# Instability of plasma waves caused by incoherent photons in dense plasmas

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**Abstract.** We consider the nonlinear instability of modified Langmuir and ion–sound waves caused by partially coherent photons in dense quantum plasmas. In our model, the dynamics of the photons is governed by a wave kinetic equation. The evolution equations for the Langmuir and ion–sound waves are deduced from the quantum hydrodynamic equations accounting for the incoherent photon pressure, the quantum statistical electron pressure, and the quantum Bohm force acting on the degenerate electrons. The governing equations are Fourier analyzed to obtain nonlinear dispersion relations. The latter are analyzed to predict instability of the modified Langmuir and ion–sound waves in the presence of partially coherent photons. Possible applications of our investigation to the next generation of intense laser–solid dense plasma experiments and compact dense astrophysical bodies are mentioned.

## 1. Introduction

It is well known that a large-amplitude coherent electromagnetic (EM) wave propagating through a classical electron–ion plasma can initiate a great variety of nonlinear effects. The latter include wave–wave and wave–particle interactions in which coherent EM waves are either scattered off plasma eigenmodes or plasma quasi-modes (electron and ion bunches). The nonlinear wave–wave and wave–particle interactions belong to a class of parametric instabilities that are referred to as stimulated Raman/Brillouin scattering (e.g. Sjölund and Stenflo 1967a, b; Gorbunov 1973; Drake et al. 1974; Yu et al. 1974; Shukla et al. 1975), stimulated Compton scattering (Drake et al. 1974; Lin and Dawson 1975, 1977), and modulational/filamentational (e.g. Bingham et al. 2004) instabilities of EM waves. Stimulated scattering instabilities of EM waves play a very essential role in the anomalous absorption of EM wave energy and heating of magnetically and inertially confined fusion plasmas, as well as in the ionospheric plasma modification by radio waves (e.g. Stenflo 1990; Shukla 2004; Stenflo 2004).

In reality, however, EM waves can have a broadband spectrum. The nonlinear propagation of broadband partially coherent or incoherent 'white' light reveals new interesting features (e.g. Bingham et al. 1977; Mendonça et al. 2003; Marklund and Shukla 2006; Shukla and Stenflo 2006; Santos et al. 2007) of parametric interactions

in classical plasmas. Specifically, in their classic paper, Bingham et al. (1997) considered nonlinear couplings between partially coherent EM waves (photons) and electrostatic Langmuir waves, and demonstrated anomalous damping of Langmuir waves due to wave–quasi-particles (photons) interactions. The work of Bingham et al. (1997) has been further extended by Santos et al. (2007) to include relativistic electron mass variations and relativistic light pressure effects.

Recently, there has been an emerging interest (e.g. Shukla and Stenflo 2006; Stenflo and Shukla 2009) in investigations of parametric instabilities of coherent EM waves in dense plasmas where the electrons are degenerate. Due to the electron degeneracy, one has to account for the forces (Manfredi 2005; Shukla 2006; Shukla 2009; Serbeto et al. 2009; Shukla and Eliasson 2010) associated with the quantum statistical electron pressure and tunneling of electrons through the quantum Bohm potential arising from the overlapping of the electron wave functions owing to the Heisenberg uncertainty principle at nanoscales. It then turns out that the inclusion of these quantum forces gives rise to new dispersive properties of electrostatic Langmuir waves and electrostatic ion–sound oscillations in dense quantum plasmas. Accordingly, the growth rates of stimulated Raman and Brillouin instabilities of EM waves in dense quantum plasmas are drastically modified (Shukla and Stenflo 2006; Stenflo and Shukla 2009).

