romer (1990)–Jones (1995) in which time $t = 1, 2, \ldots \infty$ evolves discretely. There are five sectors: final goods production, intermediate goods production, R&D, education, and health in which the two production factors physical capital and human capital are employed. Physical capital is accumulated according to the savings and investment decisions of households and it is used to produce machines in the intermediate goods sector. Human capital is available in four different forms: as "workers" in the final goods sector for the production of the consumption aggregate, as "teachers" in the education sector for the production of the knowledge and the skills of the next generation, as "healthcare personnel" in the health sector for the improvement of the health condition of the next generation (this could refer to doctors, nurses, and midwives, but also to employees of public health projects, for example, to improve sanitation), and as "scientists" for the production of the new blueprints of machines in the R&D sector.

The consumption side of the economy consists of overlapping generations of households who live for two time periods. Households consume, save, and choose the number of children on the one hand, and how much to invest in education and health of each child, on the other hand. The household's expenditures on education are used to hire the teachers to educate the children, while the household's expenditures on health are used to hire the healthcare personnel to improve children's health.

2.1. Households

We follow Prettner et al. (2013) and Strulik et al. (2013) in assuming that the utility function of households is given by

$$u_t = \log\left(c_{1,t}\right) + \beta \log\left(R_{t+1}s_t\right) + \xi \log\left(n_t\right) + \theta \log\left(e_t\right) + \zeta \log\left(f_t\right), \qquad (\mathbf{1})$$

where $c_{1,t}$ is first period consumption of the generation born at time t, R_{t+1} is the capital rental rate, s_t denotes savings such that $c_{2,t} = R_{t+1}s_t$ refers to consumption in the second period of life, n_t is the fertility rate, e_t refers to education investments per child, f_t refers to health investments per child, $\beta \in (0, 1)$ is the discount factor, $\xi \in (0, 1)$ denotes the utility weight of children, $\theta \in (0, 1)$ refers to the utility weight of children's education, and $\zeta \in (0, 1)$ is the utility weight of children's health. To avoid nonsensical solutions in which parents would either aim for (i) a negative fertility rate or (ii) for not having children, while investments in education and health of the children that they do not want to have tend to infinity, we need to employ the parameter restriction $\xi > \theta + \zeta$. The utility function without the health component of children is frequently used in the literature (cf. Prettner et al., 2013; Strulik et al., 2013; Bloom et al., 2015; Prettner and Strulik, 2016) because it operationalizes the "warm-glow motive of giving" as described by Andreoni (1989) and because it is the special case of logarithmic utility of the more general specification employed by Galor and Weil (2000) and Galor (2011). To see this, consider the formulation of Galor and Weil (2000), where parental utility depends positively on the consumption possibilities of children as approximated by their total income $n_t h_{t+1} w_{t+1}$ with w_{t+1} being the wage rate per unit of human capital of the next generation. Computing the logarithm yields $\log(n_t) + \log(h_{t+1}) + \log(w_{t+1})$, where the wage rate per unit of human capital of the next generation is a constant to the optimizing parent such that it drops out of the first-order conditions. If h_{t+1} is a multiplicative function of education and health, then our formulation in the utility function as represented by $\xi \log(n_t) + \theta \log(e_t) + \zeta \log(f_t)$ captures all the tradeoffs that parents face when deciding on the number of children and the parental expenditures for children's education and health. Observe that we assume exogenous mortality of parents. Otherwise there would be an additional endogenous choice of healthcare for longevity-extending investments and this would make the model much more complicated without providing additional insights in our context.[3]

The budget constraint of the household is given by

$$(1 - \psi n_t)\, h_t w_t = \eta e_t n_t + \kappa n_t f_t + c_{1,t} + s_t, \tag{2}$$

where $\psi \in (0, 1)$ measures the unit cost of rearing each child, $\eta > 0$ measures the unit cost of the investment in education per child, $\kappa > 0$ measures the unit cost of the investment in health per child, h_t refers to the human capital level of an adult, which is tantamount to her productivity and is itself determined by the education and health investments of her own parents, and w_t is the wage rate per unit of human capital of the parent generation. We model the costs of children as time costs (and, thus, opportunity costs of households) because otherwise, fertility would rise with the wage rate w_t which is difficult to reconcile with the experience in industrialized countries.

The result of the utility maximization problem associated with equations (1) and (2) is represented by optimal consumption, savings, fertility, education investments, and health investments as given by

$$c_{1,t} = \frac{h_t w_t}{1 + \beta + \xi}, \quad s_t = \frac{\beta h_t w_t}{1 + \beta + \xi}, \quad n_t = \frac{\xi - \zeta - \theta}{\psi\,(1 + \beta + \xi)}, \quad e_t = \frac{\theta \psi h_t w_t}{\eta\,(\xi - \zeta - \theta)},$$
$$f_t = \frac{\zeta \psi h_t w_t}{\kappa\,(\xi - \zeta - \theta)}. \tag{3}$$

Considering these optimal expressions from the household perspective, we observe the following relations. If households want to have more children (ξ is higher), the fertility rate (n_t) is higher, while consumption (c_t), savings (s_t), investments in children's education (e_t) and investments in children's health (f_t) are lower. If households want to have better educated children (θ is higher), parental investments in both education and health are higher, while fertility is lower. Finally, if households want to have healthier children (ζ is higher), parental investments in both education and health are higher, while fertility is lower. Altogether, we observe that parents who invest more in their children's education also invest more in their children's health and vice versa. At the same time, higher investments in education and health imply that parents have fewer children. These effects are fully consistent with the evidence on the relation between health and education (cf. Perri, 1984; Behrman and Rosenzweig, 2004; Currie, 2009; Behrman, 2009; Case et al., 2005) and they are also consistent with the child quality–quantity trade-off as described by Becker and Lewis (1973).

