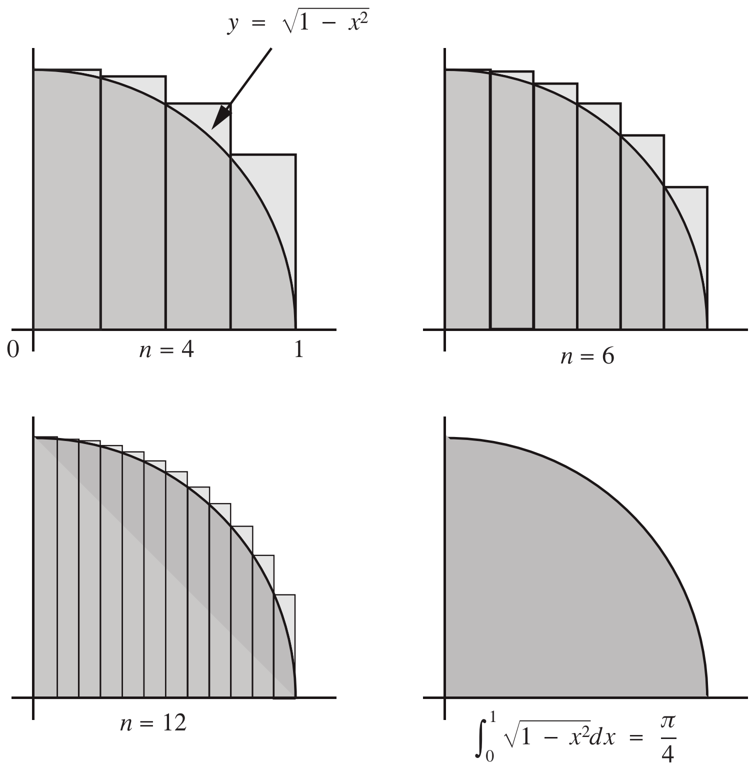
106.24 Proof without words: a Riemann sum

Claim: $\lim_{n \to \infty} \sum_{k=0}^{n-1} \frac{\sqrt{n^2 - k^2}}{n^2} = \frac{\pi}{4}$.

Proof: The limit may be seen as a Riemann sum with a geometric meaning.



$$\lim_{n \to \infty} \sum_{k=0}^{n-1} \frac{\sqrt{n^2 - k^2}}{n^2} = \lim_{n \to \infty} \sum_{k=0}^{n-1} \frac{\sqrt{1 - \left(\frac{k}{n}\right)^2}}{n} = \int_0^1 \sqrt{1 - x^2}\, dx = \frac{\pi}{4}.$$

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