In this paper, we consider nonlinear interactions between partially coherent EM waves (incoherent photons) and electrostatic dispersive Langmuir and ion–sound perturbations in an unmagnetized dense quantum plasma. Since partially coherent photons behave like quasi-particles, their dynamics in the presence of slowly varying electron density perturbations is governed by a wave-kinetic equation. The evolution equations for the electrostatic density perturbations in the presence of the photon pressure light are then deduced from the quantum hydrodynamic equations. The latter are composed of the continuity and momentum equations for the electrons and ions, as well as the Poisson equation. The governing nonlinear equations for partially coherent photon-electrostatic plasma density fluctuations are Fourier analyzed to obtain the nonlinear dispersion relations. The latter admit instabilities of the modified dispersive Langmuir and ion–sound waves due to wave–quasi-particle (photon) resonant interactions. Explicit expressions for the growth rates are presented. Our results may be relevant to understand the nonlinear propagation of partially coherent EM waves through the dense plasmas, such as those in compact astrophysical objects (Shapiro and Teukolsky 1983; Harding and Lai 2006) and in the next generation of high-energy density plasmas created by powerful laser beams (Glenzer and Redmer 2009).

## 2. Basic equations

We consider an unmagnetized dense electron–ion plasma in the presence of incoherent photons. The electric fields of the latter are

$$\mathbf{E} = \mathbf{E_k} \exp(-i\omega_k t + i\mathbf{k} \cdot \mathbf{r}) + \text{c.\,c.}, \tag{1}$$

where $\omega$ and $\mathbf{k}$ are the frequency and the propagation wave vector, respectively, related by the linear dispersion relation

$$\frac{k^2 c^2}{\omega_k^2} = 1 - \frac{\omega_{pe}^2}{\omega_k^2}. \tag{2}$$

Here $c$ is the speed of light in vacuum, $\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency, $n_e$ is the electron number density, $e$ is the magnitude of the electron charge, and $m_e$ is the rest mass of the electrons. Let us introduce the energy density

$$\mathscr{E}_{\mathbf{k}} = \frac{\omega_{pe}^2}{\omega_k^2} \frac{|\mathbf{E}_{\mathbf{k}}|^2}{4\pi} \tag{3}$$

and the action

$$N_k = \frac{\mathscr{E}_{\mathbf{k}}}{\omega_k} = \frac{\omega_{pe}^2}{\omega_k^3} \frac{|\mathbf{E}_{\mathbf{k}}|^2}{4\pi}. \tag{4}$$

The nonlinear interaction between the random phase incoherent photons is governed by a wave-kinetic equation (Kadomtsev 1965; Bingham et al. 1997)

$$\frac{\partial N_k}{\partial t} + \mathbf{V}_g \cdot \nabla N_k + \mathbf{F}_k \cdot \frac{\partial N_k}{\partial \mathbf{k}} = 0, \tag{5}$$

where $\mathbf{V}_g = \mathbf{k}c^2/\omega_k$ is the group velocity (Mendonça 2001) of incoherent photons, and $\omega_k = (k^2c^2 + \omega_{pe}^2)^{1/2}$ the photon frequency. The force acting on the photon quasi-particles due to the presence of the electron density fluctuation $n_{e1}$ ($\ll n_0$) is

$$\mathbf{F}_k = -\nabla \omega_k = -\frac{\omega_{p0}^2}{2\omega_k n_0} \nabla n_{e1}, \tag{6}$$

where $\omega_{p0} = (4\pi n_0 e^2/m_e)^{1/2}$ is the unperturbed electron plasma frequency and $n_0$ the equilibrium electron number density.

The electrostatic density perturbations associated with modified dispersive Langmuir and ion–sound disturbances in the presence of the photon pressure is governed by the quantum hydrodynamic equations for non-relativistic degenerate electrons. We have the electron continuity equation

$$\frac{\partial n_{e1}}{\partial t} + n_0 \nabla \cdot \mathbf{u}_e = 0, \tag{7}$$

the electron momentum equation

$$m_e \frac{\partial \mathbf{u}_e}{\partial t} + \frac{e^2}{2m_e\omega_k^2} \nabla \sum_{\mathbf{k}} |\mathbf{E}|_{\mathbf{k}}^2 = e\nabla\phi - \frac{\nabla p_e}{n_0} + \frac{\hbar^2}{4m_e n_0} \nabla \nabla^2 n_{e1}, \tag{8}$$

the Poisson equation

$$\nabla^2 \phi = 4\pi e(n_{e1} - n_{i1}), \tag{9}$$

where $\mathbf{u}_e$ is the electron fluid velocity, $\phi$ is the scalar potential, $\hbar$ is the Planck constant divided by $2\pi$, and the quantum statistical electron pressure for the Fermi–Dirac plasma is

$$p_e = \gamma_e k_B T_{Fe} n_{e1}, \tag{10}$$

where $\gamma_e$ is the adiabatic index, $k_B$ is the Boltzmann constant, and $T_{Fe}$ is the electron Fermi temperature.

