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Excitation of nonlinear ion acoustic wave and stimulated Brillouin scattering of hollow Gaussian beam in relativistic plasma

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Abstract

In the present work, excitation of nonlinear ion acoustic wave (IAW) in collisionless plasma by laser beam having null intensity at the center is examined considering relativistic nonlinearity. The differential equation for beam-width parameter is determined considering relativistic nonlinearity using the paraxial and Wentzel–Kramers–Brillouin approximations by the parabolic equation method. The propagation features of the IAW are found to be modified due to the nonlinearity present in the system. The hollow Gaussian beam (HGB) gets nonlinearly coupled with the seed IAW, results in excitation of nonlinear IAW. The interaction of nonlinear IAW with pump beam demonstrated stimulated Brillouin scattering (SBS) of HGB. It is found that the power of IAW and power of SBS is affected with the order of HGB. The power of IAW and backscattered power of SBS is determined analytically and numerically for various orders of HGB. It is found that the power of IAW and the backscattering is diminished for higher order of HGB.

Keywords: Brillouin scattering; hollow Gaussian beam; nonlinear ion acoustic wave; relativistic nonlinearity

1. INTRODUCTION

The laser–plasma interaction is continuously being the thrust area of research in which the excitation of plasma modes during laser propagation is extensively investigated. The study of ion acoustic wave (IAW) is important due to its role in stimulated Brillouin scattering (SBS), decay instabilities, particle heating, and other nonlinear phenomenon in the laser–plasma interaction as well as in the space plasma. In particular, the IAW nonlinearities are considered to be responsible for the fact that the SBS of the laser light in the present day experiments develops at a much lower level than predicted by the linear theory. The experimental and theoretical studies have been widely performed about the excitation of the IAW in plasma. In the collisionless plasma, the excitation of the IAW was considered by taking two laser beams (Sodha *et al.*, 1979, 1980) and whistler waves (Sodha *et al.*, 1981; Willett, 1982). The slow ion beam caused the excitation of IAW has been reported by (Pottelette & Illiano, 1982). In excitation of the IAW is also presented by the beating of two kinetic Alfven waves (Malik *et al.*, 2007). The excitation of IAW by two microwave beams has been studied in a semiconductor plasma with and without static magnetic field by Salimullah (1981). The influence of laser-excited IAW on the nonlinear absorption and reflection in an inhomogeneous plasma layer has been studied by Tsytovich *et al.* (1973). Recently, the excitation of nonlinear IAW has been studied in CH plasmas by Feng *et al.* (2016).

All the above-mentioned theoretical analysis on IAW excitation is done using the Gaussian profile of the beams; but in recent years various spatial profile of laser beam such as super Gaussian beams (Fibich, 2007), Bessel beams (Johannisson *et al.*, 2003), elliptical Gaussian beams (Cornolti *et al.*, 1990), hollow elliptical Gaussian beams (Cai & Lin, 2004), cosh Gaussian beams (Singh & Gupta, 2016), and hollow Gaussian beams (HGBs) have been used in various laser–plasma theories and experiments. Recently, numerous works have been reported considering the hollow Gaussian laser beams (HGLBs) with central shadow. Sodha *et al.* (2009*a*) have examined the focusing of dark hollow Gaussian beams (DHGBs) in the collisionless plasma. Sodha *et al.* (2009*b*) have analyzed the focusing of a DHGB in collisionless magnetoplasma. With relativistic–ponderomotive regime focusing of DHGBs in a plasma has been observed by Misra & Mishra (2009). Gill *et al.* (2010) have studied the relativistic

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and ponderomotive effects on propagation of DHGBs in collsionless plasma. Terahertz field generation by relativistic self-focusing of HGLB in magnetoplasma is investigated by Hussain *et al.* (2015). Second-harmonic generation using the HGB in relativistic plasma has been studied by Purohit *et al.* (2016).

The nonlinear evolution of IAW would have been playing an important measure in the SBS process (Mahmoud & Sharma, 2001; Chauhan *et al.*, 2010). The SBS instability which results in efficiently coupling of large section of high-power laser energy with plasma also modifies the intensity distribution that would affect the uniformity of energy deposition (Vyas *et al.*, 2014). The experimental study of SBS did not come to an agreement with the theoretical prediction for the reason that almost all theoretical study, the researchers have considered uniform or Gaussian profile with TEM00 mode, but in the experiments the pump beams can be superposition of the higher order modes. The SBS has widely been studied both theoretically and experimentally considering uniform Gaussian beam (Froula *et al.*, 2003; Kong *et al.*, 2007). The numerical simulation of Raman and Brillouin laser-pulse amplification in the magnetized plasma has been studied by Shoucri (2016). For the precise understanding of the SBS process, the analytical work using higher order mode of the waves is essential. Studies are also being done considering different intensity shapes of pump laser beam in the theoretical model of the laser plasma interaction. Self-pumped SBS effect of high-power super-Gaussian-shaped laser pulses has been investigated by Wang *et al.* (2015). Stimulated Raman backscattering of filamented HGLB has been reported by (Singh & Sharma, 2013). Stimulated Brillouin backscattering of HGLB has been investigated by Sharma & Singh (2013). Stimulated Raman scattering considering HGLB has been studied by Sharma (2015) incorporating relativistic effects. The cross-focusing of two HGLBs have also studied considering relativistic and ponderomotive effects by Sharma (2015). Edwards *et al.* (2016) have examined amplification of short-pulse laser through coupled stimulated Brilliouin scattering. HGBs are used for various experimental work and practical purpose, as this beam is used in the study of fractional Fourier transform for HGB (Lu *et al.*, 2014), optical trapping and use of neutral particles (Ashkin, 1997), and study the laser cooling of atoms (Ping *et al.*, 2002).

