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Effective interaction potential of dust particles in a plasma from experimental pair correlation functions

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Abstract. Interaction between dust particles in a plasma is investigated on the basis of experimental pair correlation functions of dust formation in a dc glow discharge and the Poisson equation. It is shown that the calculated effective potentials have an oscillating character and depend rather weakly on macroscopic parameters of the plasma. Existence of an attractive component in the interaction of dust particles is confirmed within the validity range of our model.

1. Introduction

Experimental observation of Coulomb dust crystals, first, in high-frequency discharge [1–4] and, later, in glow discharge [5] introduced a new area in the physics of strongly coupled systems. Due to ease of fabrication in laboratory conditions, parameter control and observation methods, dust crystals offer a robust tool to study a variety of collective effects like instabilities, waves and self-organization (see [6, 7] for an overview).

Despite intensive experimental, theoretical and numerical research devoted to investigation of charged dust grains in a plasma environment in the past, several fundamental questions regarding such a system remain to be opened. One of the most striking phenomena still requiring further exploration is the nature and the form of attractive forces acting between like-charged dust particles. The existence of an attractive component in the force between dust particles was experimentally studied and verified by several methods in different setups [8–10].

Interaction of dust particles in plasma is traditionally described by the screened Coulomb or the Yukawa potential:

$$\Phi(r) = \frac{Z_{\mathrm{d}}^2 e^2}{r} \exp(-r/r_{\mathrm{D}}), \tag{1.1}$$

where $r_{\mathrm{D}} = \sqrt{k_{\mathrm{B}} T_{\mathrm{e}} T_{\mathrm{i}} / (4\pi n_0 e^2 (T_{\mathrm{e}} + T_{\mathrm{i}}))}$ is the Debye radius, Z_{d} is the charge number of dust particles, k_{B} is the Boltzmann constant, n_0 is the number density of charged particles and T_{e} and T_{i} are temperatures of electrons and ions, respectively. In laboratory conditions $T_{\mathrm{e}} \gg T_{\mathrm{i}}$, so that the Debye radius is determined by the ionic component. The Yukawa potential (1.1) is barely repulsive and fully screened only at $r \to \infty$. It also assumes spatial homogeneity of the system.

Experiments with dust particles in plasma discharges, however, are characterized by the existence of directed ion flows near to the surface of dust grains. As consequence, the form of the interaction between dust particles changes in a way that the Coulomb potential is fully screened at finite distances. At larger distances, different physical effects may dominate in the interaction between dust particles and, under certain conditions, lead to attraction among them: interaction with low-frequency electrostatic waves [11–13], interaction due to polarization of dust–ion clouds [14] and interaction due to ion shadowing [15, 16].

Regardless of the exact mechanism behind the attraction of dust particles, determination of the form of the interaction potential between them is of great interest because its features like position and depth of the first minima could have an influence on properties of the system, like excess energy and pressure. Therefore, several methods were developed to determine the form of the interaction potential from time series of particles' configurations that can be captured in experiments: on the basis of dust–dust scattering [17] and simultaneous measurement of particles' spatial positions [18].

In the present paper we propose a different approach to the calculation of the interaction potential between dust particles. Our approach is based on a pair correlation function (PCF) that defines the probability of finding a dust particle within a given distance from a test particle and can be measured precisely in experiments. The measured PCF is then used to solve the Poisson equation that links the charge distribution and the effective field around the dust particle in a general form. Solution of the Poisson equation is considered as the effective interaction potential of dust particles.

2. Experimental pair correlation functions

PCFs of dust particles were determined from the experiments carried out in a dc glow discharge setup [19]. A schematic view of the setup is shown in Fig. 1.

The core of the experimental setup is formed by a glass tube of 50 cm in length and 6 cm in diameter positioned vertically. A dc power supply is used to apply voltage to the electrodes and create standing strata along the vertical axis of the tube. The experiments were carried out under the following conditions: working gas (argon) pressure varies in the range of 0.08–0.2 torr; discharge current varies in the range of 0.5–1.5 mA.

