J. Plasma Physics (2002), *vol.* 67, *parts* 2&3, *pp.* 99–114. © 2002 Cambridge University Press 99
DOI: 10.1017/S0022377801001532 Printed in the United Kingdom

Regularization of superdrift magnetic islands for finite electron temperature

A. B. MIKHAILOVSKII,[1] S. V. KONOVALOV,[1] G. I. SURAMLISHVILI[2] and V. S. TSYPIN[3]

[1]Institute of Nuclear Fusion, Russian Research Centre 'Kurchatov Institute', Kurchatov Square, 1, Moscow, 123182, Russia

[2]Andronikashvili Institute of Physics, Georgian Academy of Sciences, Tamarashvili Street 6, Tbilisi, 380077, Georgia

[3]Institute of Physics, University of São Palo, Rua do Matão, Travessa R, 187, 05508-900 São Paulo, Brasil

(Received 20 November 2000 and in revised form 5 April 2001)

Abstract. The regularization of magnetic islands is studied for the case when the electron temperature is larger than the ion temperature. The slab approximation is used. Drift effects are neglected, i.e., the case of superdrift magnetic islands, SDMIs, is analyzed. Then the regularization problem reduces to, first, a spreading of the step-functional velocity profile and the conventional delta-functional polarization current profile in the region near the island separatrix; second, finding the dispersion terms of the polarization current in this region; and, third, calculating the total polarization current contribution to the generalized Rutherford equation for the island width. It is shown that this problem can be solved if one allows for the effects of the electron pressure gradient in the parallel Ohm's law. The polarization current contribution in the case of islands regularized due to these effects proves to be the same as that in the case of nonregularized islands.

1. Introduction

Development of magnetic island theory is important for the physics of tokamaks, since islands can lead to a plasma pressure limitation in long-pulse discharges (Wilson et al. 1996a, b; Sauter et al. 1997). To find this limitation, one should allow for the polarization current effect (Smolyakov 1989; Wilson et al. 1996a). To describe this effect, one should take into account the perturbed perpendicular electric field. In turn, this field results in cross-field plasma motion, so that one should know the dependence of the velocity of this motion on the island magnetic flux, i.e., the velocity profile. This dependence is found using the ambipolarity equation. Up to now, the problem of finding the velocity profile has mainly been studied in the slab geometry approximation, neglecting drift and resistive effects. The history of this study and the related analysis of the polarization current problem are summarized below.

According to Waelbroeck and Fitzpatrick (1997), and corresponding to general plasma-physical notions (Tamm 1961), the ambipolarity equation is determined by the perpendicular plasma viscosity. Mikhailovswkii et al. (2000a) (cited below as paper I) have shown (see also Waelbroeck and Fitzpatrick 1997) that if one uses the conventional expression for the perpendicular viscosity (Braginskii 1965) and neglects the electron dynamics (such neglect implies, in particular, that the electron temperature is assumed to be zero and finite radial particle diffusion is ignored), one does not find an acceptable velocity profile: near the island separatrix, this profile proves to be of a step-function form. Then, following paper I, one should regularize the velocity profile, i.e., 'spread' the step function. This problem was considered in paper I. Similarly to Waelbroeck and Fitzpatrick (1997), the cold-electron approximation was used in paper I. Then, according to paper I, one should introduce the hyperviscosity, which spreads the step-like jump of the velocity near the separatrix over a region with a characteristic scale of the order of the ion Larmor radius ρ_i, i.e., the so-called Larmor transition layer. Note that the notion of the hyperviscosity was used, in particular, by Smith and Hammett (1997) in spectral simulation of two-dimensional drift turbulence. As usual, the hyperviscosity is introduced in numerical simulations artificially (see, e.g., Kuvshinov et al. 2001). In contrast to this, paper I found the hyperviscosity by solving the transport equations of Grad type given by Mikhailovskii and Tsypin (1971) and Mikhailovskii (1992). The results of this solution were qualitatively confirmed by Mikhailovskii et al. (2001a), who calculated the hyperviscosity by means of a reduced Boltzmann kinetic equation taken from Lakhin et al. (1987) and complemented by a model collisional term.

It was also known that, if one uses the conventional expression for the polarization current and the velocity profile of step-like form, one finds that the current profile is of delta-function form (Waelbroeck and Fitzpatrick 1997). Such a form of the current profile is a consequence of the fact that the above expression is proportional to the first derivative of the velocity with respect to the island magnetic flux. According to Waelbroeck and Fitzpatrick (1997), the contribution of the delta-function part of the polarization current, i.e., the surface current, to the generalized Rutherford equation for the island width evolution is larger than that of the volume current, and the sign of the surface current contribution is opposite to that of the volume current. As a result, allowing for this surface current reverses the sign of the polarization current term in the above equation obtained in the preceding studies where the surface current was ignored. This result is the basis for discussions regarding whether the polarization current stabilizes the neoclassical tearing modes (NTMs).

Meanwhile, the conventional transport equations, used in deriving this result, are invalid in the problems with a delta-function polarization current. Therefore, the question arose whether the above sign reversal of the polarization current contribution (the Waelbroeck–Fitzpatrick effect) is physically correct. This question was studied by Mikhailovskii et al. (2000b) (cited below as paper II). It was shown in paper II that, on regularizing the velocity profile by the hyperviscosity and assuming that the polarization current is given by the conventional expression, one regularizes, at the same time, the current profile by spreading the delta-function over the above Larmor transition layer. The contribution of the spread-out delta-function proves to coincide with that of the exact delta function.

However, according to paper II, in addition to the conventional term, the polarization current also includes so-called dispersion terms. In the Larmor transition layer, these terms are of the same order as the conventional term. Then, the question arises as to what is the contribution of these terms to the generalized Rutherford equation. The answer to this question has been given in paper II. According to

https://doi.org/10.1017/S0022377801001532 Published online by Cambridge University Press

paper II, the dispersion terms can be expressed in the form of the full derivatives of functions localized in the Larmor transition layer with respect to the island magnetic flux, while the contribution of the surface current to the above equation is the integral from these terms with the constant weighting factor. Therefore, this contribution turns out to be zero.

As a whole, the analysis of paper II resulted in two important consequences. First, the Waelbroeck–Fitzpatrick effect is correct, and, second, for description of this effect, one can restrict oneself to the conventional expression for the polarization current only.

Like paper I, paper II used the cold-electron approximation. In turn, the hyperviscosity and the dispersion terms in the polarization current, studied in papers I and II, are determined by finite-ion-temperature effects, so that they become weaker with decreasing ion temperature. Then the question arises as to whether the islands can be regularized due to finite-electron-temperature effects. The elucidation of this question is the goal of the present paper. As in papers I and II, we neglect drift and toroidicity effects, so that our subject is superdrift magnetic islands (SDMIs) in the slab geometry approximation. The conditions under which SDMIs can exist were discussed by Mikhailovskii et al. (2000c). The term 'superdrift magnetic islands' was introduced by Mikhailovskii et al. (2000d).