In the present paper, SBS of DHGBs is investigated considering relativistic nonlinearity. The pump HGB (ω_0, $\mathbf{k}_0$) interacted with the seed IAW (ω, k) and generate a scattered wave ($\omega_s = \omega_0 - \omega$ and $\mathbf{k}_s = \mathbf{k}_0 - k$), where ω_0 and $\mathbf{k}_0$ are the frequency and wave vector of initial beam, whereas ω_s and $\mathbf{k}_s$ are the frequency and wave vector of the scattered beam. The interaction of the beam with the plasma electrons become relativistic that leads to redistribution of carriers. The dispersion relation of IAW is also significantly modified. In the case of Gaussian beams, the phase velocity of IAW becomes less at the axis and increases away from it, but in the present study, it is not the case. Since the intensity of scattered beam depends on the intensity of the pump and amplitude of IAW, so it is expected that the different orders of the pump and modified IAW should lead to modified scattering.

In Section 2, a modified equation of IAW and scattered wave is derived with the self-focusing of pump beam. Also, the expression for power of IAW has been accessed. In Section 3, the expression for total backscattered power has obtained. Section 4 contains the discussion of numerical results. Conclusions of the present work have been presented in the last section.

2. MODIFIED EQUATION FOR ION ACOUSTIC AND BRILLOUIN SCATTERED WAVES

Let us consider a hollow Gaussian pump wave of frequency ω_0 and the wave vector $\mathbf{k}_0$ is propagating in isotropic and homogeneous plasma in the z-direction, its equation can be written as

$$\nabla^2 \vec{E}_0 - \nabla(\nabla.\vec{E}_0) + \frac{\omega_0^2 \varepsilon(r,z)}{c^2}\frac{\partial^2 \vec{E}_0}{\partial t^2}. \quad (1)$$

The irradiance distribution at ($z > 0$) of the HGLB is given as

$$\mathbf{E_0}.\mathbf{E_0^*}|_{z=0} = E_{00}^2 \left\{\frac{r^2}{2r_0^2 f_0^2}\right\}^{2m} \exp\left\{-\frac{r^2}{r_0^2 f_0^2}\right\} \quad (2)$$

where the positive integer number m gives order of HGB, which symbolizes the shape of the HGB and place of its irradiance maximum, r_0 refers to the spot size of the beam, r is the radial coordinate of the cylindrical coordinate system, E_0 is a real constant describing the amplitude of the HGB, E_{00} is the complex amplitude of the beam, which denotes the electric field maximum at $r_{\max} = r_0\sqrt{2m}$, corresponding to $z = 0$ and f_0 is the dimensionless beam-width parameter satisfying the boundary conditions as

$$f_0 = 1 \text{ and } \left.\frac{df_0}{dz}\right|_{z=0} = 1. \quad (3)$$

Equation (1) signifies Gaussian beam with width r_0 for $m = 0$. Further, in the case of Gaussian beams, all the parameters are expanded around central point, that is, $r = 0$ at which the intensity is maximum. Though, for HGB, the irradiance is not maximum at the midpoint. Hence, it is good to have a point (alongside the radial distance of the beam) at which all the power of the beam is thought to be focused, let it be $r = r_0 f_0(z)\sqrt{2m}$, which certainly justified in the paraxial-like approximation. We will expand all the related parameters about this point and describe this point η as $\eta = [(r/r_0 f_0) - \sqrt{2m}]$ Here, the limit $\eta \ll \sqrt{2m}$ is suitable like the case of paraxial theory. By means of the conversion and following (Sharma, 2015) one can obtain,

$$\frac{\partial^2 f_0}{\partial \xi^2} = \frac{4}{f_0^3} - \frac{R_{d0}^2 \omega_p^2 \alpha E_0^2}{\omega_0^2 k_0^2 \varepsilon_0 f_0^3} m^{2m} \exp(-2m) \left[1 + \left(\frac{\alpha E_0^2}{f_0^2} m^{2m} \exp(-2m)\right)\right]^{-3/2}, \quad (4)$$

where $R_{d0} = \mathbf{k}_0 r_0^2$ is the dimensionless initial beam width, $\xi = z/R_{d0}$ is the dimensionless distance of propagation, the dielectric constant is εdefined as

$$\varepsilon = 1 - \frac{\omega_p^2}{\gamma \omega_0^2}, \quad (5)$$

where the relativistic factor γ is given by,

$$\gamma = \left[1 + \frac{e^2}{c^2 m_0^2 \omega_0^2} \mathbf{E_0}.\mathbf{E_0^*}\right]^{1/2}. \quad (6)$$

