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Nonlinear ion-acoustic waves in weak magnetic fields with vortex-like electron distribution

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Abstract. The two-dimensional dynamics of nonlinear ion-acoustic waves in a weakly magnetized plasma comprising cold ions and trapped as well as free electrons is considered. It is shown that owing to departure from the Boltzmann electron distribution to a vortex-like one, the dynamics of small but finite-amplitude ion-acoustic waves is governed by a new nonlinear equation which is valid for both unmagnetized and magnetized plasmas. For exactly vanishing magnetic fields the modified Kadomtsev–Petviashvili (mKP) equation is recovered. For weak magnetic fields, however, the dynamics is mainly different from the mKP equation, depending on the amplitude. By increasing the magnetic field, the new equation becomes similar (but not identical) to the modified Zakharov–Kuznetsov (mZK) equation, which is fulfilled for very strong magnetic fields. The plane periodic and solitary wave solutions of this equation are obtained using the appropriate scalings.

1. Introduction

It is well known that the weakly nonlinear one-dimensional description of ion-acoustic waves in plasmas is given by the Korteweg–de Vries (KdV) equation (Washimi and Taniuti 1966). This equation has solitary wave and periodic wave solutions, both of which have been shown to be stable (Benjamin 1972). The KdV equation in two dimensions, known as the Kadomtsev–Petviashvili (KP) equation (Kadomtsev and Petviashvili 1970), was derived for ion-acoustic waves in non-magnetized plasma comprising cold ions and hot isothermal electrons by Kako and Rowlands (1976). On the other hand, if the plasma is magnetized, the governing equations are the Zakharov–Kuznetsov (ZK) equation (Zakharov and Kuznetsov 1974) in strong magnetic fields and the Laedke–Spatschek equation (Laedke and Spatschek 1982a) in weak magnetic fields. The stability of solutions of the KP, ZK and the Laedke–Spatschek equations to two-dimensional long wavelength perturbations has been investigated (Infeld et al. 1978; Laedke and Spatschek 1982a and 1982b; Allen and Rowlands 1993). In the more realistic situation in which electrons are non-isothermal (vortex-like electron distribution), Schamel (1973) showed that the equivalent governing equation for one dimension is a modified form of the

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KdV equation, and is known as the Schamel equation. So far, two equations are known for the propagation of a two-dimensional perturbation with non-isothermal electrons. The modified Kadomtsev–Petviashvili (mKP) equation is found for a weak transverse coordinate dependence in a non-magnetized plasma. O'Keir and Parkes (1997) investigated the stability solutions of mKP to two-dimensional long-wavelength perturbations. On the other hand, Munro and Parkes (1999, 2000) showed that in very strong external magnetic fields, the governing ZK equation has a modified form, referred to as the modified Zakharov–Kuznetsov (mZK) equation. It is shown that for small but finite amplitudes, only with extreme values of $\Omega_c\,(\Omega_c = 0, \Omega_c/\delta \to \infty)$, The equations have been derived for non-isothermal electrons. Here Ω_i/ω_{pi} is the ratio of the ion cyclotron frequency to the ion plasma frequency and is the amplitude parameter of the solitary waves. For small but finite Ω_c, which occur in most experiments (Laedke and Spatschek 1982a), neither limit is adequate. In this paper, we try to fill out this gap and to derive a new model equation, which is valid for finite Ω_c/δ values. In the presence of trapped electrons and weak magnetic fields, we show that, the governing equation is a modified form of the Laedke–Spatschek equation and using appropriate scalings obtain exact solutions with solitons and periodic structures.

This paper is organized as follows. The basic equations governing plasma model under investigation are given in Sec. 2. Derivation of the mKP and mZK equations is briefly given by the reductive perturbation method in Sec. 3. The new scaling leading to a new model equation for ion-acoustic waves is obtained in Sec. 4. Finally, a brief discussion is presented in Sec. 5.

2. Basic equations

We consider a plasma, which consists of positively charged cold ions and hot electrons, in the presence of an external magnetic field ($B_0 \| \hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$ is a unit vector along the x direction). The nonlinear behavior of ion-acoustic waves in this plasma system may be described by the following set of fluid equations:

$$\frac{\partial n}{\partial t} + \nabla \cdot (nV) = 0, \tag{1}$$

$$\frac{\partial V}{\partial t} + (V \cdot \nabla)V + \nabla\varphi + \Omega_c \hat{\mathbf{x}} \times V = 0, \tag{2}$$

