Robotica (2006) volume 24, pp. 211–220. © 2005 Cambridge University Press
doi:10.1017/S0263574705002043 Printed in the United Kingdom

Robotic yo-yo: modelling and control strategies

Leon Žlajpah

Robotics Laboratory, Jožef Stefan Institute, Jamova 39, 1000 Ljubljana (Slovenia)
E-mail: leon.zlajpah@ijs.si

(Received in Final Form: May 19, 2005, first published online 31 October 2005)

SUMMARY

In the paper we address a problem of controlling an oscillating motion with a robot. As the object we have selected a yo-yo. First we have measured and analysed the motion of different yo-yos. We have developed a simplified model of a yo-yo which has one degree-of-freedom, and the behaviour at the end of the string is modelled as an impact. Next, we discuss the control strategy. Our results show, that for playing a yo-yo it is important to start the upward motion before the yo-yo reaches the bottom position and the acceleration has to be reversed after the bottom impact. We present two control strategies: one based on predefined hand motion pattern and and the other generating the hand motion on-line. Both allow playing the yo-yo at a selected top height. The theoretical results have been proven by experiments on a real robot system.

KEYWORDS: Robotic yo-yo; Oscillating motion; Control strategies; Real robot.

I. INTRODUCTION

This paper presents an approach to robot arm control that is capable of performing rhythmic tasks. In the last years there has been a growing interest in robot systems that are capable of performing rhythmic tasks. One of the exciting tasks is juggling,[1–3] or playing with different toys,[3–5] among which is also a yo-yo.[6,7] Common to all of them is that playing with them is usually more or less an easy task for a human, but a complex task for a robot. Namely, the dexterity of the system and the synchronization with the toy are required. A human can use his senses to learn how to operate a toy. However, developing a robotic system that can perform the same job requires complex sensory systems and advanced control strategies.

Yo-yo is a toy made of two discs connected with a thin short axle. A string is tied to the axle and the operator controls the motion of the yo-yo by moving it up and down. The objective is to attain a periodic motion of the yo-yo. For an efficient robotic yo-yo a corresponding model is needed. There are only a few models of yo-yo available in literature. A good insight into the behaviour of the yo-yo is given by Jin.[7] The motion of the yo-yo is divided into four phases, and each of them is analysed. As the derived model is very complex, the authors propose a simplified model. However, some of their assumptions are too restrictive, especially neglecting the diameter of the string although the control relies on the cycle time which may depend on the diameter of the string. In reference [6] a simplified model is given, but the authors assume that the energy loss is only due to the friction and they neglect the bottom impact. To develop an adequate model we have analysed the behaviour of the yo-yo. Based on our previous work[8–10] we propose a one-DOF model which captures all important features of the yo-yo necessary for designing the control strategy for robotic yo-yo. We also explain which parameters influence the operability of the yo-yo.

There are different ways to generate the hand motion. For example, the rhythmic motion pattern can be predefined. In the first case, a nominal motion pattern can be learned by human demonstration[5] or it can be composed of smooth functions.[6,8] Then, the on-line control algorithm changes the amplitude and the starting time of the hand motion cycle depending on the desired height of the motion. The next possibility is that the controller generates hand trajectories on-line,[11] depending on the state of the yo-yo. The modification of this approach is that instead of hand position trajectories the accelerations are generated.[9,12]

When juggling an object the hand motion has to be synchronized with the object. For many juggling tasks there exists a stable open loop control strategy.[3] However, playing yo-yo is a representative example of a rhythmic task that is not stable under open-loop control strategy. For stable motion, the action of the controller must be in a proper phase with the motion of the yo-yo. To synchronize the controller actions and yo-yo motion Jin[11] proposed a neural network control which relies on phase-locked coupled oscillators. Using the vision system the yo-yo peak position is detected and after some predefined time, the hand motion cycle starts. Also, Hashimoto[6] has done the synchronization in the same way. However, such an approach is applicable only if the initial yo-yo peak height is already near the desired one. Therefore, we have proposed to initiate the hand motion at a certain yo-yo height without any delay and reset when the bottom position is reached.[9] A similar strategy has been used by Jin.[12]

In the paper we deal with the modelling of a yo-yo and control strategy for the robotic yo-yo. First, we analyse the behaviour of the yo-yo and then we propose a simplified one-DOF model. Next, we describe some control strategies which enable a robot to operate a yo-yo, and in the last section we give some experimental results.

