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ARTICLES

PATENT LENGTH AND PRICE REGULATION IN AN R&D GROWTH MODEL WITH MONOPOLISTIC COMPETITION

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This paper considers the effects of patent length and price regulation in an R&D growth model with variety expansion. Innovation requires lower bounds on patent length and price. Increasing patent duration promotes growth; increasing the cap on the price of patented products promotes growth below the monopoly-pricing level. Each policy instrument can raise welfare unless excessively used, and their welfare ranking depends on parameterizations. It is desirable, on welfare grounds, to limit patent protection along both dimensions, namely by limiting patent length and capping the price of patented products. Such limits raise welfare despite reducing the growth rate.

Keywords: Innovation, Growth, Patents, Price Regulation

1. INTRODUCTION

R&D activities for innovation have emerged as the major driving force for improvements in standards of living and have been the focus of a large number of studies in the literature on economic growth, especially since Schumpeter (1942). A key challenge in dealing with costly innovation is to balance dynamic vs. static efficiency when granting market powers to innovators in the form of patent protection. At one extreme, with permanent patent protection and the resulting monopoly pricing, there are strong incentives for innovation, but pricing above

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marginal costs discourages the demands for goods and services originating from innovation. At the other extreme, without patent protection, competition would lead to marginal-cost pricing and there would be no incentive to carry out costly innovations in the first place.

Numerous studies have attempted to find the optimal patent length. Without long-run growth, early studies on the optimal patent length typically used either partial equilibrium frameworks [e.g., Nordhaus (1969); Gilbert and Shapiro (1990); Klemperer (1990)] or general equilibrium models [e.g., Judd (1985); Veall (1992)]. More recent studies examine the optimal patent length in R&D-based endogenous growth models [e.g., Horowitz and Lai (1996); Michel and Nyssen (2002); Kwan and Lai (2003); Futagami and Iwaisako (2007)].[1] However, the findings are inconclusive. For example, Klemperer (1990) shows in a partial equilibrium model that, depending on the structure of preferences, the length of patents should be either very long or very short. In general equilibrium models, Judd (1985) finds that the optimal patent length is infinite, whereas Veall (1992) shows that it is finite. With endogenous growth, Horowitz and Lai (1996) find that both the growth- and welfare-maximizing patent lengths should be finite in a quality-ladder model. Similarly, Kwan and Lai (2003) and Futagami and Iwaisako (2007) find that there exists a finite welfare-maximizing patent length in a variety-expansion model. However, Michel and Nyssen (2002) show that these lengths are finite in a variety-expansion model only if the knowledge spillover effect in the R&D sector is weak enough.

Compared to patent length, price regulation of patented products has received little attention in these R&D-based growth models.[2] Rare exceptions include Evans et al. (2003), in which a permanent monopoly right is granted to all innovators over the supply of their invented goods and services. The literature has been silent on whether or not capping prices of patented products is better than limiting patent duration, or whether both should go hand in hand to maximize social welfare.

Theoretically speaking, price regulation and patent length should be closely related to each other as two dimensions of patent protection when they are used to maximize social welfare. On one hand, under patent protection, monopoly pricing would emerge and might thus call for price regulation; the optimal level of the regulated price might vary with the duration of patents. On the other hand, the extent to which prices of patented products are regulated would affect the rate of return on R&D investment and thus influence the optimal patent duration.

In reality, we do observe that governments in many countries use limits on patent length and price regulation together in certain industries. For example, in the United States, Canada, and many European countries, finite patent lengths and price caps are used simultaneously in the pharmaceutical industries.[3] Therefore, it is intriguing as well as relevant to theoretically investigate whether combining price regulation and patent duration can do a better job in terms of growth and welfare than using them separately. To the best of our knowledge, no studies have considered the interactions of price regulation and patent duration for promoting growth or improving welfare.

The purpose of this paper is to fill in this gap between theory and practice. We consider the effects of patent length and price regulation in an R&D growth model with variety expansion along the balanced growth path. Innovation requires lower bounds on patent length and the price. Beyond these lower bounds, increasing patent duration always promotes growth; increasing the cap on the price of patented products promotes growth below the monopoly-pricing level. Each policy instrument can raise welfare unless excessively used, and their welfare ranking depends on parameterizations. It is desirable, on welfare grounds, to limit patent protection along both dimensions, namely by limiting patent length and capping the price of patented products. Such limits raise welfare despite reducing the growth rate.

The remainder of this paper proceeds as follows. The next section sets up the model, solves the problems faced by innovators, firms, and households, and defines the general equilibrium. Section 3 derives the growth-maximizing patent length and regulated price level. Section 4 provides the welfare-maximizing patent length and regulated price level. Section 5 concludes.

2. THE MODEL

The basic framework with continuous time in this paper builds on a stylized R&D model in Barro and Sala-i-Martin (1995, Ch. 6), in which economic growth is driven by technological progress resulting from intentional investment in R&D that creates new varieties of intermediate goods. We extend it by splitting the intermediate-goods sector into two parts: the one under patent protection behaves as in the original model, whereas the other with expired patents becomes perfectly competitive. The price level of intermediate goods under patent protection may be regulated, whereas the price level of intermediate goods without patent protection is determined competitively by the market.

The household sector is standard. It consists of a continuum of identical infinitely lived households with a mass $L \in (0, \infty)$. Every household derives utility from consumption $c \in R_+$ in an infinite horizon:

$$U_0 = \int_0^\infty \left(\frac{c^{1-\theta} - 1}{1 - \theta} \right) e^{-\rho t} dt, \quad \theta > 0, \quad \rho > 0, \tag{1}$$

where ρ is the rate of time preference, $1/\theta$ is the elasticity of intertemporal substitution, and t represents time. Throughout the paper, all time subscripts are omitted for ease of notation. Each household uses one unit of labor time inelastically to work and has a budget constraint

$$\dot{a} = ar + w - c, \tag{2}$$

where $a \in R$ is the amount of asset, $\dot{a}$ the rate of change in a over time, $r \in R_+$ the interest rate, and $w \in R_+$ the wage rate. Solving this standard utility-maximization problem yields the growth rate of consumption:

$$g \equiv \dot{c}/c = (r - \rho)/\theta. \tag{3}$$

The transversality condition is $\lim_{t\to\infty} a \cdot \exp(-rt) = 0$; i.e., neither debt nor asset should be left at the end of the planning horizon.

Partitioning the intermediate sector into monopolistic and competitive subsets (with or without valid patents respectively) will differentiate the final and intermediate sectors in our model from their usual environment in the literature. We describe these sectors in the following.

2.1. Final-Good Production

A final good is produced competitively by a large number of identical firms using labor and intermediate goods. At the beginning of each period, there are $N \in R_+$ types of intermediate goods available. A firm i uses $X_{ij} \in R_+$ units of intermediate good j and $L_i \in R_+$ units of labor to produce $Y_i \in R_+$ units of the final good according to

$$Y_i = AL_i^{1-\alpha} \int_0^N X_{ij}{}^{\alpha} dj, \quad A > 0, \quad 0 < \alpha < 1, \tag{4}$$

where A is a productivity parameter and α measures the importance of intermediate good j relative to labor in the final production. There are two types of intermediate goods: a competitive type without patent protection X_{ij}^c with $j \in [0, N^c]$ and a monopolized (or regulated) type under patent protection X_{ij}^m with $j \in [N^c, N]$, where $N \geq N^c \geq 0$. Because by symmetry $X_{ij}^c = X_i^c$, $\forall j \in [0, N^c]$, and $X_{ij}^m = X_i^m$, $\forall j \in [N^c, N]$, in equilibrium the production function in (4) becomes

$$Y_i = AL_i^{1-\alpha} \left[N^c \left(X_i^c\right)^{\alpha} + (N - N^c)\left(X_i^m\right)^{\alpha}\right], \tag{5}$$

where output growth is driven by expanding the variety of intermediate goods N (to be described later).