Magnetic islands with account of finite-electron-temperature effects were earlier studied by Smolyakov (1989, 1993); however, the spatial structure of the perturbed plasma velocity was not analyzed. (The velocity profile was there taken in a model form.) According to the above papers, to describe these effects, one should use the parallel Ohm's law, including the term with electron pressure gradient. We follow this approach in the present paper.

Using the parallel Ohm's law with the electron-pressure-gradient contribution, one needs, generally speaking, to allow for perturbations of both plasma density and electron temperature, i.e., to deal with the electron continuity equation and the electron heat balance equation (Smolyakov 1989, 1993). However, for simplicity, we neglect the perturbations of the electron temperature. For this reason, we do not use the electron heat balance equation.

According to Smolyakov (1989, 1993), taking into account the electron pressure gradient in the parallel Ohm's law allows us to describe two effects: first, the effect of a finite ratio of the 'effective' ion Larmor radius (i.e., that defined by the electron temperature), denoted below by ρ_s, to the characteristic scale of the problem, and, second, electron drift effects. However, according to the above discussion, we do not study drift effects.

Note that, from the above discussion, when the hyperviscosity is introduced (see paper I), the characteristic scale in the near-separatrix region is of the order of the ion Larmor radius ρ_i. Apparently, in this case, one can use only model expressions for the hyperviscosity, since there is no small parameter allowing one to solve the Grad-type transport equations or the Boltzmann kinetic equation by expansion in series in this parameter in calculating the hyperviscosity. Such model expressions are obtained by keeping only the first few terms of an infinite series in $\rho_i^2\nabla_\perp^2$ (where $\nabla_\perp$ is the perpendicular gradient). In the case of finite electron temperature considered in the present paper, we start with exact (non-model) MHD equations, so that the initial stage of our analysis does not require expansion in $\rho_s^2\nabla_\perp^2$. However, in the following steps, we need to replace the infinite series in $\rho_s^2\nabla_\perp^2$ by a finite series, so that, as in the problem with hyperviscosity (see in detail paper I), our procedure is a model one. This modeling is based on the following reasoning. Our problem does not have an exact analytical solution, so that one should consider our results as not exact but rather as very close approximations. Actually, the recent magnetic island theory is based on a series of other reliable but not absolutely exact results, so that the level of reliability of our modeling approach is typical of recent studies. At the same time, undoubtedly, our problem can be analyzed numerically. Then our analytical results can be used as benchmarks for numerical calculations.

It is well known that one of the obligatory equations of magnetic island theory is the current continuity equation or, in other terminology, the vorticity equation (see paper I). Thus, our basic MHD equations are the parallel Ohm's law, the electron continuity equation, and the vorticity equation. These equations are given in Sec. 2. Instead of the vorticity equation, one can use the ion continuity equation. This equation is also given in Sec. 2.

In addition to the basic MHD equations, Sec. 2 contains explanation of the magnetic island geometry, transition to the 'island variables', the procedure of reducing the MHD equations in the approximation of weak dissipation, and an explanation of the structure of the generalized Rutherford equation for the island width in the slab approximation considered. Let us recall that the island variables are the magnetic flux of the island and the island cyclic variable. Note also that in reducing the MHD equations, we introduce the reduced electrostatic potential, which differs from the conventional electrostatic potential by a term proportional to x, where x is the distance from the 'centering' rational magnetic surface. This function is an important element of Smolyakov's (1989, 1993) approach used in the present paper.

In Sec. 3, we integrate the ideal (nondissipative) parts of the MHD equations following the integration procedure developed by Smolyakov (1989, 1993) and called there the vector integration. The most important difference of this procedure from the standard integration procedure used in paper II is that, when integrating the ion continuity equation, one has to deal with an 'integration constant' that is an arbitrary function of the reduced electrostatic potential, while the standard integration procedure leads to integration constants that are functions of the island magnetic flux. This difference is not trivial for finite electron temperature, i.e., for finite $\rho_s\nabla_\perp$, since in this case the reduced electrostatic potential depends on both the island magnetic flux and the island cyclic variable. Therefore, it is impossible to obtain an exact analytical expression for the above integration constant for finite $\rho_s\nabla_\perp$, and one needs to construct model expressions for it. This is the first step of our modeling mentioned in the above discussion. This modeling is one of the goals of Sec. 3.

Having constructed the model integration constant of the ion continuity equation, one can obtain a closed equation for the difference between the reduced electrostatic potential and the electrostatic potential profile function. This difference is, physically, the oscillatory part of the reduced electrostatic potential describing the parallel electric field. This part is determined by the oscillatory part of the integrated ion continuity equation. The exact solution of this equation can be expressed in terms of an infinite series in the parameter $\rho_s^2\nabla_\perp^2$. However, such a series is unacceptable for analytical calculations. Therefore, we have to model it by a finite series in the above parameter. Thereby, the reduced electrostatic potential is expressed in terms of the electrostatic potential profile function as a finite series in $\rho_s^2\nabla_\perp^2$. In a similar way, we model also the expression for the polarization current. The modeling of all these functions is given in Sec. 3.

Section 4 is devoted to the regularization of the electrostatic potential profile function. The starting equation for this function is obtained from the condition that the fourth derivative of the reduced electrostatic potential with respect to x averaged over the island cyclic variable vanishes. Since, as was explained above, the reduced electrostatic potential is a series in $\rho_s^2\nabla_\perp^2$ for finite electron temperature, this equation contains, in addition to the fourth derivative of the profile function, also terms with its higher derivatives. These terms play a role similar to that of the hyperviscosity. Thereby, they allow one to regularize the profile function. Note, however, that the regularization mechanism in the considered case is more complicated than in the case with hyperviscosity. The fact is that allowing only for the term of order $\rho_s^2\nabla_\perp^2$ does not lead to regularization, since the sign of this term is opposite to the sign of the hyperviscous term of order $\rho_i^2\nabla_\perp^2$. Therefore, allowing for this term only, one would obtain a nonlocalized addition to the profile function predicted by conventional transport theory. Thus, for the regularization, one should also allow for the term of order $\rho_s^4\nabla_\perp^4$, which is given in Sec. 4.

In Sec. 5, we discuss the polarization current contribution to the generalized Rutherford equation for the considered case of magnetic islands regularized due to the finite-electron-temperature effect. It is shown in this section that, similarly to the case of finite ion temperature and vanishing electron temperature considered in paper II, in our case, first, the polarization current is destabilizing, and, second, its contribution to the generalized Rutherford equation for regularized islands proves to be the same as that for non-regularized islands.