Fig. 1. Measurement of yo-yo motion with optical system SMART.

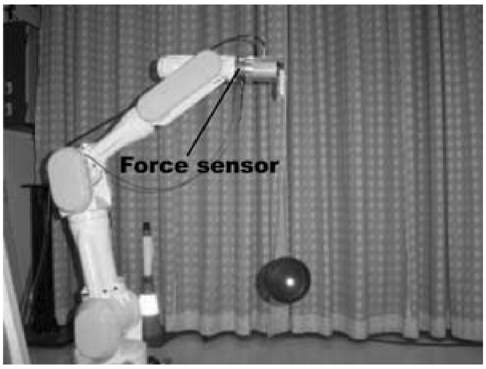


Fig. 2. Experimental setup for force measurement.

II. ANALYSIS OF YO-YO MOTION

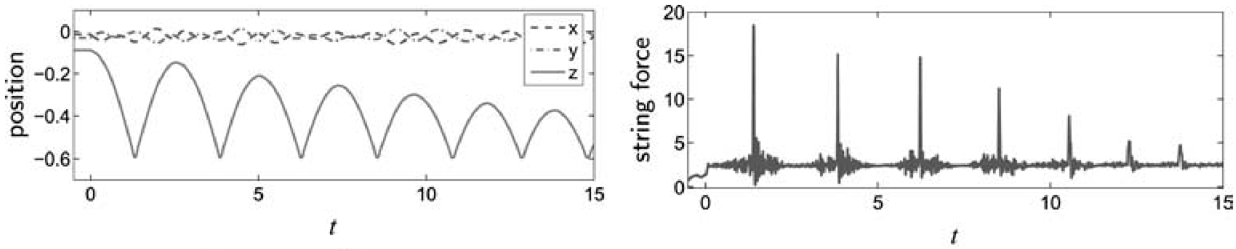
Before modelling we have analysed the motion of the yo-yo without human interaction and when a human has played yo-yo. We have measured the position trajectories of the yo-yo and hand, and the forces in the string. The parameters of measured yo-yos are given in the Appendix.

The experimental setup for the motion analysis consists of an optical system SMART which can measure 3D positions using passive markers. Figure 1 shows infrared cameras and a yo-yo with a marker attached to the center of a disc. The system operates at 66Hz and the position accuracy is around 1 mm. The forces in the string attached to the axle of the yo-yo have been measured with a 6-dimensional force/torque sensor JR3 and a PC computer (Figure 2).

We have analysed two situations: the motion without moving the hand (string is rigidly restrained) and when a human plays with the yo-yo. Figure 3(a) shows the motion of the yo-yo without human interaction ("free" motion). We can notice that the amplitude decreases with each period. Furthermore, some small disturbing oscillations in x and y direction can be seen. The string forces during free motion are shown in Figure 3(b). We can see that when the yo-yo reaches the bottom position the string forces are significantly larger than the gravity forces due to the weight of the yo-yo. This indicates that an impact occurs when the yo-yo reaches the bottom position.

The amplitude of the impact forces significantly depends on the properties of the string. We have compared two strings which differ in their elasticity. The string forces during free motion are shown in Figure 4. We can see that before the yo-yo reaches the bottom position the string force is rather small (proportional to the yo-yo weight). However, when the yo-yo reaches the bottom position an impact occurs. After the impact, there is a short period when the string is not under tension (yo-yo is flying free) followed by a series of smaller impacts. Comparing both figures we can see that the amplitude of the impact force is greater when the string is not elastic and that the free flying period is longer when the string is elastic.

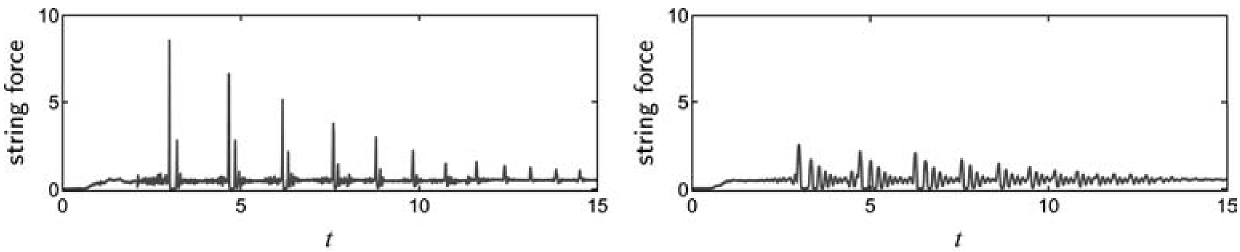
Additionally, we have investigated the forces during down/up motion. As the string winds around the axle in several layers, some jerks occur when the string layer



(a) yo-yo positions

(b) string forces

Fig. 3. Position and forces during yo-yo free motion (Yo-yo A).