Taking into account the expression for fertility in equation (3), the evolution of the population size is governed by the difference equation

$$N_{t+1} = n_t N_t = \frac{\xi - \zeta - \theta}{\psi \, (1 + \beta + \xi)} N_t \tag{4}$$

and the labor force participation rate can be calculated as

$$lpr = 1 - \psi n_t = \frac{1 + \beta + \zeta + \theta}{1 + \beta + \xi}.$$

Naturally, the labor force participation rate is smaller than one because of the time parents spend on rearing children.

Due to the derived expressions for the number of children (n_t) and our parameter restrictions, it is clear that fertility is bounded. It could only tend to infinity if the time cost of children in the budget constraint of households (ψ) tends to zero. Mathematically, this would not have implications on education or health investments because the higher number of children is then—as far as the budget constraint is concerned—fully compensated by the lower time cost of children. In a calibration of the model, the parameter (ψ) is crucial to obtain a fertility rate that fits to the stylized facts.

2.2. Production

The production side of the economy consists of five sectors: final goods production, intermediate goods production, R&D, education, and health. The description of the first three sectors follows the standard R&D-based growth literature with the only difference being that human capital (as determined by the number of people, their eduction level, and their health condition) is used instead of raw labor as a factor of production.

The final goods sector produces a consumption good Y_t with human capital $H_t = h_t N_t$ and machines $x_{t,i}$ as inputs according to the production function

$$Y_t = H_{t,Y}^{1-\alpha} \int_0^A x_{t,i}^{\alpha} \, di, \tag{5}$$

where A is the technological frontier and $\alpha \in (0, 1)$ denotes the elasticity of output with respect to machines of type i. Profit maximization implies

$$w_t = (1 - \alpha) \frac{Y_t}{H_{t,Y}}, \qquad p_{t,i} = \alpha H_{t,Y}^{1-\alpha} x_{t,i}^{\alpha - 1}, \tag{6}$$

where $p_{t,i}$ is the price of machines.

The intermediate goods sector is monopolistically competitive as in Dixit and Stiglitz (1977). Firms in the intermediate goods sector have access to the production technology $x_{t,i} = k_{t,i}$, where $k_{t,i}$ denotes physical capital employed by each firm. Operating profits of intermediate goods producers are then given by

$\pi_{t,i} = p_{t,i}x_{t,i} - R_t k_{t,i} = \alpha H_{t,Y}^{1-\alpha} k_{t,i}^{\alpha} - R_t k_{t,i}$, such that profit maximization yields the optimal price of a machine as $p_{t,i} = R_t/\alpha$ for all i. In this context, $1/\alpha$ is the markup over marginal cost. Due to symmetry with respect to the pricing policy of individual firms, we know that the aggregate capital stock is $K_t = A_t k_t$ such that we can write the aggregate production function as

$$Y_t = \left(AH_{t,Y}\right)^{1-\alpha} K_t^{\alpha}. \tag{7}$$

The R&D sector employs scientists $H_{t,A}$ to discover new blueprints A_t according to the production technology

$$A_{t+1} - A_t = \delta A_t^{\phi} H_{t,A}, \tag{8}$$

where $\delta > 0$ refers to the productivity of scientists and $\phi < 1$ to the intertemporal spillover effects of technologies that raise the productivity of human capital employed in the research sector (cf. Jones, 1995). R&D firms maximize profits $\pi_{t,A} = p_{t,A}\delta A_t^{\phi} H_{t,A} - w_{t,A}H_{t,A}$, with $p_{t,A}$ being the price of blueprints. From the first-order condition we get

$$w_{t,A} = p_{t,A}\delta A_t^{\phi}, \tag{9}$$

where $w_{t,A}$ refers to the wage rate per unit of human capital of scientists. The interpretation of this equation is straightforward: wages of scientists increase with their productivity as measured by δA_t^{ϕ} and with the price that a research firm can charge for the blueprints that it sells to the intermediate goods producers.

The education sector employs teachers with human capital $H_{t,E}$ to produce the knowledge and the skills of the next generation.[4] Employment in the education sector is determined by the equilibrium condition that household expenditures for teachers are equal to the total wage bill of teachers, that is,

$$\eta e_t n_t N_t = H_{t,E} w_t \Leftrightarrow H_{t,E} = \frac{\theta H_t}{1+\beta+\xi}.$$

Similarly, the health sector employs healthcare personnel with human capital $H_{t,F}$ to improve the health condition of the next generation. Employment in the health sector is therefore determined by the equilibrium condition that household expenditures for health are equal to the total wage bill of healthcare personnel, that is,

$$\kappa f_t n_t N_t = H_{t,F} w_t \Leftrightarrow H_{t,F} = \frac{\zeta H_t}{1+\beta+\xi}.$$

Individual human capital is a Cobb–Douglas aggregate of the education level and the health condition such that

$$h_{t+1} = \left(\mu \frac{H_{t,E}}{N_{t+1}}\right)^{\nu} \left(\omega \frac{H_{t,F}}{N_{t+1}}\right)^{1-\nu} \tag{10}$$

where $H_{t,E}/N_{t+1}$ measures the education intensity per child, $\mu > 0$ is the productivity in the schooling sector, $H_{t,F}/N_{t+1}$ measures the healthcare intensity per

https://doi.org/10.1017/S136510051900035X Published online by Cambridge University Press

child, $\omega > 0$ is the productivity in the healthcare sector, and $\nu \in (0, 1)$ denotes the elasticity of individual human capital with respect to education.