After stabilization of strata, poly-disperse dust particles of Al_2O_3 with average diameter b_d of 5 μm and average mass m_d of 0.26×10^{-9} g were injected into the discharge volume using a dispenser at the top of the tube. Falling dust particles are trapped in the electric field of the discharge, thus enabling investigation of various dust formations.

To illuminate the trapped particles, a He–Ne laser with 50 mW is used; optical lenses are used to widen a laser beam. Particle configurations illuminated in the horizontal plane are captured by a CCD camera at a rate of 25 frames per second. Video data are permanently stored in computer memory for further processing.

Table 1 presents summarized data of four experiments performed. The following parameters are presented: working gas pressure, P; discharge current, I; number density of electrons, n_{e0}; number density of dust particles, n_{d0}.

Table 1. Summary of the experiments with dusty plasma in dc discharge.

Experiment No.	Discharge parameters	Buffer plasma parameters	Dust particle parameters	Dimensionless parameters
1	$P = 0.152$ torr $I = 0.992$ mA	$n_{e0} = 2.96 \times 10^{10}$ cm^{-3}	$Z_d = 2.1 \times 10^4$ $n_{d0} = 14300$ cm^{-3}	$\Gamma = 121$ $k = 1.79$
2	$P = 0.182$ torr $I = 0.775$ mA	$n_{e0} = 2.1 \times 10^{10}$ cm^{-3}	$Z_d = 2.1 \times 10^4$ $n_{d0} = 3940$ cm^{-3}	$\Gamma = 239.7$ $k = 2.67$
3	$P = 0.11$ torr $I = 1.8$ mA	$n_{e0} = 2.4 \times 10^{10}$ cm^{-3}	$Z_d = 2.1 \times 10^4$ $n_{d0} = 10712$ cm^{-3}	$\Gamma = 68.50$ $k = 1.69$
4	$P = 0.13$ torr $I = 1.44$ mA	$n_{e0} = 2.4 \times 10^{10}$ cm^{-3}	$Z_d = 2.1 \times 10^4$ $n_{d0} = 20995$ cm^{-3}	$\Gamma = 40.67$ $k = 1.49$

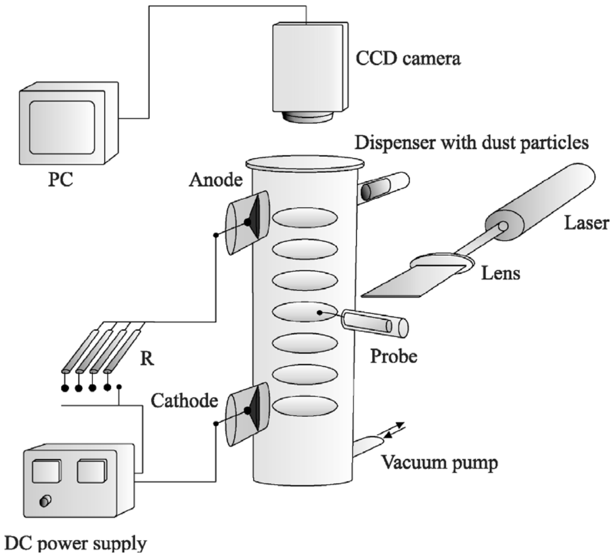

Figure 1. Schematic view of the experimental setup for the investigation of dusty plasma properties in dc glow discharge.

Electron density n_{e0} and electron temperature T_e were obtained from the probe measurements [20]. Temperature of ions was assumed to be equal to room temperature $T_i = 300$ K due to the relatively high pressure of the buffer gas.