This dielectric constant may be expanded around the maximum of the HGLB. Hence,

$$\varepsilon(\eta, z) = \varepsilon_0(z) + \phi(E.E^*), \quad (7)$$

where ε_0 (z) is linear dielectric constant given by Eq. (5) and $\phi(E.E^*)$ is nonlinear dielectric constant given as

$$\phi(E.E^*) = \frac{\omega_p^2}{\omega_0^2}\left(1 - \frac{1}{\{1 + (e^2/c^2 m_0^2 \omega_0^2) E_0.E_0^*\}}\right). \quad (8)$$

Moreover, expanding the effective dielectric function using Taylor series around the irradiance maximum $\eta = 0$ of the HGB as

$$\varepsilon(\eta, z) = \varepsilon_f(\eta = 0) + \eta^2 \varepsilon_2(\eta = 0), \quad (9)$$

where ε_2 is defined as $\partial\varepsilon/\partial\eta^2$. To derive the value of the coefficients ε_f and ε_2 given in Eq. (9). Following the paraxial-like approximation one can expand the dielectric function in axial and radial parts around the maximum of the HGB. Thus, from the set of Eqs. (6–9), one finds

$$\varepsilon_f(\eta = 0) = 1 - \frac{\omega_{p0}^2}{\omega_0^2}\left[1 + \frac{\alpha E_0^2}{f_0^2} m^{2m} \exp(-2m)\right]^{-1/2}, \quad (10)$$

$$\varepsilon_2(\eta = 0) = \frac{\omega_{p0}^2}{\omega_0^2} \frac{\alpha E_0^2}{f_0^2} m^{2m} \exp(-2m)\left[1 + \frac{\alpha E_0^2}{f_0^2} m^{2m} \exp(-2m)\right]^{-3/2}. \quad (11)$$

Further, the propagation of the laser beam through the plasma leads the variation of density of the plasma channel. The seed IAW exists in the plasma nonlinearly interacts with the propagating laser beam and as a result the nonlinear IAW gets excited. The excitation process of IAW in the presence of relativistic nonlinearity can be described as follows.

The general equation governing the ion density variation

$$\frac{\partial^2 n_{is}}{\partial t^2} + 2\Gamma_i \frac{\partial n_{is}}{\partial t} - \gamma' v_{thi}^2 \nabla^2 n_{is} + \frac{\omega_{pi}^2}{\gamma} \frac{k^2 \lambda_d^2}{1 + k^2 \lambda_d^2} n_{is} = 0, \quad (12)$$

where $v_{thi} = (k_\beta T_i/M)^{1/2}$ is the ion thermal speed, $\lambda_d = (k_B T_0/4\pi n_0 e^2)^{1/2}$ the Debye length. In Eq. (12), the Landau damping coefficient is

$$2\Gamma_i = \frac{k}{(1 + k^2\lambda_d^2)}\left(\frac{\pi k_B T_e}{8 m_i}\right)^{1/2} \times \left[\left(\frac{m}{m_i}\right)^{1/2} + \left(\frac{T_e}{T_i}\right)^{3/2} e^{-\left\{\frac{T_e}{T_i(1 + k^2\lambda_d^2)}\right\}}\right].$$

The solution of the above equation can be written as $n_{is} = n(r, z)\exp\{i[\omega t - k(z + s(r, z))]\}$, here "$s$" is the eikonal for IAW. The frequency and wave number of the IAW satisfy the following dispersion relation in the presence of laser beam $\omega^2 = k^2 c_s^2/1 + k^2\lambda_d^2(1/\gamma)^{-1}$. Substituting the value of n_{is} into Eq. (12) and separating the real and imaginary parts, one obtains, the real part as

$$2\left(\frac{\partial s}{\partial z}\right) + \left(\frac{\partial s}{\partial \eta}\right)^2 = \frac{1}{nk^2}\left[\frac{1}{(\sqrt{2m} + \eta)} \frac{\partial n}{\partial \eta} + \frac{\partial^2 n}{\partial \eta^2}\right] + \eta^2 \frac{\omega^2}{k^2 v_{thi}^2}\left[1 - \frac{1}{\gamma 1 + k^2\lambda_d^2}\right] \quad (13)$$

and imaginary part as

$$\frac{\partial n^2}{\partial z} + \left(\frac{1}{(\sqrt{2m} + \eta)} \frac{\partial s}{\partial \eta} + \frac{\partial^2 s}{\partial \eta^2}\right) n^2 + \frac{\partial n^2}{\partial r}\frac{\partial s}{\partial r} + \frac{2\Gamma_i}{v_{thi}^2}\frac{\omega n^2}{k} = 0. \quad (14)$$

To deal the HGB the transformation of (r, z) coordinate into (η, z) coordinate is required.