2.3. Market Clearing

Labor markets are assumed to clear such that $L_t = L_{t,Y} + L_{t,A} + L_{t,E} + L_{t,F}$, where L_t is total employment and $L_{t,j}$ for $j = Y, A, E, F$ refers to employment in the four different sectors that use human capital. This implies that $H_t = H_{t,Y} + H_{t,A} + H_{t,E} + H_{t,F}$ because human capital is embodied. Since there is free movement of labor in the economy, wages in the final goods sector and in the R&D sector will be equal in equilibrium. Inserting (6) into (9) therefore yields the following equilibrium condition that equates the marginal value product of a worker in the final goods sector and of a scientist in the R&D sector

$$p_{t,A}\delta A_t^{\phi} = (1-\alpha)\frac{Y_t}{H_{t,Y}}. \tag{11}$$

We follow Strulik et al. (2013) and assume that patent protection lasts for one generation, which is reasonably in line with the duration of patents in reality (cf. The United States Patent and Trademark Office, 2016). After a patent expires, the right to sell the blueprint is handed over to the government that consumes the associated proceeds.[5] As a consequence, the patent price is given by the one-period profits of the intermediate goods sector, which can be written as

$$\pi_{t,i} = p_{t,A} = (1-\alpha)\,\alpha k_t^{\alpha} H_{t,Y}^{1-\alpha} = \left(\alpha - \alpha^2\right)\frac{Y_t}{A_t}.$$

Plugging this into (11) and solving for the human capital employed in the final goods sector yields $H_{t,Y} = A_t^{1-\phi}/(\alpha\delta)$. Now we can use the relation $H_{t,A} = H_t - H_{t,Y} - H_{t,E} - H_{t,F}$, which is implied by the labor market clearing condition, to solve for human capital employment in the R&D sector as

$$H_{t,A} = \frac{(1+\beta)\,h_t N_t}{1+\beta+\xi} - \frac{A_t^{1-\phi}}{\alpha\delta}. \tag{12}$$

Plugging the resulting employment level of human capital of scientists into the production function of the R&D sector [equation (8)], yields the following law of motion for blueprints:

$$A_{t+1} = \frac{(1+\beta)\,\delta h_t N_t A_t^{\phi}}{1+\beta+\xi} - \frac{(1-\alpha)\,A_t}{\alpha}. \tag{13}$$

We immediately see that, *ceteris paribus*, a higher productivity of scientists (δ), a higher employment level of human capital in the R&D sector [$H_{t,A}$ as defined in equation (12)], and stronger intertemporal knowledge spillovers (ϕ) all lead to a faster accumulation of patents between time t and $t+1$.

Capital market clearing requires that total savings $s_t N_t$ are either used for investment in physical capital K_{t+1} or for buying newly developed blueprints to establish an intermediate goods producer. Given that the price of a patent is $p_{t,A}$, the value of savings in the form of new patents amounts to $p_{t,A}\,(A_{t+1} - A_t)$. Thus, the stock of physical capital at time $t+1$ is equal to aggregate savings net of savings invested in the shares of intermediate goods producers such that

$$K_{t+1} = s_t N_t - p_{t,A}\,(A_{t+1} - A_t) = Y_t - c_{1,t} N_t - c_{2,t-1} \frac{N_t}{n_{t-1}} - G_t, \qquad \textbf{(14)}$$

where G_t are governmental expenditures financed by the proceeds of expired patents and the second equality follows from the national accounts identity $Y_t = C_t + K_{t+1} + G_t$ for a closed economy with $C_t = c_{1,t} N_t - c_{2,t-1} N_t / n_{t-1}$ being aggregate consumption. In this expression, $c_{2,t-1} N_t / n_{t-1}$ refers to total consumption of the generation born at time $t-1$, which is in the second phase of its life cycle in year t and is of size N_t / n_{t-1}. Consequently, we have total output net of consumption expenditures by households and the government, that is, total investment in terms of physical capital, on the right-hand side of equation (14). Solving the resulting equation for K_{t+1} as a function of K_t, H_t, and A_t yields

$$K_{t+1} = (1-\alpha)\,K_t^{\alpha} \left(\frac{A_t^{2-\phi}}{\alpha\delta} \right)^{1-\alpha} - \frac{(1-\alpha)\,A_t h_t N_t K_t^{\alpha} \left(\frac{A_t^{2-\phi}}{\alpha\delta} \right)^{-\alpha}}{1+\beta+\xi}. \qquad \textbf{(15)}$$

Finally, we solve for the evolution of individual human capital as determined by parental investments in education and health. Plugging human capital employment in education and healthcare ($H_{t,E}$ and $H_{t,F}$), which result from the household maximization problem into the production function of human capital [equation (10)] yields

$$h_{t+1} = \frac{(\theta\mu)^{\nu}\,(\zeta\omega)^{1-\nu}\,\psi}{\xi - \zeta - \theta} h_t. \qquad \textbf{(16)}$$

Observe that, if parents want to have better educated children (higher θ) or if parents want to have healthier children (higher ζ), individual human capital accumulation increases *ceteris paribus*. By contrast, if parents want to have more children (higher ξ), individual human capital accumulation decreases because of the quality–quantity trade-off. The main question that arises regarding *aggregate* human capital accumulation is whether the increase in individual human capital accumulation due to a stronger preference for children's health and education can overcompensate the negative effect of the associated reduction in the population growth rate.

