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Collisional effects on the electrostatic dust cyclotron instability

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A kinetic analysis of the electrostatic dust cyclotron instability in a weakly ionized collisional dusty plasma is presented. In a plasma with negatively charged dust and a current along the magnetic field **B**, it is found that the instability can be excited by ions drifting along **B**. The effect of dust–neutral collisions is stabilizing, while the effect of ion–neutral collisions can be destabilizing. Possible applications to laboratory environments are discussed.

1. Introduction

The electrostatic ion cyclotron (EIC) instability is a low-frequency field-aligned current-driven instability that has one of the lowest threshold drift velocities among current-driven instabilities (Rasmussen 1988). The EIC wave has a frequency of the order of the ion cyclotron frequency Ω_i, propagates nearly perpendicular to the magnetic field **B**, and has a small but finite wavenumber along **B**, so that it can be destabilized by electrons drifting along **B** (see e.g. Rasmussen and Schrittwieser 1991). The effect of charged dust grains in a plasma on the behaviour of the EIC instability has been investigated both theoretically (Chow and Rosenberg 1995, 1996a) and experimentally (Barkan et al. 1995). Both theory and experiment show that as more negative charge is carried by dust, frequencies and growth rates increase. Estimates of the critical drift show that the instability is easier to excite with higher concentration of negatively charged dust (Chow and Rosenberg 1996a). Moreover, the effect of electron–neutral and ion–neutral collisions on the EIC in dusty plasmas has recently been studied by Bharuthram et al. (1998).

In a plasma with two species of ions, such as a negative-ion plasma, there can be an EIC wave associated with each ion component (see e.g. Song et al. 1989; Chow and Rosenberg 1996b). Similarly, in a dusty plasma, there can be electrostatic dust cyclotron (EDC) waves associated with the charged dust. These waves have a very low frequency: of the order of the dust cyclotron frequency $\Omega_d = Z_d eB/m_d c$, which is smaller than Ω_i by a factor $(Z_d/m_d)\, m_i/Z_i$. (The charge-to-mass ratio of dust is typically orders of magnitude smaller than that of the ions. For example, the charge state of a 1 μm sized dust grain in a 1 eV Ar plasma, Z^d, is of the order of $Z_d \approx 10^3$, while its mass m_d is of the order of $\gtrsim 10^{12}$ times the proton mass.) In a collisional dusty plasma in the presence of an external electric field $\mathbf{E}_0 \parallel \mathbf{B}$, for example, electrons and ions drift along **B** relative to the massive dust (see e.g. D'Angelo and Merlino 1996), and this may lead to an EDC instability (D'Angelo 1998).

For the EIC instability, the critical drift is obtained by balancing growth arising from electron drift along **B** with damping due to parallel electron Landau damping and ion cyclotron damping. However, for the EDC instability, the analogous collisionless damping effects may be so small that collisional damping effects may become more important. Since the dust is very massive, the dust thermal speed v_d can be orders of magnitude smaller than the ion thermal speed v_i. In addition, the wave phase speed along **B** can be orders of magnitude smaller than both the electron and ion thermal speeds v_e and v_i (corresponding to small Landau damping), while being much larger than the dust thermal speed v_d (corresponding to small dust cyclotron damping). Therefore, we investigate a collisional EDC instability using a kinetic analysis. We note that a fluid analysis of the EDC instability in a collisional dusty plasma, in the regime where the wavelength parallel to **B** is larger than the electron or ion mean free path, is given in D'Angelo (1998). The analysis is given in Sec. 2, and both analytical and numerical results are given in Sec. 3. Section 4 gives a discussion of possible applications.

2. Analysis

We consider a weakly ionized plasma composed of negatively charged dust, electrons and singly charged positive ions, in a neutral background. The dust particles for simplicity are assumed to have uniform mass m_d and charge state Z_d (and thus uniform radius a). The plasma is homogeneous, with a magnetic field **B** in the positive z direction, and there is a constant current along **B**. We consider the frame where the dust is stationary, so that the electrons and ions that comprise the current drift in opposite directions along **B** relative to the dust. We assume that the ions (electrons) drift in the positive (negative) z direction.

We use drifting Maxwellian distributions for the electrons and ions given by

$$f_{0j} = n_j\left(\frac{m_j}{2\pi T_j}\right)^{3/2} \exp\left\{-\frac{1}{v_j^2}[v_x^2+v_y^2+(v_z\hat{\mathbf{z}}-\mathbf{u}_{0j})^2]\right\}, \tag{1}$$

where $v_j = (T_j/m_j)^{1/2}$ is the thermal speed of species $j = e$ and i, and a Maxwellian distribution with temperature T_d for the dust. The condition of overall charge neutrality is given by

$$n_e + Z_d n_d = n_i, \tag{2}$$

where n_α is the density of charged species α ($\alpha = e$, i and d for electrons, ions and dust). This can also be written as

$$\frac{Z_d n_d}{n_e} = \delta - 1, \tag{3}$$

where $\delta = n_i/n_e$, which gives charge density carried by the dust.