Charge of the dust grain Z_d was determined experimentally from the balance of gravity and electric forces [21]:

$$Z_d = \frac{m_d g}{e_0 E(z)}, \tag{2.1}$$

where g is free-fall acceleration, e_0 is electron charge and $E(z)$ is axial electric field that is experimentally determined by Nowak's rule:

$$E_{av} = \frac{\phi}{\zeta}, \tag{2.2}$$

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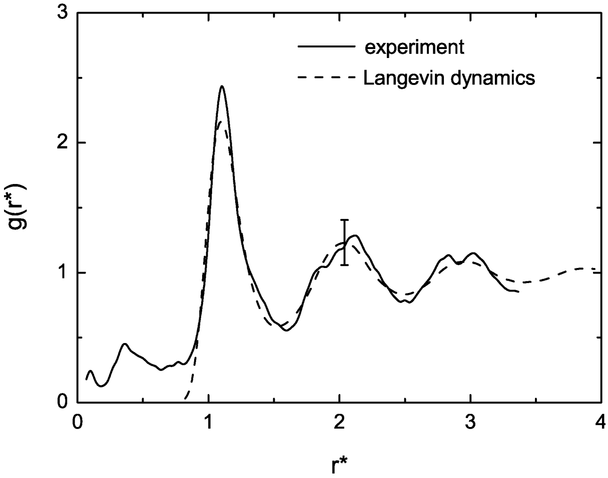

Figure 2. Experimental and calculated PCFs. Experiment No. 1.

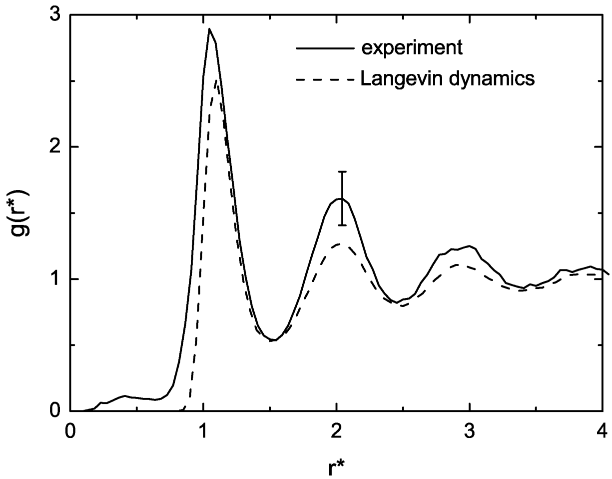

Figure 3. Experimental and calculated PCFs. Experiment No. 2.

where $E_{\rm av}$ is the average field, ϕ is the first threshold for argon excitation and ζ is the stratum period.

The coupling parameter $\Gamma = (Z_{\rm d}e)^2/(ak_{\rm B}T_{\rm d})$ was determined from the following expression:

$$\Gamma^* = \Gamma(1 + k + k^2)\exp(-k), \tag{2.3}$$

where Γ^* is the effective coupling parameter; a is the average distance between dust particles. It can be taken from the calibration curve from [22] using the ratio of maxima and minima of the pair correlation function. In (2.3), $k = a/r_{\rm D}$ is the screening parameter. Since data for $T_{\rm e}$ and $n_{\rm e}$ are known from the probe diagnostic, it is possible to calculate the Debye radius and the screening parameter.

Experimental PCFs for four experiments listed in Table 1 are presented in Figs 2–5. The forms of the curves follow the expected behavior with subsequent maxima

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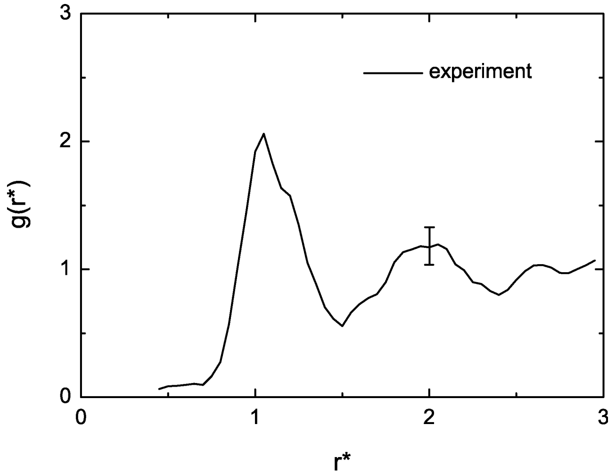

Figure 4. Experimental PCF. Experiment No. 3.