To estimate the importance of the first-mentioned fact, let us turn to Waelbroeck and Fitzpatrick (1997), who also discussed the role of the polarization current. These authors calculated the polarization current contribution for magnetic islands with continuous but nonlocalized velocity profile, and concluded that this current is destabilizing. However, such a profile is beyond the scope of the generally accepted magnetic island theory. In addition, they considered islands with discontinuous localized velocity profile, and suggested that in the case of such islands, the polarization current also should be destabilizing. It was explained in paper II that this suggestion should be considered as nothing but a likely hypothesis, which needs justification by calculations with adequate velocity profile function and an adequate starting expression for the perpendicular current. The analysis of paper II has justified this hypothesis for the case of finite ion temperature and vanishing electron temperature. Because of its importance, this hypothesis has been called in paper II the Waelbroeck–Fitzpatrick (WF) rule. The results given in Sec. 5 show that the WF rule is valid also in the opposite case when the electron temperature is larger than the ion temperature.

Let us now comment on the above-mentioned result given in Sec. 5 that the polarization current contribution is the same for both regularized and nonregularized magnetic islands. This result is important owing to the fact that to find the polarization current contribution one should not perform the rather complicated calculations typical of the exact theory of regularized magnetic islands, since the same results can be obtained in the scope of the rough but simple approach of the theory of non-regularized magnetic islands. Paying tribute with respect to paper II, this fact can be called Mikhailovskii's rule.

General conclusions are given in Sec. 6.

Generalizations of our theory by incorporating electron drift effects and the effect of finite radial particle diffusion are presented in Mikhailovskii et al. (2001b) (see also Wilson et al. 2000).

2. Basic equations and their reduction in the weak-dissipation approximation

2.1. *Basic MHD equations*

We describe the electrons by the continuity equation and the parallel Ohm's law:

$$\frac{d_0 n}{dt} - \frac{1}{e}\nabla_\parallel J = 0, \tag{2.1}$$

$$- en_0 E_\parallel - T_e \nabla_\parallel n + \frac{en_0}{\sigma_\parallel} J = 0. \tag{2.2}$$

Here n is the plasma number density, n_0 is the unperturbed plasma density at the resonant magnetic surface, J is the parallel electric current, $E_\parallel = \mathbf{E}\cdot\mathbf{b}$ is the parallel electric field, $\mathbf{b} = \mathbf{B}/B$, $\mathbf{B}$ is the magnetic field, $\nabla_\parallel = \mathbf{b}\cdot\boldsymbol{\nabla}$ is the parallel gradient, T_e is the electron temperature (assumed to be constant),

$$\frac{d_0}{dt} = \frac{\partial}{\partial t} + \mathbf{V}_E \cdot \boldsymbol{\nabla}, \tag{2.3}$$

$\mathbf{V}_E = c\mathbf{b} \times \boldsymbol{\nabla}\phi/B$ is the cross-field velocity, ϕ is the electrostatic potential (related to the perpendicular electric field $\mathbf{E}_\perp$ by $\mathbf{E}_\perp = -\nabla_\perp \phi$), $\nabla_\perp = \boldsymbol{\nabla} - \mathbf{b}\nabla_\parallel$ is the perpendicular gradient, e is the ion charge, c is the speed of light, and $\sigma_\parallel$ is the parallel electric conductivity. The term with T_e in (2.2) describes the effect of the electron pressure gradient.

The ions are described by the ion continuity equation:

$$\frac{d_0 n}{dt} - \frac{c^2}{4\pi e V_A^2}\left(\frac{d_0}{dt}\nabla_\perp^2 \phi - \mu \nabla_\perp^4 \phi\right) = 0. \tag{2.4}$$

Here V_A is the Alfvén velocity, the term with $\nabla_\perp^4 \phi$ describes the effect of ion perpendicular viscosity, $\mu = 0.3\nu_i \rho_i^2$ is the viscosity coefficient (Braginskii 1965), ν_i is the ion collision frequency, and ρ_i is the ion Larmor radius.

Note that on subtracting (2.1) from (2.4), one can obtain the vorticity equation (the current continuity equation) in the form

$$\frac{d_0}{dt}\nabla_\perp^2 \phi - \mu \nabla_\perp^4 \phi - \frac{4\pi V_A^2}{c^2}\nabla_\parallel J = 0. \tag{2.5}$$

Thus, our basic MHD equations are (2.1), (2.2), and (2.5) (or (2.4)).

2.2. *Description of the island magnetic field*

As in paper I, the magnetic island is assumed to be localized near some rational magnetic surface $r = r_s$, where r is the 'radial' coordinate labeling the magnetic surface in a tokamak. The total magnetic field $\mathbf{B}$ is taken in the form

$$\mathbf{B} = B_0 \mathbf{z} - \boldsymbol{\nabla}\psi \times \mathbf{z}, \tag{2.6}$$

where $B_0\mathbf{z}$ is the main (equilibrium) magnetic field at $r = r_s$ and $\mathbf{z}$ is the unit vector determined by $\mathbf{z} = (B_{0\zeta}\boldsymbol{\zeta} + B_{0\theta}\boldsymbol{\theta})/B_0$, with $\boldsymbol{\theta}$ and $\boldsymbol{\zeta}$ the unit vectors along gradients of the poloidal and toroidal angles θ and ζ, respectively; $B_{0\theta}$ and $B_{0\zeta}$ are the poloidal and toroidal components of the equilibrium magnetic field at the magnetic surface $r = r_s$. The magnetic island flux function ψ is given by

$$\psi = \tilde{\psi}\cos\xi - \frac{x^2 B_0}{2L_s}, \tag{2.7}$$

https://doi.org/10.1017/S0022377801001532 Published online by Cambridge University Press

where $x = r - r_s$, L_s is the shear length, $\tilde{\psi}$ is a positive constant characterizing the perturbation amplitude, $\xi = m\theta - l\zeta - \omega t$ is the island cyclic variable, m and l are the poloidal and toroidal mode numbers, and ω is the island rotation frequency.