The dispersion relation for electrostatic waves with $E_1 \sim \exp(i\mathbf{k}\cdot\mathbf{x}-\omega t)$, frequency $\omega \sim \Omega_d \ll \Omega_i$, and wavevector components $k_\perp$ and k_z, which are perpendicular and parallel to **B**, taking into account collisions with neutrals via a Krook-type collisions model, is given by (see e.g. Kindel and Kennel 1971; Satyanarayana et al. 1985) is

$$D(\omega, k) = 1 + \sum_\alpha \chi_\alpha = 0, \tag{4}$$

where

$$\chi_e = \frac{k_{De}^2}{k^2}[1+\zeta_e\,\Gamma_0(\mu_e)\,Z(\zeta_e)]\left[1+\frac{i\nu_e}{\sqrt{2}k_z v_e}\Gamma_0(\mu_e)\,Z(\zeta_e)\right]^{-1}, \tag{5}$$

$$\chi_i = \frac{k_{Di}^2}{k^2}[1+\zeta_i\,\Gamma_0(\mu_i)\,Z(\zeta_i)]\left[1+\frac{i\nu_i}{\sqrt{2}k_z v_i}\Gamma_0(\mu_i)\,Z(\zeta_i)\right]^{-1}, \tag{6}$$

$$\chi_d = \frac{k_{Dd}^2}{k^2}\left[1+\frac{\omega+i\nu_d}{\sqrt{2}k_z v_d}\sum_{m=-\infty}^{\infty}\Gamma_m(\mu_d)\,Z(\zeta_d)\right]\left[1+\frac{i\nu_d}{\sqrt{2}k_z v_d}\sum_{m=-\infty}^{\infty}\Gamma_m(\mu_d)\,Z(\zeta_d)\right]^{-1}. \tag{7}$$

Here

$$\zeta_e = \frac{\omega+k_z u_{0e}+i\nu_e}{\sqrt{2}k_z v_e}, \tag{8}$$

$$\zeta_i = \frac{\omega-k_z u_{0i}+i\nu_i}{\sqrt{2}k_z v_i}, \tag{9}$$

$$\zeta_d = \frac{\omega-m\Omega_d+i\nu_d}{\sqrt{2}k_z v_d}, \tag{10}$$

and $\mu_\alpha = k_\perp^2\rho_\alpha^2$, with $\rho_\alpha = v_\alpha/\Omega_\alpha$ being the gyroradius of each charged species, $\alpha = e$, i and d denoting electrons,, ions and dust. In the above equations, $\Gamma_m(x) = I_m(x)\exp(-x)$, where I_m is the modified Bessel function of order m, $Z(\zeta)$ is the plasma dispersion function (Fried and Conte 1961), u_{0e} and u_{0i} are the electron and ion drift speeds, and $k_{D\alpha} = (4\pi n_\alpha Z_\alpha^2 e^2/T_\alpha)^{1/2}$. Here ν_α is the collision frequency for each charged species, which we assume to be due primarily to collisions with neutrals.

Assuming the wave is either weakly damped or growing, the solution for the real part of the frequency ω_r is given by

$$D_r(\omega_r, k) = 0, \tag{11}$$

where D_r is the real part of (4). The imaginary part of the frequency is then given by (see e.g. Krall and Trivelpiece 1973)

$$\gamma = -\frac{D_i(\omega_r, k)}{\partial D_r(\omega_r, k)/\partial\omega_r}, \tag{12}$$

where D_i is the imaginary part of (4).

3. Results

3.1. Analytical results

Making some simplifying assumptions, it is possible to solve (4) analytically in certain regimes. For the EDC instability, we are interested in the phase-velocity regime $\zeta_d \gg 1$ where dust cyclotron damping is small. We give analytical results for several different regimes for the electrons and ions. Initially, we consider a kinetic analysis for the collisionless case, in order to compare with the subsequent collisional results. Then we consider the effects of collisions using a kinetic analysis for two cases. First, we consider the case in which the wavelength component along **B** ($\lambda_z \sim 1/k_z$) is much smaller than the electron and ion mean free paths, so that collisional effects are relatively small for these latter species. Secondly, we consider the regime where the wavelength

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component along **B** is much larger than the electron and ion mean free paths, where collisional effects are strong for these species.

3.1.1. Collisionless case. We first consider the case where $\nu_\alpha = 0$, in order to compare with the collisional cases to be discussed below. In the regime $\zeta_e \ll 1$, $\zeta_i \ll 1$, and assuming μ_e, $\mu_i \ll 1$, we have

$$\chi_e \approx \frac{k_{De}^2}{k^2}\left(1 + i\sqrt{\pi}\frac{\omega + k_z u_{0e}}{\sqrt{2}k_z v_e}\right), \tag{13}$$

$$\chi_i \approx \frac{k_{Di}^2}{k^2}\left(1 + i\sqrt{\pi}\frac{\omega - k_z u_{0i}}{\sqrt{2}k_z v_i}\right). \tag{14}$$

In the phase-velocity regime $\zeta_d \gg 1$, and retaining only the $m = 0$ and 1 terms for the dust, since $\omega \sim \Omega_d$, we have

$$\chi_d \approx \frac{k_{Dd}^2}{k^2}\left\{1 - \Gamma_0(\mu_d) - \Gamma_1(\mu_d)\frac{\omega}{\omega - \Omega_d} + i\sqrt{\pi}\Gamma_1(\mu_d)\frac{\omega}{\sqrt{2}k_z v_d}\exp\left[-\left(\frac{\omega - \Omega_d}{\sqrt{2}k_z v_d}\right)^2\right]\right\}. \tag{15}$$

