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Electron acceleration in a plasma filled rectangular waveguide under obliquely applied magnetic field

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Abstract. Analytical expressions for the TE_{10} mode fields and the acceleration gradient for an electron are obtained in a plasma filled rectangular waveguide under the effect of an obliquely applied external magnetic field. The lower and upper cutoff frequencies for the mode propagation and the acceleration gradient for the electron are evaluated under the effect of various parameters. The acceleration gradient is increased with the microwave frequency and angle of magnetic field. It becomes reduced for the higher plasma density and stronger magnetic field. The larger acceleration of electron is achieved when $\omega_{\mathrm{p}} > \omega_{\mathrm{c}}$.

1. Introduction

The plasma being ionized can sustain large electric fields and therefore plasma based accelerators have received a lot of interest for producing extremely large acceleration gradients. Since the waveguides can guide the electromagnetic waves for longer distances, they can be potential candidates for the purpose of particle acceleration. In view of this, researchers have studied the modes in waveguides filled partially or fully with plasma [1–3]. Also the microwave field has been employed to accelerate the electrons in different mechanisms [4–7].

Since the magnetic field can help divert the electron motion and plays an important role, in the present analysis we attempt the electron acceleration in a plasma filled rectangular waveguide when the electron is injected along the direction of mode propagation (z-axis) under the effect of an obliquely (angle θ with z-axis) applied external magnetic field.

2. Mode field components and electron acceleration

In the present model we realize that the dielectric constant of the plasma becomes a tensor quantity and anisotropy is introduced through

$$\overline{\varepsilon_{\mathrm{eff}}} = \begin{pmatrix} \epsilon_{11} & -i\epsilon_{12} & i\epsilon_{13} \\ i\epsilon_{21} & \epsilon_{22} & -\epsilon_{23} \\ -i\epsilon_{31} & -\epsilon_{32} & \epsilon_{33} \end{pmatrix}. \tag{2.1}$$

The elements of the above tensor are given as

$$\epsilon_{11} = 1 - \varsigma^2/(1-\tau^2), \quad \epsilon_{12} = \epsilon_{21} = \varsigma^2 \tau \cos\theta/(\tau^2 - 1),$$

$$\epsilon_{13} = \epsilon_{31} = \varsigma^2 \tau \sin\theta/(\tau^2 - 1), \quad \epsilon_{22} = 1 - \varsigma^2(\tau^2 \sin^2\theta - 1)/(1-\tau^2),$$

$$\epsilon_{23} = \epsilon_{32} = \varsigma^2 \tau^2 \sin\theta\cos\theta/(\tau^2 - 1), \quad \epsilon_{33} = 1 - \varsigma^2(\tau^2 \cos^2\theta - 1)/(1-\tau^2)$$

together with $\varsigma = \omega_{\mathrm{p}}/\omega$ and $\tau = \omega_{\mathrm{c}}/\omega$.

From the Maxwell's equations with due consideration of plasma and magnetic field effects, we obtain

$$\nabla \times \bar{\epsilon}_{\mathrm{eff}}^{-1}(\nabla \times \mathbf{H}) = \frac{\omega^2}{c^2}\mathbf{H}, \tag{2.2}$$

where

$$\bar{\epsilon}_{\mathrm{eff}}^{-1} = \frac{\mathrm{adj}(\bar{\epsilon}_{\mathrm{eff}})}{\mid \bar{\epsilon}_{\mathrm{eff}} \mid}.$$

Finally the field components of TE_{10} mode can be calculated for the time dependence of $e^{-i\omega t}$ as

$$E_y = \frac{i\omega\mu_0 M_+^2}{[M_+^2(M_+^2 - P_+^2) - Q_+^4]}\left[A_0 \frac{\pi}{a}\sin\frac{\pi x}{a} e^{ik_{\mathrm{g}}z}\right],$$

$$H_x = -\frac{ik_{\mathrm{g}} M_+^2}{[M_+^2(M_+^2 - P_+^2) - Q_+^4]}\left[A_0 \frac{\pi}{a}\sin\frac{\pi x}{a} e^{ik_{\mathrm{g}}z}\right],$$

$$H_y = -\frac{k_{\mathrm{g}} Q_+^2}{[M_+^2(M_+^2 - P_+^2) - Q_+^4]}\left[A_0 \frac{\pi}{a}\sin\frac{\pi x}{a} e^{ik_{\mathrm{g}}z}\right],$$