3. DYNAMICS AND LONG-RUN EQUILIBRIUM

We summarize the model dynamics defined by (4), (13), (15), and (16) in the following four-dimensional system of difference equations:

$$A_{t+1} = \frac{(1+\beta)\delta h_t N_t A_t^{\phi}}{1+\beta+\xi} - \frac{(1-\alpha)A_t}{\alpha}, \qquad \textbf{(17)}$$

https://doi.org/10.1017/S136510051900035X Published online by Cambridge University Press

$$K_{t+1} = (1-\alpha)K_t^{\alpha}\left(\frac{A_t^{2-\phi}}{\alpha\delta}\right)^{1-\alpha} - \frac{(1-\alpha)A_t h_t N_t K_t^{\alpha}\left(\frac{A_t^{2-\phi}}{\alpha\delta}\right)^{-\alpha}}{1+\beta+\xi}, \tag{18}$$

$$N_{t+1} = \frac{\xi-\zeta-\theta}{\psi(1+\beta+\xi)}N_t, \tag{19}$$

$$h_{t+1} = \frac{(\theta\mu)^{\nu}\psi(\zeta\omega)^{1-\nu}}{\xi-\zeta-\theta}h_t. \tag{20}$$

It follows that the variables A, N, and h grow at the following rates:

$$g_A = \frac{(1+\beta)\,\delta h_t N_t A_t^{\phi-1}}{1+\beta+\xi} - \frac{1}{\alpha}, \tag{21}$$

$$g_N = \frac{\xi-\zeta-\theta}{\psi\,(1+\beta+\xi)} - 1, \tag{22}$$

$$g_h = \frac{(\theta\mu)^{\nu}\,\psi\,(\zeta\omega)^{1-\nu}}{\xi-\zeta-\theta} - 1. \tag{23}$$

While, from a mathematical point of view, it is possible to have negative growth of technology, this would require negative human capital employment in R&D according to equation (12). Since this is not an economically meaningful solution, we focus on the parameter values and initial conditions under which this case does not occur, that is, we look at a situation in which h_t and N_t both grow and their initial levels together with the productivity of researchers (δ) are large enough for g_A to be positive.

Note that, in principle, the growth rates of individual human capital and of the population could become negative. However, for a large enough productivity in the schooling sector (μ) and in the health sector (ω), the following relation holds for the time cost of children:

$$\frac{\xi-\zeta-\theta}{(\theta\mu)^{\nu}\,(\zeta\omega)^{1-\nu}} < \psi < \frac{\xi-\zeta-\theta}{1+\beta+\xi}.$$

If this relation is fulfilled, both the growth rate of individual human capital and the growth rate of the population are positive. If $\psi > (\xi-\zeta-\theta)/(1+\beta+\xi)$, the population would shrink and converge to zero, while individual human capital would grow. If $\psi < (\xi-\zeta-\theta)/[(\theta\mu)^{\nu}\,(\zeta\omega)^{1-\nu}]$, population growth would be high but individual human capital would decline and converge to zero. In both of these cases the economy would cease to exist for $t\to\infty$ and no balanced growth path—along which the growth rate of technology stays constant—could be reached.

In the following, we focus on the case of a growing population and a growing individual human capital stock in which a balanced growth path exists. It is obvious from equation (21) that such a balanced growth path has to fulfill

$$\frac{h_t}{h_{t-1}}\frac{N_t}{N_{t-1}}\left(\frac{A_t}{A_{t-1}}\right)^{\phi-1}=1.$$

From this we can infer the long-run growth rate of technology as

$$g_A^*=\left[(1+g_h)\,(1+g_N)\right]^{\frac{1}{1-\phi}}-1=\left[\frac{\zeta\,(\theta\mu)^{\nu}\,\omega\,(\zeta\omega)^{-\nu}}{1+\beta+\xi}\right]^{\frac{1}{1-\phi}}-1.$$

From this result and equation (7) we know that the long-run growth rate of per capita GDP that is associated with a constant capital-to-output ratio is given by

$$g_y^*=\left[(1+g_h)\,(1+g_A)\right]-1=\frac{(1+\beta+\xi)\,\psi\left[\frac{\zeta(\theta\mu)^{\nu}\omega(\zeta\omega)^{-\nu}}{1+\beta+\xi}\right]^{1+\frac{1}{1-\phi}}}{\xi-\zeta-\theta}-1. \quad (\mathbf{24})$$

While the dependence of the growth rate of per capita GDP on the parameters that are related to fertility, education investments, and health investments is analyzed below in formal propositions, it is immediately clear that one result of the Jones (1995) model, that the long-run growth rate of the economy depends positively on the extent of intertemporal knowledge spillovers ϕ, carries over to our framework. The intuition for this effect is that higher intertemporal knowledge spillovers *ceteris paribus* raise the productivity of scientists in developing new technologies.

For the sake of completeness, the growth rates of aggregate GDP and of aggregate physical capital are given by

$$g_Y^*=g_K^*=(1+g_N)\,(1+g_h)\,(1+g_A)-1=\left[\frac{\zeta\,(\theta\mu)^{\nu}\,\omega\,(\zeta\omega)^{-\nu}}{1+\beta+\xi}\right]^{1+\frac{1}{1-\phi}}-1.$$

Next, we state our central results regarding the differential evolution of fertility, education, and health and their corresponding effects on long-run economic growth.