Solving (4) for the real, ω_r, and imaginary, γ, parts of the frequency gives

$$\omega_r \approx \Omega_d(1 + \Delta), \tag{16}$$

$$\Delta \approx \frac{\Gamma_1(\mu_d)}{1 - \Gamma_0(\mu_d) - \Gamma_1(\mu_d) + \dfrac{T_d}{T_e}\dfrac{1}{Z_d(\delta - 1)}\left(1 + k^2\lambda_{De}^2 + \delta\dfrac{T_e}{T_i}\right)}, \tag{17}$$

$$\gamma \approx -\sqrt{\pi}\frac{(\omega_r - \Omega_d)^2}{\Gamma_1(\mu_d)\,\Omega_d}\frac{1}{Z_d(\delta - 1)}\frac{T_d}{T_e}\left(\frac{\omega_r + k_z u_{0e}}{\sqrt{2}k_z v_e} + \frac{\delta T_e}{T_i}\frac{\omega_r - k_z u_{0i}}{\sqrt{2}k_z v_i}\right) - \sqrt{\pi}\frac{(\omega_r - \Omega_d)^2}{\sqrt{2}k_z v_d}(1 + \Delta)\exp\left[-\left(\frac{\omega_r - \Omega_d}{\sqrt{2}k_z v_d}\right)^2\right]. \tag{18}$$

It can be seen from (17) that the mode can have an EDC character (with $\Delta < 1$) as long as $Z_d(\delta - 1)\,T_e/T_d$ and $Z_d(\delta - 1)\,T_i/\delta T_d$ are roughly $\gtrsim 1$. It is seen from (18) that growth can be driven by ion drift, and in this case the presence of electrons reduces the growth rate. However, when $u_{0i}/v_i = u_{0e}/v_e$, the ion growth term would be larger than the electron damping term by a factor $\delta > 1$ (for $T_e = T_i$).

Maximum growth occurs for μ_d of the order of unity (in analogy with the EIC), so that $\zeta_d \approx \Delta\Omega_d/k_z v_d \sim \Delta k_\perp/k_z$. Combining the condition $\zeta_d \gg 1$ (for small dust cyclotron damping) with the condition $\omega/k_z v_i \ll 1$ (for small ion Landau damping) leads to the condition

$$\frac{1}{\Delta} \ll \frac{k_\perp}{k_z} \ll \frac{v_i}{v_d}. \tag{19}$$

Now $\Delta \lesssim \frac{1}{2}$ for the first dust cyclotron harmonic, while the ratio v_i/v_d can be many orders of magnitude larger than unity (for example, this ratio is of order 10^3 for a 0.01 μm grain in a helium plasma, with $T_i \approx T_d$). Thus there can be a relatively large window in $k_\perp/k_z$ where both dust cyclotron damping and ion Landau damping could be very weak. It follows that when $k_\perp\rho_d \approx 1$, wave growth can occur for relatively small ion drifts $u_{0i} > (k_\perp/k_z)\,v_d$. This is illustrated by the numerical results that will be given in Sec. 3.2.

3.1.2. Collisional case. In this section, we consider the stabilizing effect of dust–neutral collisions, and also the destabilizing effect of electron and ion collisions with neutrals. The electron–neutral and ion–neutral collision rates are given by (see e.g. Marconi and Mendis 1983)

$$\nu_{e,i} \sim \sigma n_n [v_{e,i}^2 + (u_{0e,i} - v_n)^2]^{1/2}, \tag{20}$$

where the cross-section σ is typically about 5×10^{-15} cm^2 (Book 1990), v_α is the thermal speed of species α, and n_n and v_n are the neutral density and thermal speed. For the dust–neutral collision rate, we use the hard-sphere rate

$$\nu_d \sim \pi a^2 n_n v_n \frac{m_n}{m_d}, \tag{21}$$

where m_n is the mass of the neutrals. We first consider the 'weakly collisional' regime where the parallel (to **B**) wavelength is much smaller than the mean free path of the ions or electrons (see e.g. Kindel and Kennel 1971; Satyanarayana *et al.* 1985). Thus, in this regime, $\zeta_e \ll 1$ and $\zeta_i \ll 1$, and, with μ_e and $\mu_i \ll 1$, we have

$$\chi_e \approx \frac{k_{De}^2}{k^2}\left[1 + i\sqrt{\pi}\frac{\omega + k_z u_{0e}}{\sqrt{2}k_z v_e}\left(1 + \sqrt{\frac{\pi}{2}}\frac{\nu_e}{k_z v_e}\right)\right], \tag{22}$$

$$\chi_i \approx \frac{k_{Di}^2}{k^2}\left[1 + i\sqrt{\pi}\frac{\omega - k_z u_{0i}}{\sqrt{2}k_z v_i}\left(1 + \sqrt{\frac{\pi}{2}}\frac{\nu_i}{k_z v_i}\right)\right]. \tag{23}$$