Two comments are in order. First, the second term in the left-hand side of (8) is the photon pressure arising from the ensemble average (over the period $2\pi/\omega_k$) of the advection and nonlinear Lorentz forces involving the photon induced electron quiver velocity and the photon magnetic field. Second, the third term in the right-hand side of (8) represents the contribution of electron tunneling through the quantum

Bohm potential (Manfredi 2005; Shukla and Eliasson 2010). This term arises owing to the overlapping electron wave functions due to the Heisenberg uncertainty principle.

The ion density perturbation is obtained from the ion continuity equation

$$\frac{\partial n_{i1}}{\partial t} + n_0 \nabla \cdot \mathbf{u}_i = 0, \tag{11}$$

where the ion fluid velocity $\mathbf{u}_i$ is given by the momentum equation

$$m_i \frac{\partial \mathbf{u}_i}{\partial t} = -Z_i e \nabla \phi, \tag{12}$$

where $m_i$ is the ion mass. In dense quantum plasmas, non-degenerate ions can in general be treated classically. The ion temperature is much smaller than the electron Fermi temperature.

We now obtain the governing equations for the driven (by the pressure of incoherent photons) modified dispersive Langmuir and ion–sound perturbations in quantum plasmas. First, we consider modified Langmuir waves, which occur on a time scale much larger than the ion plasma period, so that the ions do not respond. Therefore, the ion number density perturbation is neglected. Accordingly, combining (7), (8) and (9), we obtain

$$\left( \frac{\partial^2}{\partial t^2} + \omega_{p0}^2 - \frac{3}{5} V_{Fe}^2 \nabla^2 + \frac{\hbar^2}{4m_e^2} \nabla^4 \right) \frac{n_{e1}}{n_0} = \frac{2\pi e^2}{m_e^2 \omega_{p0}^2} \nabla^2 \sum_{\mathbf{k}} (\omega_k N_{k1}), \tag{13}$$

where $V_{Fe} = (k_B T_{Fe}/m_e)^{1/2}$ is the electron Fermi speed and $N_{k1}$ is a small perturbation in the equilibrium action $N_{k0}(= N_k - N_{k1})$. In the absence of incoherent photons, we neglect the right-hand side of (13) and obtain, by assuming that $n_{e1} \sim \exp(-i\Omega t + i\mathbf{K} \cdot \mathbf{r})$, the frequency of the modified Langmuir waves (Manfredi 2005; Shukla and Eliasson 2010):

$$\Omega = \left[ \omega_{p0}^2 + (3/5)K^2 V_{Fe}^2 + \hbar^2 K^4 / 4m_e^2 \right]^{1/2} \equiv \Omega_L. \tag{14}$$

Second, we consider the low-phase velocity (in comparison with $V_{Fe}$) modified ion–sound waves driven by the incoherent photons. Here we neglect the electron inertia in (8) and combine the resultant equation with (9) and use

$$\frac{\partial^2 n_{i1}}{\partial t^2} - \frac{Z_i e n_0}{m_i} \nabla^2 \phi = 0, \tag{15}$$

which is deduced from (11) and (12), to obtain the driven ion–sound wave equation with $n_{i1} = n_{e1}$

$$\left( \frac{\partial^2}{\partial t^2} - \frac{m_e}{m_i} V_{Fe}^2 \nabla^2 + \frac{\hbar^2}{4m_e m_i} \nabla^4 \right) \frac{n_{e1}}{n_0} = \frac{2\pi e^2}{m_e m_i \omega_{p0}^2} \nabla^2 \sum_{\mathbf{k}} (\omega_k N_{k1}). \tag{16}$$

We note that the quasi-neutrality condition $n_{i1} = n_{e1}$ holds for long wavelengths (in comparison with the Fermi electron Debye radius $V_{Fe}/\omega_{p0}$). Neglecting the right-hand side in (16), we obtain after a Fourier analysis the frequency of the dispersive

ion–sound waves (Shukla and Eliasson 2010):

$$\Omega = K \left( C_s^2 + \hbar^2 K^2/4m_e m_i \right)^{1/2} \equiv \Omega_I, \tag{17}$$

where $C_s = (k_B T_{Fe}/m_i)^{1/2}$ is the ion–sound Fermi speed.