Using paraxial-like approximation Eqs (13) and (14) can be solved and for that we assume the initial radial variation of density perturbation and the solution as

$$n^2 = \frac{N_{e00}^2}{2^{2m} f_i^2}(\sqrt{2m} + \eta)^{4m}\left(\frac{r_0 f_0}{a f_i}\right)^{4m} \exp\left[-\left(\frac{r_0 f_0}{a f_i}\right)^2 (\sqrt{2m} + \eta)^2\right] \exp[-2k_i(z)], \quad (15)$$

$$s(\eta, z) = \frac{(\sqrt{2m} + \eta)^2}{2} r_0^2 f_0^2 \beta(z) + \phi(z), \quad \beta(z) = \frac{1}{f_i}\frac{df_i}{dz}, \quad (16)$$

where a is the initial beam width and f_i is a dimensionless parameter of the IAW, respectively, $k_i = 2\Gamma_e \omega / k v_{th}^2$ is the damping factor and N_{e00}^2 is the magnitude of the excited ion wave. Moreover, we have used the boundary conditions (for an initially plane wave front): at $df_i/dz = 0, f_i = 1, s = 0$ and $z = 0$. Substituting, Eq. (15 and 16) in Eq. (13) and equating the coefficients of η^2 on both sides, we get the equation for f_i as

$$\frac{\partial^2 f_i}{\partial \xi^2} = \frac{R_{d0}^2 f_i}{f_0^2}\left(\frac{1}{k^2 r_0^4 f_0^2}\left\{3 + \left(\frac{r_0 f_0}{a f_i}\right)^4\right\}\right) - \frac{R_{d0}^2 f_i \omega_p^4 k^2 \lambda_d^2}{2\omega_0^2 f_0^2 k^2 v_{th}^2 (1 + k^2\lambda_d^2)^2}\left[1 + \left(\frac{\alpha E_0^2}{f_0^2} m^{2m} e^{-2m}\right)\right]^{-3/2}, \tag{17}$$

where $R_{d0} = ka^2$ is the diffraction length of the IAW. When the coupling between propagating HGB and IAW is taken into consideration, Eq. (17) signifies the profile of IAW under the influence of relativistic nonlinearity.

Excitation of IAW, used in the laser-heating experiments is governed by Eq. (17), where the last term represents the coupling of IAW to the modified background density. Excitation of IAW is significantly affected by the order of hollow Gaussian electromagnetic beam, so the power of IAW is will also be affected by the same. The total power of IAW in the entire plane at any point on the z-axis may be given

$$P_I = \frac{c}{8\pi}\int_0^\infty \varepsilon^{1/2} E.E^* 2\pi r dr, \tag{18}$$

$$P_I = \frac{c\varepsilon^{1/2}}{8}\frac{(4\pi e N_{e00})^2 r_0^2 f_0^2 |2n}{2^{2n} f^2}\left(\frac{r_0^2 f_0^2}{a^2 f^2}\right)^{2n} \exp\left(-\frac{r_0^2 f_0^2}{a^2 f^2} - 2k_i z\right) \tag{19}$$

The phenomena of Brillouin scattering of a HGB in plasma with relativistic nonlinearity is to be studied. In this direction, we have studied excitation process of IAW, as the scattering process depends on the amplitude of the electrostatic mode generated. So here we present the process of SBS through the nonlinear IAW. For this purpose, we assume that the high-electric field $(E_0 + E_s)$, satisfies the wave equation

$$\nabla^2(E_0 + E_s) - \nabla\{\nabla.(E_0 + E_s)\} = \frac{1}{c^2}\frac{\partial^2(E_0 + E_s)}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial \mathbf{J}_T}{\partial t}. \tag{20}$$

The current density vector $\mathbf{J}_T$ corresponding to high-frequency electric field has two components, one oscillating at the pump wave frequency ω_0 and the other at the frequency of scattered wave ω_s. Consequently the scattered wave satisfies the equation

$$\nabla^2 E_s + \frac{\omega_s^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega_s^2 \gamma}\right)E_s = \frac{\omega_p^2}{2c^2}\frac{\omega_s}{\omega_0}\frac{n_s^*}{N_0}E_0. \tag{21}$$