PROPOSITION 1. *A reduction in the population growth rate is associated with an increase in the rate of long-run economic growth.*

Proof. The derivative of equation (24) with respect to ξ is

$$\frac{\partial g_y^*}{\partial \xi}=\frac{[\zeta+\theta+\xi\,(\phi-2)+\beta\,(\phi-1)+\phi-1]\,\psi\left[\frac{\zeta(\theta\mu)^{\nu}\omega(\zeta\omega)^{-\nu}}{1+\beta+\xi}\right]^{1+\frac{1}{1-\phi}}}{(\zeta+\theta-\xi)^2\,(1-\phi)}.$$

Recalling the parameter restriction $\xi>\zeta+\theta$ and noting that the term $\phi-2$ is smaller than -1 because $\phi<1$, the numerator of this expression is always negative. Since the denominator is always positive, the proof of the proposition is established. ■

The intuition for this finding is that parents who prefer to have fewer children reduce fertility. This allows them—for a given income level—to spend more on education and health for each child. In addition, the reduction in fertility allows parents to supply more time on the labor market such that their disposable income rises. Part of this additional income is spent on children's education and health. While the reduction in fertility reduces the growth rate of aggregate human capital, the reverse holds true for increases in educational investments and health investments. Since the fall in fertility unleashes additional resources that can be spent on education and health, this effect is so strong that it overcompensates the negative effect of the reduction in fertility. Consequently, aggregate human capital accumulates faster and economic growth increases in case of lower fertility. This is a similar mechanism as in the partial equilibrium framework of Prettner et al. (2013). The implied negative association between fertility and long-run economic growth is consistent with the empirical evidence for modern economies (see, for example, Brander and Dowrick, 1994; Ahituv, 2001; Herzer et al., 2012).

Next, we obtain the following result.

PROPOSITION 2. *Higher parental investments in education lead to an increase in the rate of long-run economic growth.*

Proof. Taking the derivative of equation (24) with respect to θ yields

$$\frac{\partial g_y^*}{\partial \theta} = \frac{(\beta+\xi+1)\left\{\theta\left[\nu\left(\phi-2\right)-\phi+1\right]+\nu\left(\zeta-\xi\right)\left(\phi-2\right)\right\}\psi\left[\frac{\zeta(\theta\mu)^{\nu}\omega(\zeta\omega)^{-\nu}}{1+\beta+\xi}\right]^{1+\frac{1}{1-\phi}}}{\theta\left(\zeta+\theta-\xi\right)^2\left(1-\phi\right)}.$$

To see that this expression is positive, we first observe that the denominator is always positive. Next, we inspect the following part of the numerator: $\theta[\nu\,(\phi-2)+1]+\nu\,(\zeta-\xi)\,(\phi-2) = \theta\nu\,(\phi-2)-\theta\phi+\theta+\nu(\zeta-\xi)(\phi-2)$. This is unambiguously positive because (i) $\nu\,(\zeta-\xi)\,(\phi-2)$ is positive, (ii) $|\,\theta\nu\,(\phi-2)\,|<|\nu\,(\zeta-\xi)\,(\phi-2)\,|$ since $\xi>\zeta+\theta$, and (iii) $-\theta\phi+\theta$ is positive. Consequently, the numerator and the whole derivative are both positive. ■

The intuition behind this result is that parents who want to have better educated children do not only increase their educational investments but they also reduce fertility due to the quality–quantity substitution described in Becker and Lewis (1973). This implies in turn that they supply more of their time on the labor market and partly spend the additional income on education and health of their children. The additional investments in the quality of children are greater than the reductions in the investments in their quantity. Consequently, aggregate human capital growth increases, despite the fact that population growth decreases. Due to this increase in the rate of aggregate human capital accumulation, technological progress and economic growth gain momentum.

In our model, mortality is exogenous. In case that mortality depended negatively on education, there would be an additional transmission channel by which better education affected population growth. On the one hand, the rise in education would reduce mortality and thereby foster population growth for a constant fertility rate. In this case, the negative effect of more education on population growth could be partly compensated by the associated endogenous reduction in mortality. On the other hand, it could be that the decrease in mortality by itself leads to a reduction in fertility. In this case, the population growth rate could be reduced by even more than the initial effect of education alone. Which of the two effects is stronger depends on the extent to which the reduction in mortality occurs at young or at old age. If mortality is predominantly reduced at older ages, it is conceivable that fertility even rises because then there are more grandparents available who could support child care and, thus, reduce the time cost of children to households (cf. Fanti and Gori, 2014b).

Finally, we obtain the following result.

PROPOSITION 3. *Higher parental investments in children's health lead to an increase in the rate of long-run economic growth.*

Proof. The derivative of equation (24) with respect to ζ is given by

$$\frac{\partial g_y^*}{\partial \zeta} = \frac{(\beta + \xi + 1)\left\{\zeta\left[\nu\left(\phi - 2\right) + 1\right] + \left(\nu - 1\right)\left(\theta - \xi\right)\left(\phi - 2\right)\right\}\psi\left[\frac{\zeta(\theta\mu)^{\nu}\omega(\zeta\omega)^{-\nu}}{1+\beta+\xi}\right]^{1+\frac{1}{1-\phi}}}{\zeta\left(\zeta + \theta - \xi\right)^2\left(\phi - 1\right)}.$$

To see that this expression is positive, we first observe that the denominator is negative. Next, we inspect the following part of the numerator: $\zeta\left[\nu\left(\phi - 2\right) + 1\right] + \left(\nu - 1\right)\left(\theta - \xi\right)\left(\phi - 2\right) = \zeta + \zeta\nu\left(\phi - 2\right) + \left(\nu - 1\right)\left(\theta - \xi\right)\left(\phi - 2\right)$. This expression is negative because $\xi > \zeta + \theta$, which implies that the numerator is negative such that the whole derivative is positive. ■

The intuition behind this result is similar to that of Proposition 2 and it is again rooted in the quality–quantity substitution. Parents who want to have healthier children do not only increase their health investments but they also reduce fertility. Again, this allows them to work more and spend part of the additional income on education and health of their children. Analogous to the intuition behind the previous result, this leads to faster human capital accumulation, technological progress, and economic growth. Note that, due to our assumption $\xi > \theta + \zeta$, education and health investments cannot become infinite. Considering the possible range of the other parameters, the same holds true for the growth rate of per capita GDP (g_y^*).