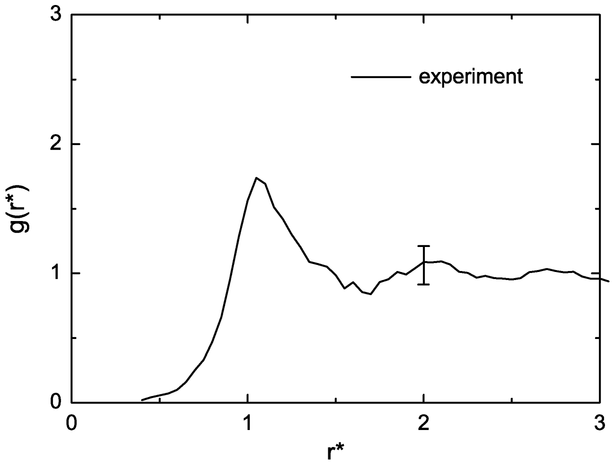

Figure 5. Experimental PCF. Experiment No. 4.

and minima reflecting the existing structural order in the system. According to the determined dusty plasma parameters, dust systems observed in Experiment Nos. 1 and 2 correspond to solid state, and in Experiment Nos. 3 and 4 correspond to liquid state. Errors in PCFs are shown for all four cases at the second maxima of the respective functions. The values of the errors lie within the range of 10–15% and are mainly determined by the precision of the processing methods.

A short note is to be made on the non-zero values of correlation functions at the distances prior to the first maximum. Such unusual behavior has its origin in the inefficient particle-detection algorithm applied during the video-processing stage. It was observed that the image of a dust-cloud layer under observation often contains particles from bottom and top layers. Although these particles have smaller brightness than the particles from the original layer, they are recognized as separate bodies by the processing software and, thus, contribute to an artificial rise of the radial PCF at small distances.

In order to understand the real behavior of the radial distribution functions, we performed computer simulation experiments on the basis of Langevin dynamics [23] at the same set of macroscopic parameters as provided in Table 1, so that the comparison of both data sets can be performed. Correlation functions from these experiments are also shown in Figs 2 and 3 for comparison. In the calculations discussed below, we interpolate calculated correlation functions with experimental correlation functions at the distances closer than the first maximum in order to obtain physically meaningful results.

3. Poisson equation

The general relationship between the electrostatic interaction potential and the charge distribution around a dust particle is represented by a Poisson equation. Assuming single ionization of ions in the plasma, this equation can be written in the following form:

$$\nabla^2\varphi(r) = -4\pi e[n_{\rm i}(r) - n_{\rm e}(r) - Z_{\rm d}n_{\rm d}(r)], \tag{3.1}$$

where $\varphi(r), n_{\rm i}(r), n_{\rm e}(r), n_{\rm d}(r)$ are, correspondingly, the electrostatic potential in the vicinity of the dust particle and the distributions of electron, ion and dust number densities as functions of distance to the center of mass of a test dust particle.

Assuming local equilibrium and weak interaction of ions and electrons with the dust particle, the distributions of number densities for electrons and ions can be taken in the form of linearized Boltzmann distributions:

$$n_{\rm i}(r) = n_{\rm i0}\left(1 - \frac{e\varphi(r)}{k_{\rm B}T_{\rm i}}\right), \tag{3.2}$$

$$n_{\rm e}(r) = n_{\rm e0}\left(1 + \frac{e\varphi(r)}{k_{\rm B}T_{\rm e}}\right). \tag{3.3}$$