(a) string is not elastic

(b) elastic string

Fig. 4. Forces in the string attached to yo-yo axle for different strings (Yo-yo B).

https://doi.org/10.1017/S0263574705002043 Published online by Cambridge University Press

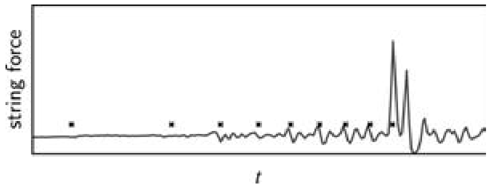


Fig. 5. Forces due to string unwinding (markers show the time when yo-yo is turned for one turn).

changes. Figure 5 shows string forces during unwinding, i.e. yo-yo is moving from top to bottom position. The markers indicate the time when the yo-yo has turned for one turn. We can observe small force pulses which are "synchronized" with the turns of the yo-yo. Note that the time between two markers correspond to one turn of the yo-yo.

III. MODELLING OF THE YO-YO

In general, a yo-yo is a free flying object constrained by a string attached to it. It has 6 degrees-of-freedom (DOF). However, for the bouncing motion only two DOFs are used: one for the vertical motion and the other for the rotation of the yo-yo around the axle. The other four DOFs allow motion in the remaining directions and this motion represents the disturbances. The complete mathematical model of a yo-yo (considering all DOF) would be very complicated.[7] However, when modelling a system it is important to know for what purposes the model will be used. Namely, although the model could describe all features of the system it is reasonable to consider only those features which are important for the purpose of the model. To model the bouncing motion of the yo-yo it is essential to observe the up and down motion.

Figure 6 shows a detailed picture of a yo-yo. When the string is stretched the relation between the vertical position of the yo-yo y and the rotational angle φ is

$$y = \begin{cases} h - l, & |\varphi| \geq \frac{\pi}{2} \\ h - L - r_o \cos(\varphi), & |\varphi| < \frac{\pi}{2} \end{cases} \tag{1}$$

where h is the height of the top end of the string (hand position), L is the total length of the string, and l is the length of the unwinded part of the string. Note that l depends

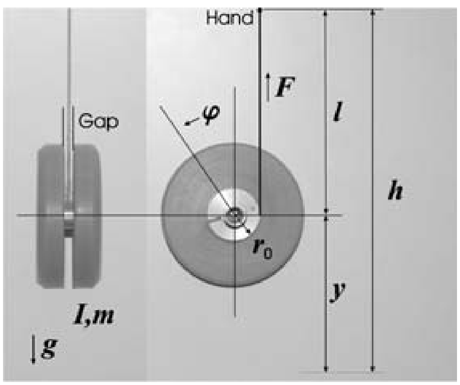


Fig. 6. Schematic picture of a yo-yo.

on the winding angle φ and on the winding radius r. The winding radius r depends on how the string is winded on the axle. For example, if the gap between the discs is greater than the diameter of the string then r does not change until the string starts winding on the next layer. This dependency can be approximated by the following relation

$$r(\varphi) = r_o + k_r\left(|\varphi| - \frac{\pi}{2}\right) \quad \text{for} \quad |\varphi| \geq \frac{\pi}{2} \tag{2}$$

where r_o is the radius of the axle and k_r is the effective radius of the string. Using this in Eq. (1) yields

$$y = \begin{cases} h - L + \left(r_o + k_r\left(|\varphi| - \frac{\pi}{2}\right)\right)\left(|\varphi| - \frac{\pi}{2}\right), & |\varphi| \geq \frac{\pi}{2} \\ h - L - r_o \cos(\varphi), & |\varphi| < \frac{\pi}{2} \end{cases} \tag{3}$$

Next, the motion of the yo-yo can be described by the following equation

$$\begin{aligned} I\ddot{\varphi} + B\dot{\varphi} &= -r_a F \\ m\ddot{y} &= F - mg \end{aligned} \tag{4}$$

where I, m and B are the inertia, the mass and viscose friction coefficient of the yo-yo, respectively, g is gravity constant, F is the string force, and r_a is the force moment arm

$$r_a = \begin{cases} r(\varphi)\,\text{sign}(\varphi), & \|\varphi\| \geq \frac{\pi}{2} \\ r_o \sin(\varphi), & \|\varphi\| < \frac{\pi}{2} \end{cases} \tag{5}$$