Next, we illustrate the transitional dynamics and the long-run solution of our model by solving the four-dimensional system of difference equations (17)–(20) for the parameter values displayed in Table 1. In this numerical exercise, which, of course, does not represent a full-fledged calibration, the discount factor β is computed based on a discount rate ρ that is equal to 2% and considering that each period lasts for 25 years in our OLG structure. The elasticity of output with respect to physical capital, α, and the knowledge spillover, ϕ, attain the values of 0.33 and 0.7, respectively (cf. Jones, 1995; Jones and Williams, 2000). The other parameters are chosen such that we obtain—along the balanced growth path, where the growth rates do not change anymore over time—annualized values of the growth rates of per capita GDP and of the population that are consistent with the US experience averaged over the years 2006-2015 according to the World Bank (2016)

TABLE 1. Parameter values for the numerical exercise

Parameter	Value	Parameter	Value
β	0.6	δ	7
ϕ	0.7	α	0.33
ξ	0.85	ζ	0.3
θ	0.4	ψ	0.05
μ	8.68	ω	8.65
ν	0.5		

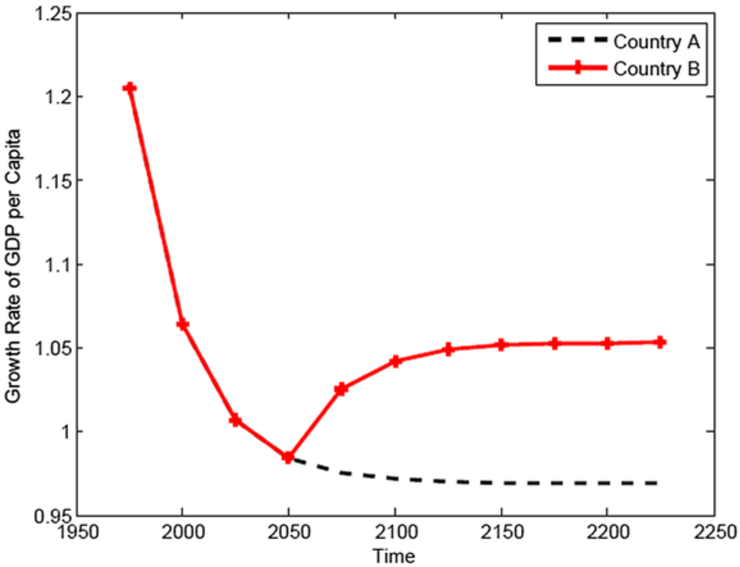

FIGURE 1. Growth rates of countries A and B over 15 periods. After the fifth period in the numerical exercise, the weight of health in parental utility (ζ) increases by 1% in country B.

data. Figure 1 displays the convergence of economic growth from above toward its long-run rate. The dashed line (Country A) represents the baseline case. We observe that the long-run growth rate of per capita GDP almost reaches the intergenerational growth rate of per capita GDP of the US, which is 14.59%. The growth rate of the population is constant [see equation (22)] and in our simulations we obtain a value of 22.45% which is a reasonable approximation of the US intergenerational population growth rate of 23.26%.

After the fifth period in the numerical exercise, we increase the value of the weight of children's health in the parental utility function (ζ) by 1% in an alternative scenario (Country B). We observe that, after the increase in the parameter ζ, country B shows a higher growth rate as compared to country A. This is exactly what we stated in Proposition 3.

4. CONCLUSIONS

We set up a framework of R&D-based economic growth in which the stock of human capital is determined by parental education and health investments. Due to the quality–quantity tradeoff, an increase in fertility leads to a reduction in education and health investments to the extent that the growth rate of overall human capital slows down. The reverse holds true for falling fertility. Altogether, this generates a pattern in which a lower population growth rate is associated with faster economic growth. This pattern is consistent with the empirical findings for modern economies in the second half of the twentieth century (Brander and Dowrick, 1994; Ahituv, 2001; Herzer et al., 2012). If parents prefer to have better educated children, they do not only increase educational investments but also health investments and if parents put more weight on their children's health, they do not only raise health investments but also educational investments. This formally shows that there is a complementarity between health and education as emphasized in the literature.

We show that a better health condition of children raises the growth rate of human capital and therefore the growth rate of the central input in the R&D sector. As a consequence, technological progress increases, which in turn raises economic growth. This provides a mechanism based on R&D-driven endogenous economic growth to explain the positive effect of health on growth that is found for modern economies (Cervellati and Sunde, 2011). This mechanism is likely to complement the ones that are based on the neoclassical capital dilution effect (Cervellati and Sunde, 2011) and on the Ben–Porath mechanism that a higher life expectancy implies a stronger incentive for education investments (Ben-Porath, 1967; Cervellati and Sunde, 2005, 2013).

To focus on the most important transmission channels of the effects of children's health on economic growth, we abstracted from some aspects that would be present in a more realistic setting but which would make the model more complicated such that analytical closed-form solutions for the long-run growth rates could not be obtained. For example, i) health might not only be represented by physical well-being but also by longevity, ii) the function by which health and education investments translate into human capital might have a more general form than the currently used Cobb–Douglas specification. While we do not find any reason to believe that generalizations along these lines would render our central results invalid, a consideration of these aspects is surely a promising avenue for further research.