Including dust–neutral collisions and neglecting dust cyclotron damping for reasons discussed above, we have in the phase-velocity regime $\zeta_d \gg 1$ (retaining only the $m = 0$ and 1 terms)

$$\chi_d \approx \frac{k_{Dd}^2}{k^2}\frac{(\omega + i\nu_d)[1 - \Gamma_0(\mu_d) - \Gamma_1(\mu_d)] - \Omega_d[1 - \Gamma_0(\mu_d)]}{(\omega - \Omega_d) - i\nu_d\left\{\dfrac{\omega - \Omega_d}{\omega}\Gamma_0(\mu_d) - [1 - \Gamma_1(\mu_d)]\right\}}, \tag{24}$$

where it has been assumed that $\nu_d/\omega \ll 1$. Using these expressions for χ_α, we can solve (4) for the real and imaginary parts of the frequency, using $\omega = \omega_r + i\gamma$, with $\omega_r = (1+\Delta)\,\omega_d$, and $|\gamma| \ll \Omega_r$, and assuming weak dust collisions such that $\nu_d/(\omega - \Omega_d) < 1$. The result for the real part of the frequency is approximately the same as that in the collisionless regime given in (16) and (17). The imaginary part of the frequency is

$$\gamma \approx -\frac{(\omega - \Omega_d)^2}{\Omega_d \Gamma_1(\mu_d)}\frac{\sqrt{\pi}}{Z_d(\delta - 1)}\frac{T_d}{T_e}\left[\frac{\omega_r + k_z u_{0e}}{\sqrt{2}k_z v_e}\left(1 + \sqrt{\frac{\pi}{2}}\frac{\nu_e}{k_z v_e}\right)\right.$$
$$\left. + \frac{\delta T_e}{T_i}\frac{\omega_r - k_z u_{0i}}{\sqrt{2}k_z v_i}\left(1 + \sqrt{\frac{\pi}{2}}\frac{\nu_i}{k_z v_i}\right)\right] - \nu_d[1 - \Gamma_1(\mu_d)]. \tag{25}$$

It can be seen from this equation that dust–neutral collisions (given by the last term) are stabilizing, while ion–neutral collisions can slightly increase the growth rate of the instability. The stabilizing effect of dust collisions may provide a more stringent requirement on the ion drift than the stabilizing effect of dust cyclotron damping. As discussed in Sec. 3.1.1, dust cyclotron damping may be negligibly small over a wide range of $k_\perp/k_z$. Since the frequency of the dust cyclotron wave is very low because of the small charge-to-mass ratio of dust, even a low dust collision rate can be important. This can be seen by

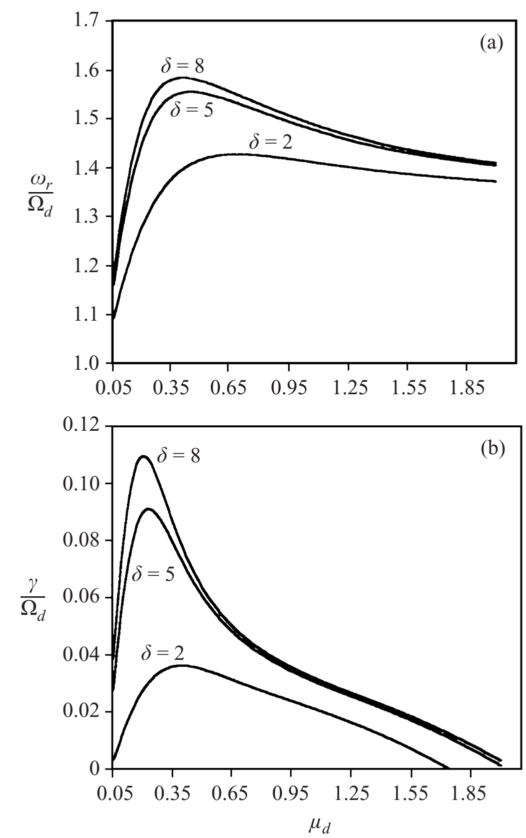

Figure 1. Frequency (a) and growth rate (b) as functions of μ_d for various values of δ, with $u_{0i}/v_i = u_{0e}/v_e = 0.5$, $T_e = T_i = T_d$, $m_i/m_p = 40$, $m_d/m_p = 2.0\times10^6$, $k_\perp/k_z = 10.0$, $\omega_{pi}/\Omega_i = 1.0$, $Z_d = 10$ and $\nu_d = \nu_i = \nu_e = 0$.

comparing the last term in (18), which represents dust cyclotron damping, with the last term in (25), which represents damping due to dust collisions. For $k\rho_d \approx 1$, the last term in (18) is of order $-\Delta\zeta_d\Omega_d\exp(-\zeta_d^2)$, while the last term in (25) is of order ν_d. Since, as discussed after (18), ζ_d can be much greater than unity over a wide range of wavenumbers, the collisional damping term can be larger than dust cyclotron damping even for relatively small values of ν_d/Ω_d. The effect of dust–neutral collisions will be illustrated in the numerical results in Sec. 3.2, which show the stabilizing effect of dust–neutral collisions on the growth rate, even for relatively small values of ν_d/Ω_d.