The distribution of dust particles can be expressed through the PCF $g(r)$ according to the following expression:

$$n_{\rm d}(r) = n_{\rm d0}g(r). \tag{3.4}$$

In (3.2)–(3.4) the number densities of ions, electrons and dusty particles—$n_{\rm i0}$, $n_{\rm e0}$ and $n_{\rm d0}$—were introduced. They are coupled together by the condition of quasi-neutrality:

$$n_{\rm i0} = Z_{\rm d}n_{\rm d0} + n_{\rm e0}. \tag{3.5}$$

Introducing a dimensionless potential through $\varphi^*(r^*) = Z_{\rm d}e\varphi(r^*)/(k_{\rm B}T_{\rm e})$ and $r^* = r/a$, (3.1) can be rewritten as follows:

$$\nabla^2\varphi^*(r^*) - k^2(\delta\tau + 1)\varphi^*(r^*) = -k^2\left[(\delta - 1)Z_{\rm d} - Z_{\rm d}^2\frac{n_{\rm d0}}{n_{\rm e0}}g(r^*)\right], \tag{3.6}$$

where $\delta = n_{\rm i0}/n_{\rm e0}$, $\tau = T_{\rm e}/T_{\rm i}$ and k is the screening parameter.

Two types of boundary conditions for (3.6) were considered in our calculations. The first set of boundary conditions is characterized by the limit of the Yukawa

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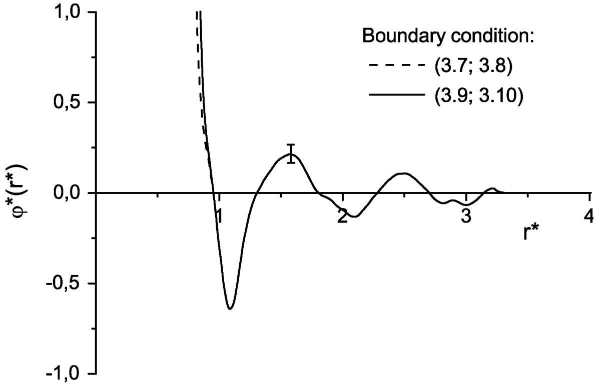

Figure 6. Effective interaction potential for dusty particles. Experiment No. 1.

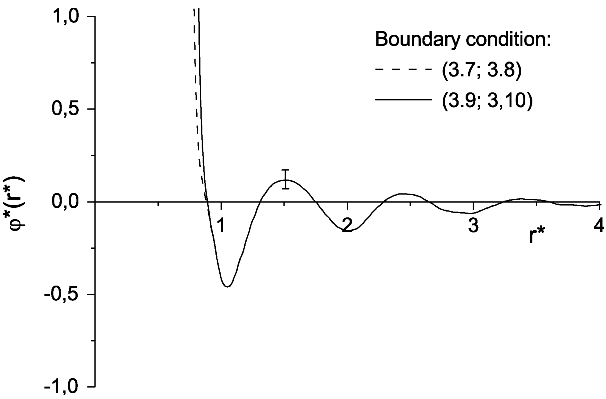

Figure 7. Effective interaction potential for dusty particles. Experiment No. 2.

potential as $r \to 0$:

$$\varphi|_{r^*=r_0} = \frac{(Z_\mathrm{d}e)^2}{r_0}\exp(-r_0/r_\mathrm{D}), \tag{3.7}$$

$$\varphi|_{r^*\to\infty} = 0. \tag{3.8}$$

The second set of boundary conditions is characterized by the limit of the Coulomb potential as $r \to 0$:

$$\varphi|_{r^*=r_0} = \frac{(Z_\mathrm{d}e)^2}{r_0}, \tag{3.9}$$

$$\varphi|_{r^*\to\infty} = 0, \tag{3.10}$$

where r_0 is an artificial truncation distance, $a \gg r_0 > b_\mathrm{d}$.

Numerical solutions of the system of coupled equations represented by (3.6), (3.7) and (3.8) are shown in Figs 6–9 together with the solutions of (3.6), (3.9) and (3.10).