The profit function of firm i in the final-good sector is defined as

$$\Pi_i = AL_i^{1-\alpha} \int_0^N X_{ij}{}^{\alpha} dj - wL_i - \int_0^N P_j X_{ij} dj, \tag{6}$$

where the price of the final good is normalized to unity and $P_j \in R_+$ stands for the price of intermediate good j measured in units of the final good. Also, one unit of an available intermediate good can be produced from one unit of the final good (i.e., a unit marginal cost).

In the competitive final sector, factors are paid their marginal products: $\partial Y_i/\partial X_{ij} = P_j$ and $\partial Y_i/\partial L_i = w$. Theoptimality condition $\partial Y_i/\partial X_{ij} = P_j$

gives firm i's demand and the aggregate demand for intermediate good j, X_{ij} and X_j, respectively:

$$X_{ij}^k = L_i(\alpha A/P_j)^{1/(1-\alpha)} \quad \text{and} \quad X_j^k = L(\alpha A/P_j)^{1/(1-\alpha)}, \quad \text{for } k = c, m, \tag{7}$$

where

$$P_j = \begin{cases} 1, & \text{if } j \in [0, N^c] \\ P \geq 1, & \text{if } j \in [N^c, N]. \end{cases}$$

For $P_j < 1$, the unit marginal cost would not be compensated in the production of intermediate goods and thus the economy would have no production at all, a trivial case that will be ignored in this paper.

Similarly, $\partial Y_i/\partial L_i = w$ gives firm i's demand for labor L_i,

$$L_i = (1-\alpha)Y_i/w, \tag{8}$$

where the aggregate demand for labor should be equal to the aggregate supply of labor, i.e., $\sum_i L_i = L$. As a result, we have

$$L = (1-\alpha)Y/w. \tag{9}$$

2.2. Expansion of the Variety of Intermediate Goods

Technologies for new intermediate goods are discovered through R&D investment. To simplify our analysis, we adopt some standard assumptions from related work. First, investing η units of the final good creates a new type of intermediate good. Once the innovation is made, a patent with duration T is granted, during which the innovator has a monopoly right over the production and sale of his newly invented intermediate good. Finally, there is free entry in the R&D sector.

With the monopoly right for duration T, the value of a new technology equals the discounted present value of the gross profit from producing a new intermediate good:

$$V_t(P_j) = \int_t^{t+T} (P_j - 1)X_j^m e^{-r(v-t)}dv = \frac{(P_j-1)X_j^m(1-e^{-rT})}{r}, \tag{10}$$

where r is the (stationary) interest rate.[4] Without any state variable involved in (7) and (10), the problem $\max_{P_j} V_t$ subject to price regulation $P_j \leq P$ is solved by

$$P_j = \min\left\{P, \max_{P_j}\left[(P_j-1)X_j^m\right]\right\} = \min\left\{P, \max_{P_j}\left[(P_j-1)L(\alpha A/P_j)^{\frac{1}{1-\alpha}}\right]\right\}. \tag{11}$$

Equation (11) leads to a stationary monopoly pricing rule:

$$P_j = \begin{cases} P & \text{if } P \leq 1/\alpha, \\ 1/\alpha & \text{if } P > 1/\alpha. \end{cases} \tag{12}$$

That is, if the price cap P on patented intermediate goods is set at or below the monopoly price $1/\alpha$ in (12), then it is binding; if the price cap is set above the monopoly price, then it is not binding any more and the monopoly price applies. For the purpose of our analysis, we ignore the latter case in the rest of the paper. So long as the price cap is set above the marginal cost ($P > 1$), there is a positive markup on the unit marginal cost.

From (7) and (12), the equilibrium quantity of an intermediate good under patent protection, X_j^m, with binding price regulation is determined as

$$X_j^m = X^m \equiv L(\alpha A/P)^{1/(1-\alpha)} \quad (P \text{ binding}). \qquad \textbf{(13)}$$

This equilibrium quantity of X^m is constant over time and the same for all patented-protected intermediate goods with binding price regulation. With free entry in the R&D sector, we have a zero-profit condition for R&D: $\eta = V_t = (P-1)X^m(1-e^{-rT})/r$ (with binding price regulation). This condition requires

$$r = \frac{(P-1)X^m(1-e^{-rT})}{\eta} = \frac{(P-1)L(\alpha A/P)^{1/(1-\alpha)}(1-e^{-rT})}{\eta}. \qquad \textbf{(14)}$$

We can now see that the rate of return on R&D investment depends on the duration of patents (T) as well as on the regulated level of prices for intermediate goods under patent protection (P). Moreover, given stationary patent instruments (P, T), a time-invariant interest rate r can indeed be determined in (14), as expected.

2.3. Equilibrium

The general equilibrium of the model with patent protection is defined as follows:[5]

DEFINITION 1. *A competitive equilibrium given patent protection* (P, T), *initial stock* a_0, *and initial variety* N_0 *is a set of allocations* $(a, c, N^c, N, X^m, X^c, L, Y)$ *and prices* (r, w) *that satisfy the household budget constraint (2), utility maximization (3), the final goods technology (4) or (5), firm profit maximization (7) and (8), innovator profit maximization under patent protection and free entry (12) and (14), competitive pricing* $P_j = 1$ *for old intermediate goods* X_j *without patent protection with* $j \in [0, N^c]$, *and markets clearing under symmetry across intermediate goods* $\sum_i L_i = L$; $\sum_i X_{ij}^m = X_j^m = X^m$ *for* $j \in [N^c, N]$; $\sum_i X_{ij}^c = X_j^c = X^c$ *for* $j \in [0, N^c]$, *whereby* $N \geq N^c$; *and* $Lc = Y - \eta\dot{N} - N^c X^c - (N - N^c)X^m$.

In the next two sections, we will investigate the patent length and price regulation that maximize the growth rate and welfare, respectively.

3. GROWTH-MAXIMIZING PATENT LENGTH AND PRICE REGULATION

To investigate the growth-maximizing patent length and price regulation, we first characterize the balanced growth equilibrium path, where output Y, consumption

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C, and the number of intermediate goods N all grow at the same constant rate g. In addition, the proportional allocations of labor and output are stationary.

The balanced growth rate is determined by rewriting (3) as $r = \theta g + \rho$ and substituting it into (14) for r:

$$\frac{\theta g + \rho}{1 - e^{-(\theta g+\rho)T}} = \frac{(P-1)L(\alpha A/P)^{1/(1-\alpha)}}{\eta}, \qquad \textbf{(15)}$$

where the growth rate g is an implicit function of the patent length T and the price of patent-protected intermediate goods P, namely $g(P, T)$.

We now examine the growth effects of patent duration and price regulation. For $P \geq 1$, the marginal cost of producing intermediate goods can be fully compensated for, and thus all existing intermediate goods remain in production given any $T \geq 0$, implying that $g \geq 0$. The boundary between the combinations of (P, T) with positive and zero growth can be found by setting $r = \rho$ in (14), because $g = (r - \rho)/\theta$ in (3):

$$\mathcal{B} = \left\{(P,T) \mid (P-1)P^{1/(\alpha-1)}(1-e^{-\rho T}) = \eta\rho/[L(\alpha A)^{1/(1-\alpha)}]\right\}.$$

Using this relationship between P and T for any point in $\mathcal{B}$, we can define the lower bound of patent length $\hat{T}(P)$ for a given price cap $P > 1$ as

$$\hat{T}(P) = \frac{1}{\rho}\ln\left[\frac{L(P-1)P^{1/(\alpha-1)}(\alpha A)^{1/(1-\alpha)}}{L(P-1)P^{1/(\alpha-1)}(\alpha A)^{1/(1-\alpha)} - \rho\eta}\right]. \qquad \textbf{(16)}$$

It will soon become clear that positive growth will occur beyond this lower bound.