We next consider the 'strongly collisional limit' for the electrons and ions, where the electron and ion mean free paths are much smaller than the wavelength component along $\mathbf{B}$; that is, where $\nu_e/k_z v_e \gg 1$ and $\nu_i/k_z v_i \gg 1$ (see e.g. Satyanarayana et al. 1985). In this limit, $\zeta_e \gg 1$ and $\zeta_i \gg 1$, and we have for the electron and ion susceptibilities

$$\chi_e \approx -\frac{k_{De}^2}{k^2}\Gamma_0(\mu_e)\left[2\zeta_e^2 - \frac{i\nu_e(\omega+k_z u_{0e}+i\nu_e)}{k_z^2 v_e^2}\Gamma_0(\mu_e)\left(1+\frac{1}{2\zeta_e^2}\right)\right]^{-1}, \quad (26)$$

$$\chi_i \approx -\frac{k_{Di}^2}{k^2}\Gamma_0(\mu_i)\left[2\zeta_i^2 - \frac{i\nu_i(\omega-k_z u_{0i}+i\nu_i)}{k_z^2 v_i^2}\Gamma_0(\mu_i)\left(1+\frac{1}{2\zeta_i^2}\right)\right]^{-1}. \quad (27)$$

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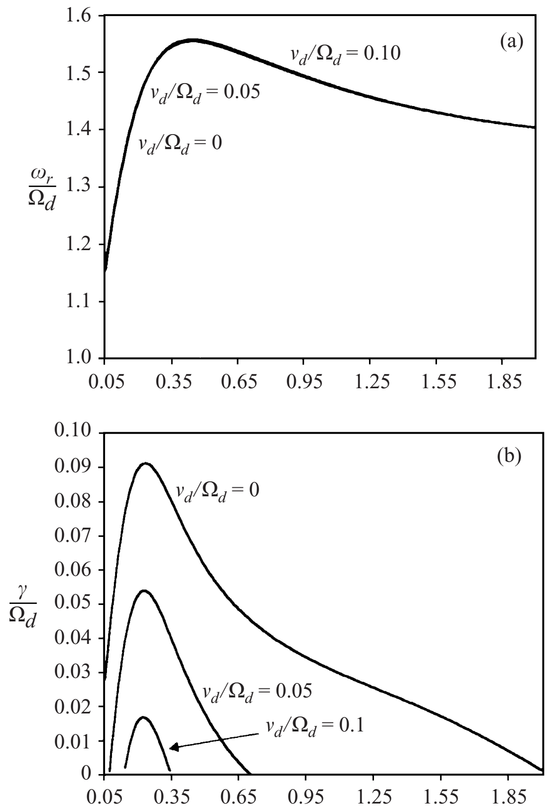

Figure 2. Frequency (a) and growth rate (b) as functions of μ_d for various values of ν_d/Ω_d, with $u_{0i}/v_i = u_{0e}/v_e = 0.5$, $T_e = T_i = T_d$, $m_i/m_p = 40$, $m_d/m_p = 2.0 \times 10^6$, $k_\perp/k_z = 10.0$, $\omega_{pi}/\Omega_i = 1.0$, $Z_d = 10$, $\delta = 5$ and $\nu_i = \nu_e = 0$.

In the limit that $\nu_e \gg (\omega + k_z u_{0e})$, $\nu_i \gg |\omega - k_z u_{0i}|$ and $\Gamma_0(\mu_e) \approx \Gamma_0(\mu_i) \approx 1$,

$$\chi_e \approx -\frac{k_{De}^2}{k^2}\left[\frac{i\nu_e(\omega + k_z u_{0e})}{k_z^2 v_e^2} - 1\right]^{-1}, \tag{28}$$

$$\chi_i \approx -\frac{k_{Di}^2}{k^2}\left[\frac{i\nu_i(\omega - k_z u_{0i})}{k_z^2 v_i^2} - 1\right]^{-1}. \tag{29}$$

We use (24) for χ_d, which takes into account only dust collisional damping. We again assume that $\omega = \omega_r + i\gamma$, with $|\gamma| \ll \omega_r = (1+\Delta)\Omega_d$, and that $\nu_d/(\omega - \Omega_d) < 1$. In the parameter regime where

$$\frac{\nu_e(\omega + k_z u_{0e})}{k_z^2 v_e^2} \ll 1,$$

$$\left|\frac{\nu_i(\omega - k_z u_{0i})}{k_z^2 v_i^2}\right| \ll 1,$$

the real part of the frequency is given approximately by the expressions (16)

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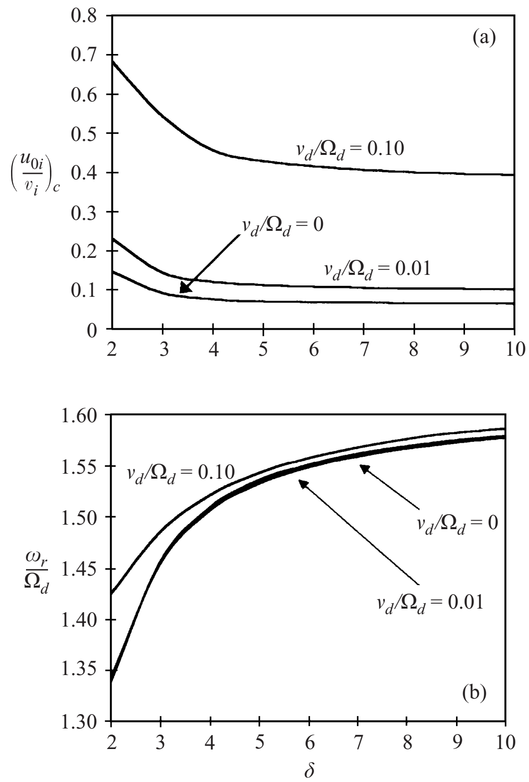