2.3. *Transition to the 'island variables'*

Instead of the variables $(\mathbf{r}, t)$, let us use the 'island variables' (ψ, ξ, t). In terms of these variables,

$$\nabla_{\parallel} = \frac{k_y}{B_0}\psi_x \frac{\partial}{\partial \xi}, \tag{2.8}$$

$$\frac{d_0}{dt} = \frac{\partial}{\partial t} - \omega\left(\frac{\partial}{\partial \xi} + \frac{\partial \psi}{\partial \xi}\frac{\partial}{\partial \psi}\right) + \frac{c k_y}{B_0}\psi_x\left(\frac{\partial \phi}{\partial \psi}\frac{\partial}{\partial \xi} - \frac{\partial \phi}{\partial \xi}\frac{\partial}{\partial \psi}\right), \tag{2.9}$$

where

$$\psi_x \equiv \frac{\partial \psi}{\partial x} = -\sigma_x\left(\frac{2B_0}{L_s}\right)^{1/2}(\tilde{\psi}\cos\xi - \psi)^{1/2}, \tag{2.10}$$

and

$$k_y = \frac{m}{r_s}, \qquad \sigma_x = \operatorname{sgn} x = \pm 1, \qquad \frac{\partial \psi}{\partial \xi} \equiv \left(\frac{\partial \psi}{\partial \xi}\right)_x.$$

Note also that, starting with the formula

$$E_{\parallel} = -\nabla_{\parallel}\phi + \frac{1}{c}\frac{\partial \psi}{\partial t}, \tag{2.11}$$

one can show that (Smolyakov 1989, 1993)

$$E_{\parallel} = -\nabla_{\parallel}\tilde{\phi}, \tag{2.12}$$

where

$$\tilde{\phi} = \phi - \frac{B_0 \omega x}{c k_y}, \tag{2.13}$$

and x is expressed in terms of (ψ, ξ) by means of (2.7). The function $\tilde{\phi}$ can be called the reduced electrostatic potential. According to Wilson et al. (2000), this function can be interpreted as the electrostatic potential in the island rest frame.

Using (2.13), we reduce (2.9) to (Smolyakov 1989, 1993)

$$\frac{d_0}{dt} = \frac{\partial}{\partial t} + \frac{c k_y}{B_0}\psi_z\left(\frac{\partial \tilde{\phi}}{\partial \psi}\frac{\partial}{\partial \xi} - \frac{\partial \tilde{\phi}}{\partial \xi}\frac{\partial}{\partial \psi}\right). \tag{2.14}$$

2.4. *MHD equations in the approximation of weak dissipation*

Assuming the dissipative terms in (2.1), (2.2), (2.4), and (2.5) to be small, one can develop the method of successive approximations similar to Smolyakov (1989, 1993) based on expansion in series in small dissipation. Then, each perturbed function X entering (2.1)–(2.5) is represented in the form

$$X = X^{(0)} + X^{(1)}, \tag{2.15}$$

where $X^{(0)}$ and $X^{(1)}$ are the ideal and dissipative parts, respectively.

The ideal parts of (2.1), (2.2), and (2.4) are

$$\left(\frac{d_0}{dt}\right)^{(0)} n^{(0)} - \frac{1}{e}\nabla_{\parallel}J^{(0)} = 0, \tag{2.16}$$

https://doi.org/10.1017/S0022377801001532 Published online by Cambridge University Press

$$\nabla_\parallel \left(n^{(0)} - \frac{en_0}{T_e} \tilde{\phi}^{(0)} \right) = 0, \tag{2.17}$$

$$\left(\frac{d_0}{dt} \right)^{(0)} n^{(0)} - \frac{c^2}{4\pi e V_A^2} \left(\frac{d_0}{dt} \right)^{(0)} \nabla_\perp^2 \tilde{\phi}^{(0)} = 0, \tag{2.18}$$

where

$$\left(\frac{d_0}{dt} \right)^{(0)} = \frac{ck_y}{B_0} \psi_x \left(\frac{\partial \tilde{\phi}^{(0)}}{\partial \psi} \frac{\partial}{\partial \xi} - \frac{\partial \tilde{\phi}^{(0)}}{\partial \xi} \frac{\partial}{\partial \psi} \right). \tag{2.19}$$

Similarly, the dissipative parts of (2.2) and (2.5) are of the form

$$-T_e \nabla_\parallel \left(n^{(1)} - \frac{en_0}{T_e} \tilde{\phi}^{(1)} \right) + \frac{en^{(0)}}{\sigma_\parallel} J^{(0)} = 0, \tag{2.20}$$

$$\left(\frac{d_0}{dt} \right)^{(0)} \nabla_\perp^2 \tilde{\phi}^{(1)} + \left(\frac{d_0}{dt} \right)^{(1)} \nabla_\perp^2 \tilde{\phi}^{(0)} - \mu \nabla_\perp^4 \tilde{\phi}^{(0)} - \frac{4\pi V_A^2}{c^2} \nabla_\parallel J^{(1)} = 0, \tag{2.21}$$

where

$$\left(\frac{d_0}{dt} \right)^{(1)} = \frac{\partial}{\partial t} + \frac{ck_y}{B_0} \psi_x \left(\frac{\partial \tilde{\phi}^{(1)}}{\partial \psi} \frac{\partial}{\partial \xi} - \frac{\partial \tilde{\phi}^{(1)}}{\partial \xi} \frac{\partial}{\partial \psi} \right). \tag{2.22}$$

In addition, one can write the dissipative part of (2.1), but it is not needed here.

Let us introduce the operator of averaging over the island magnetic surface $\langle \ldots \rangle$ defined by

$$\langle \ldots \rangle = \oint (\ldots) \frac{d\xi}{\psi_x} \Big/ \oint \frac{d\xi}{\psi_x}. \tag{2.23}$$

We will use this operator outside the separatrix (for $\psi < -\tilde{\psi}$). In this case, $\oint (\ldots)\, d\xi$ means integration over the island cyclic variable equal to $\int_{-\pi}^{\pi} (\ldots)\, d\xi$. Then the averaged parts of (2.20) and (2.21) are

$$\langle J^{(0)} \rangle = 0, \tag{2.24}$$

$$\left\langle \left(\frac{d_0}{dt} \right)^{(0)} \nabla_\perp^2 \tilde{\phi}^{(1)} + \left(\frac{d_0}{dt} \right)^{(1)} \nabla_\perp^2 \tilde{\phi}^{(0)} \right\rangle - \mu \langle \nabla_\perp^4 \tilde{\phi}^{(0)} \rangle = 0. \tag{2.25}$$

These equation are the so-called orthogonality conditions.

2.5. Generalized Rutherford equation

One of our goals is calculation on the polarization current contribution to the stationary generalized Rutherford equation for the island width. According to paper II, in the slab approximation considered, this equation can be represented in the form

$$\tfrac{1}{4} \Delta' + \Delta_p = 0, \tag{2.26}$$

where

$$\Delta_p = \frac{1}{c\tilde{\psi}} \left(\frac{2L_s}{B_0} \right)^2 \int_{\tilde{\psi}+\Delta}^{-\infty} d\psi \oint \frac{J^{(0)} \cos\xi \, d\xi}{(\tilde{\psi} \cos\xi - \psi)^{1/2}}, \tag{2.27}$$

Δ' is the standard parameter of the linear tearing mode theory, and Δ is a positive infinitesimal. Equation (2.27) has the property that the polarization current density is symmetric about the rational surface.