Some features of $\mathcal{B} \subset R^2_+$ are given as follows:

LEMMA 1. *Suppose that* $\rho < (L/\eta)(1/\alpha - 1)\alpha^{1/(1-\alpha)}(\alpha A)^{1/(1-\alpha)}$. *For* $(P, T) \in \mathcal{B} \subset R^2_+$ *and for a binding price cap* $1 \leq P \leq 1/\alpha$, (i) *$\mathcal{B}$ is nonempty;* (ii) $P > 1$*;* (iii) $dT/dP < 0$ *and* $d^2T/dP^2 > 0$ *for* $1 < P < 1/\alpha$*;* (iv) *the minimum length of patents, denoted as* $T_{\min}$*, is associated with* $P = 1/\alpha$ *as follows:*

$$T_{\min} \equiv \hat{T}\left(\frac{1}{\alpha}\right) = \frac{1}{\rho}\ln\left[\frac{L(1/\alpha-1)\alpha^{1/(1-\alpha)}(\alpha A)^{1/(1-\alpha)}}{L(1/\alpha-1)\alpha^{1/(1-\alpha)}(\alpha A)^{1/(1-\alpha)} - \rho\eta}\right] \in (0,\infty);$$

and (v) *for any given* $T \in (T_{\min}, \infty]$*, there exists a* $\underline{P}(T) \in (1, 1/\alpha)$ *such that* $(\underline{P}(T), T) \in \mathcal{B}$.

Proof. From (16) and by the construction of $\mathcal{B}$, at $P = 1/\alpha$ we have $\tilde{T}(1/\alpha) > 0$ under the condition $\rho < (L/\eta)(1/\alpha - 1)\alpha^{1/(1-\alpha)}(\alpha A)^{1/(1-\alpha)}$. This is a valid point in $\mathcal{B}$, establishing (i). Define $F(P, T) \equiv (P-1)P^{1/(\alpha-1)}(1-e^{-\rho T})$ and rewrite

$$\mathcal{B} = \left\{(P,T) \mid F(P,T) = \eta\rho/[L(\alpha A)^{1/(1-\alpha)}]\right\}.$$

Because $F(P, T)$ has to be positive for any pair $(P, T) \in \mathcal{B}$ and because $T \geq 0$, we must have $P > 1$, establishing (ii).

Because $F(P, T)$ is equal to a constant for any $(P, T) \in \mathcal{B}$, $dT/dP = -F_P/F_T$. Because $P > 1$ and because $T \geq 0$, we have $F_T = (P-1)P^{1/(\alpha-1)}\rho e^{-\rho T} > 0$, $1 - e^{-\rho T} > 0$, and sign F_P = sign $(1/\alpha - P)$. Thus, sign dT/dP = sign $[-(1/\alpha - P)/F_T]$ = sign $(P - 1/\alpha) < 0$ for $P \in (0, 1/\alpha)$. Equivalently, $\tilde{T}'(P) < 0$ for $P \in (1, 1/\alpha)$ from (16):

$$\hat{T}'(P) = \frac{\eta(\alpha P - 1)}{(1-\alpha)(P-1)P[L(P-1)P^{1/(\alpha-1)}(\alpha A)^{1/(1-\alpha)} - \rho\eta]} < 0, \quad \text{if } 1 < P < \frac{1}{\alpha}. \tag{17}$$

Further, $d^2T/dP^2 > 0$ for $P \in (1, 1/\alpha)$ because

$$\begin{aligned}\hat{T}''(P) &= \frac{\eta}{(1-\alpha)\{(P-1)P[L(P-1)P^{1/(\alpha-1)}(\alpha A)^{1/(1-\alpha)} - \rho\eta]\}^2} \\ &\quad \times \{[(2P-1)(1-\alpha P) + \alpha(P-1)P] \\ &\quad \times [L(P-1)P^{1/(\alpha-1)}(\alpha A)^{1/(1-\alpha)} - \rho\eta] \\ &\quad + (2P-1)(1-\alpha P)^2 L(P-1)P^{1/(\alpha-1)}(\alpha A)^{1/1-\alpha}/(1-\alpha)\} \\ &> 0, \quad \text{if } 1 < P < \frac{1}{\alpha}.\end{aligned} \tag{18}$$

These complete Part (iii).

Following (iii), at $P = 1/\alpha$ we must have the smallest T, denoted as $T_{\min}$, that can maintain $F(1/\alpha, T) = \rho/[(L/\eta)(\alpha A)^{1/(1-\alpha)}]$ such that $(1/\alpha, T_{\min})$ lies in $\mathcal{B}$. The condition $\rho < (L/\eta)(1/\alpha - 1)\alpha^{1/(1-\alpha)}(\alpha A)^{1/(1-\alpha)}$ implies that $T_{\min} > 0$. Obviously, $T_{\min} < \infty$. Part (iv) follows. For any $T \in (T_{\min}, \infty)$, from (16) or from parts (i)–(iv) there exists a unique $1 < \underline{P} < 1/\alpha$ such that $(\underline{P}(T), T) \in \mathcal{B}$. See Figure 1 for illustration. ■

The condition $\rho < (L/\eta)(1/\alpha - 1)\alpha^{1/(1-\alpha)}(\alpha A)^{1/(1-\alpha)}$ means that positive growth is possible, so that $\mathcal{B}$ is not empty, at least at the monopoly price. According to Lemma 1, when the length of patents is at its minimum level to maintain $r = \rho$ (i.e., zero growth), the corresponding price has a unique value equal to the monopoly price. The minimum patent length is essential because if it is too low then there is no incentive to create and produce new intermediate goods. Beyond its minimum level, the patent duration depends negatively on the capped price in such a way as to maintain zero growth $r = \rho$. Intuitively, a decline in the price cap from the monopoly pricing level reduces the rate of return on R&D investment, r, whereas a rise in patent duration raises the rate of return. Starting from the situation with zero growth, a decline in the price cap from the monopoly level $1/\alpha$ reduces the rate of return, and requires a counteracting rise in patent duration at a monotonically increasing rate. The exact responses of r to changes in both the price level and patent length will be given later.

With the aid of Lemma 1, we also define a subset of R_+^2: $\mathcal{S} = \{(P, T)|\ T > T_{\min}$ and $\underline{P}(T) < P(T) \leq 1/\alpha\}$ for positive growth. The features of $\mathcal{S}$ and the

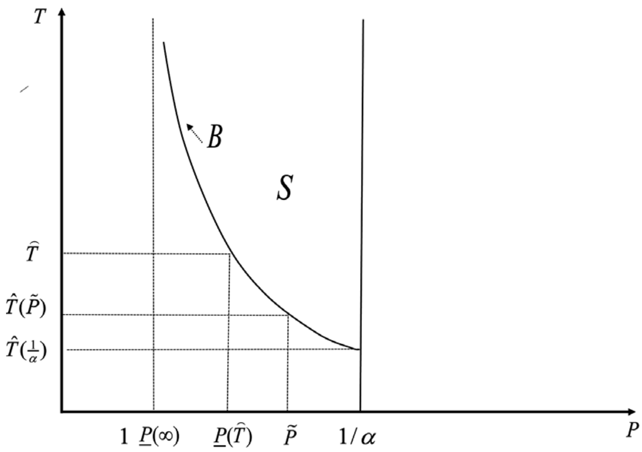

FIGURE 1. Domain for positive growth.

growth effects of patent duration and price regulation are given in Propositions 1 and 2.

PROPOSITION 1. *Suppose that* $\rho < (L/\eta)(1/\alpha - 1)\alpha^{1/(1-\alpha)}(\alpha A)^{1/(1-\alpha)}$. *For* $(P, T) \in \mathcal{S} \subset R^2_+$, (i) $\mathcal{S}$ *is nonempty and convex with the boundary* $\mathcal{B}$ *and* $P = 1/\alpha$*;* (ii) $g(P, T) > 0$*;* (iii) $\partial g/\partial T > 0$*; and* (iv) sign $\partial g/\partial P =$ sign $(1 - \alpha P) \geq 0$*. The growth-maximizing combination of* $(P, T) \in \mathcal{S}$ *is* $P = 1/\alpha$ *and* $T = \infty$.