Figure 3. Critical drift (a) and the corresponding frequency (b) as functions of δ for various values of ν_d/Ω_d, with $\mu_d = 0.3$, $T_e = T_i = T_d$, $m_i/m_p = 40$, $m_d/m_p = 2.0 \times 10^6$, $\omega_{pi}/\Omega_i = 1.0$, $Z_d = 10$ and $u_{0i}/v_i = u_{0e}/v_e$.

and (17) for the collisionless case, while the imaginary part of the frequency is given by

$$\gamma \approx -\frac{(w-\Omega_d)^2}{\Omega_d Z_d(\delta-1)\Gamma_1(\mu_d)}\frac{T_d}{T_e}\left[\frac{\nu_e(\omega+k_z u_{0e})}{k_z^2 v_e^2}+\delta\frac{T_e}{T_i}\frac{\nu_i(\omega-k_z u_{0i})}{k_z^2 v_i^2}\right]-\nu_d[1-\Gamma_1(\mu_d)]. \tag{30}$$

We note that since $\nu_i/k_z v_i > 1$ here, the growth rate would be larger by this factor than the growth rate in the 'weakly collisional' case given by (25).

3.2. *Numerical results*

Figure 1 shows the numerical solution of (4) for the collisionless case, for various values of δ, using the following parameters: $m_i/m_p = 40$, $m_d/m_p = 2 \times 10^6$ (corresponding to a dust grain of radius $a \sim 0.01$ μm and mass density of about 1 g cm^{-3}, $T_i = T_e = T_d$, $Z_d = 10$, $\omega_{pi}/\Omega_i = 1.0$ and $u_{0i}/v_i = u_{0e}/v_e = 0.5$, with $k_\perp/k_z = 10$. It can be seen that the growth rate increases as $\delta = n_i/n_e$ increases, since the ion driving term becomes larger. Figure 2 shows how the frequency and growth rate vary when only dust collisions are included; parameters here are the same as in Fig. 1, but $\delta = 5$ is kept fixed. It can be seen that the growth rate decreases as ν_d/Ω_d increases. The stabilizing effect of dust collisions can also

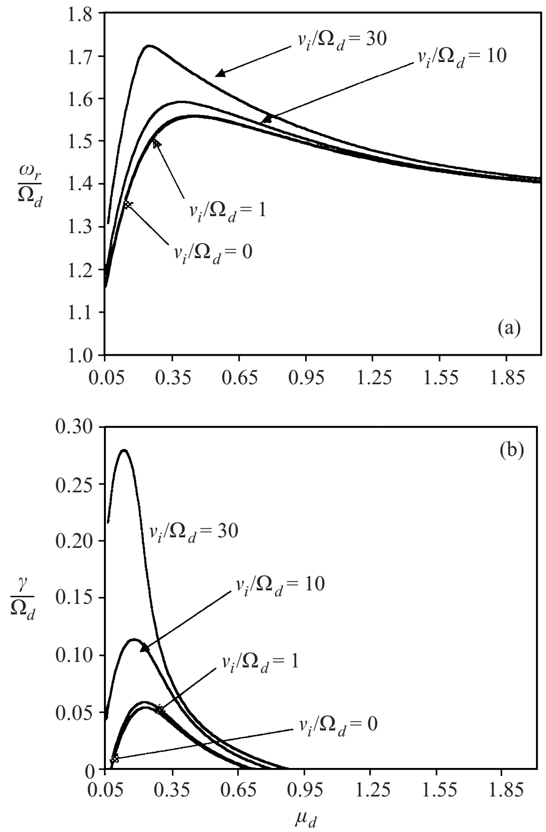

Figure 4. Frequency (a) and growth rate (b) as functions of μ_d for various values of ν_i/Ω_d (and $\nu_e/\Omega_d = \nu_i/\Omega_d\,(m_i/m_e)^{1/2}$), with $u_{0i}/v_i = u_{0e}/v_e = 0.5$, $T_e = T_i = T_d$, $m_i/m_p = 40$, $m_d/m_p = 2.0 \times 10^6$, $k_\perp/k_z = 10.0$, $\omega_{pi}/\Omega_i = 1.0$, $Z_d = 10$, $\delta = 5$ and $\nu_d/\Omega_d = 0.05$.

be seen in Fig. 3, which shows the critical ion drift, $(u_{0i}/v_i)_c$, corresponding to marginal stability, versus δ as a function of the dust–neutral collision frequency. The parameters here are the same as in Fig. 1, but $\mu_d = 0.3$ is kept fixed. It should also be noted that $k_\perp/k_z \sim 5$ in Fig. 3, and varies as δ and ν_d/Ω_d vary. Figure 4 shows the effects of collisions of all three species, with the same parameters as in Fig. 1, but with $\delta = 5$ kept fixed, $u_{0e}/v_i = u_{0e}/v_e = 0.5$ and $\nu_d/\Omega_d = 0.05$. It can be seen that, compared with Fig. 2, the effect of ion collisions is destabilizing. We note that $\nu_d/\Omega_d = 30$ in this figure does not correspond to the parameter regime used to derive (30) analytically for the strongly collisional case. This case, where

$$\frac{\nu_e(\omega + k_z u_{0e})}{k_z^2 v_e^2} \gg 1,$$

$$\left|\frac{\nu_i(\omega - k_z u_{0i})}{k_z^2 v_i^2}\right| \gg 1,$$

is left for further investigation.