Let the solution of Eq. (21) may be written as

$$E_s = E_{s0}(r, z)e^{ik_{s0}z} + E_{s1}(r, z)e^{-ik_{s1}z} \text{ with } E_{s0} = E_{s00}e^{ik_{s0}s_z}, \tag{22}$$

where $k_{s0}^2 = \omega_s^2/c^2(1 - \omega_p^2/\omega_s^2) = (\omega_s^2/c^2)\varepsilon_{s0}^2$ and k_{s1} and ω_s follows $\omega_s = \omega_0 - \omega$, and $k_{s1} = k_0 - k$. Substituting Eq. (22) into (21), one obtains

$$\left(\frac{\partial^2 E_{s0}}{\partial r^2} + \frac{1}{r}\frac{\partial E_{s0}}{\partial r}\right) + 2ik_{s0}\frac{\partial E_{s0}}{\partial z} - k_{s0}^2 E_{s0} + \frac{\omega_s^2}{c^2}\left[\varepsilon_{s0} + \frac{\omega_p^2}{\omega_s^2}\left(1 - \frac{1}{\gamma}\right)\right]E_{s0} = 0, \tag{23}$$

$$\left(\frac{\partial^2 E_{s1}}{\partial r^2} + \frac{1}{r}\frac{\partial E_{s1}}{\partial r}\right) - 2ik_{s1}\frac{\partial E_{s1}}{\partial z} - k_{s1}^2 E_{s1} + \frac{\omega_s^2}{c^2}\left[\varepsilon_{s0} + \frac{\omega_p^2}{\omega_s^2}\left(1 - \frac{1}{\gamma}\right)\right]E_{s1} = \frac{\omega_p^2}{2c^2}\frac{\omega_s}{\omega_0}\frac{n^* E_0}{N_0}e^{-ik_0 s_0}. \tag{24}$$

Substituting $E_{s0} = E_{s00}\ (r, z)e^{iks_c}$ into Eq. (23) and separating the real and imaginary parts, we get two equations as

$$2\frac{\partial s_c}{\partial z} + \left(\frac{\partial s_c}{\partial r}\right)^2 = \frac{1}{k_{s0}^2 E_{s00}}\left\{\frac{\partial^2 E_{00}}{\partial r^2} + \frac{1}{r}\frac{\partial E_{s00}}{\partial r}\right\} + \frac{\omega_p^2}{\varepsilon_{s0}\omega_s^2}\left\{1 - \frac{1}{\gamma}\right\}, \tag{25}$$

$$\frac{\partial E_{s00}^2}{\partial z} + \frac{\partial s_c}{\partial r}\frac{\partial E_{s00}^2}{\partial r} + E_{s00}^2\left(\frac{1}{r}\frac{\partial s_c}{\partial r} + \frac{\partial^2 s_c}{\partial r^2}\right) = 0. \tag{26}$$

Solution of Eqs (25) and (26) can be written as

$$E_{s00}^2 = \frac{B^2}{f_s^2 2^{2n}}(\eta + \sqrt{2n})^{4n}\left(\frac{r_0 f_0}{b f_s}\right)^{4n}\exp\left\{-\left(\frac{r_0 f_0}{b f_s}\right)^2(\eta + \sqrt{2n})^2\right\}, \tag{27}$$

$$s_c = \frac{(\eta + \sqrt{2n})^2}{2}\frac{r_0^2 f_0^2}{f_s}\frac{df_s}{dz} + \phi_s(z). \tag{28}$$

Using Eqs (27) and (28) in Eq. (25) and equating η^2 coefficients on both sides, one gets

$$\frac{d^2 f_s}{d\xi^2} = \frac{f_s \rho_0^2}{f_0^2}\left\{\frac{1}{k^2 r_0^2 f_0^2}\left[3 + \left(\frac{r_0 f_0}{b f_s}\right)^4\right]\right\} - \frac{f_s \rho_0^2 \omega_p^2}{f_0^2 \omega_s^2 \varepsilon_{s0}}\left\{\frac{\alpha E_0^2}{f_0^2 \gamma^{\frac{3}{2}}} n^{2n} e^{-2n}\right\}, \tag{29}$$

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where b is a radius of scattered beam and f_s is the dimensionless parameter of the backscattered beam and B is the constant.

3. EXPRESSION OF BACKSCATTERED POWER

It can be clearly seen that IAW is modified as it moves along the z-axis. Consequently, the scattered wave amplitude should also modify with z. Thus, expression for B' and b may be obtained on applying suitable boundary conditions $E_s = E_{s0}\exp(ik_{s0} z) + E_{s1}\exp(-ik_{s1} z) = 0$ *at* $z = z_c$; here z_c is the place at which the amplitude of scattered wave is zero. This gives

$$B = \frac{\omega_P^2 \omega_s n_0}{2c^2 \omega_0 N_0} \frac{f_s}{f} \left(\frac{bf_s}{af}\right)^{2n} \frac{\begin{array}{c} E_{00} e^{i(\omega t - k(z+S(r,z)))} \exp[-(r_0 f_0/af)^2 \\ (\eta + \sqrt{2n})^2 - 2ik_i z/2] \end{array}}{\begin{array}{c} \exp[-(r_0 f_0/bf)^2 (\eta + \sqrt{2n})^2/2] \\ [k_{s1}^2 - k_{s0}^2 - \omega_P^2/c^2(1 - 1/\gamma)] \end{array}} \frac{e^{-i(k_0 s_0 + k_{s1} z_c)}}{e^{i(k_{s0} s_c + k_{s0} z_c)}} \tag{30}$$