NOTES

1. See also de la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Boucekkine et al. (2002), Boucekkine et al. (2003), Lagerlöf (2003), Bar and Leukhina (2010), and Gehringer and Prettner (2017).

2. For notable examples of R&D-based growth models, see among many others, Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995), Peretto (1998), and Segerström (1998). For frameworks that explicitly model human capital as a result of schooling

investments, see, for example, Funke and Strulik (2000), Strulik (2005), Grossmann (2007), Bucci (2008, 2013), Strulik et al. (2013), and Prettner (2014).

3. For models with endogenous life expectancy, see, for example, Blackburn and Cipriani (2002), Chakraborty (2004), Cervellati and Sunde (2005), Hashimoto and Tabata (2005), Bhattacharya and Qiao (2007), Castelló-Climent and Doménech (2008), de la Croix and Licandro (2013), Fanti and Gori (2014*a*), and Chakraborty et al. (2016).

4. Berk and Weil (2015) emphasize the problem of older teachers in the context of population aging. Children who have older teachers might study outdated knowledge. This observation could be considered in an extension of our model that allows for this type of the "vintage effect".

5. For the long-run balanced growth rate of the economy it would make no difference if the government were allowed to invest part of (or even all) of these proceeds.

REFERENCES

Acemoglu, D. and Johnson, S. (2007) Disease and development: The effect of life expectancy on economic growth. *Journal of Political Economy* 115(6), 925–985.

Aghion, P. and Howitt, P. (1992) A model of growth through creative destruction. *Econometrica* 60(2), 323–351.

Aghion, P., Howitt, P., and Murtin, F. (2011) The relationship between health and growth: When Lucas meets Nelson-Phelps. *Review of Economics and Institutions* 2(1), 1–24.

Ahituv, A. (2001) Be fruitful or multiply: On the interplay between fertility and economic development. *Journal of Population Economics* 14, 51–71.

Andreoni, J. (1989) Giving with impure altruism: Applications to charity and Ricardian equivalence. *Journal of Political Economy* 97, 1447–1458.

Bar, M. and Leukhina, O. (2010) The role of mortality in the transmission of knowledge. *Journal of Economic Growth* 15(4), 291–321.

Becker, G. S. (2007) Health as human capital: Synthesis and extensions. *Oxford Economic Papers* 59(3), 379–410.

Becker, G. S. and Lewis, H. G. (1973) On the interaction between the quantity and quality of children. *Journal of Political Economy* 81, 279–288.

Behrman, J. R. (2009) Early life nutrition and subsequent education, health, wage, and intergenerational effects. *Health and Growth* 6, 167–183.

Behrman, J. R. and Rosenzweig, M. R. (2004) Returns to birthweight. *Review of Economics and statistics* 86(2), 586–601.

Ben-Porath, Y. (1967) The production of human capital and the life cycle of earnings. *Journal of Political Economy* 75(4), 352–365.

Berk, J. and Weil, D. N. (2015) Old teachers, old ideas, and the effect of population aging on economic growth. *Research in Economics* 69(4), 661–670.

Bhattacharya, J. and Qiao, X. (2007) Public and private expenditures on health in a growth model. *Journal of Economic Dynamics and Control* 31(8), 2519–2535.

Blackburn, K. and Cipriani, G. P. (2002) A model of longevity, fertility and growth. *Journal of Economic Dynamics and Control* 26(2), 187–204.

Bloom, D. E., Cafiero-Fonseca, E. T., McGovern, M. E., Prettner, K., Stanciole, A., Weiss, J., Bakkila, S., and Rosenberg, L. (2014) The macroeconomic impact of non-communicable diseases in China and India: Estimates, projections, and comparisons. *The Journal of the Economics of Ageing* 4, 100–111.

Bloom, D. E. and Canning, D. (2000) The health and wealth of nations. *Science* 287(5456), 1207–1209.

Bloom, D. E., Kuhn, M., and Prettner, K. (2015) The Contribution of Female Health to Economic Development. NBER Working Paper: No. 21411.

Boucekkine, R., de la Croix, D., and Licandro, O. (2003) Early mortality declines at the dawn of modern growth. *The Scandinavian Journal of Economics* 105(3), 401–418.

Boucekkine, R., de la Croix, D., and Licandro, O. (2002) Vintage human capital, demographic trends, and endogenous growth. *Journal of Economic Theory* 104(2), 340–375.

Brander, J. A. and Dowrick, S. (1994) The role of fertility and population in economic growth. *Journal of Population Economics* 7(1), 1–25.

Bucci, A. (2008) Population growth in a model of economic growth with human capital accumulation and horizontal R&D. *Journal of Macroeconomics* 30(3), 1124–1147.

Bucci, A. (2013) Returns to specialization, competition, population, and growth. *Journal of Economic Dynamics and Control* 37, 2023–2040.

Case, A., Fertig, A., and Paxson, C. (2005) The lasting impact of childhood health and circumstance. *Journal of Health Economics* 24(2), 365–389.

Castelló-Climent, A. and Doménech, R. (2008) Human capital inequality, life expectancy and economic growth. *The Economic Journal* 118(528), 653–677.

Cervellati, M. and Sunde, U. (2005) Human capital formation, life expectancy, and the process of development. *American Economic Review* 95(5), 1653–1672.

Cervellati, M. and Sunde, U. (2011) Life expectancy and economic growth: The role of the demographic transition. *Journal of Economic Growth* 16, 99–133.