Proof. By construction, it is clear that $\mathcal{B}$ and $P = 1/\alpha$ form the boundaries around $\mathcal{S}$. With $\rho < (L/\eta)(1/\alpha - 1)\alpha^{1/(1-\alpha)}(\alpha A)^{1/(1-\alpha)}$ assumed, $\underline{P}(T) < 1/\alpha$ and $0 < T_{\min} < \infty$, by Lemma 1. Thus, $\mathcal{S}$ is nonempty. Because $\hat{T}'(P) < 0$ and because $\hat{T}''(P) > 0$ in Lemma 1, $\mathcal{S}$ is convex as illustrated in Figure 1. For part (ii), we first rewrite (15) as

$$\frac{r}{1 - e^{-rT}} = (L/\eta)(\alpha A)^{1/(1-\alpha)}(P - 1)P^{1/(\alpha-1)}. \tag{19}$$

Differentiating (19) with respect to T yields

$$\frac{\partial r}{\partial T} = \frac{r^2 e^{-rT}}{1 - e^{-rT} - rTe^{-rT}} > 0, \quad \forall\, T \in (0, \infty),$$

because $1 - e^{-rT} - rTe^{-rT} > 0, \forall\, T \in (0, \infty)$. Thus, starting from any $(P, T) \in \mathcal{B}$ with $r = \rho$ (i.e., $g = 0$), increasing T (holding P constant) increases the growth rate from zero to a positive level by increasing r.[6] This movement enters $\mathcal{S}$ from the boundary $\mathcal{B}$. Similarly, starting from any point in $\mathcal{S}$, increasing T alone always increases the growth rate by increasing r. Then, parts (ii) and (iii) hold. For part (iv), differentiating (19) with respect to P, we have

$$\frac{\partial r}{\partial P} = \frac{(1-\alpha P)r(1-e^{-rT})}{(1-\alpha)P(P-1)(1-e^{-rT}-rTe^{-rT})}.$$

Because $P > 1$, $1 - e^{-rT} > 0$, and $1 - e^{-rT} - rTe^{-rT} > 0 \ \forall\ (P, T) \in \mathcal{B}$ or $\mathcal{S}$, sign $\partial g/\partial P$ = sign $\partial r/\partial P$ = sign $(1 - \alpha P) \geq 0$ under $P \leq 1/\alpha$. The last result follows parts (iii) and (iv). ■

PROPOSITION 2. *For* $(P, T) \notin \mathcal{S}$ *with* $1/\alpha \geq P \geq 1$, *we have* (i) $g = 0$; (ii) $\partial g/\partial P = \partial g/\partial T = 0$ *outside* $\mathcal{B}$; (iii) sign $\partial g/\partial P$ *and* sign $\partial g/\partial T$ *inside* $\mathcal{B}$ *are the same as inside* $\mathcal{S}$.

Proof. Part (i) follows from our earlier discussion and the construction of $\mathcal{B}$ and $\mathcal{S}$. For $(P, T) \notin \mathcal{B} \cup \mathcal{S}$, an (infinitely small) change in P or T toward $\mathcal{B}$ or elsewhere such that $P \in [1, 1/\alpha]$ still retains $g = 0$. This completes part (ii). The proof of part (iii) is similar to the counterpart in the proof of Proposition 1. Because there is no incentive to set a price of patented intermediates above $1/\alpha$, we ignore cases with $P > 1/\alpha$. ■

The results in Propositions 1 and 2 are intuitive. Overall, whether there is new R&D investment for growth or how it responds to changes in patent length and price regulation depends on whether the rate of return on R&D investment exceeds the rate of time preference. When the rate of return is dominated by, or equal to, the rate of time preference, because of patents being too short or the regulated price being too low, there will be no new R&D investment for growth. In this lack-of-incentive scenario, a small change in the patent length or price level does not create any new investment for growth before they reach their critical bounds. In contrast, when the rate of return dominates the rate of time preference, because of patents being long enough and the regulated price lying between its lower and upper bounds, there will be new innovation and growth. In addition, in this growing economy, changing the length of patents or the level of the regulated price will influence the rate of innovation or growth according to how the rate of return on R&D investment responds.

On one hand, when the price level remains constant within its bounds, increasing the length of patents at or beyond its lower bound for new R&D investment will always increase the rate of return on R&D investment, leading to more R&D investment and higher growth rates. This accords with the result in Futagami and Iwaisako (2007) for discrete time. From (15), we can see that for any given price P, the maximum growth rate is achieved at $T = \infty$, i.e., $g_{\max}(P, \infty) = \frac{1}{\theta}[(L/\eta)(\alpha A)^{1/(1-\alpha)}(P-1)P^{1/(\alpha-1)} - \rho]$. This result differs from those of both Horowitz and Lai (1996) and Michel and Nyssen (2002), because of the differences in the modeling of the R&D sector (i.e., the nature of innovations and the specification of R&D costs).[7]

On the other hand, when the patent length exceeds its lower bound for new R&D investment, increasing the price level of patent-protected intermediate goods

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at or beyond its lower bound will increase the rate of return on R&D investment before the monopoly level is reached, leading to more R&D investment and higher growth rates. Above the monopoly price level, however, a further increase in the price level will decrease the rate of return on R&D investment and thus reduce R&D investment and the growth rate. At the monopoly price level $P = 1/\alpha$, the maximum growth rate $g_{\max}(1/\alpha, T)$ is implicitly determined by (15); i.e., $\theta g_{\max}(1/\alpha, T) + \rho = (L/\eta)(1-\alpha)(A\alpha^{1+\alpha})^{1/(1-\alpha)}[1 - e^{-(\theta g_{\max}(1/\alpha,\, T)+\rho)T}]$.

The combination of (P, T) that has monopoly pricing and infinite patent length, $P = 1/\alpha$ and $T = \infty$, obtains the maximum growth rate, $g_{\max} = \lim_{T\to\infty} g_{\max}(1/\alpha, T) = \lim_{P\to 1/\alpha} g_{\max}(P, \infty) = \frac{1}{\theta}[(L/\eta)(1-\alpha)(A\alpha^{1+\alpha})^{1/(1-\alpha)} - \rho]$. Around this growth-maximizing combination of patent duration and price regulation, the marginal growth effect of any further increase in either of the two directions should approach zero. This observation will be helpful when we consider optimal patent policy to maximize welfare. We now turn to the welfare analysis.

4. WELFARE-MAXIMIZING PATENT LENGTH AND PRICE REGULATION

To assess the welfare implications of patent length and price regulation, we derive the equilibrium welfare level as a function of these policy instruments, starting with the solution for the equilibrium paths of per capita output and consumption. The aggregate final output in (5) can be rewritten as

$$Y = AL^{1-\alpha}N\left[\frac{N^c}{N}(X^c)^\alpha + \left(1 - \frac{N^c}{N}\right)(X^m)^\alpha\right]. \tag{20}$$

The ratio N^c/N in (20) can be determined as follows. Suppose that the economy starts at time t_0 with an initial total number of intermediate goods N_{t_0} and an initial number of competitive intermediate goods $N_{t_0}^c$. At time $t_0 + T$, all the intermediate goods produced at time t_0, with a measure N_{t_0}, become competitive when their patents expire. Thus, the number of competitive intermediate goods at time $t_0 + T$ is $N_{t_0+T}^c = N_{t_0}$. At the same time, the total number of intermediate goods grows to the level $N_{t_0+T} = N_{t_0}e^{gT}$ because the growth rate of the total number of intermediate goods equals g. As a result, we have $N_{t_0+T}^c/N_{t_0+T} = N_{t_0}/(N_{t_0}e^{gT}) = e^{-gT}$. According to this, a longer patent reduces the number of competitive intermediate goods, N^c, relative to the total number of intermediate goods, N^c/N, i.e., a smaller fraction of intermediate goods in the competitive sector. In addition, a higher growth rate g increases the total number of intermediate goods N relative to the number of competitive intermediate goods N^c, also resulting in a smaller fraction of intermediate goods in the competitive sector N^c/N.