and

$$\frac{1}{b^2 f_s^2(z_c)} = \frac{1}{r_0^2 f_0^2(z_c)} + \frac{1}{a^2 f^2(z_c)},$$

which implies that at $z = 0$

$$\frac{1}{b^2} = \frac{1}{r_0^2} + \frac{1}{a^2} \tag{31}$$

the above condition is obtained on using the following boundary conditions at $z = 0$:

$$f_s, f_0, f = 1 \text{ and } \frac{df_0}{\partial z}, \frac{df_s}{\partial z}, \frac{df}{\partial z} = 1. \tag{32}$$

The total scattered power at any plane $z =$ constant may be given

$$P_s = \frac{c\varepsilon_{s0}^{1/2}}{8\pi} \int_0^\infty E_s . E_s^* 2\pi r dr. \tag{33}$$

The value of $E_s.E_s^*$ is calculated as

$$\begin{aligned} E_s.E_s^* &= E_{s00}^2 \exp(2ik_{s0}(S_c + z)) \\ &\quad + 2E_{s0}E_{s1} \exp(ik_{s0} z) \exp(-ik_{s1} z) \\ &\quad + \frac{1}{4}\left(\frac{\omega_p^2 n_{es}^* \omega_s}{c^2 N_0 \omega_0}\right)^2 \left(\frac{r_0 f_0}{af}\right)^{4n} \\ &\quad \exp\left(-\left(\frac{r_0 f_0}{af}\right)^2 (\sqrt{2n} + \eta)^2 - 2k_i z\right) \\ &\quad \left(\frac{E_{00} e^{i[\omega t - k(z+S)]}}{\left[k_{s1}^2 - k_{s0}^2 - \omega_p^2/c^2(1 - 1/\gamma_0)\right]}\right)^2. \end{aligned}$$

The total time averaged backscattered power is

$$\begin{aligned} P_s = \frac{c\varepsilon_{s0}^{1/2} r_0^2 f_0^2 |2n}{8} \Bigg[& \frac{B'^2}{2^{2n} f_s^2} \left(\frac{r_0 f_0}{bf_s}\right)^{4n} \\ & \exp\left(-\left(\frac{r_0 f_0}{bf_s}\right)^2\right) e^{2ik_{s0}(s_c + z)} \\ & - \frac{B' \omega_p^2 n_{es}^* \omega_c}{2^n f_s c^2 N_0 \omega_0} \left(\frac{r_0 f_0}{bf_s}\right)^{2n} \\ & \frac{E_{00} e^{i[\omega t - k(z+s)]}}{\left[k_{s1}^2 - k_{s0}^2 - \omega_p^2/c^2(1 - 1/\gamma_0)\right]} \left(\frac{r_0 f_0}{af}\right)^{2n} \\ & \times \exp\left(-\frac{1}{2}\left(\frac{r_0^2 f_0^2}{b^2 f_s^2} + \frac{r_0^2 f_0^2}{a^2 f^2}\right) - 2k_i z\right) \\ & e^{ik_{s0}(s_c + z) - ik_0 s_0 - ik_{s1} z} + \frac{1}{4}\left(\frac{\omega_p^2 n_{es}^* \omega_c}{c^2 N_0 \omega_0}\right)^2 \\ & \times \left[\frac{E_{00} e^{i[\omega t - k(z+s)]}}{k_{s1}^2 - k_{s0}^2 - \omega_p^2/c^2(1 - 1/\gamma_0)}\right]^2 \\ & \left(\frac{r_0 f_0}{af}\right)^{4n} \exp\left(-\left(\frac{r_0 f_0}{af}\right)^2 - 2k_i z\right) e^{2ik_{s0}(s_c + z)} \Bigg]. \end{aligned} \tag{34}$$

The pump-wave power at $z = 0$ is defined as ratio the of P_s-to-P_0 is calculated and can be formulated as $P = P_s/P_0$, where $P_0 = (c/8\pi)\pi . r_0^2 . E_0 . E_0^*$. The ratio calculated gives the value 4.2×10^4.

4. NUMERICAL RESULTS AND DISCUSSION

The results of the present analysis may be appreciated through the numerical computation of the backscattered power and the variation of the beam-width parameter f with dimensionless distance of propagation ξ, for a chosen set of parameters for relativistic nonlinearity. The computations have also been made to find the dependence of the beam-width parameter associated with the propagation of the HGB (for various values of densities) on the dimensionless distance of propagation ξ in homogeneous plasmas. Variation in ion acoustic density profile and IAW power with normalized distance and radial distance has also been observed for different orders of HGB. Numerical results are presented for the beam-width parameter of backscattered wave as well as the backscattered power has also been estimated. In conclusion, we have derived the focusing equation for HGLB (considering relativistic nonlinearity) in the cylindrical coordinate system. It is good to observe the nonlinear excitation of IAW and corresponding backscattering of the pump HGLB.