Actually, F represents forces in the dynamic model of the string. In our case (the design of robot control) it is important to consider energy balance and hence, it is not so important to include forces in the model output. Of course, if we want to use the model to simulate the yo-yo which is operated with a haptic interface, then it is important to know the forces in the string which occur during the up and down motion.[10]

When a yo-yo is bouncing up and down, the kinetic energy is converted to potential energy and *vice versa*. Additionally, it dissipates the energy at the bottom impact, due to the friction between the string and the yo-yo. To obtain an oscillatory motion it is necessary to supply energy to the system. This can be done via the string by moving the hand up and down. Note that we will use the model to select the control strategy for the robot and that the model will not be directly included in the control loop. Therefore, when modelling we can use some assumptions which simplify the analysis.

Assumption 1. The center the yo-yo mass is moving only in the vertical direction and the yo-yo is rotating only along the axle. The rotational axis is always perpendicular to the vertical axis.
Assumption 2. The string is flexible but not extensible; the mass of the string can be neglected.
Assumption 3. All dissipative forces are due to the viscous friction which is proportional to the rotational velocity.
Assumption 4. The time needed for the rotation for π at the bottom can be neglected.

https://doi.org/10.1017/S0263574705002043 Published online by Cambridge University Press

Assumption 5. The string is always stretched and the restitution coefficient is zero.
Assumption 6. Motion of the hand is smooth, i.e. hand velocities are continuous.

The first assumption enables us to model the yo-yo as a two-DOF system. When playing the yo-yo undesired motion like swinging, yawing and pitching are present. Although they disturb the primary bouncing motion, in some cases even significantly, we neglect them in the model because the model is primary intended for the selection of the control strategy.

Assumption 2 allows us to neglect the dynamics of the string. Namely, as the motion of the yo-yo can be controlled only by the motion of the top end of the string, it is necessary that the string is always under tension, otherwise the controllability of the yo-yo is lost. Therefore, the string should not be extensible. Note that when string forces are important, then this assumption is not correct.[10] Furthermore, some authors neglect also the influence of the diameter of the string.[6,7] However, the tests on different yo-yos have shown that the diameter of the string (actually the change of the winding radius r) influences the cycle time of the yo-yo and it cannot be neglected in modelling if the control strategy relies on the yo-yo cycle timing.

Using assumption 4 the motion of the yo-yo at the bottom position (when no string is wrapped around the axle) is modelled as an impact. As the rotation for π at the bottom is neglected we will use in the following $|\varphi|$ instead of $(|\varphi| - \pi/2)$.

Assumption 5 allows further simplifications of the model. If the string is stretched, then the vertical motion and rotation are dependent (constrained) and the yo-yo can be modelled as a one-DOF system. In reference [7] it is explained that the free motion can occur only after an impact when the yo-yo is set off with extra free string. The impact usually occurs when the yo-yo reaches the bottom position and the whole string is unwinded. If the restitution coefficient is greater then zero, then after the impact the vertical velocity is greater than the winding of the string due to the rotation. Hence, the string is loose and the yo-yo is free flying. Neglecting the friction, the rotation of the yo-yo during the free motion is constant, but the translational motion is changing due to the gravity. However, the energy loss after the bottom impact is independent of the restitution coefficient if the transition phase (series of minor impacts after the bottom impact) is completed. Therefore, it is reasonable to set for restitution coefficient to zero.
Using these assumptions Eq. (3) simplifies to

$$y = h - L + (r_o + k_r|\varphi|)\,|\varphi| \tag{6}$$

and Eq. (5) to

$$r_a = (r_o + k_r|\varphi|)\,\mathrm{sign}(\varphi) \tag{7}$$

The velocities and accelerations can be derived by differentiating Eq. (6)

$$\dot{y} = \dot{h} + r\,\mathrm{sign}(\varphi)\dot{\varphi} + k_r\,\varphi\,\dot{\varphi} \tag{8}$$

$$\ddot{y} = \ddot{h} + (r\mathrm{sign}(\varphi) + k_r\varphi)\,\ddot{\varphi} + 2k_r\,\dot{\varphi}^2 \tag{9}$$

Note that $\frac{\partial \mathrm{sign}(\varphi)}{\partial \varphi} = 0$ for $\varphi \neq 0$.