$$\nabla^2\varphi = n_e - n, \tag{3}$$

where n is the ion number density normalized to the equilibrium plasma density n_0; V is the ion fluid velocity normalized to the ion speed of sound $c_s = (T_{ef}/m_i)$, with T_{ef} being the constant temperature of the free electrons and m_i being the mass of positively charged ions; φ is the electrostatic wave potential normalized to T_{ef}/e, with e being the magnitude of the electron charge. The time and space variables are given in the units of the ion plasma period $\omega_{pi}^{-1} = (m_i/4\pi n_0 e^2)^{1/2}$ and the Debye length $\lambda_D = c_s\omega_{pi}$, respectively. $\Omega_c = B_0/\sqrt{4\pi n_0 m_i}$ is the ion cyclotron frequency normalized to ω_{pi} and can vary from small values to $\sim$1 for practical applications.

To model an electron distribution with trapped particles, we employ a vortex-like electron distribution function after Schamel (1972, 1973) that solves the electron

Vlasov equation. Thus, we have

$$f_{\mathrm{ef}} = \frac{1}{\sqrt{2\pi}} e^{-(v^2-2\varphi)/2} \quad (|v| > \sqrt{2\varphi}), \tag{4a}$$

$$f_{\mathrm{et}} = \frac{1}{\sqrt{2\pi}} e^{-\beta(v^2-2\varphi)/2} \quad (|v| \leqslant \sqrt{2\varphi}), \tag{4b}$$

where the subscripts f and t indicate the free and trapped electron contributions, respectively. It may be noted here that the distribution function, as presented above is continuous in velocity space and satisfies the regularity requirements for an admissible BGK solution [Schamel 1973]. It should be mentioned that the velocity v is normalized to the electron thermal velocity v_{te} and β which is the ratio of the free electron temperature (T_{ef}) to the trapped electron temperature (T_{et}), is a parameter determining the number of trapped electrons. It has been assumed that the velocity of nonlinear ion-acoustic waves is small in comparison with the electron thermal velocity. Integrating the electron distribution functions over the velocity space, we readily obtain the electron number density n_{e} as

$$n_{\mathrm{e}} = e^{\varphi}\,\mathrm{erf}\,c(\sqrt{\varphi}) + \frac{e^{\beta\varphi}}{\sqrt{|\beta|}}\,\mathrm{erf}(\sqrt{\beta\varphi}) \quad (\beta \geqslant 0), \tag{5a}$$

$$n_{\mathrm{e}} = e^{\varphi}\,\mathrm{erf}\,c(\sqrt{\varphi}) + \frac{e^{\beta\varphi}}{\sqrt{\pi|\beta|}} W(\sqrt{-\beta\varphi}) \quad (\beta < 0), \tag{5b}$$

where W is the Dawson integral. If we expand this n_{e} for the small-amplitude limit and keep the terms up to φ^2, it is found that n_{e} is the same for both $\beta > 0$ and $\beta < 0$ and is finally given by

$$n_{\mathrm{e}} = 1 + \varphi - \tfrac{4}{3} b \varphi^{3/2} + \tfrac{1}{2}\varphi^2, \tag{6}$$

where $b = (1-\beta)\pi^{1/2}$ measures the deviation from isothermality. We assume that $b \succ 0$, which is suggested by experiment (Schamel 1973). It should be noted here that if we neglect resonant effects ($b = 0$), n_{e} is reduced to the Maxwellian electron distribution.

3. Derivation of the mKP and mZK equations

To derive the mKP equation, we use the standard reductive perturbation method and in order to find a suitable choice of scalings for the independent variables, we use a linear dispersion argument, similar to that used by Infeld and Rowlands (1990) in their derivation of the ZK equation. Accordingly, we choose the following scalings for the independent variables:

$$\xi = \varepsilon^{1/4}(x - t), \quad \sigma = \varepsilon^{1/2} y, \quad \tau = \varepsilon^{3/4} t. \tag{7}$$

From the basic equations (1)–(3) and (5a), we expand the density, fluid velocities and electrical potential asymptotically by a smallness parameter ε as

$$n = 1 + \varepsilon n^{(1)} + \varepsilon^{3/2} n^{(2)} + \cdots, \tag{8a}$$

$$\varphi = \varepsilon\varphi^{(1)} + \varepsilon^{3/2}\varphi^{(2)} + \cdots, \tag{8b}$$

$$\nu_x = \varepsilon\nu_x^{(1)} + \varepsilon^{3/2} v_x^{(2)} + \cdots, \tag{8c}$$

$$\nu_y = \varepsilon^{1/4}(\varepsilon\nu_y^{(1)} + \varepsilon^{3/2}\nu_y^{(2)} + \cdots). \tag{8d}$$