3. Integration and reduction of the ideal parts of the MHD equations

3.1. *Integration of the ideal parts of the MHD equations*

According to Smolyakov (1989, 1993), (2.16)–(2.18) can be integrated. The integration procedure starts with (2.17). One can see that this equation is satisfied if

$$n^{(0)} = \frac{en_0}{T_e}\tilde{\phi}^{(0)} + F(\psi), \tag{3.1}$$

where $F(\psi)$ is an arbitrary function of ψ. Using (3.1) and the expressions (2.8) and (2.10) for the operators $\nabla_\parallel$ and $(d_0/dt)^{(0)}$, we transform (2.16) to

$$\nabla_\parallel (J^{(0)} + ecF'\tilde{\phi}^{(0)}) = 0, \tag{3.2}$$

where the prime indicates the derivative with respect to ψ. Allowing for the orthogonality condition (2.24), we find from (3.2) that the function $J^{(0)}$ is given by

$$J^{(0)} = -ceF'(\tilde{\phi}^{(0)} - \langle\tilde{\phi}^{(0)}\rangle). \tag{3.3}$$

Now we turn to (2.18), and note that this equation is satisfied if

$$\frac{c^2}{4\pi e V_A^2}\nabla_\perp^2\tilde{\phi}^{(0)} + G(\tilde{\phi}^{(0)}) = n^{(0)}, \tag{3.4}$$

where $G(\tilde{\phi}^{(0)})$ is an arbitrary function of $\tilde{\phi}^{(0)}$.

3.2. *Transformation of the integrated ideal equations*

Let us use the definitions

$$\langle n^{(0)}\rangle \equiv N(\psi), \tag{3.5}$$

$$\tilde{n}^{(0)} = n^{(0)} - N(\psi), \tag{3.6}$$

where $N(\psi)$ is the density profile function. In addition, we introduce the electrostatic potential profile function $h(\psi)$ defined by

$$\langle\tilde{\phi}^{(0)}\rangle = -\frac{B_0\omega}{ck_y}h(\psi), \tag{3.7}$$

and the function $\alpha^{(0)}$ characterizing the ideal oscillatory part of the electrostatic potential,

$$\alpha^{(0)} = -\frac{ck_y}{B_0\omega}(\tilde{\phi}^{(0)} - \langle\tilde{\phi}^{(0)}\rangle). \tag{3.8}$$

Then

$$\tilde{\phi}^{(0)} = -\frac{B_0\omega}{ck_y}(h + \alpha^{(0)}). \tag{3.9}$$

It then follows from the averaged part of (3.1) that the function F is related to the profile functions N and h by

$$F = N + \frac{eB_0 n_0\omega}{k_y c T_e}h. \tag{3.10}$$

On the other hand, the oscillatory part of (3.1) yields

$$\tilde{n}^{(0)} = -\frac{eB_0 n_0\omega}{k_y c T_e}\alpha^{(0)}. \tag{3.11}$$

Now we assume that the equilibrium plasma number density is homogeneous and restrict ourselves to the model assumption

$$N = n_0. \tag{3.12}$$

Physically, such an assumption means that we neglect drift effects (cf. Sec. 1).

Allowing for (3.3), (3.10), and (3.12), in terms of $\alpha^{(0)}$ and h, the ideal parallel current is

$$J^{(0)} = \frac{cMn_0\omega^2 h'\alpha^{(0)}}{k_y^2\rho_s^2}, \tag{3.13}$$

where ρ_s is the effective ion Larmor radius given by $\rho_s^2 = T_e/M\omega_B^2$, M is the ion mass, and ω_B is the ion cyclotron frequency.

Taking the averaged part of (3.4) and using (3.5) and (3.9), we find that the function G satisfies the equation

$$\left\langle G\left[-\frac{B_0\omega}{ck_y}(h+\alpha^{(0)})\right]\right\rangle = -n_0 - \frac{cB_0\omega}{4\pi eV_A^2k_y}\langle\nabla_\perp^2(h+\alpha^{(0)})\rangle. \tag{3.14}$$

Evidently, by redefining the argument of G, one can omit the factor $(-B_0\omega/ck_y)$ in this argument.

The oscillatory part of (3.4) leads to the following equation for the function $\alpha^{(0)}$:

$$\alpha^{(0)} - \rho_s^2(\nabla_\perp^2\alpha^{(0)} - \langle\nabla_\perp^2\alpha^{(0)}\rangle) = \rho_s^2(\nabla_\perp^2 h - \langle\nabla_\perp^2 h\rangle) - \frac{k_y cT_e}{eB_0n_0\omega}(G - \langle G\rangle). \tag{3.15}$$

Let us turn to (3.1), (3.3), and (3.4). These equations and all other equations of Secs 3.1 and 3.2 are rigorous. However, (3.14) contains an as-yet unknown function G, which should be found and excluded from our remaining equations by substituting G into (3.15). In this stage, one of the key problems arises: How can this function be found?

One can see that a solution to (3.14) can be found by expanding in a series in the oscillatory part of the electrostatic potential with several terms in the series. On the other hand, one can see from (3.15) that the oscillatory part of the electrostatic potential is a series in $(\rho_s\nabla_\perp)^2$. This is the ground for our statement in Sec. 1 that the solution can be expressed as an infinite series in the above parameter.

3.3. Model expression for G

Having arrived at the conclusion that one cannot find an exact analytical solution of (3.14), we have two possibilities: (1) to abandon completely an attempt to study the effects of finite electron temperature, or (2) to perform such analysis by using some models for the function G. We follow the second approach. The general logic of our modeling is the following: we initially find an expression by expanding in a series in a small parameter, and then consider this expression as a model valid when the expansion parameter may be of the order of unity.

The first step of such a modeling is the following procedure of obtaining a model expression for the function G.

According to (3.15), for small $\rho_s^2\nabla_\perp^2$, the function $\alpha^{(0)}$ is small compared with h. In this case, (3.14) can be solved by an expansion in a series in $\alpha^{(0)}$. In the zeroth-order approximation in $\alpha^{(0)}$, it follows from (3.14) that

$$G = -n_0 - \frac{cB_0\omega}{4\pi eV_A^2k_y}\langle\nabla_\perp^2 h\rangle. \tag{3.16}$$

This equation can be considered as a convenient model for G.