Based on Assumption 5 the string is always stretched and $F > 0$. Therefore, Eqs (7) can be combined and substituting Eq. (7) into Eq. (4) yields

$$I\ddot{\varphi} + B\dot{\varphi} = -(r_o + k_r|\varphi|)\,\mathrm{sign}(\varphi)\,m(\ddot{y} + g) \tag{10}$$

Substituting Eq. (9) into Eq. (10) yields after some cal culations

$$\ddot{\varphi} = -\frac{(r_o + k_r|\varphi|)\mathrm{sign}(\varphi)m(\ddot{h} + 2k_r\dot{\varphi}^2 + g) + B\dot{\varphi}}{I + mr^2 + mrk_r|\varphi|} \tag{11}$$

Eq. (11) describes the motion of the yo-yo during up an down motion. The complicated part is the motion at the bottom where motion direction changes. We have modelled this phase as an impact. Let $(\cdot)^-$ denote the states immediately before the impact and $(\cdot)^+$ after the impact. Applying Assumption 2 and 4, we neglect the highly complex dynamic motion which depends mainly on the properties of the string. We assume that during this negligible short period of time when the yo-yo is rotated for π, no change in the yo-yo velocity occurs. The impact occurs after the rotation. Note that during this period the radius r is constant, $r = r_o$, and Assumptions 4 and 6 imply $\varphi^+ = \varphi^- = 0$ and $\dot{h}^+ = \dot{h}^- = \dot{h}$, respectively. Note also that after the impact φ changes its sign.

The velocities after the impact can be obtained by using the principles of impact dynamics. The angular momentum before and after the impact is conserved

$$I\dot{\varphi}^+ + mr_o\,\mathrm{sign}(\varphi^+)\dot{y}^+ = I\dot{\varphi}^- + mr_o\,\mathrm{sign}(\varphi^+)\dot{y}^- \tag{12}$$

Substituting Eq. (8) for $\dot{y}^-$ and $\dot{y}^+$ yields

$$\begin{aligned} &I\dot{\varphi}^+ + mr_o\,\mathrm{sign}(\varphi^+)(\dot{h} + r_o\,\mathrm{sign}(\varphi^+)\dot{\varphi}^+) \\ &\quad = I\dot{\varphi}^- + mr_o\,\mathrm{sign}(\varphi^+)\,(\dot{h} + r_o\,\mathrm{sign}(\varphi^-)\dot{\varphi}^-) \end{aligned} \tag{13}$$

After some calculations we obtain

$$\dot{\varphi}^+ = \frac{1}{I + mr_o^2}\big(I\,\dot{\varphi}^+ + mr_o^2\,\mathrm{sign}(\varphi^+)\,\mathrm{sign}(\varphi^-)\,\dot{\varphi}^-\big) \tag{14}$$

Since φ changes the sign at the bottom

$$\mathrm{sign}(\varphi^+)\,\mathrm{sign}(\varphi^-) = -1$$

Eq. (14) can be simplified

$$\dot{\varphi}^+ = \frac{I - mr_o^2}{I + mr_o^2}\,\dot{\varphi}^- \tag{15}$$

The vertical velocity can be easily obtained by combining Eqs. (15) and (8)

$$\dot{y}^+ = \dot{h} + r_o\,\mathrm{sign}(\varphi^+)\dot{\varphi}^+ \tag{16}$$

https://doi.org/10.1017/S0263574705002043 Published online by Cambridge University Press

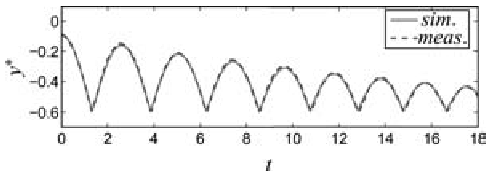


Fig. 7. Yo-yo free motion: comparison of simulation results and measured responses (Yo-yo A).

As $\text{sign}(\varphi^+) = \text{sign}(\dot{\varphi}^+)$ we get

$$\dot{y}^+ = \dot{h} + r_o|\dot{\varphi}^+| \tag{17}$$

which shows that after the impact the yo-yo is moving up if the string is stretched (Assumption 5).

Figure 7 shows the simulation response (the relative yo-yo height y^*, $y^* = y - h$) compared with the measured motion of a yo-yo. As one can see, the motion of the yo-yo is almost equal in both cases. To obtain such results we had to consider the change in effective inner radius due to the string thickness.