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For $\Omega_c = 0$ and using the scalings above, the basic equation is simplified to the modified Kadomtsev–Petviashvili equation (O'Keir and Parkes 1997),

$$(\partial_\tau n + bn^{1/2}\partial_\xi n + \tfrac{1}{2}\partial_\xi^3 n)_\xi + \tfrac{1}{2}\partial_\sigma^2 n = 0. \tag{9}$$

Here, we have set $n^{(1)} = n$. The corresponding one-dimensional version of the mKP equation was derived by Schamel (1973), and is known as the Schamel equation. The mKP equation has solutions that represent plane periodic and solitary travelling waves (O'Keir and Parkes 1997; Chakraberty and Das 1998); these are, of course, also solutions of the Schamel equation. On the other hand, for $\Omega_c \sim 1$ the mZK equation can be derived. To include the $E \times B$ and polarization drifts, we replace in (5b) and (6) the following variables:

$$\sigma = \varepsilon^{1/4}y, \quad \eta = \varepsilon^{1/4}z, \quad \nu_\perp = \varepsilon^{5/4}\nu_\perp^{(1)} + \varepsilon^{3/2}\nu_\perp^{(2)} + \cdots . \tag{10}$$

The longitudinal dynamics and therefore the longitudinal scaling is unchanged compared with (5b) and (6). Straightforward but lengthy calculations lead to the mZK equation (Munro and Parkes 2000)

$$\partial_\tau n + bn^{1/2}\partial_\xi n + \tfrac{1}{2}\partial_\xi^3 n + \tfrac{1}{2}(1 + \Omega_c^{-2})\partial\xi(\partial_\sigma^2 + \partial_\eta^2)n = 0. \tag{11}$$

The stability of plane periodic and solitary traveling wave solutions of (7) and (8b) to two-dimensional long-wavelength perturbations has been investigated. As discussed in O'Keir and Parkes (1997) and Munro and Parkes (2000), the solitary wave solutions are stable with respect to transverse perturbations when $\Omega_c = 0$, on the other hand, for $\Omega_c \sim 1$, soliton solutions are unstable. We can see that for small but finite Ω_c the situation is unknown so far. While the mKP equation suggests stability, the mZK equation predicts instability. But one should realize that actually both derivations break down in the intermediate region, where Ω_c is small but finite.

4. Effect of weak magnetic fields on ion-acoustic waves

In Sec. 3 we have shown that in the two limits $\Omega_c = 0$, and $\Omega_c \sim 1$, qualitatively different results occur for the two-dimensional dynamics. Most particle applications are in the region where $\Omega_c \ll 1$ (Laedke and Spatschek 1982a). The behavior of ion-acoustic waves in that region is correctly described by neither the mKP equation nor by the mZK equation. Therefore, we now derive a new nonlinear equation for long-wavelength ion-acoustic waves propagating in a magnetized plasma, which is valid in this intermediate region. Since we want to cover the transitions from stable to unstable behavior, we are guided by the mKP equation scalings and take care of the drift term, which dominates in the mZK scalings. Thus, we choose the following scaling for the independent variables:

$$\xi = \varepsilon^{1/4}(x - t), \quad \sigma = \varepsilon^{1/2}y, \tag{12a}$$

$$\eta = \varepsilon^{1/2}z, \quad \tau = \varepsilon^{3/4}t, \tag{12b}$$

together with the additional condition, $\Omega_c \sim \varepsilon^{1/4}$. For the dependent variables we shall assume expansions to be of the form

$$n = 1 + \varepsilon^{r_{n1}}n^{(1)} + \varepsilon^{r_{n2}}n^{(2)} + \cdots, \tag{13a}$$

$$\varphi = \varepsilon^{r_{\varphi 1}}\varphi^{(1)} + \varepsilon^{r_{\varphi 2}}\varphi^{(2)} + \cdots, \tag{13b}$$

$$\nu_x = \varepsilon^{r_{x1}}\nu_x^{(1)} + \varepsilon^{r_{x2}}\nu_x^{(2)} + \cdots, \tag{13c}$$

$$\nu_\perp = \varepsilon^{r_{\perp 1}}\nu_\perp^{(1)} + \varepsilon^{r_{\perp 2}}\nu_\perp^{(2)} + \cdots. \tag{13d}$$