Cervellati, M. and Sunde, U. (2013) Life expectancy, schooling, and lifetime labor supply: Theory and evidence revisited. *Econometrica* 81(5), 2055–2086.

Chakraborty, S. (2004) Endogenous lifetime and economic growth. *Journal of Economic Theory* 116(1), 119–137.

Chakraborty, S., Papageorgiou, C., and Sebastián, F. P. (2016) Health cycles and health transitions. *Macroeconomic Dynamics* 20 (1), 189–213

Currie, J. (2009) Healthy, wealthy, and wise: Socioeconomic status, poor health in childhood, and human capital development. *Journal of Economic Literature* 47(1), 87–122.

de la Croix, D. and Licandro, O. (1999) Life expectancy and endogenous growth. *Economics Letters* 65(2), 255–263.

de la Croix, D. and Licandro, O. (2013) The child is father of the man: Implications for the demographic transition. *The Economic Journal* 123(567), 236–261.

Dixit, A. K. and Stiglitz, J. E. (1977) Monopolistic competition and optimum product diversity. *American Economic Review* 67(3), 297–308.

Fanti, L. and Gori, L. (2014a) Endogenous fertility, endogenous lifetime and economic growth: The role of child policies. *Journal of Population Economics* 27(2), 529–564.

Fanti, L. and Gori, L. (2014b) An OLG model of growth with longevity: When grandparents take care of grandchildren. *Portuguese Economic Journal* 13(1), 39–51.

Funke, M. and Strulik, H. (2000) On endogenous growth with physical capital, human capital and product variety. *European Economic Review* 44, 491–515.

Galor, O. (2011) *Unified Growth Theory*. Princeton, NJ, USA: Princeton University Press.

Galor, O. and Weil, D. (2000) Population, technology, and growth: From Malthusian Stagnation to the demographic transition and beyond. *The American Economic Review* 90(4), 806–828.

Gehringer, A. and Prettner, K. (2017) Longevity and technological change. *Macroeconomic Dynamics* (forthcoming) DOI: https://doi.org/10.1017/S1365100517000293

Grossman, G. M. and Helpman, E. (1991) Quality ladders in the theory of economic growth. *Review of Economic Studies* 58(1), 43–61.

Grossmann, V. (2007) How to promote R&D-based growth? Public education expenditure on scientists and engineers versus R&D subsidies. *Journal of Macroeconomics* 29(4), 891–911.

Hashimoto, K.-i. and Tabata, K. (2005) Health infrastructure, demographic transition and growth. *Review of Development Economics* 9(4), 549–562.

Herzer, D., Strulik, H., and Vollmer, S. (2012) The long-run determinants of fertility: One century of demographic change 1900-1999. *Journal of Economic Growth* 17(4), 357–385.

Jones, C. I. (1995) R&D-based models of economic growth. *Journal of Political Economy* 103(4), 759–783.

Jones, C. I. and Williams, J. C. (2000) Too much of a good thing? The economics of investment in R&D. *Journal of Economic Growth* 5(1), 65–85.

Kalemli-Ozcan, S., Ryder, H. E., and Weil, D. N. (2000) Mortality decline, human capital investment, and economic growth. *Journal of Development Economics* 62(1), 1–23.

Kuhn, M. and Prettner, K. (2016) Growth and welfare effects of health care in knowledge-based economies. *Journal of Health Economics* 46, 100–119.

Lagerlöf, N.-P. (2003) From Malthus to modern growth: Can epidemics explain the three regimes? *International Economic Review* 44(2), 755–777.

Lorentzen, P., McMillan, J., and Wacziarg, R. (2008) Death and development. *Journal of Economic Growth* 13, 81–124.

Madsen, J.B. (2016) Health, human capital formation and knowledge production: Two centuries of international evidence. *Macroeconomic Dynamics* 20(4), 909–953

Peretto, P. F. (1998) Technological change and population growth. *Journal of Economic Growth* 3(4), 283–311.

Perri, T. J. (1984) Health status and schooling decisions of young men. *Economics of Education Review* 3(3), 207–213.

Preston, S. H. (1975) The changing relation between mortality and level of economic development. *Population Studies* 29(2), 231–248.

Prettner, K. (2014) The non-monotonous impact of population growth on economic prosperity. *Economics Letters* 124, 93–95.

Prettner, K., Bloom, D. E., and Strulik, H. (2013) Declining fertility and economic well-being: Do education and health ride to the rescue? *Labour Economics* 22, 70–79.

Prettner, K. and Strulik, H. (2016) Technology, trade, and growth: The role of education *Macroeconomic Dynamics* 20, 1381–1394.

Romer, P. (1990) Endogenous technological change. *Journal of Political Economy* 98(5), 71–102.

Schultz, T. W. (1961) Investment in human capital. *The American Economic Review* 51(1), 1–17.

Segerström, P. S. (1998) Endogenous growth without scale effects. *American Economic Review* 88(5), 1290–1310.

Solow, R. M. (1956) A contribution to the theory of economic growth. *The Quarterly Journal of Economics* 70(1), 65–94.

Strulik, H. (2005) The role of human capital and population growth in R&D-based models of economic growth. *Review of International Economics* 13(1), 129–145.

Strulik, H., Prettner, K., and Prskawetz, A. (2013) The past and future of knowledge-based growth. *Journal of Economic Growth* 18(4), 411–437.

The United States Patent and Trademark Office (2016) Url: http://www.uspto.gov [accessed on 11/06/2012].

World Bank (2016) World Development Indicators & Global Development Finance Database. Available at: http://databank.worldbank.org/data/reports.aspx?source=world-development-indicators#.