$$E_x = -\frac{\omega\mu_0 Q_+^2}{[M_+^2(M_+^2 - P_+^2) - Q_+^4]}\left[A_0 \frac{\pi}{a}\sin\frac{\pi x}{a} e^{ik_{\mathrm{g}}z}\right],$$

$$H_z = A_0 \cos\tfrac{\pi x}{a} e^{ik_{\mathrm{g}}z},$$

where

$$M_+^2 = \left[\epsilon_{11}\left(\frac{\omega^2}{c^2}\right) - k_{\mathrm{g}}^2\right], \quad Q_+^2 = \frac{\omega^2}{c^2}\left[\frac{\omega_{\mathrm{p}}^2\omega_{\mathrm{c}}\cos\theta}{\omega(\omega_{\mathrm{c}}^2 - \omega^2)}\right] \quad \text{and} \quad P_+^2 = \left[\frac{\omega_{\mathrm{p}}^2\omega_{\mathrm{c}}^2\sin^2\theta}{c^2(\omega_{\mathrm{c}}^2 - \omega^2)}\right].$$

Here it is clear that the effect of plasma density n_0, magnetic field B_0 and angle θ are entered through the terms M_+, Q_+ and P_+. Moreover, it can be seen that the original TE_{10} mode carries additional components H_y and E_x and therefore is modified. Finally, the following dispersion relation is obtained for the mode

$$\omega^2 = \frac{c^2(\epsilon_{22}\epsilon_{33} - \epsilon_{23}^2)}{\mid \epsilon_{\mathrm{eff}} \mid}\left[k_{\mathrm{g}}^2 + \frac{\pi^2}{a^2}\right]. \tag{2.3}$$

By putting $k_{\mathrm{g}} = 0$ in (2.3) we obtain the following equation for ω_{cutoff}

$$\omega_{\mathrm{cutoff}}^{10} - \kappa_1\omega_{\mathrm{cutoff}}^8 + \kappa_2\omega_{\mathrm{cutoff}}^6 - \kappa_3\omega_{\mathrm{cutoff}}^4 + \kappa_4\omega_{\mathrm{cutoff}}^2 - \kappa_5 = 0, \tag{2.4}$$

where the constants are given by

$$\kappa_1 = 3(\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2) + \frac{c^2\pi^2}{a^2}, \quad \kappa_2 = 3(\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2)^2 + \omega_{\mathrm{p}}^2\omega_{\mathrm{c}}^2 + 3(\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2)\frac{c^2\pi^2}{a^2},$$

$$\kappa_3 = (\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2)^3 + 2\omega_{\mathrm{p}}^2\omega_{\mathrm{c}}^2(\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2) + \omega_{\mathrm{p}}^4\omega_{\mathrm{c}}^2 + [(\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2)^2 + 2\omega_{\mathrm{c}}^2(\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2) + \omega_{\mathrm{p}}^2\omega_{\mathrm{c}}^2]\frac{c^2\pi^2}{a^2},$$

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Table 1. Effect of plasma density n_0, magnetic field B_0 and waveguide width a on lower (f_L) and upper (f_U) cutoff frequencies and the frequency band $\triangle f$.

$B_0(T)$	n_0 ($\times 10^{15}$ m^{-3})	$a = 4.0$ cm f_{L} (GHz)	$a = 4.2$ cm f_{L} (GHz)	($a = 4.0$ cm, 4.2 cm) f_{U} (GHz)	$a = 4.0$ cm $\triangle f$ (GHz)	$a = 4.2$ cm $\triangle f$ (GHz)
0.135	5	3.8252	3.8150	6.0704	2.2452	2.2554
0.135	8	3.8701	3.8627	6.1104	2.2403	2.2477
0.145	5	4.0757	4.0241	6.0851	2.0094	2.0610
0.145	8	4.1440	4.1344	6.1307	1.9867	1.9963

$$\kappa_4 = \omega_{\mathrm{p}}^2\omega_{\mathrm{c}}^2(\omega_{\mathrm{p}}^2+\omega_{\mathrm{c}}^2)^2 + \omega_{\mathrm{p}}^4\omega_{\mathrm{c}}^2(\omega_{\mathrm{p}}^2+\omega_{\mathrm{c}}^2) + [\omega_{\mathrm{c}}^2(\omega_{\mathrm{p}}^2+\omega_{\mathrm{c}}^2)^2 + \omega_{\mathrm{p}}^2\omega_{\mathrm{c}}^2 + \omega_{\mathrm{p}}^2\omega_{\mathrm{c}}^2(\omega_{\mathrm{p}}^2+\omega_{\mathrm{c}}^2)]\frac{c^2\pi^2}{a^2},$$