Summarizing, the yo-yo can be modelled as a one-DOF system consisting of Eqs. (6), (11) and (15). The "efficiency" of the yo-yo can be described by the factor

$$\zeta = \frac{I - mr_o^2}{I + mr_o^2} \tag{18}$$

From Eqs. (15) and (17) we can easily conclude that the energy loss during the impact is proportional to ζ^2

$$E^+ = \zeta^2 E^- \quad \text{and} \quad \Delta E = (\zeta^2 - 1)E^- \tag{19}$$

Because increasing r_o implies that the energy loss during impact ζ also increases, it explains why it is harder to play the yo-yo with large r_o.

From Eqs. (18) and (19) it follows that if $I \leq mr_o^2$ it is impossible to operate the yo-yo.[7] Our remark here is that this conclusion is valid only if the string is not extensible (Assumption 2). Namely, during the testing of different yo-yos we have found out that if the string is very elastic (not the case for the most yo-yos in practice), it is possible to play the yo-yo even when $I \leq mr_o^2$. Therefore, if a more accurate model of the yo-yo is required the string dynamics should be incorporated into the model.

Furthermore, it is also practically impossible to play a yo-yo if r_o is too small. Namely, from Eq. 11 it is evident that $\ddot{\varphi}$ is proportional to r_o and for small r_o the operator's influence on the yo-yo motion is small. Of course, small r_o yields also small energy loss during impact and the energy loss due to the friction becomes significant. As the loss of the energy due to the friction cannot be compensated for very small r_o, the yo-yo cannot be operated.

IV. CONTROL STRATEGY

The objective of playing the yo-yo is to keep the amplitude of the yo-yo at a desired level. It is evident that the motion of the yo-yo can be controlled only by moving the free end of the string (i.e. hand) up and down.

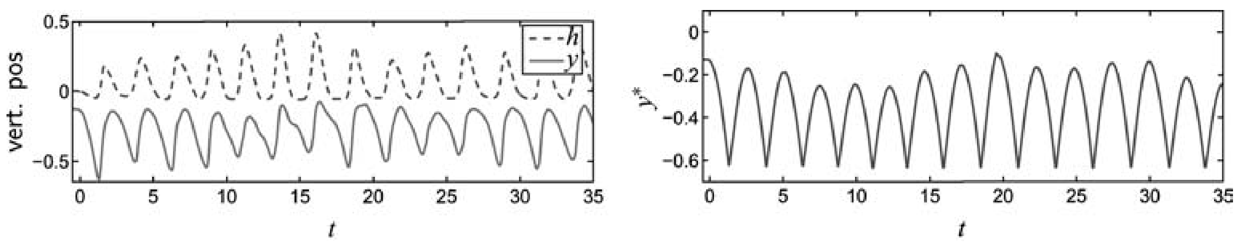
For the design of the robot control it is important to understand energy transfer. The potential energy stored in the yo-yo at the top position is sum of the potential energy in the previous top position, the energy lost during the last impact and the energy supplied by the hand during the last cycle. Neglecting the friction and when hand is not moved, $\ddot{h} = 0$, the energy between two successive bottom impacts is constant

$$E_o = mg\Delta y = \frac{1}{2}(I + mr_o^2)\dot{\varphi}_o^2 \tag{20}$$

where Δy is the difference between the peak height and the bottom height of the yo-yo, and $\dot{\varphi}_o$ is the velocity before the impact.

From Eq. (11) it follows that only the hand acceleration can influence the yo-yo motion. Based on measurements and using the derived model, we have found out that the most efficient transfer of the energy from the hand to the yo-yo is if the major upward acceleration is performed just before the bottom impact.[8]