From the presented results, one can conclude that the difference between the introduced boundary conditions is relatively small at distances larger than the distance to the closest neighbor grain. This is not surprising, as in all four

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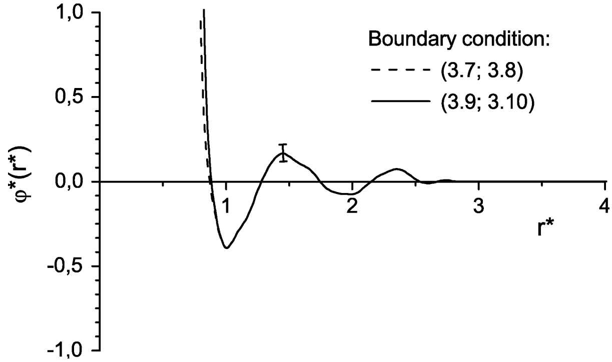

Figure 8. Effective interaction potential for dusty particles. Experiment No. 3.

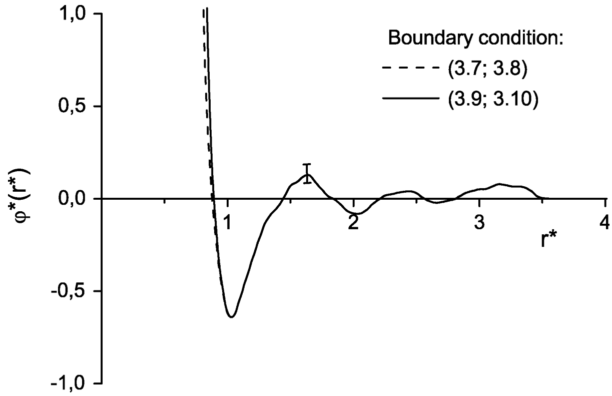

Figure 9. Effective interaction potential for dusty particles. Experiment No. 4.

experiments $k \geqslant 1$, i.e. the Couloumb part of the interaction is not fully screened at $r^* \approx 1$.

One can also note that the interaction potentials exhibit oscillations for all four data sets. These oscillations are connected to the respective oscillations of pair correlation functions. This can be verified, for example, by examination of positions of maxima and minima of the corresponding functions. It is also remarkable that, whereas the first maximum of the pair correlation functions varies between 2.9 for Experiment No. 2 ($\Gamma = 240$) and 1.9 for Experiment No. 4 ($\Gamma = 69$), the corresponding depths of the first minimum of the calculated potentials are −0.45 and −0.42. We leave, however, the question on the weak dependence, if any, of the interaction potential within the assumptions of our model open and will address the issue in future work.

It is to be noted that the results of the present paper do not fully coincide with the conclusions presented in [24]. It may be the consequence of usage of different methods for reconstruction of the interaction potential from experimental data. In contrast to [24] where the integral equations of Ornstein–Zernike were used, our results were obtained by the application of the Poisson equation which links the charge distribution in the plasma and the effective field around the dust particle in a more general form.

Existence of an attractive component in the interaction among dust particles as implied by results of the current paper requires further thorough investigation, both experimental and theoretical, to provide the differentiation between the proposed theoretical concepts. For example, one should understand the influence of a confinement (trap) potential that may contribute significantly under conditions of the experiment. Several generalizing modifications of the presented numerical model can also be made, like the rejection of the assumption of weak interparticle interaction in buffer plasma as implied by (3.2) and (3.3). The driving forces behind these future investigations can be the determination of nonlinear screening parameters [25] or Mach number [26].

4. Conclusion

In the present work, the effective interaction potential of dust particles was calculated on the basis of the experimental PCF using the Poisson equation. Calculations were performed for two types of boundary conditions: the Coulomb- and the Yukawa-type potentials at $r \to 0$. For both cases the interactive potential has an oscillating character which is the consequence of oscillations of the PCF. A remarkable feature of the calculated potentials is a weak dependence on the macroscopic parameters.

Within the validity range of the current model, we can confirm the existence of an attractive component in the interaction of dust particles. Further investigation is required to obtain insights into the origin of the attraction and nonlinearity of Coulomb interaction screening.

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