3.4. Model expressions for $\alpha^{(0)}$ and $J^{(0)}$

Substituting (3.16) into (3.15), we find the following equation for $\alpha^{(0)}$:

$$\alpha^{(0)} - \rho_s^2(\nabla_\perp^2 \alpha^{(0)} - \langle \nabla_\perp^2 \alpha^{(0)} \rangle) = \rho_s^2(\nabla_\perp^2 h - \langle \nabla_\perp^2 h \rangle). \tag{3.17}$$

Let us emphasize that (3.17) should be considered as a model equation but not as a rigorous one! Moreover, one can show that this model seems to be reasonable only in neglecting the drift effects. If one allows for drift effects, (3.17) becomes a priori invalid! This equation can be solved by an expansion in a series in ρ_s^2. Keeping only the first two terms of this expansion, one can obtain the following model expression for $\alpha^{(0)}$:

$$\alpha^{(0)} = \alpha^{(0,0)} + \alpha^{(0,2)}, \tag{3.18}$$

where

$$\alpha^{(0,0)} = \rho_s^2(\nabla_\perp^2 h - \langle \nabla_\perp^2 h \rangle), \tag{3.19}$$

$$\alpha^{(0,2)} = \rho_s^4[\nabla_\perp^2(\nabla_\perp^2 h - \langle \nabla_\perp^2 h \rangle) - \langle \nabla_\perp^2(\nabla_\perp^2 h - \langle \nabla_\perp^2 h \rangle)\rangle]. \tag{3.20}$$

We are interested in the case when $\partial/\partial x \gg k_y$. Then (3.19) reduces to

$$\alpha^{(0,0)} = \frac{2B_0\tilde{\psi}}{L_s}\rho_s^2 h''(\cos\xi - \langle \cos\xi \rangle). \tag{3.21}$$

We will use the function $\alpha^{(0,2)}$ only near the separatrix. In this case, one can take

$$\nabla_\perp^2 = \psi_x^2 \frac{\partial^2}{\partial \psi^2}. \tag{3.22}$$

Then (3.20) reduces to

$$\alpha^{(0,2)} = \rho_s^2 h^{IV} \psi_x^4. \tag{3.23}$$

Here we have taken into account that near the separatrix, $\langle \psi_x^2 \rangle$ and $\langle \psi_x^4 \rangle$ vanish.

By means of (3.18), (3.21), and (3.23), we represent (3.13) into the form

$$J^{(0)} = J^{(0,0)} + J^{(0,2)}, \tag{3.24}$$

where

$$J^{(0,0)} = \frac{cB_0^3\omega^2\tilde{\psi}}{2\pi V_A^2 L_s k_y^2} h'h''(\cos\xi - \langle \cos\xi \rangle), \tag{3.25}$$

$$J^{(0,2)} = \frac{1}{4\pi}\rho_s^2 \frac{cB_0^2\omega^2}{V_A^2 k_y^2} h'h^{IV}\psi_x^4. \tag{3.26}$$

Equation (3.25) is the standard expression for the polarization current in the approximation of cold ions and electrons, while (3.26) describes the part of the polarization current due to finite electron temperature. This part has the same structure as that due to finite ion temperature (cf. paper II).

4. Electrostatic potential profile function

Now we turn to analysis of the orthogonality condition (2.25). In the operator $(d_0/dt)^{(0)}$, we neglect the term with $\alpha^{(0)}$ (see (2.19)). This operator then proves to be proportional to $\psi_x \partial/\partial\xi$, so that the contribution of the first term in the angular brackets of (2.25) vanishes. In addition, we neglect the contribution of the term with the operator $(d_0/dt)^{(1)}$ in (2.25), since, first, we use the stationary approximation,

https://doi.org/10.1017/S0022377801001532 Published online by Cambridge University Press

$\partial/\partial t = 0$, and, second, the resistivity is assumed to be sufficiently small. Then (2.25) reduces to

$$\langle \nabla_\perp^4 \tilde{\phi}^{(0)} \rangle = 0. \tag{4.1}$$

Allowing for (3.9) and the above discussion on calculating the function $a^{(0)}$, one can see that (4.1) can be analyzed only by means of a model approach, i.e., by expanding in a series in the small parameter $\rho_s^2 \nabla_\perp^2$. Next, we assume that physically correct results can be obtained by considering this parameter to be of the order of unity in the corresponding terms of the series. Then, using (3.9), (3.18), (3.21), and (3.23), one can transform (4.1) to

$$(A_3 h'')'' + \rho_s^2 A_{5s} h^{VI} + \rho_s^4 A_{7s} h^{VIII} = 0, \tag{4.2}$$

where,

$$A_l = \oint \psi_x^l \, d\xi, \qquad l = 3, 5, 7, \tag{4.3}$$

while the subscript s means that the value A_l is taken at $\psi = -\tilde{\psi}$, i.e., at the separatrix.

We need a solution of (4.2) for h' vanishing for both $\psi \to -\infty$ and $\psi \to -\tilde{\psi}$. We represent

$$h' = \bar{h}' + \hat{h}'. \tag{4.4}$$

The functions $\bar{h}'$ and $\hat{h}'$ are the 'slow' and 'fast' parts of h', respectively. They are defined by

$$(A_3 \bar{h}'')'' = 0, \tag{4.5}$$

$$A_{3s} \hat{h}^{IV} + \rho_s^2 A_{5s} \hat{h}^{VI} + \rho_s^4 A_{7s} \hat{h}^{VIII} = 0. \tag{4.6}$$

The solution of (4.5) vanishing for $\psi \to -\infty$ is

$$\bar{h}' = D \tilde{g}(\psi)/\tilde{g}_s, \tag{4.7}$$

where

$$\tilde{g}(\psi) = \int_{-\infty}^{\psi} \frac{d\psi}{A_3}, \tag{4.8}$$

$\tilde{g}_s \equiv \tilde{g}(-\tilde{\psi})$, and D is the constant of integration. Using the condition that electrostatic potential ϕ is finite as $|x| \to \infty$, one can obtain $D = (2\pi B_0/L_s)^{1/2} \tilde{g}_s$ (see in detail paper II). For $\psi \to -\tilde{\psi}$, the solution (4.7) is finite, $h' \to D$. Then, one should obtain the solution for $\hat{h}'$ vanishing for $\psi \to -\infty$ and tending to $-D$ for $\psi \to -\tilde{\psi}$. Thereby, the function h' will be regularized. In addition, in accordance with (4.6), one can regularize also the first derivative of the function h', obtaining $h'' \to 0$ for $\psi \to -\tilde{\psi}$.