$$\kappa_5 = \omega_{\mathrm{p}}^6\omega_{\mathrm{c}}^2.$$

We have prepared Table 1 for analyzing the lower (f_{L}) and upper (f_{U}) cutoff frequencies and the frequency band $\triangle f$ for the mode propagation. Here it is evident that f_{L} and f_{U} frequencies increase and the band $\triangle f$ decreases for the higher n_0 and B_0, and the effect of n_0 (B_0) is more significant for the stronger magnetic field (plasma density). Also the lower-frequency f_{L} is reduced for the increasing width of the waveguide, which broadens the allowed frequency band $\triangle f$.

We can obtain from the equation of momentum for the electrons (electron bunch) *that the electrons will be deflected* at an angle ϕ when injected along z-axis under the effect of mode fields. This angle is given by [7]

$$\tan\phi = -\frac{eE_0}{\omega k_{\mathrm{g}} m_{\mathrm{e}} v_z^2}\left[\omega + v_z k_{\mathrm{g}}\sqrt{\frac{M_+^2+Q_+^2}{Q_+^2-M_+^2}}\right]\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{2\pi}{\lambda_{\mathrm{g}}}z\right). \tag{2.5}$$

Above relation infers that the electron will execute sinusoidal oscillations in the waveguide and the amplitude of oscillations depends on the microwave intensity I_0 ($\propto E_0^2$) and microwave frequency f in addition to n_0, a, B_0 and θ.

The acceleration gradient for the electron (or electron bunch) can be obtained by using the momentum and energy equations under the effect of mode fields as

$$m_{\mathrm{e}}\frac{d(\gamma\mathbf{v})}{dt} = -e(\mathbf{E}+\mathbf{v}\times\mathbf{B}) \quad \text{and} \quad m_{\mathrm{e}}\frac{d(\gamma c^2)}{dt} = -e(\mathbf{v}\,.\,\mathbf{E}),$$

respectively. Now changing the coordinates to $\xi = v_{\mathrm{g}}t - z$ and solving for the velocity components v_x, v_y and v_z, we can obtain $\gamma\gamma_{\xi\xi} + K_1\gamma_\xi^2 - K_2\gamma_\xi = 0$ from the energy equation, where the subscript ξ represents the differentiation. Here

$$K_1 = 1 - K_0/\gamma^2 \quad \text{and} \quad K_2 = \left[\frac{e\mu_0 H_z}{m_{\mathrm{e}}(v_{\mathrm{g}}-v_z)}\right]\left(\frac{c^2\mu_0 H_x + E_y v_z}{c^2\mu_0 H_y - E_x v_z}\right)$$

together with

$$K_0 = \frac{c^2 E_x}{v_z(c^2\mu_0 H_y - E_x v_z)}.$$

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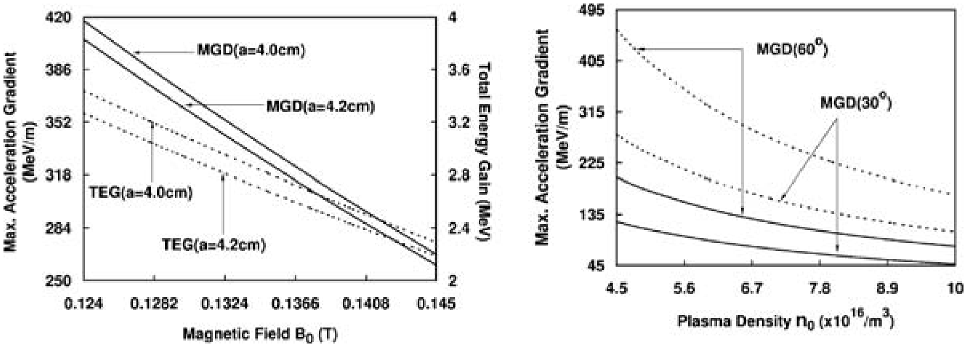