Letting $J \equiv N_0(A\alpha^\alpha)^{1/(1-\alpha)}$ and substituting $N^c/N = e^{-gT}$, $X^c = L(\alpha A)^{1/(1-\alpha)}$, and $X^m = L(\alpha A/P)^{1/(1-\alpha)}$ into (20), we obtain the equilibrium path of per capita output:

$$y = Y/L = y_0 e^{gt}, \quad \text{where } y_0 = J\left[e^{-gT} + (1 - e^{-gT})P^{\alpha/(\alpha-1)}\right]. \tag{21}$$

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Obviously, the level of per capita output y depends positively on the growth rate g and the initial level of per capita output y_0. From (21), the initial level of per capita output is increasing with the initial number of intermediate goods N_0 and the fraction of intermediate goods outside patent protection e^{-gT}. However, it is decreasing with the price level of intermediate goods under patent protection P because under $0 < \alpha < 1$ the demand for patent-protected intermediate goods X^m is decreasing with the price P. To see how changes in the patent length and price level affect the initial per capita output, we differentiate (21) with respect to T and P, respectively:

$$\frac{\partial y_0}{\partial T} = \underbrace{-J\left[1 - P^{\alpha/(\alpha-1)}\right]\left(g + T\frac{\partial g}{\partial T}\right)e^{-gT}}_{\Omega^T<0}, \tag{22}$$

$$\frac{\partial y_0}{\partial P} = \underbrace{-J\left(\frac{\alpha}{1-\alpha}\right)P^{1/(\alpha-1)}(1-e^{-gT})}_{\Omega_1^P<0} \underbrace{-J\left[1 - P^{\alpha/(\alpha-1)}\right]T\frac{\partial g}{\partial P}e^{-gT}}_{\Omega_2^P<0\ \ \forall P\in(\underline{P}(T),1/\alpha)}. \tag{23}$$

As shown in Proposition 1, $\partial g/\partial T > 0$ if T is above its lower bound for positive growth, and $\partial g/\partial P > 0$ if P is above its lower bound for positive growth and below the monopoly price level.

From (21) and (22), increasing patent duration has opposing effects on per capita output. On one hand, an increase in the patent length raises the growth rate and hence the number of intermediate goods over time, generating a positive effect on final output as time unfolds. On the other hand, it has a direct negative effect on final output (Ω^T) because increasing patent duration (T) reduces the fraction of intermediate goods in the competitive sector over time, i.e., reducing the weight e^{-gT} relative to $(1-e^{-gT})$ as time unfolds. For any price level of the intermediate goods above the marginal cost in the patent-protected sector ($P > 1$), such a switch from e^{-gT} toward $(1 - e^{-gT})$ reduces per capita output by reducing the demand for intermediate goods via worsening the price distortion.

Similarly, from (21) and (23), an increase in the price level of intermediate goods under patent protection (between the lower bound and the monopoly price level given in Proposition 1) also raises the growth rate and thus the number of intermediate goods, leading to a higher level of output over time. However, increasing the price level has a negative effect on per capita output, in part by directly magnifying the price distortion for patent-protected intermediate goods (Ω_1^P) and in part by reducing the fraction of intermediate goods in the competitive sector indirectly through accelerating growth (Ω_2^P).

To derive per capita consumption, we use the aggregate resource constraint given in Definition 1:

$$C = Lc = Y - \eta g N - N^c X^c - (N - N^c)X^m. \tag{24}$$

Letting $D \equiv N_0(A\alpha^{\alpha})^{1/(1-\alpha)}(1-\alpha) > 0$ and $G(P) \equiv (P-\alpha)P^{1/(\alpha-1)}/(1-\alpha) < 1$, combining (21) and (24), and using $X^c = L(\alpha A)^{1/(1-\alpha)}$, $X^m = L(\alpha A/P)^{1/(1-\alpha)}$, and $N = N_0 e^{gt}$, we have

$$c = c_0 e^{gt}, \quad \text{where } c_0 = D[e^{-gT} + G(P)(1-e^{-gT})] - \eta g N_0/L. \tag{25}$$

The restriction $G(P) < 1$ arises from $G(1) = 1$ and $G'(P) = \alpha(1-P)/[(1-\alpha)^2 P^{(2-\alpha)/(1-\alpha)}] < 0, \forall P > 1$. Here, the initial level of per capita consumption is increasing with the initial number of available intermediate goods (N_0) and the fraction of intermediate goods outside patent protection (e^{-gT}). But it is decreasing with the price level of intermediate goods under patent protection (P) and the R&D investment being carried out in the same period ($\eta g N_0/L$). To see more clearly how the patent length and price level affect the initial level of per capita consumption, we differentiate (25) with respect to T and P, respectively:

$$\frac{\partial c_0}{\partial T} = \underbrace{-D\left(g + T\frac{\partial g}{\partial T}\right)e^{-gT}[1-G(P)]}_{\Delta_1^T<0} \underbrace{-\left(\frac{\eta N_0}{L}\right)\frac{\partial g}{\partial T}}_{\Delta_2^T<0}, \tag{26}$$

$$\frac{\partial c_0}{\partial P} = \underbrace{-DT\frac{\partial g}{\partial P}e^{-gT}[1-G(P)]}_{\Delta_1^P<0 \ \forall P\in(\underline{P}(T),1/\alpha)} \underbrace{-D\left(1-e^{-gT}\right)P^{1/(\alpha-1)}\frac{\alpha(P-1)}{P(1-\alpha)^2}}_{\Delta_2^P<0}$$
$$\underbrace{-\left(\frac{\eta N_0}{L}\right)\frac{\partial g}{\partial P}}_{\Delta_3^P<0 \ \forall P\in(\underline{P}(T),1/\alpha)} . \tag{27}$$

From (25) and (26), increasing patent duration increases per capita consumption c over time by raising the growth rate. In the initial period, however, it also reduces per capita consumption c_0 by strengthening the price distortion effect (i.e., more intermediate goods are subject to price distortions, Δ_1^T) and by increasing R&D investment (Δ_2^T). Similarly, according to (25) and (27), raising the price level between its lower bound and the monopoly price level increases per capita consumption c over time by accelerating growth. But a higher price level also reduces per capita consumption in the initial period c_0 both by worsening the price distortion (i.e., more intermediate goods are under patent protection Δ_1^P and there is a greater price distortion for each of these goods Δ_2^P) and by stimulating R&D investment (Δ_3^P).

Finally, we solve (1) by using $c = c_0 e^{gt}$ to obtain the welfare function

$$U_0 = \max_c \int_0^{\infty}\left(\frac{c^{1-\theta}-1}{1-\theta}\right)e^{-\rho t}dt = \frac{c_0^{1-\theta}}{(1-\theta)[\rho-(1-\theta)g]} - \frac{1}{\rho(1-\theta)}, \tag{28}$$

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where g is given by (15) and c_0 is given by (25). The transversality condition implies that $\rho - (1-\theta)g > 0$. The dynamic effects of patent length and price regulation on welfare are channeled mainly through the growth rate g, although they are also reflected in the trade-off between consumption and R&D investment in the initial period. The static effects of patent length and price regulation on welfare are channeled mainly through the price distortion factor in the determination of consumption in the initial period c_0.