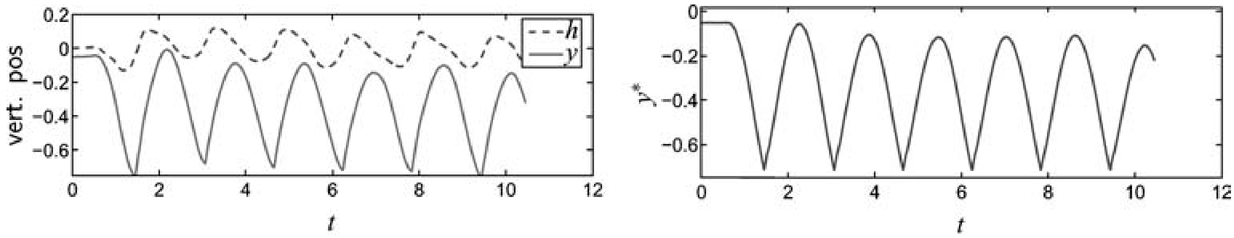
Before selecting the control strategy for the robot we have studied the strategies of human operators. Figures 8 and 9 show some examples of the hand motion and the yo-yo motion when a human is operating the yo-yo. We have found out that the "playing" strategies depend mainly on the skills of the operator. One operator prefers smooth hand motion with larger amplitudes and another more "jerky" motion. Common to all of them is that for the successful playing they had to synchronize the hand motion with the motion of the yo-yo. For synchronization some information about the state of the yo-yo is needed. The basic question is what is more important "seeing" the yo-yo motion or "feeling" the string forces. Playing with the yo-yo can reveal which information is important. First we have tested the importance of "seeing". The operators closed their eyes, and it has turned out that is was practically impossible to play the yo-yo. The reason for



(a) hand and yo-yo position (b) yo-yo relative height

Fig. 8. Hand and yo-yo motion when human is operating the yo-yo (Yo-yo A).

https://doi.org/10.1017/S0263574705002043 Published online by Cambridge University Press



(a) hand and yo-yo position (b) yo-yo relative height

Fig. 9. Hand and yo-yo motion when human is operating the yoyo (Yo-yo C).

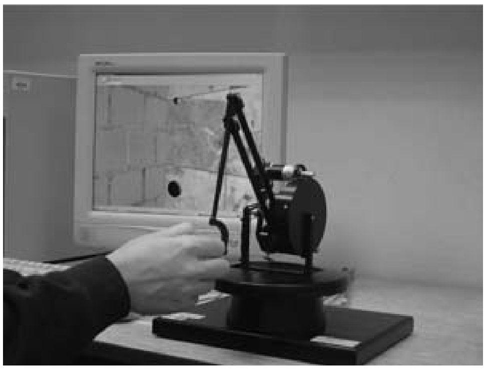


Fig. 10. Operator playing a virtual yo-yo.

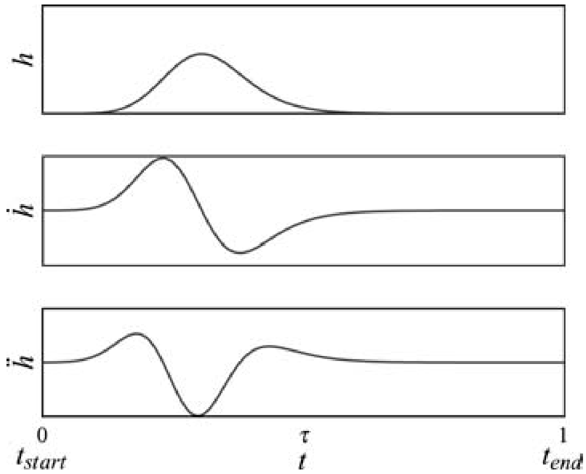


Fig. 11. Hand motion pattern.

unsuccessful playing is that the hand has to move upward before the yo-yo is reaches the bottom position. Of course, the operator can sense the bottom position because there occurs an impact and he can "feel" the force in the string. However, for playing it is necessary to predict the time of the next bottom impact. As the time interval between the two consecutive bottom impacts is relatively long, a human can not predict precisely enough the moment when the upward motion should start and he starts the motion at the wrong moment. The other situation to be verified is that the operator cannot "feel" the forces in the string. However, as it is practically impossible to prevent a player to feel the force in the string (he has to hold the yo-yo), the role of "feeling" the string force can not be determined by experiments with a real yo-yo. Therefore, we have developed a virtual yo-yo where the player uses a haptic device to play the yo-yo[10] (see Figure 10). As the virtual yo-yo enables us to select which feedback information gets the operator, we can easily check how the yo-yo is played with or without visual or force feedback. Our tests have shown that the yo-yo can be played without many problems when the operator does not "feel" the force. So, we can conclude that knowing the position (height) of the yo-yo is more important than "feeling" the force in the string. Hence, a vision system is crucial for the robotic yo-yo control.