By balancing the effects of nonlinearity and dispersion (Benjamin 1972), we can find the appropriate values for $r_n, r_\varphi, r_\perp$ and r_x as follows. Using the scalings (8c) and the expansions (8d) with the set of non-dimensional equations (1)–(3) and (5a), we equate coefficients of like powers of ε. At the lowest order, meaningful equations can be obtained only if

$$\frac{\partial n^{(1)}}{\partial \xi} = \frac{\partial v_x^{(1)}}{\partial \xi} \quad r_{n1} = r_{x1} \tag{14}$$

$$\frac{\partial v_x^{(1)}}{\partial \xi} = \frac{\partial \varphi^{(1)}}{\partial \xi} \quad r_{x1} = r_{\varphi 1} \tag{15}$$

$$\frac{\partial v_y^{(1)}}{\partial \xi} = \frac{\partial \varphi^{(1)}}{\partial \sigma} - \Omega_c v_z^{(1)} \quad r_{\perp 1} = r_{\varphi 1} + \frac{1}{4} \tag{16}$$

$$\frac{\partial v_z^{(1)}}{\partial \xi} = \frac{\partial \varphi^{(1)}}{\partial \eta} - \Omega_c v_y^{(1)} \quad r_{\varphi 1} = r_{\perp 1} - \frac{1}{4} \tag{17}$$

$$\varphi^{(1)} = n^{(1)} \quad r_{\varphi 1} = r_{n1}. \tag{18}$$

To balance the nonlinearity with the dispersion, we require that $r_{x1} = 1$. It follows from the last five equations that $r_{n1} = r_{\varphi 1} = r_{x1}$ and $r_{\perp 1} = \frac{5}{4}$. Note that $v_y^{(1)}$ and $v_z^{(1)}$ are the components of the $E \times B$ drift. Therefore we can also find the value of $r_{\perp 1}$ by looking at the $E \times B$ drift. At the next order we obtain

$$\frac{\partial n^{(1)}}{\partial \tau} - \frac{\partial n^{(2)}}{\partial \xi} + \frac{\partial v_x^{(2)}}{\partial \xi} + \frac{\partial v_y^{(1)}}{\partial \sigma} + \frac{\partial v_z^{(1)}}{\partial \eta} = 0 \quad r_{x2} = r_{n2}, \tag{19}$$

$$\frac{\partial v_x^{(1)}}{\partial \tau} + \frac{\partial \varphi^{(2)}}{\partial \xi} - \frac{\partial v_x^{(2)}}{\partial \xi} = 0 \quad r_{\varphi 2} = r_{\varphi 1} + \frac{1}{2}, \tag{20}$$

$$\frac{\partial v_y^{(2)}}{\partial \xi} = \frac{\partial v_y^{(1)}}{\partial \tau} + \frac{\partial \varphi^{(2)}}{\partial \sigma} - \Omega_c v_z^{(2)} \quad r_{\perp 2} = r_{\perp 1} + \frac{1}{2}, \tag{21}$$

$$\frac{\partial v_z^{(2)}}{\partial \xi} = \frac{\partial v_z^{(1)}}{\partial \tau} + \frac{\partial \varphi^{(2)}}{\partial \eta} + \Omega_c v_y^{(2)} \quad r_{\perp 2} = r_{\varphi 2} + \frac{1}{4}, \tag{22}$$

$$\frac{\partial^2 \varphi^{(1)}}{\partial \xi^2} = \varphi^{(2)} - \frac{4}{3} b {\varphi^{(1)}}^{3/2} - n^{(2)} \quad r_{\varphi 2} = r_{n2}, \tag{23}$$

where $v_y^{(1)}$ and $v_z^{(1)}$ are the components of the ion polarization drift. From equations (9), (10) and (12b) we obtain

$$n^{(1)} = \varphi^{(1)} = v_x^{(1)}. \tag{24}$$

Taking the derivative (14) with respect to ξ, and using equations (13b) and (15), we obtain from (13a)

$$2\frac{\partial n^{(1)}}{\partial \tau} + 2b{\varphi^{(1)}}^{1/2}\frac{\partial n^{(1)}}{\partial \xi} + \frac{\partial^3 n^{(1)}}{\partial \xi^3} + \frac{\partial v_y^{(1)}}{\partial \sigma} + \frac{\partial v_z^{(1)}}{\partial \eta} = 0, \tag{25}$$

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whereas equations (9) and (10) yield

$$\left(\frac{\partial^2}{\partial\xi^2}+\Omega_c^2\right)v_y^{(1)}=\frac{\partial^2\varphi^{(1)}}{\partial\sigma\partial\xi}-\Omega_c\frac{\partial\varphi^{(1)}}{\partial\eta}, \tag{26a}$$

$$\left(\frac{\partial^2}{\partial\xi^2}+\Omega_c^2\right)v_z^{(1)}=\frac{\partial^2\varphi^{(1)}}{\partial\eta\partial\xi}+\Omega_c\frac{\partial\varphi^{(1)}}{\partial\sigma}. \tag{26b}$$