Figure 1. (a) Variation of the total energy gain (TEG) and maximum acceleration gradient (MGD) with B_0 and waveguide width a when $n_0 = 5 \times 10^{15}\,\mathrm{m}^{-3}$, $f = 5.4$ GHz, $B_0 = 0.135$ T, $\theta = 20^0$, $I_0 = 10^{10}$ Wm^{-2} and initial energy of the electron = 500 keV. (b) Effect of plasma density n_0 on the gradient MGD for two different angles θ of magnetic field in a 4.0 cm × 2.5 cm rectangular waveguide when $f = 6.3$ GHz, $I_0 = 10^{10}$ Wm^{-2}. Solid (dotted) curves are for $\omega_\mathrm{p} < \omega_\mathrm{c}$ ($\omega_\mathrm{p} > \omega_\mathrm{c}$) when $B_0 = 0.095$ T ($B_0 = 0.045$ T).

Finally we get the following expression for the acceleration gradient (or energy gain) in megaelectronvolts per meter from this equation of γ

$$\frac{dV}{dz} = \frac{c^2 E_0[M_+^2(M_+^2 - P_+^2) - Q_+^4]}{\omega(v_\mathrm{g} - v_\mathrm{z})\sqrt{(Q_+^4 - M_+^4)}} \frac{a}{\pi} \frac{M_+^2}{Q_+^2} \times \left[1 - \frac{\omega c^2}{\gamma^2 v_z(\omega v_z - c^2 k_\mathrm{g})}\right] \cos\left(\frac{\pi}{a}x\right) \sin\left(\frac{2\pi}{\lambda_\mathrm{g}}z\right). \tag{2.6}$$

3. Results

The expression (2.6) reveals that the maximum gradient is attained when the electron completes the distance of $\lambda_g/4$. Therefore, we restrict our calculations to $z = \lambda_g/4$ and consider the point that the electron never collides with the walls of the waveguide. Figure 1(a) shows that the effects of the magnetic field and waveguide width are to reduce the maximum gradient and the total energy gain. When we analyze Fig. 1(b) we see that the maximum acceleration gradient decreases with plasma density n_0, but it gets larger with angle θ. Moreover, when we compare the cases of $\omega_\mathrm{p} > \omega_\mathrm{c}$ and $\omega_\mathrm{p} < \omega_\mathrm{c}$, it is concluded that the effective acceleration is possible when n_0 and B_0 are taken such that $\omega_\mathrm{p} > \omega_\mathrm{c}$ (dotted curves). Our numerical calculations infer that the gradient (gain) increases from 390 MeV m^{-1} (3.07 MeV) to 448 MeV m^{-1} (3.32 MeV) when the operational frequency f is increased from 5.6 to 5.8 GHz. Also the larger gradient can be obtained for the higher intensity of the microwave and larger initial energy of the electron.

4. Conclusions

To obtain a higher level of cutoff frequency for the mode propagation we need a larger plasma density and/or magnetic field in the waveguide. Although the allowed frequency band broadens with increasing waveguide width, it becomes smaller for increasing n_0 and B_0. Larger acceleration gradients are obtained for the increasing microwave frequency f and angle θ of magnetic field. However, the effect of n_0 and B_0 are to reduce the gradient and the energy gain of the electron. The present analysis infers that larger acceleration is achieved when n_0 and B_0 are employed such that ω_p remains larger than ω_c.

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References

[1] Trivelpiece, A. W. and Gould, R. W. 1959 Space charge waves in cylindrical plasma coloumns. *J. Appl. Phys.* **30**, 1784–1793.

[2] Maraghechi, B. et al. 1994 High-frequency waves in a plasma waveguide. *Phys. Plasmas* **1**, 3181–3188.

[3] Ivanov, S. T. and Nikolaev, N. I. 1998 The spectrum of electromagnetic waves in planar gyrotropic plasma waveguide. *Japan. J. Appl. Phys.* **37**, 5033–5038.

[4] Hirshfield, J. L. et al. 1996 Multimegawatt cyclotron autoresonance accelerator. *Phys. Plasmas* **3**, 2163–2168.

[5] Yoder, Y. et al. 2001 Energy-gain measurement from a microwave inverse free-electron–laser accelerator. *Phys. Rev. Lett.* **86**, 1765–1768.

[6] Jing, C. 2003 Dipole-mode wakefields in dielectric-loaded rectangular waveguide accelerating structures. *Phys. Rev. E* **68**, 016502.

[7] Malik H. K. 2003 Energy gain by an electron in the fundamental mode of a rectangular waveguide by microwave radiation. *J. Plasma Phys.* **69**, 59–67.

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