The differential equation for the beam-width parameters are solved by the Taylor series technique employing the boundary conditions given by Eq. (31). The other parameters are chosen as laser beam Nd: YAG of wavelength 1064 nm with laser intensity 10^{22} W/cm^2, electron density $n_0 = 10^{19}$ cm^{-3}, $r_0 = 15$μm, $\omega_0 = 1.778 \times 10^{15}$rad/s, $n_0/N_{00} = 0.01$ $a = 10$ μm and $\omega_p^2/\omega_0^2 = 0.2$. Figure 1 describes the

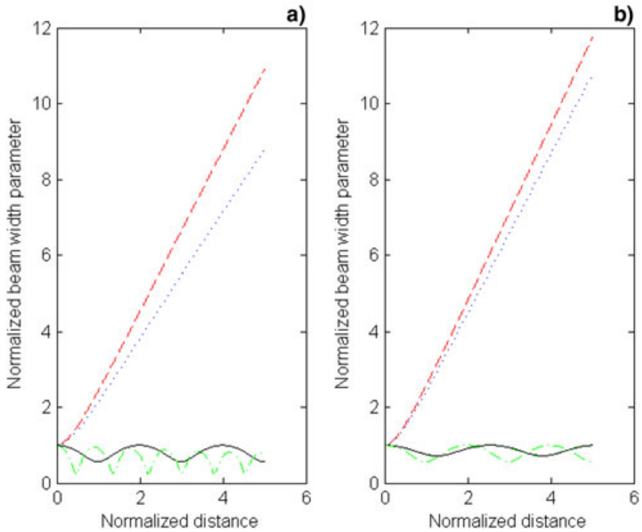

Fig. 1. Variation in HGLB beam-width parameter with normalized distance and radial distance (r): (a) for $\omega_p^2/\omega_0^2 = 0.2$ 2 laser power $\alpha E_{00}^2 = 2.5$, (b) for $\omega_p^2/\omega_0^2 = 0.4$ laser power $\alpha E_{00}^2 = 3.4$

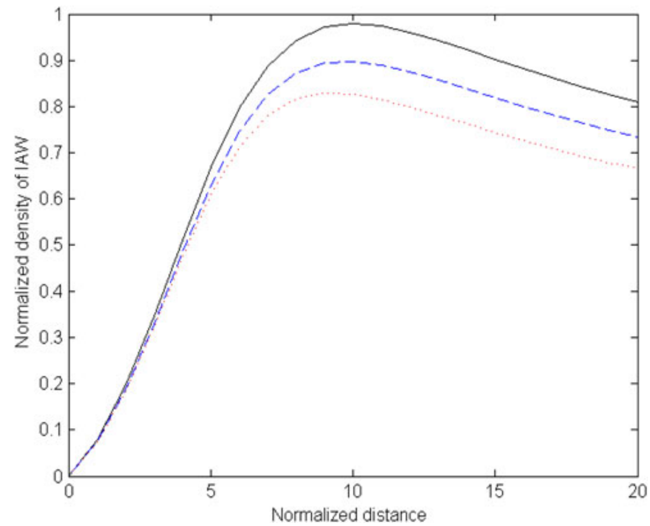

Fig. 3. Variation in normalized power of IAW with normalized distance when relativistic nonlinearity is operative for different orders of propagation of HGLB: (a) black solid line is for $m = 1$, (b) $m = 2$, (c) $m = 3$, (d) $m = 4$.

dependence of the beam-width parameter f on the dimensionless distance of propagation ξ for plasma with dominant relativistic nonlinearity. It denotes the influence of plasma density on the self-focusing beam (HGB) in nonlinear unmagnetized plasma. It displays the effect of order of the beam on self-focusing for various values plasma density, it can clearly be seen that for the same order of the beam focuses fast for the lower densities in comparison to higher density. The black color solid curve shows the zero order of HGB, the red color dashed line shows the first order of HGB, the blue color dotted line shows the second order of HGB and the green color dashed curve shows the third order of HGB, respectively. On comparing the plot in Figure 1a, and 1b we can conclude that on increasing plasma density the beam focused lately. However, the focusing of different orders of the beam considering relativistic nonlinearity can also be noted by the results as self-focusing of the laser beam gets decreases when the order of the beam increases. The set of Figure 2 reveals the dependence of the normalized intensity of IAW on normalized distance of propagation ξ in the presence of relativistic nonlinearity. In Figure 2a, the order of HGB is zero where the intensity of IAW is 10^{-21}, in Figure 2b the intensity gets increased, while the order of HGB increases. Thus, it is clear from the sets of graphs, the intensity increases with increase in order of HGB. However, Figure 3 shows the power of excited IAW with the order of HGB and it clearly shows that as the order of HGB increases the power content with the corresponding IAW increases. The set of Figure 4 shows the normalized intensity of backscattered wave against the normalized distance of propagation ξ. In Figure 4a, the order of HGB is $m = 0$, where the scattering