We take

$$\hat{h}' = \exp\left(\frac{\hat{\psi}\kappa_L}{\rho_s}\right), \tag{4.9}$$

where $\hat{\psi} \equiv \psi + \tilde{\psi}$ and κ_L is an as-yet unknown number. Then we obtain from (4.6) that κ_L satisfies the 'characteristic equation'

$$A_{7s}\kappa_L^4 + A_{5s}\kappa_L^2 + A_{3s} = 0. \tag{4.10}$$

Hence it follows that

$$\kappa_{L,\pm}^2 = \frac{A_{5s}}{2A_{7s}}\left[-1 \pm \left(1 - 4\frac{A_{3s}A_{7s}}{A_{5s}}\right)^{1/2}\right]. \tag{4.11}$$

Using (4.3), we find the value $A_{ls}(l = 3, 5, 7)$, and reduce (4.11) to

$$\kappa_{L,\pm}^2 = \frac{7}{48}\frac{L_s}{B_0\tilde{\psi}}\left[-1 \pm i\sqrt{\frac{23}{7}}\right]. \tag{4.12}$$

Since $\mathrm{Im}\,\kappa_{L,\pm}^2 \neq 0$, there are two localized solutions of the form (4.9). By combining these solutions, one can construct the function h' satisfying the conditions

$$\lim_{\psi\to-\tilde{\psi}} h' = 0, \qquad \lim_{\psi\to-\tilde{\psi}} h'' = 0. \tag{4.13}$$

Similarly to paper II, in this case, the function $\hat{h}'$ is given by

$$\hat{h}' = -\left\{D_1 \exp\left[\frac{\hat{\psi}(\hat{\kappa}_L + ik_L)}{\rho_s}\right] + D_2 \exp\left[\frac{\hat{\psi}(\hat{\kappa}_L - ik_L)}{\rho_s}\right]\right\}. \tag{4.14}$$

Here, D_1 and D_2 are given by

$$D_1 = \frac{D}{2}\left[1 + \frac{i}{k_L}\left(\hat{\kappa}_L - \frac{1}{\tilde{g}_s A_{3s}}\right)\right], \tag{4.15}$$

$$D_2 = \frac{D}{2}\left[1 - \frac{i}{k_L}\left(\hat{\kappa}_L - \frac{1}{\tilde{g}_s A_{3s}}\right)\right], \tag{4.16}$$

$$k_L = \kappa_{L0}\cos\tfrac{1}{2}\delta, \qquad \hat{\kappa}_L = \kappa_{L0}\sin\tfrac{1}{2}\delta,$$

where

$$\kappa_{L0} = \frac{7}{48}\frac{L_s}{B_0\tilde{\psi}}, \qquad \delta = \cos^{-1}\left[-\left(\frac{7}{30}\right)^{1/2}\right].$$

Equations (4.4), (4.7), and (4.14) describe the regularized velocity profile, i.e., the regularized electrostatic potential profile function.

5. Polarization current contribution to the generalized Rutherford equation

For $J^{(0)}$ given by (3.24)–(3.26), the value of Δ_p defined by (2.27) can be calculated similarly to paper II. Then, one can find that the value $J^{(0,2)}$ does not contribute to Δ_p. Therefore, the resulting expression for Δ_p proves to be the same as in the case of nonregularized magnetic islands. It is given by (for details, see paper II)

$$\Delta_p = \frac{4\pi}{3}\frac{\omega^2 L_s^2}{V_A^2 k_y^2 w^3}[g^2(1) - I], \tag{5.1}$$

where $w = 2(\tilde{\psi}L_s/B_0)^{1/2}$ is the magnetic island halfwidth, $g(1) = 0.869$, and $I = 0.229$. According to (5.1),

$$\Delta_p > 0, \tag{5.2}$$

i.e., the polarization current is destabilizing.

https://doi.org/10.1017/S0022377801001532 Published online by Cambridge University Press

6. Conclusions

Neglecting drift effects, we have shown that, in addition to the hyperviscosity, the effect of a parallel electron pressure gradient can lead to regularization of rotating magnetic islands. This effect seems to be more important than that of hyperviscosity in the case when the electron temperature is larger than the ion temperature.

The above discussion, together with papers I and II, indicates that the magnetic island theory generally accepted at present and dealing with a discontinuous localized velocity profile does not need to be revised in the sense that would follow from the paper by Waelbroeck and Fitzpatrick (1997), who rejected such profiles and suggested an alternative concept of magnetic islands with nonlocalized velocity profiles. The latter profiles could be of interest in the problem of magnetic islands in a plasma flow with sheared velocity characterized by some Alfvén Mach number. It is known that a significant shear of the equilibrium plasma velocity appears in the internal transport barrier (ITB) zone (see, e.g., Staebler et al. 1997). In this respect, an investigation of the possibility of magnetic islands existing in this zone seems to be important. Apparently, investigations of such a kind have not yet been carried out. Note also that initially a nonlocalized velocity profile was predicted in a numerical simulation by Parker (1992).

We have considered the polarization current contribution to the generalized Rutherford equation for the island width, assuming the island to be regularized by the above effect of a parallel electron pressure gradient, and have shown that this contribution is the same as that for an island with a nonregularized (discontinuous) velocity profile. Thereby, we have shown that the above-mentioned rule of paper II is also valid in the case considered. As in the case of hot ions, this contribution is destabilizing.

Note that the case where the electron temperature is essentially larger than the ion temperature can be realized in electron cyclotron resonance heating (ECRH). Such heating is used, in particular, in the TCV tokamak (Sauter et al. 2000). Observation of magnetic islands in TCV was recently reported by Reimerdes et al. (2000), while initially the neoclassical tearing modes in discharges with much higher electron temperature were observed in COMPASS-D (Gates et al. 1997; Zohm et al. 1997).

Finally, let us discuss the situation where the electron and ion temperatures, T_e and T_i are of the same order. One can find that, to generalize (4.2) for the case of arbitrary T_i/T_e, one should substitute in this equation

$$\rho_s^2 \rightarrow \rho_s^2 \left(1 - \frac{T_i}{T_e} c_L\right), \tag{6.1}$$

$$\rho_s^4 \rightarrow \rho_s^4 \left(1 - \frac{T_i^2}{T_e^2} d_L\right), \tag{6.2}$$

where, according to equation (79) of paper I, $c_L = \frac{5}{3}$ and $d_L = \frac{553}{300}$. Similarly, in the characteristic equation (4.10), one should change

$$A_{7s} \rightarrow A_{7s} \left(1 + \frac{T_i^2}{T_e^2} d_L\right), \tag{6.3}$$

$$A_{5s} \rightarrow A_{5s} \left(1 + \frac{T_i}{T_e} c_L\right). \tag{6.4}$$

https://doi.org/10.1017/S0022377801001532 Published online by Cambridge University Press

Then, one can make sure that a localized function h' satisfying the conditions (4.13) exists for arbitrary T_i/T_e.

For a finite ratio T_i/T_e, one should also take into account the dispersion additions to the polarization current dependent on T_i. Then, allowing for equation (37) of paper II, the relation (3.26) for $J^{(0,2)}$ is modified as follows:

$$\rho_s^2 \to \rho_s^2 \left(1 + \frac{3}{4}\frac{T_i}{T_e}\right). \tag{6.5}$$

This means that the structure of $J^{(0,2)}$ for finite T_i/T_e is the same as for $T_i = 0$. Meanwhile, according to Sec. 5, the function $J^{(0,2)}$ of the form (3.26) does not contribute to Δ_p. Therefore, the same conclusion is valid for arbitrary T_i/T_e.

