EPSTEIN, BERNARD, *Partial Differential Equations—An Introduction* (McGraw-Hill, New York, 1962), x+273 pp., 74s.

The subject of Partial Differential Equations is greatly neglected in Britain. Most of the workers in this field are Applied Mathematicians, interested in obtaining an analytical or numerical solution of some physical problem in which a partial differential equation arises. It is therefore a pleasure to welcome a book in English whose object is to present a rigorous modern treatment of the subject.

Although there is one chapter (Chapter 2) which gives the classical theory of partial differential equations of the first order, the book is almost wholly concerned with the quasi-linear equation of the second order, usually with two independent variables.

The book is intended for first-year graduate students, in the American terminology. The first chapter of 27 pages contains an account of the parts of the theory of functions of a real variable which the reader will need in the sequel.

Chapter 3 is concerned primarily with the Cauchy Problem for the quasi-linear equation $ar+2bs+ct+f(x, y, z, p, q) = 0$ in the usual notation, where a, b, c are functions of x, y alone. It introduces the idea of characteristics, the classification into equations of elliptic, parabolic and hyperbolic types, and the solution of the problem of Cauchy by Riemann's method. All this is classical, and can be found equally well in Goursat.

Newer ideas are introduced in Chapters 4 and 5, which are concerned with the Fredholm Alternative in a Banach or Hilbert Space, the principal object being to develop the theory of linear integral equations of Fredholm type. Instead of dealing explicitly with integral equations, the author presents a more abstract approach in terms of normed linear spaces. These chapters provide an excellent elementary introduction to the subject of abstract linear algebra.

Chapter 6 gives an introduction to the Elements of Potential Theory (Logarithmic Potential), followed by a long chapter of 50 pages on the Problem of Dirichlet. This problem is treated vigorously in five ways; (i) Poincaré's method of balayage, (ii) the method of Perron and Remak, (iii) the method of integral equations, (iv) the Dirichlet principle method, (v) the Courant-Friedrichs-Lewy method of finite differences. The chapter concludes with a brief account of the Riemann conformal mapping theorem.

The remaining two chapters deal with the Equation of Heat and Green's Functions and Separation of Variables, and are in more classical style.

In each chapter, there are many interesting exercises, and, at the end of the book will be found solutions to many of these.

Throughout the book, emphasis is laid on existence and uniqueness theorems, rather than on the effective solutions of specific classes of particular problems. The author intends it as a complement to other books which emphasise the more practical or applied aspects of the subject.

I can heartily recommend this book to the reader who wants a sound introduction to the rigorous theory of partial differential equations.

E. T. COPSON

Essays on the Foundations of Mathematics, edited by Y. BAR HILLEL, E. I. J. POZNANSKI, M. O. RABIN and A. ROBINSON (North Holland Publishing Co., 1962), 351 pp.

This handsome volume of essays is dedicated to A. A. Fraenkel, a pioneer in the logical study of set theory, in honour of his seventieth birthday on February 12, 1961. The collection is divided into four parts reflecting Fraenkel's interests in foundation studies, preceded by a bibliography of Fraenkel's works. Part I contains six papers on axiomatic set theory amongst which are a new codification of set theory by Paul Bernays. Part II contains an important paper by Th. Skolem on the interpretation of mathematical theories in first order predicate calculus. It has long been known that a theory with a finite number of axioms formulated in PC (first order predicate calculus) can be absorbed into PC (it was in this way that the negative solution of the decision problem for PC was deduced from that of a finitely axiomatisable fragment of arithmetic) but it is also known that arithmetic with induction is not finitely axiomatisable. Skolem describes a way of translating a theory with axiom schemata (infinite bundles of axioms) into a theory with finitely many axioms having the same deductive power. The third part of the volume contains six papers on the foundations of arithmetic and analysis. Heyting discusses the descriptive role of axioms in intuitionistic mathematics and gives axiom systems for an intuitionistic theory of vector spaces. Mostowski shows that in weak second order logic (with only *finite* subsets of the set of individuals as values of the second order variables) there is no (finite or recursively enumerable) set X of axioms such that the set of all true formulas in the field of real numbers is exactly the class of consequences of X. Sierpinski (in a paper rather outside the field of this collection) gives a delightfully simple proof that if the numbers m^n, where m and n run through all positive integers, are arranged in increasing order then the difference of consecutive terms is unbounded.

Part IV contains four papers on the philosophy of logic and mathematics, the last of which is a very easy to read account by Hao Wang of such fundamental questions as the reduction of mathematics to logic, the nature of number and existence in mathematics.

R. L. GOODSTEIN

FUCHS B. A. AND LEVIN, V. I., *Functions of a complex variable and some of their applications*, translated by J. BERRY and edited by T. KÖVARI (International series of monographs on pure and applied mathematics Volume 21, Pergamon Press, 1961), 296 pp., 50s.

This book, which is intended for engineers and technologists, is a translation of a book published in Russia in 1951 and is a sequel to one with the same title by B. A. Fuchs and B. V. Shabat which covers the basic theory of functions of a complex variable.

The present volume contains five chapters entitled: I Algebraic functions, II Differential equations, III The Laplace transformation and its inversion, IV Contour integration and asymptotic expansions, V Hurwitz's problem for polynomials. It may be remarked that Chapter IV contains none of the elementary theory of contour integration but is concerned with applications of the inversion formula for the Laplace transform and with the derivation of asymptotic expansions. Chapter V is concerned with the problem of determining conditions under which the zeros of a polynomial should all have negative real parts, and should be particularly useful to workers in stability theory.

Throughout the book there are many worked examples (though there are none for the reader to work out) and the exposition would be very clear were it not for the large number of printer's errors.

D. MARTIN

JEFFREYS, HAROLD, *Asymptotic Approximations* (Clarendon Press: Oxford University Press, 1962), 144 pp., 30s.

In his preface, Sir Harold remarks that great advances have been made in the theory and use of asymptotic approximations during the last few decades. Many of these advances are due to Sir Harold himself, and the reader will find the present monograph a valuable and stimulating account of recent work in this field, written