Suppose that $N_k = N_{k0} + N_{k1} \exp(-i\Omega t + i\mathbf{K} \cdot \mathbf{r})$, we obtain from (5),

$$N_{k1} = -\frac{\omega_{p0}^2 n_{e1}}{2n_0 \omega_k (\Omega - \mathbf{K} \cdot \mathbf{V}_g)} \mathbf{K} \cdot \frac{\partial N_{k0}}{\partial \mathbf{k}}. \tag{18}$$

Furthermore, Fourier analyzing (13) and (16) we obtained, respectively,

$$\frac{n_{e1}}{n_0} = \frac{2\pi e^2 K^2}{m_e^2 \omega_{p0}^2 (\Omega^2 - \Omega_L^2)} \sum_{\mathbf{k}} (\omega_k N_{k1}) \tag{19}$$

and

$$\frac{n_{e1}}{n_0} = \frac{2\pi e^2 K^2}{m_e m_i \omega_{p0}^2 (\Omega^2 - \Omega_I^2)} \sum_{\mathbf{k}} (\omega_k N_{k1}). \tag{20}$$

Inserting (18) into (19) and (20) we obtain the desired nonlinear equations for the modified Langmuir and ion–sound waves in the presence of incoherent photons. We have

$$\Omega^2 - \Omega_L^2 = -\frac{\pi e^2 K^2}{m_e^2} \int d\mathbf{k} \frac{\mathbf{K} \cdot (\partial N_{k0}/\partial \mathbf{k})}{(\Omega - \mathbf{K} \cdot \mathbf{V}_g)}, \tag{21}$$

for the modified Langmuir waves, and

$$\Omega^2 - \Omega_I^2 = -\frac{\pi e^2 K^2}{m_e m_i} \int d\mathbf{k} \frac{\mathbf{K} \cdot (\partial N_{k0}/\partial \mathbf{k})}{(\Omega - \mathbf{K} \cdot \mathbf{V}_g)}, \tag{22}$$

for the modified ion–sound waves.

The resonant type instability occurs because the resonant function $R = (\Omega - \mathbf{K} \cdot \mathbf{V}_g)^{-1} \to -i\pi\delta(\Omega - \mathbf{K} \cdot \mathbf{V}_g)$, where $\delta$ is the Dirac delta function. Letting $\Omega = \Omega_r + i\gamma_{l,i}$, where $\gamma_{l,i} \ll \Omega_r = \Omega_L, \Omega_I$, we obtain from (21) and (22) the growth rates, respectively,

$$\gamma_l = \frac{\pi^2 e^2 K^2}{2\Omega_L m_e^2} \int d\mathbf{k}\mathbf{K} \cdot (\partial N_{k0}/\partial \mathbf{k})\delta(\Omega_L - \mathbf{K} \cdot \mathbf{V}_g) \tag{23}$$

and

$$\gamma_i = \frac{\pi^2 e^2 K^2}{2\Omega_I m_e m_i} \int d\mathbf{k}\mathbf{K} \cdot (\partial N_{k0}/\partial \mathbf{k})\delta(\Omega_I - \mathbf{K} \cdot \mathbf{V}_g). \tag{24}$$

The condition $\mathbf{K} \cdot (\partial N_{k0}/\partial \mathbf{k}) > 0$ is required for instability. Equations (23) and (24) are the main results of our paper. They exhibit that both the modified dispersive Langmuir and ion–sound waves are amplified at the expense of the photon wave energy due to the resonant wave–quasi-particle interactions.