In what follows, we restrict our welfare analysis to the combinations of (P, T) with positive growth plus those in the lower boundary, i.e., $(P, T) \in \mathcal{S} \cup \mathcal{B}$.[8] To see how patent duration and price regulation affect welfare, we differentiate (28) with respect to T and P, respectively:

$$\frac{\partial U_0}{\partial T} = \frac{[\rho - (1-\theta)g](\partial c_0/\partial T) + c_0(\partial g/\partial T)}{c_0^{\theta}[\rho - (1-\theta)g]^2}, \tag{29}$$

$$\frac{\partial U_0}{\partial P} = \frac{[\rho - (1-\theta)g](\partial c_0/\partial P) + c_0(\partial g/\partial P)}{c_0^{\theta}[\rho - (1-\theta)g]^2}, \tag{30}$$

where

$$\frac{\partial g}{\partial T} = \frac{r^2 e^{-rT}}{\theta(1 - e^{-rT} - rTe^{-rT})},$$

$$\frac{\partial g}{\partial P} = \frac{(1-\alpha P)r(1-e^{-rT})}{\theta(1-\alpha)P(P-1)(1 - e^{-rT} - rTe^{-rT})},$$

and $\partial c_0/\partial T$ and $\partial c_0/\partial P$ are respectively given by (26) and (27). This system of equations pinning down the optimal policies is rather complex without explicit solutions for c_0, r, and g. We will begin with separate analyses of optimal patent duration and optimal price regulation and then their mix.

4.1. Patent Duration

For any price level within the bounds for positive growth, we have the welfare effects of patent duration as follows:

PROPOSITION 3. *Given any $P \in (\underline{P}(T), 1/\alpha]$, if T is at its lower bound for $g \geq 0$, then $\partial U_0/\partial T > 0$ for a small enough ρ; if T is large enough, then $\partial U_0/\partial T < 0$. Thus, the optimal duration of patents exceeds the lower bound of T for a small enough ρ and is finite.*

Proof. From (29), we have sign $\partial U_0/\partial T =$ sign $\Phi(T)$, where

$$\Phi(T) \equiv \underbrace{[\rho - (1-\theta)g](\partial c_0/\partial T)}_{\text{Level effect}} + \underbrace{c_0(\partial g/\partial T)}_{\text{Growth effect}}. \tag{31}$$

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If $T = \hat{T}(P)$, which was defined in (16), then we have $g|_{T=\hat{T}(P)} = 0$, $c_0|_{T=\hat{T}(P)} = D > 0$, and

$$\left.\frac{\partial g}{\partial T}\right|_{T=\hat{T}(P)} = \frac{\rho^2 e^{-\rho\hat{T}(P)}}{\theta[1 - e^{-\rho\hat{T}(P)} - \rho\hat{T}(P) - e^{-\rho\hat{T}(P)}]} > 0,$$

$$\left.\frac{\partial c_0}{\partial T}\right|_{T=\hat{T}(P)} = -\left(\left.\frac{\partial g}{\partial T}\right|_{T=\hat{T}(P)}\right)\{D\hat{T}(P)[1 - G(P)] + \eta N_0/L\} < 0.$$

Substituting these expressions into (31) gives sign $\left.\partial U_0/\partial T\right|_{T=\hat{T}(P)} =$ sign $\Phi[\hat{T}(P)]$, where

$$\Phi[\hat{T}(P)] \equiv \left(\left.\frac{\partial g}{\partial T}\right|_{T=\hat{P}}\right)\{D - \rho D\hat{T}(P)\,[(1 - G(P)] - \rho\eta N_0/L\}.$$

Because $\partial g/\partial T|_{T=\hat{T}(P)} > 0$ and $D > 0$, we have $\Phi[\hat{T}(P)] > 0$ if ρ is sufficiently small. That is, welfare rises as T increases from $\hat{T}(P)$, holding P constant, when consumers are patient enough.

We now show that if T is sufficiently large, a further increase in T reduces welfare. To do so, we rewrite (31) as $\Phi(T) \equiv \phi(T)\partial g/\partial T$, where

$$\phi(T) \equiv [\rho - (1-\theta)g]\frac{\partial c_0}{\partial T}\bigg/\frac{\partial g}{\partial T} + c_0. \tag{32}$$

Because $\partial g/\partial T$ is finite and positive for $T \in [T_{\min}, \infty)$, as seen in the proof of Proposition 1, we have sign $\Phi(T) =$ sign $\phi(T)$. Substituting the expressions for $\partial c_0/\partial T$ and $\partial g/\partial T$ into (32) gives

$$\phi(T) = -(r-g)\left\{\frac{\eta N_0}{L} + D\,[1 - G(P)]\,(Te^{-gT} + E)\right\} + c_0,$$

where $E \equiv \theta g e^{(r-g)T}(1 - e^{-rT} - rTe^{-rT})/r^2$. Note that E is strictly increasing in T if T is sufficiently large and $\lim_{T\to\infty} E = \infty$. Because $\eta N_0/L > 0$, $D > 0$, $Te^{-gT} > 0$, $G(P) < 1$, $r > g$ (given by the transversality condition), and $c_0 < \infty$, we have $\phi(T) < 0$ and thus $\Phi(T) < 0$ if T is sufficiently large. As a result, $\partial U_0/\partial T < 0$ for sufficiently large values of T. Thus, the optimal T must be finite and above the lower bound $\hat{T}$. ■

Increasing patent duration above its lower bound for positive growth exerts opposing effects on welfare. First, it strengthens the incentives for innovation and thus promotes R&D investment and growth, creating dynamic gains in efficiency. This effect is captured by the second term, $c_0\partial g/\partial T$, in (31), which signs the marginal utility of increasing the duration of patents (the *growth effect*). Second, with longer patent duration, monopoly pricing becomes more persistent in the

TABLE 1. Welfare effect of patent duration

T^a	X^c	X^m	Y_0	c_0	r	g	U_0
8.10	0.179	0.032	0.597	0.418	0.050	0.000	−27.866
10.00	0.179	0.032	0.556	0.393	0.087	0.018	−17.195
15.00	0.179	0.032	0.491	0.357	0.128	0.039	−11.536
19.00	0.179	0.032	0.460	0.342	0.139	0.044	−10.929
20.20	0.179	0.032	0.452	0.339	0.141	0.045	−**10.908**
21.00	0.179	0.032	0.448	0.337	0.142	0.046	−10.915
25.00	0.179	0.032	0.429	0.329	0.146	0.048	−11.097
30.00	0.179	0.032	0.411	0.321	0.148	0.049	−11.455
40.00	0.179	0.032	0.389	0.312	0.149	0.050	−12.141

Note: Parameters: $\alpha = 0.3$, $\theta = 2$, $\eta = 0.5$, $\rho = 0.05$, $P = 1/\alpha$, $A = L = N_0 = 1$. Boldface indicates "optimal points."
[a] The frequency with which the length of patent duration is changed each step is 0.1 (about 1%).

intermediate sector, which increases the fraction of intermediate goods sold at a price level above the marginal cost. This increased price distortion reduces final output and hence consumption, as mentioned earlier (the *level effect*). The decline in consumption due to a longer T via the price distortion is captured by the first term Δ_1^T in (26). This distortion exists as long as some intermediate goods are priced above their marginal costs (i.e., $P > 1$).[9] Consumption in the initial period also falls when increasing patent duration stimulates R&D investment in the same period, which is echoed in the second term Δ_2^T in (26). If the rate of time preference is low enough, the dynamic gain in efficiency via the growth effect dominates the static loss via the level effect at the lower bound of patent duration. As mentioned earlier, the growth effect will eventually vanish when the patent length approaches infinity. Therefore, the level effect will eventually dominate, leading to a finite optimal patent length. Our result in Proposition 3 with continuous time agrees with that in Futagami and Iwaisako (2007) with discrete time.

The quantitative implications of Proposition 3 are given in Table 1 along with the parameterization. In the parameterization, the values of the coefficient of relative risk aversion at $\theta = 2.0$ and the rate of time preference at $\rho = 0.05$ are in line with those in the literature. The values of the rest of the parameters are chosen so that the resulting values of the growth rate are close to its observed level. According to the results in Table 1, the lower bound on patent duration for positive growth is equal to $T = 8.10$, at which the interest rate equals 5%. Increasing the duration of patents drives up the interest rate and the growth rate. In this process of increasing patent duration, the welfare level first rises, peaks at $T = 20.20$, and then declines. When we reduce the rate of time preference to $\rho = 0.045$, the minimum duration of patents becomes 7.9, at which the interest rate equals 4.5%, and the welfare-maximizing patent duration becomes 21.1. It is worth mentioning that the patent length has been 20 years since 1995 according to the terms set by GATT, which is very close to our optimal patent duration with $\rho = 0.05$ in Table 1. The magnitude of the gain in welfare from choosing the optimal patent length is substantial.