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4. Applications

As a laboratory application, we consider an environment such as the Q-machine (a cylindrical plasma device), which has been used to study the EIC in a dusty plasma (Barkan et al. 1995). With an electric field $\mathbf{E}_0$ in the axial direction along the direction of $\mathbf{B}$, the equilibrium electron and ion drifts would be given by (see D'Angelo and Merlino 1996)

$$\mathbf{u}_{0e} = -\frac{eE_0}{m_e \nu_e}\hat{\mathbf{z}}, \tag{31}$$

$$\mathbf{u}_{0i} = \frac{eE_0}{m_i \nu_i}\hat{\mathbf{z}}, \tag{32}$$

in the frame where the dust is stationary. We note that roughly $u_{0i}/v_i \sim (T_e/T_i)\, u_{0e}/v_e$.

For a grain with a radius $a = 0.01\ \mu$m in Ar plasma with $T_e \sim 0.5$ eV, the charge state is approximately $Z_d \sim 700a(\mu\text{m})\,|\phi_s(V)| \sim 14$ (here ϕ_s is the grain surface potential, which is a few times T_e for an isolated grain), while the dust-grain mass is $m_d \sim 4\times10^{-18}$ g, assuming a mass density of 1 g cm^{-3}. In a magnetic field of 0.4 T, the dust cyclotron frequency would be $\Omega_d \sim 270\ \text{s}^{-1}$, while the dust gyroradius would be $\rho_d \sim 1.7$ cm, assuming that $T_d \sim T_e$. The dust–neutral collision frequency given by (21) is $\nu_d \sim 20\ \text{s}^{-1}$ for neutral density $n_n \sim 10^{13}\ \text{cm}^{-3}$ and neutral thermal speed $v_n \sim 3\times10^4\ \text{cm s}^{-1}$. Since the dust cyclotron damping term and the ion Landau damping terms may be negligibly small for $k_\perp \rho_d \sim 1$ and large $k_\perp/k_z$ as discussed in Sec. 3.1.1 after (18), damping due to dust–neutral collisions may be the most important mechanism in determining the critical drift for realistic laboratory dusty-plasma parameters.

We consider the 'weakly collisional' regime for the electrons and ions, where $1/k_z$ is smaller than the mean free path of the electrons or ions. (For example, for a neutral density $n_n \sim 10^{13}\ \text{cm}^{-3}$, the mean free path is about 20 cm). For $\omega_r/k_z \ll v_i$, and assuming that roughly $u_{0i}/v_i \sim (T_e/T_i)\ u_{0e}/v_e$,

$$\left(\frac{u_{0i}}{v_i}\right)_c \left(1-\frac{1}{\delta}\frac{T_i^2}{T_e^2}\right) \approx \sqrt{\frac{2}{\pi}}\,\frac{Z_d(\delta-1)}{\delta}\frac{T_i}{T_d}\frac{\nu_d}{\Omega_d}\frac{[1-\Gamma_1(\mu_d)]\,\Gamma_1(\mu_d)}{\Delta^2}. \tag{33}$$

For example, using the above values, and assuming $\delta = 2$ (so that half of the negative-charge density in the plasma resides on the dust), and also assuming $T_e \sim T_i \sim T_d$ and $\mu_d \sim 1$ (so that $\Gamma_1(\mu_d) \sim 0.2$), and using (17), which gives $\Delta \sim 0.2$, we have $(u_{0i}/v_i)_c \sim 3$. This is larger than the collisionless result, which shows growth for much lower values of the ion drift speed, of the order of $u_{0i}/v_i \sim (k_\perp/k_z)\, v_d/v_i \sim 0.04$, as shown in Fig. 3. Figure 5 illustrates the effect of increasing the neutral density n_n in a plasma with parameters similar to those of the Q-machine. It can be seen that the net effect of increasing n_n is stabilizing, since the collision frequencies of all three species increase.

As another possible example of laboratory dusty plasma parameters, we note that the behaviour of dust particles has been discussed in relation to a magnetized cylindrical electron cyclotron resonance plasma experiment (Nunomura et al. 1997). The plasma parameters considered were $T_e = 5$ eV, $T_i = 0.5$ eV, $n_i = 10^{10}\ \text{cm}^{-3}$ (He plasma) and $n_n = 10^{13}\ \text{cm}^{-3}$. The magnetic field strength was