## 3. Summary and conclusions

In this paper, we have considered nonlinear couplings between incoherent photons and electrostatic dispersive Langmuir and ion oscillations in dense quantum plasmas. Partially coherent photons are treated like quasi-particles and their dynamics is governed by a wave kinetic equations in which a force arising from the spatially

inhomogeneous slowly varying electron density fluctuations appear. The photon pressure, in turn, reinforces the electron density fluctuations that support modified dispersive Langmuir and ion–sound waves in dense quantum plasmas. The governing coupled equations admit the nonlinear dispersion relations, which predict nonlinear instabilities of modified dispersive Langmuir and ion–sound waves due to wave–quasi-particle (photon) resonant interactions. Our results are useful for diagnostic purposes, e.g. deducing the electron number density and the Fermi electron temperature from the frequencies of the photon driven modified dispersive Langmuir and ion–sound waves in dense plasmas, such as those in the interior of white dwarfs, magnetars (Harding and Lai 2006) and in the next generation of intense laser–solid density plasma experiments (e.g. Glenzer and Redmer 2009).

# References

Bingham, R., Mendonça, J. T. and Dawson, J. M. 1997 *Phys. Rev. Lett.* **78**, 247.

Bingham, R., Mendonça, J. T. and Shukla, P. K. 2004 *Plasma Phys. Control. Fusion* **46**, R1.

Drake, J. F., Kaw, P. K., Lee, Y. C., Schmidt, G., Liu, C. S. and Rosenbluth, M. N. 1994 *Phys. Fluids* **17**, 778.

Glenzer, S. H. and Redmer, R. 2009 *Rev. Mod. Phys.* **81**, 1625.

Gorbunov, L. M. 1973 *Sov. Phys.-Usp.* **16**, 217.

Harding, A. K. and Lai, D. 2006 *Rep. Prog. Phys.* **69**, 2631.

Kadomtsev, B. B. 1965 *Plasma Turbulence*. New York: Academic Press.

Lin, A. T. and Dawson, J. M. 1975 *Phys. Fluids* **18**, 201.

Lin, A. T. and Dawson, J. M. 1977 *Phys. Fluids* **20**, 538.

Manfredi, G. 2005 *Fields Inst. Commun.* **46**, 263.

Marklund, M. and Shukla, P. K. 2006 *Rev. Mod. Phys.* **78**, 591.

Mendonça, J. T. 2001 *Theory of Photon Acceleration*. Bristol: Institute of Physics.

Mendonça, J. T., Bingham, R. and Shukla, P. K. 2003 *Phys. Rev. E* **68**, 016406.

Santos, J. E., Silva, L. O. and Bingham, R. 2007 *Phys. Rev. Lett.* **98**, 235001.

Serbeto, A., Monteiro, L. F., Tsui, K. H. and Mendonça, J. T. 2009 *Plasma Phys. Control. Fusion* **51**, 124024.

Shapiro, S. L. and Teukolsky, S. A. 1983 *Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects*. New York: John Wiley & Sons.

Sjölund, A. and Stenflo, L. 1967a *Z. Phys.* **204**, 211.

Sjölund, A. and Stenflo, L. 1967b *Appl. Phys. Lett.* **10**, 201.

Shukla, P. K. 2004 *Nonlinear Physics in Action, Phys. Scr.* **T113**, pp. 4–111.

Shukla, P. K. 2006 *Phys. Lett.* A **357**, 229.

Shukla, P. K. 2009 *Nat. Phys.* **5**, 92.

Shukla, P. K. and Eliasson, B. 2010 *Phys.-Usp.* **53**, 51.

Shukla, P. K. and Stenflo, L. 2006 *Phys. Plasmas* **13**, 044505.

Shukla, P. K., Yu, M. Y. and Spatschek, K. H. 1975 *Phys. Fluids* **18**, 265.

Stenflo, L. 1990 *Phys. Scr.* **T30**, 166.

Stenflo, L. 2004 *Phys. Scr.* **T107**, 262.

Stenflo, L. and Shukla, P. K. 2009 In: *From Leonardo to ITER: Nonlinear and Coherence Aspects* (ed. Jan Weiland), AIP Conference Proceedings # 1177. New York: American Institute of Physics, pp. 4–9.

Yu, M. Y., Spatschek, K. H. and Shukla, P. K. 1974 *Z. Naturforsch* **29a**, 1736.