Thereby, it is clear that the analysis given in the present paper, together with that of papers I and II, is exhaustive for arbitrary T_i/T_e.

Acknowledgements

We are grateful to A. I. Smolyakov, V. D. Pustovitov, I. C. Nascimento and R. M. O. Galvão for useful discussions. We appreciate useful comments from the anonymous referee.

This work was supported in part by the Russian Foundation for Basic Research, Project No. 96-15-96815 (under the program 'Leading Scientific Schools'), Grant No. 2-23 of the Georgian Academy of Sciences, the Research Support Foundation of the State of São Paulo (FAPESP), the National Council of Scientific and Technological Development (CNPq), and an Excellence Research Programs (PRONEX) RMOG 50/70 Grant from the Ministry of Science and Technology, Brazil.

References

Braginskii, S. I. 1965 Transport processes in plasma. In: *Reviews of Plasma Physics*, Vol. 1 (ed. M. A. Leontovich). New York: Consultants Bureau, pp. 205–309.

Gates, D. A., Lloyd, B., Morris, A. W., McArdle, G., O'Brien, M. R., Valovic, M., Warrick, C. D. and Wilson, H. R. 1997 Neoclassical islands on COMPASS-D. *Nucl. Fusion* **37**, 1593–1606.

Kuvshinov, B. N., Rem, J., Schep, T. J. and Westerhof, E. 2001 Electron vortices in magnetized plasmas. *Phys. Plasmas* **8**, 3232–3250.

Lakhin, V. P., Makurin, S. V., Mikhailovskii, A. B. and Onishchenko, O. G. 1987 Dispersion ion-drift hydrodynamics. *J. Plasma Phys.* **38**, 387–405.

Mikhailovskii, A. B. 1992 *Electromagnetic Instabilities in an Inhomogeneous Plasma*. Bristol: Institute of Physics, p. 147.

Mikhailovskii, A. B. and Tsypin, V. S. 1971 Transport equations and gradient instabilities in a high pressure collisional plasma. *Plasma Phys.* **13**, 785–798.

Mikhailovskii, A. B., Pustovitov, V. D., Tsypin, V. S. and Smolyakov, A. I. 2000a Regularized magnetic islands. I. Hyperviscosity and profile functions. *Phys. Plasmas* **7**, 1204–1213.

Mikhailovskii, A. B., Pustovitov, V. D., Smolyakov, A. I. and Tsypin, V. S. 2000b Regularized magnetic islands. II. The role of polarization current. *Phys. Plasmas* **7**, 1214–1223.

Mikhailovskii, A. B., Konovalov, S. V., Pustovitov, V. D. and Tsypin, V. S. 2000c Mechanism of viscosity effect on magnetic island rotation. Report NIFS-634, National Institute for Fusion Science, Nagoya, Japan, NIFS Series.

Mikhailovskii, A. B., Konovalov, S. V., Pustovitov, V. D. and Tsypin, V. S. 2000d An approach to calculation of magnetic island rotation frequency. *Phys. Plasmas* **7**, 2530–2538.

Mikhailovskii, A. B., Konovalov, S. V. and Tsypin, V. S. 2001a Hyperviscosity of magnetized plasma. *Dokl. Phys.* **46**, 153–156.

Mikhailvoskii, A. B., Konovalov, S. V., Pustovitov, V. D., Tsypin, V. S., Galvão, R. M. O. and Nascimento, I. C. 2001b Electron drift effects on magnetic islands. *Phys. Plasmas* **8**, 4020–4029.

Parker, R. D. 1992 Nonlinear mechanism for the suppression of error field magnetic islands by plasma flow. In: *Proceedings of 19th EPS Conference on Controlled Fusion and Plasma Physics, Innsbruck*, Vol. 1, pp. 427–430.

Reimerdes, H., Goodman, T., Pochelon, A. and Sauter, O. 1998 Neoclassical tearing modes in TCV. Presented at the Joint Varenna–Lausanne International Workshop on Theory of Fusion Plasma, Villa Monastero, Varenna, Italy, 31 August–4 September 1998.

Sauter, O., LaHaye, R. J., Chang, Z., Gates, D. A., Kamada, Y. and Zohm, H. 1997 Beta limits in long-pulse tokamak discharges. *Phys. Plasmas* **4**, 1654–1664.

Sauter, O., Henderson, M. A., Hofmann, F., Goodman, T., Alberti, S. and Angioni, C. 2000 Steady-state fully noninductive current driven by electron cyclotron waves in a magnetically confined plasma. *Phys. Rev. Lett.* **84**, 3322–3325.

Smith, S. A. and Hammett, G. W. 1997 Eddy viscosity and hyperviscosity in spectral simulations of 2D drift wave turbulence. *Phys. Plasmas* **4**, 978–990.

Smolyakov, A. I. 1989 *Sov. J. Plasma Phys.* **15**, 667–670.

Smolyakov, A. I. 1993 Nonlinear evolution of tearing modes in inhomogeneous plasma. *Plasma Phys. Contr. Fusion* **35**, 657–687.

Staebler, G. M., Waltz, R. E. and Wiley, J. C. 1997 The role of rotation in tokamak internal transport barriers. *Nucl. Fusion* **37**, 287–291.

Tamm, I. E. 1961 Theory of the magnetic thermonuclear reactor. In: *Plasma Physics and the Problem of Controlled Thermonuclear Reactions*, Vol. 1 (ed. M. A. Leontovich), Oxford: Pergamon, pp. 1–20.

Waelbroeck, F. L. and Fitzpatrick, R. 1997 Rotation and locking of magnetic islands. *Phys. Rev. Lett.* **78**, 1703–1706.

Wilson, H. R., Connor, J. W., Hastie, R. J. and Hegna, C. C. 1996a Threshold for neoclassical magnetic islands in a low collision frequency tokamak. *Phys. Plasmas* **3**, 248–265.

Wilson, H. R., Alexander, M., Connor, J. W., Edwards, A. M., Gates, D., Gruber, O. et al. 1996b The collisionality dependence of tokamak beta-limits. *Plasma Phys. Contr. Fusion* **38**, A149–A163.

Wilson, H. R., Connor, J. W., Gimblett, C. G., Hastie, R. J., and Waelbroeck, F. L. 2000 Theoretical understanding of tokamak pressure limits. In: *Proceedings of 18th IAEA Fusion Energy Conference, Sorrento, Italy*, Paper IAEA-CN-77/TH3/5.

Zohm, H., Gates, D. A., Wilson, H. R., Gatenbein, G., Gruber, O., Gunter, S. et al. 1997 Neoclassical MHD in ASDEX upgrade and COMPASS-D. *Plasma Phys. Contr. Fusion* **39**, B237–B246.