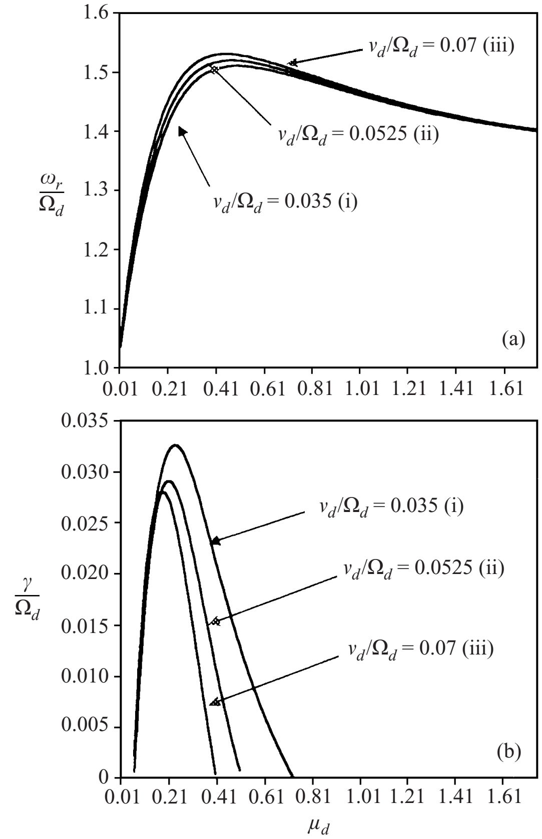

Figure 5. Frequency (a) and growth rate (b) as functions of μ_d for various values of ν_d/Ω_d, $\nu_i/\Omega_d = 180\nu_d/\Omega_d$, $\nu_e/\Omega_d = (m_i/m_e)^{1/2}\ \nu_i/\Omega_d$, $u_{0i}/v_i = u_{0e}/v_e = 0.5$, $T_e = T_i = T_d$, $m_i/m_p = 40$, $m_d/m_p = 4.0\times10^6$, $k_\perp/k_z = 10.4$, $\omega_{pi}/\Omega_i = 5.0$, $Z_d = 14$ and $n_d/n_i = 0.036$ (i.e. $\delta \sim 2$). The three values of ν_d/Ω_d correspond roughly to (i) $n_n \sim 0.5\times10^{13}$ cm^{-3}, (ii) $n_n \sim 0.75\times10^{13}$ cm^{-3} and (iii) $n_n \sim 1.0\times10^{13}$ cm^{-3} for the given parameters.

$B = 875$ G. If there were an electric field parallel to **B** in a plasma with parameters similar to these, this could lead to ion and electron drifts along **B**. The smallest grain size considered is $a = 0.01\ \mu$m. Using these parameters, the charge state would be approximately $Z_d \sim 140$, while the dust grain mass is $m_d/m_p \sim 4\times10^6$, assuming a mass density of about 2 g cm^{-3}. The dust cyclotron frequency would be $\Omega_d \sim 300$ s^{-1}, while the dust gyroradius would be $\rho_d \sim 1$ cm, assuming $T_d \sim T_i$. The dust–neutral collision frequency given by (21) is $\nu_d \sim 20$ s^{-1}, assuming $T_n \sim T_i$. Again considering the 'weakly collisional' regime for the electrons and ions, (33) would give an estimate of the critical drift, given a value of δ. Assuming low dust-charge density, with $\delta = 1.01$, this gives roughly a critical drift $(u_{0i}/v_i)_c \sim 0.5$. Figure 6 illustrates the critical drift for parameters similar to those mentioned above. It can be seen that as δ initially increases, the critical drift required to excite the instability decreases. However, as δ increases further, the critical drift increases.

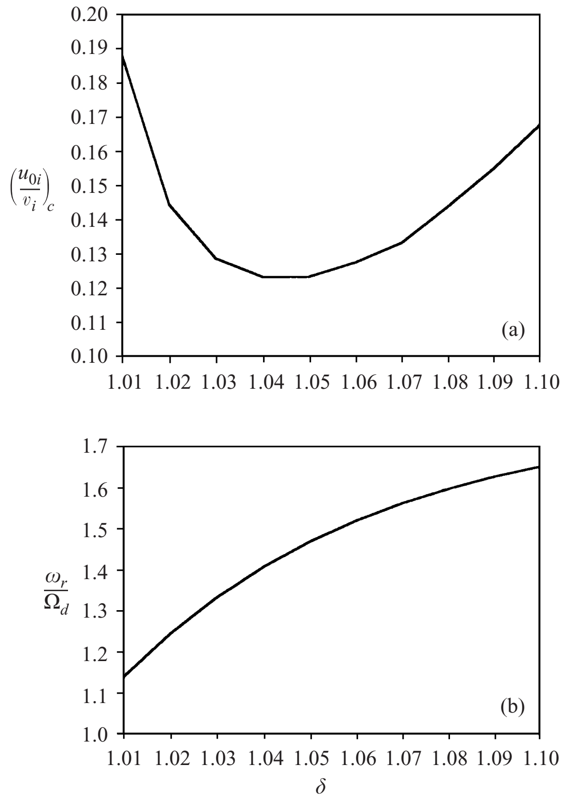

Figure 6. Critical drift (a) and the corresponding frequency (b), with $\mu_d = 0.3$, $T_e/T_d = 10$, $T_i = T_d$, $m_i/m_p = 4$, $m_d/m_p = 4.0 \times 10^6$, $\omega_{pi}/\Omega_i = 30.0$, $Z_d = 140$, $\nu_d/\Omega_d = 0.06$, $\nu_i/\Omega_d = 67.0$, $\nu_e/\Omega_d = 1.3 \times 10^4$ and $u_{09}/v_i = u_{0e}/v_e$.

5. Conclusions

A kinetic analysis of the electrostatic dust cyclotron instability has been presented in three regimes: collisionless, 'weakly collisional' and 'strongly collisional'. In all regimes, it has been shown that the critical drift of ions required to excite the instability generally decreases as the dust density (i.e. δ) increases. In both collisional regimes, it has further been shown that the effect of dust–neutral collisions is stabilizing, while the effect of ion–neutral collisions is destabilizing. It appears that it may be possible to excite the EDC instability for plasma parameters similar to those discussed in Sec. 4.

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