4.2. Price Regulation

We now investigate optimal price regulation given any duration of patents T in excess of its lower bound:

PROPOSITION 4. *Given any* $T > \hat{T}(1/\alpha)$, *there exists an optimal* $P^*(T) \in (\underline{P}(T), 1/\alpha)$ *such that if* $P^*(T) > P \geq \underline{P}(T)$, *then* $\partial U_0/\partial P > 0$ *for a small enough* ρ; *if* $P = P^*(T)$, *then* $\partial U_0/\partial P = 0$; *and if* $P^*(T) < P < 1/\alpha$, *then* $\partial U_0/\partial P < 0$.

Proof. From (30), we have sign $\partial U_0/\partial T =$ sign $\Psi(P)$, where

$$\Psi(P) \equiv \underbrace{[\rho - (1-\theta)g](\partial c_0/\partial P)}_{\text{Level effect}} + \underbrace{c_0(\partial g/\partial P)}_{\text{Growth effect}}. \tag{33}$$

If $P = \underline{P}(T)$, then we have $g|_{P=\underline{P}(T)} = 0$, $c_0|_{P=\underline{P}(T)} = D > 0$, $\partial g/\partial P|_{P=\underline{P}(T)} > 0$, and

$$\left.\frac{\partial c_0}{\partial P}\right|_{P=\underline{P}(T)} = -\left(\left.\frac{\partial g}{\partial P}\right|_{P=\underline{P}(T)}\right)\{DT\left(1 - G[\underline{P}(T)]\right) + \eta N_0/L\} < 0.$$

Substituting these expressions into (33) gives sign $\partial U_0/\partial P|_{P=\underline{P}(T)} =$ sign$\Psi[\underline{P}(T)]$, where

$$\Psi[\underline{P}(T)] \equiv \left(\left.\frac{\partial g}{\partial P}\right|_{P=\underline{P}(T)}\right)\{D - \rho DT\left(1 - G[\underline{P}(T)]\right) - \rho\eta N_0/L\}.$$

Because $\partial g/\partial P|_{P=\underline{P}(T)} > 0$ and $D > 0$, we have $\Psi[\underline{P}(T)] > 0$ if ρ is sufficiently small.

If $P = 1/\alpha$, then $g|_{P=1/\alpha} > 0$, $\partial g/\partial P|_{P=1/\alpha} = 0$, $c_0|_{P=1/\alpha} > 0$, and $\partial c_0/\partial P|_{P=1/\alpha} = -D(1 - e^{-gT})/(1-\alpha) < 0$, leading to $\partial U_0/\partial P|_{P=1/\alpha} < 0$. Thus, the critical $P^*(T) \in (\underline{P}(T), 1/\alpha)$ is determined by $\Psi(P^*) = 0$. ■

Proposition 4 has an interpretation similar to that for Proposition 3. For any given patent duration above its lower bound, a rise in the price level of intermediate goods under patent protection reduces consumption in the initial period (the *level effect*) by reducing final output due to the price distortion [Δ_1^P and Δ_2^P in (27)] and possibly by increasing R&D investment in the same period [Δ_3^P in (27)]. On the other hand, the rise in the price level accelerates growth and thus generates dynamic gains in efficiency over time when the price lies between the marginal cost and the monopoly level (the *growth effect*), which can dominate the static loss if the rate of time preference is small enough. The growth effect of increasing the price fully vanishes at or above the monopoly level. As a consequence, the optimal price level is above the marginal cost but below the monopoly level.

The quantitative implication of price regulation is given in Table 2 with the same parameterization as that in Table 1, except that the patent duration is now assumed to be infinite. The response of the growth rate to a rising price level

TABLE 2. Welfare effect of price regulation

P^a	X^c	X^m	Y_0	c_0	r	g	U_0
1.18	0.179	0.142	0.557	0.415	0.050	0.000	−28.216
1.40	0.179	0.111	0.517	0.396	0.089	0.019	−16.409
1.80	0.179	0.077	0.464	0.368	0.124	0.037	−11.264
2.00	0.179	0.067	0.444	0.356	0.133	0.042	−10.672
2.10	0.179	0.062	0.434	0.351	0.137	0.043	**−10.582**
2.20	0.179	0.058	0.426	0.345	0.139	0.045	−10.586
2.60	0.179	0.046	0.396	0.327	0.146	0.048	−11.197
3.00	0.179	0.037	0.373	0.311	0.149	0.050	−12.329
3.40	0.179	0.031	0.353	0.297	0.150	0.050	−13.709

Note: Parameters: $\alpha = 0.3$, $\theta = 2$, $\eta = 0.5$, $\rho = 0.05$, $T = \infty$, $A = L = N_0 = 1$. Boldface indicates "optimal points."
[a] The frequency at which the price is changed each step is 0.01 (less than 1%).

is positive until the price reaches the monopoly level. By contrast, with a rising price level, the welfare level first rises, peaks at $P = 2.10$, and then declines. The magnitude of the welfare gain from the optimal price regulation is also substantial, as in the case with the optimal patent. When we reduce the rate of time preference to $\rho = 0.045$, the optimal price regulation is only slightly changed to $P = 2.16$.

By comparing Tables 1 and 2, it can be seen that the optimal price regulation (with infinite patent duration) results in a higher level of welfare than the optimal patent duration (with monopoly pricing). However, for different parameterizations, the ranking order may reverse. In Table 3, we use different parameterizations by changing the values of $(\alpha, \theta, \rho, \eta)$. In this table, it becomes clear that the welfare ranking of optimal price regulation and optimal patent duration is ambiguous, depending on parameterizations. We highlight this as follows:

Simulation Result 1. Optimal patent duration with monopoly pricing may lead to a higher or lower level of welfare than optimal price regulation with infinite patent duration.

We next examine the optimal mix of price regulation and patent duration.

4.3. Optimal Combination of Patent Duration and Price Regulation

When choosing the patent duration and regulated price level simultaneously to maximize welfare, we have the following result:

PROPOSITION 5. *For $(P, T) \in \mathcal{S} \cup \mathcal{B}$, the welfare-maximizing combination (P^*, T^*) is determined by $\Phi(T^*) = \Psi(P^*) = 0$. This optimal policy exists for a small enough ρ. Also, T^* is finite and P^* is above the marginal cost but below the monopoly-pricing level.*

Proof. The necessary conditions for optimality are obtained by setting $\partial U_0/\partial T$ in (29) and $\partial U_0/\partial P$ in (30) equal to zero. The sufficient condition is argued

TABLE 3. Welfare comparisons of patent duration, price regulation, and their mixes

	Social planner	No price regulation[a]		Price regulation[b]		Combinations (T, P)	
Parametric variations	Welfare	Patent duration	Welfare	Regulated price	Welfare	Mix (T, P)	Welfare
Benchmark	9.80	20.20	−10.91	2.10	−10.58	(26.73, 2.33)	−9.69
$\alpha = 0.1$	16.16	33.82	−0.82	4.89	−0.96	(39.85, 5.72)	−0.43
$\alpha = 0.6$	−24.96	24.97	−69.83	1.34	−71.27	(34.39, 1.42)	−66.72
$\theta = 1.1$	91.02	39.34	7.69	2.67	8.48	(45.74, 2.68)	8.51
$\theta = 3.0$	−0.59	17.46	−24.70	1.89	−25.87	(23.61, 2.23)	−22.82
$\rho = 0.03$	22.66	21.40	−2.42	2.20	−1.37	(28.77, 2.32)	−0.72
$\rho = 0.10$	0.86	19.26	−12.07	2.03	−13.27	(23.99, 2.44)	−11.71
$\eta = 0.25$	14.60	10.98	1.44	2.21	2.06	(14.69, 2.32)	2.35
$\eta = 0.75$	5.53	29.06	−19.03	2.08	−19.74	(37.43, 2.36)	−17.93