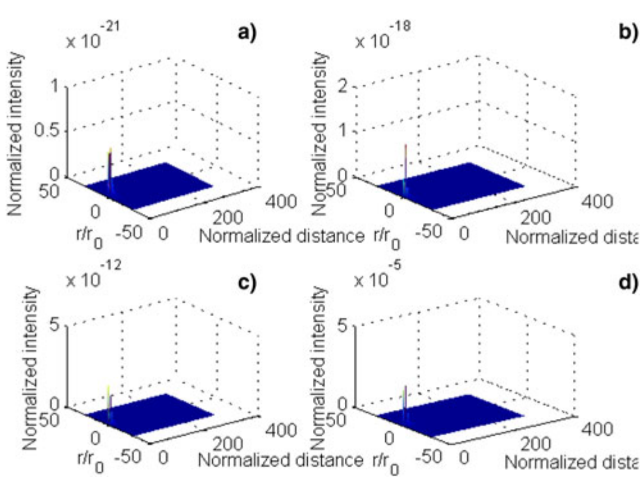

Fig. 2. Variation in normalized ion acoustic intensity profile with normalized distance and radial distance (r). (a) $m = 0$, (b) $m = 1$, (c) $m = 2$, (d) $m = 4$.

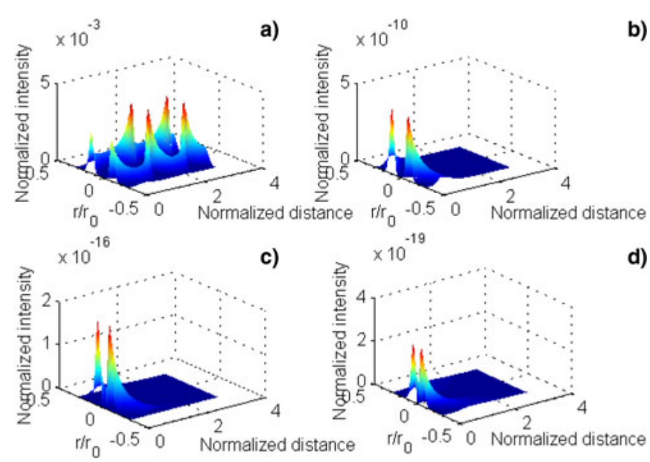

Fig. 4. Variation in scattered beam intensity with normalized distance and radial distance (r): (a) $m = 0$, (b) $m = 1$, (c) $m = 2$, (d) $m = 4$.

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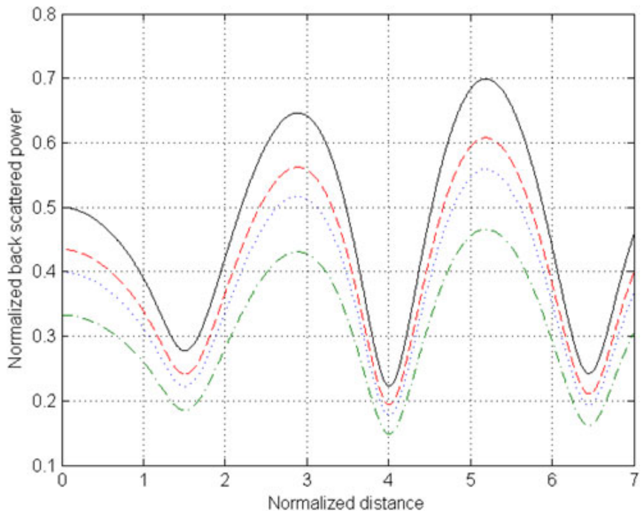

Fig. 5. Variation in normalized backscattered power of SBS, with normalized distance, (a) for, (b) for.

intensity is 10^{-3}, while in comparison with Figure 4d the order of HGB is $m = 3$, and the scattering intensity is 10^{-19}, which displays that the scattering intensity of hollow Gaussian wave decreases on increasing the order of HGB. Figure 5 illustrates the backscattered power of HGB against the normalized distance. Here, the black color solid curve is of zero order, the red color dashed curve is of first order, the blue color dotted curve is of second order, and the green color dash-dotted curve is of third order of HGB. For the maximum order the backscattered power is lower than the minimum order of HGB. In conclusion, self-focusing of HGB in collisionless, unmagnetized plasma has been studied along with the nonlinear excitation of IAW in the presence of relativistic nonlinearity. Excitation of IAW leads to stimulation of Brillouin scattering in the relativistic regime. The power of IAW and backscattered power of SBS has also been analyzed in Sections 2 and 3. The dependence of power of IAW and scattered power of order of HGB has been shown numerically and analytically. The ratio of scattered power to the initial power gives value 4.2×10^4, which is the important outcome of the study.

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