Now, using equations (16) and (17), one can eliminate $v_y^{(1)}$and $v_z^{(1)}$, and obtain

$$\partial_\tau n+bn^{1/2}\partial_\xi n+\tfrac{1}{2}\partial_\xi^3 n+\tfrac{1}{2}(\partial_\xi^2+\Omega_c^2)^{-1}\partial_\xi(\partial_\sigma^2+\partial_\eta^2)n=0, \tag{27}$$

where we have set $n^{(1)}=n$. Equation (18) is the new nonlinear equation for ion-acoustic waves in weak magnetic fields with non-isothermal electrons. It should be noted here that, for the case of isothermal electrons the equation corresponding to (18) is known as the Laedke–Spatschek equation which has the same linear terms as (18) but its nonlinear term is different (Infeld and Rowlands 1990). We can see in (18) first of all that for the one-dimensional version of this equation ($\partial_\sigma=0,\partial_\eta=0$), we recover the Schamel equation as a modified form of the KdV equation. Secondly, for $\Omega_c=0$, (18) may be regarded as a weakly two-dimensional generalization of the Schamel equation, known as the mKP equation. Finally, for $\Omega_c \gg \partial_\xi^2$, an equation similar to the mZK equation (8b) arises. However, it should not be expected that (8b) and (18) have a common region of applicability since they have been derived for complementary Ω_c regions. In this case, we can transform (18) to the standard form of the mZK equation to study plane periodic and solitary wave solutions. Therefore, we choose the following scalings for variables:

$$t=8\tau \quad x=\xi \quad \sigma=\Omega_c^{-2}y, \tag{28a}$$

$$\eta=\Omega_c^{-2}z \quad u=\frac{b^2}{225}n, \tag{28b}$$

with the new scalings (19), we obtain

$$16\partial_t u+30u^{1/2}\partial_x u+\nabla^2 u_x=0, \tag{29}$$

where $\nabla^2=\partial_x^2+\partial_y^2+\partial_z^2$ is the isotropic Laplacian. In order to study solutions (20) that represent plane periodic and solitary traveling waves propagating in the direction, we introduce the wave variable $\zeta=x-\delta t$, where δ is a positive real constant. The periodic solution of (20) in this case is

$$u_0(\zeta)=r^2, \tag{30}$$

with

$$r=r_2+(r_3-r_2)\,cn^2(p\zeta/m), \tag{31a}$$

$$p^2=r_3-r_1, \quad m=\frac{r_3-r_2}{r_3-r_1}. \tag{31b}$$

Here r_1, r_2 and r_3 are the three real roots of

$$-4r^3+6\kappa r^2+2\kappa^3 A=0, \tag{32}$$

obtained from the first equation of (21) and two integrations (20) and A is an integration constant such that $-1<A<0$, $\kappa=\frac{2}{3}\delta$ and $r_1<r_2<r_3$ (Schamel 1973). The solution has a period time of $2K(m)/p$, where $K(m)$ is a complete

elliptic integral of the first kind (Abramowitz and Stegun 1972). For $m = 1$, which corresponds to $A = 0$, the solution is a solitary wave hump given by

$$u_0(\zeta) = \delta^2 \sec h^4(\delta^{1/2}\zeta). \tag{33}$$

A detailed discussion of the solutions of (20) and the investigation of the stability of its periodic and solitary wave solutions against two-dimensional long-wavelength perturbations can be found in the works of Munro and Parkes (2000, 2004) and Wazwaz (2005).

5. Conclusions

A new nonlinear equation in two dimensions has been derived for ion-acoustic waves in a plasma consisting of cold ions and non-isothermal electrons. We have shown that this equation is a modified form of the Laedke–Spatschek equation in the presence of trapped electrons and weak magnetic fields. The new equation transforms into the mKP form for $\Omega_c = 0$, and is similar to the mZK equation for strong external magnetic fields ($\Omega_c \sim 1$). It covers the most important region $\Omega_c \sim \delta$, where the transition from stable to unstable behavior occurs. With appropriate scalings, this equation may be transformed into one form of the mZK equation. In this case, exact solutions with solitons and periodic structures are obtained. It may be stressed here that the results of this investigation should be useful in understanding the nonlinear features of electrostatic disturbances in magnetized laboratory and space plasmas where positively charged ions and free and trapped electrons are the plasma species.

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