Note: Parameters: $\alpha = 0.3$, $\theta = 2$, $\eta = 0.5$, $\rho = 0.05$, $A = L = N_0 = 1$.
[a] Without price regulation, the prices of intermediate goods are set at the monopoly price (i.e., $P = 1/\alpha$).
[b] With price regulation, the patent length is assumed to be infinite (i.e., $T = \infty$).

as follows. For a small enough ρ, according to Propositions 3 and 4, welfare is increasing with T and P at their lower bounds, but decreasing with T and P at their finite upper bounds (especially $P < 1/\alpha$). That is, the permissible mixes (P, T) for maximizing welfare form a compact subset in $\mathcal{S}$. Also, the welfare function in (28) is continuous in $(P, T) \in \mathcal{S}$. Thus, by the Weierstrass theorem, there must exist at least one mix of (P, T) that maximizes the welfare level in the compact subset in $\mathcal{S}$ (particularly $1 < P^* < 1/\alpha$ and $\hat{T}(1/\alpha) < T^* < \infty$). ■

In general, the optimal mix of patent duration and price regulation can always do better than either of them alone. The crucial point is by how much the optimal mix can improve welfare over the situations where only one of the instruments is used to maximize welfare. For this purpose, we report the simulated welfare levels for the optimal mixes of the two instruments in Table 3, along with the results of using each instrument at one time. In some cases of the results in Table 3, the gain in welfare from optimally blending the two instruments can be substantial, whereas in other cases the gain can be moderate or marginal.

Although the existence of the optimal mix of (P, T) is offered here, we find it cumbersome to establish the uniqueness. To ease concern about the uniqueness, Figure 2 illustrates the simulated relationship of welfare with patent length and price regulation, based on the parameterization of the benchmark case in Table 3. According to this figure (and many other unreported ones), the welfare level is single-peaked in the set $\mathcal{S}$.

The result that the welfare-maximizing combination has a finite length of patents and a price level below the monopoly pricing level emerges from two reasons. First, we have noted that the marginal growth effect eventually becomes zero. Second,

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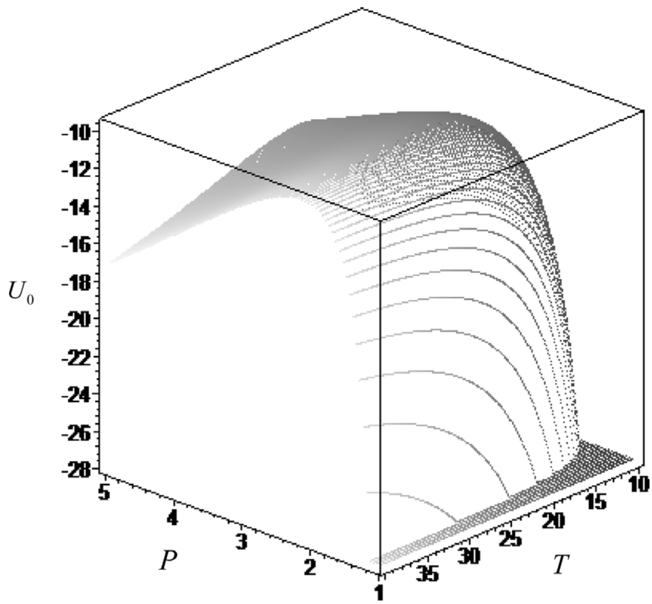

FIGURE 2. Welfare in relation to price and patent length.

the marginal level effect of pricing above the marginal cost is increasing with either longer patent duration or a higher price cap (starting below the monopoly price). Therefore, the marginal welfare gain from stronger patent protection in either direction through the growth effect must be eventually dominated by the marginal welfare loss through the level effect before the strongest patent protection is granted.

Finally, note that all the optimal policies here are the second best. The social planner's solution is given by

$$Y_{\text{sp}} = LN(A\alpha^{\alpha})^{1/(1-\alpha)},$$
$$X_{\text{sp}} = L(A\alpha)^{1/(1-\alpha)},$$
$$g_{\text{sp}} = \frac{1}{\theta}\left[(L/\eta)(1-\alpha)\,(A\alpha^{\alpha})^{1/(1-\alpha)} - \rho\right],$$

and $c_{\text{sp}} = c_0 e^{gt}$ with $c_0 \equiv N_0[(A\alpha^{\alpha})^{1/(1-\alpha)}(1-\alpha) - \eta g_{\text{sp}}/L]$. Because the maximum growth rate in the decentralized equilibrium, i.e., $g_{\max} = \frac{1}{\theta}[(L/\eta)(1-\alpha)(A\alpha^{1+\alpha})^{1/(1-\alpha)} - \rho]$, is lower than the socially optimal growth rate g_{sp}, none of the optimal policies consisting of patent duration and/or price regulation can be the first-best policy.

5. CONCLUSION

This paper has investigated the growth and welfare effects of patent duration and price regulation in an extended version of the Barro and Sala-i-Martin model (1995). The extension takes the form of splitting the intermediate sector into two subsets: a competitive one with expired patents and a monopolized (or regulated) one with valid patents. We have shown that limiting patent duration and capping prices of patented products, or their mix, can improve social welfare substantially, despite reducing the growth rate. But none of them can achieve the first-best outcome that would be chosen by a social planner. Also, in welfare terms, there is no clear ranking between optimal patent duration (with monopoly pricing) and price regulation (with infinite patent duration). However, using their optimal mix is always better than using them separately. The magnitudes of the welfare gains of the optimal mix over separate uses of the instruments can be substantial.

To the best of our knowledge, this paper is the first attempt to investigate patent duration and price regulation jointly in an endogenous growth model. We believe that the analysis in this paper can help understand why governments in many countries use combinations of various policy instruments rather than a single instrument to regulate certain industries. In particular, our results may justify the popular use of setting limits on patent duration and on prices in the pharmaceutical and telecommunication industries in both North America and Europe.

NOTES

1. Several other studies [e.g., Hunt (1995); Aghion and Howitt (1998)] also use endogenous growth models to analyze optimal patent policy, but their focus is not on patent length.
2. The traditional analysis of price regulation aims at the static distortion of pricing goods above their marginal costs.
3. Quite a number of studies have attempted to investigate the impact of patent duration or price regulation on competition, R&D investment, and pricing behavior in the pharmaceutical industries [e.g., Anis and Wen (1998); Pazderka (1999); Danzon and Chao (2000); Jones et al. (2001); Troyer and Krasnikov (2002)]. However, none of them focuses on the welfare implications of these policies.
4. Futagami and Iwaisako (2007) analyze transitional dynamics in this type of model with discrete time. They show that it converges to a unique balanced growth path in a finite number of periods. For our purpose in this model, we focus only on the balanced growth path.
5. For general equilibrium with price regulation, see, e.g., Anderson and Enomoto (1986), Kelly (2005), and some other related studies cited therein.
6. For different purposes, Boucekkine et al. (2005) and Bambi (2008) deal with equations similar to our (19) and derive some analytical and graphical representations of the space of roots of such quasi-polynomials.
7. Horowitz and Lai (1996) use a quality-ladder model. Also, rather than a physical input for innovation in our model, labor and knowledge are used for innovation in Michel and Nyssen (2000) with a possible spillover from knowledge.
8. Outside $\mathcal{S} \cup \mathcal{B}$ with $\dot{N} = g = 0$, the only local optimum is obviously at $P = 1$ and $T \geq 0$. This optimum with zero growth and marginal-cost pricing would become global if households were extremely impatient (i.e., if ρ were very large). The meaningful case we focus on is a local optimum with positive growth, which becomes global for a low enough rate of time preference ρ.
9. Note that the term Δ_1^T would become zero if the prices of all intermediate goods were set at their marginal costs (i.e., $P = 1$).

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