IV.1. Predefined motion pattern

The control strategy can be based on predefined hand motion patterns. The nominal motion pattern can be learned by imitating the human motion,[5] or it can be composed of smooth functions.[6,8] We have selected the nominal hand motion pattern h_n as shown in Figure 11. This pattern satisfies all previously mentioned requirements. Then, the actual hand motion is generated from the nominal one considering the state of the yo-yo.

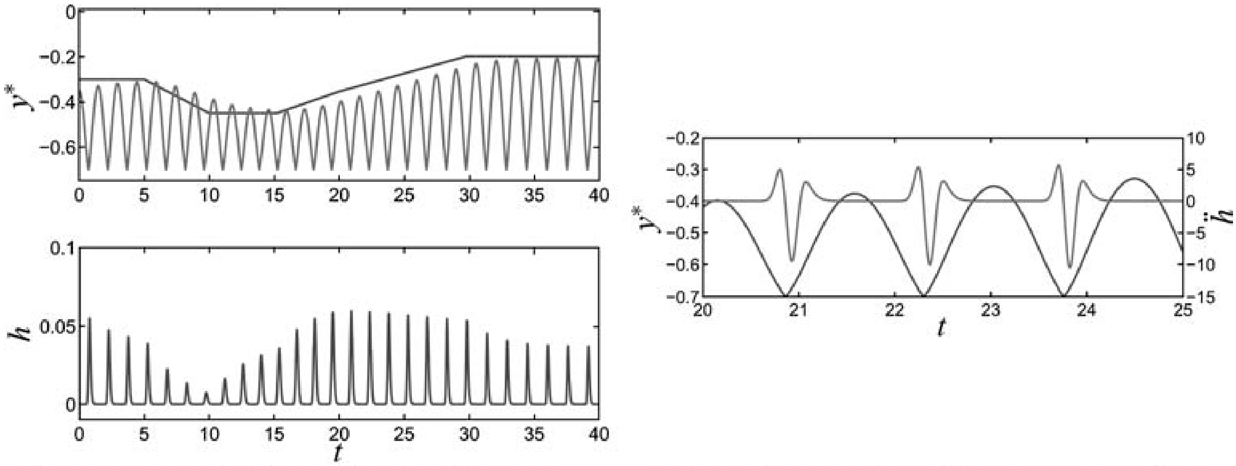
$$h = k_a\, h_n(k_t \tau), \quad 0 \le \tau \le 1 \tag{21}$$

where τ is the nominal pattern time, k_a is the gain to adjust the amplitude of the hand motion, and k_t is the scaling factor between the real and nominal time; $\tau = 0$ and $\tau = 1$ indicate the time when the motion starts (t_{start}) and when one cycle is completed (t_{end}), respectively. Between two cycles, $t_{end,i} < t < t_{start,i+1}$, hand is not moving, $h = 0$, $\dot{h} = 0$ and $\ddot{h} = 0$. It is obvious that $t_{end,i} - t_{start,i}$ must be less than the cycle of the yo-yo. The moment t_{start} must occur before the impact. As it is impossible to predict t_{start} directly, the hand motion is started at a certain height before the yo-yo reaches bottom position. By changing the amplitude (gain k_a) and the duration of the motion (the factor k_t) the yo-yo peak height can be controlled. The peak height can be increased by increasing k_h or decreasing k_t and vice versa.

To illustrate the capabilities of the proposed control strategy we present some simulation results. We have selected the nominal hand motion pattern as shown in Figure 11. The objective has been to play the yo-yo at the desired peak height $\hat{y}_d^*$ which has been changing during the motion. The amplitude of the yo-yo motion can be influenced by changing the gains k_h and k_t. So, we have used the following control algorithm

$$k_h = k_1(\hat{y}_d^* - \hat{y}^*) + k_2 \quad \text{and} \quad k_t = \frac{1}{k_3\, \hat{y}_d^* + k_4}$$

https://doi.org/10.1017/S0263574705002043 Published online by Cambridge University Press



a) desired yo-yo top height and actual yo-yo height and hand motion

b) yo-yo height and hand acceleration when top height should be increased

Fig. 12. Simulation of robotic yo-yo (Yo-yo B).

where $\hat{y}^*$ is actual peak height, and k_1, k_2, k_3 and k_4 are positive constants. Note that k_h and k_t have been changed only in the moment when the peak yo-yo height has been reached and that their value has been constant during the remaining time.

The simulation results are shown in Figure 12. We can see that the proposed control strategy ensures a stable yo-yo motion and that the peak height tracks the desired height.

IV.2. On-line generated motion

The alternative to the predefined hand motion pattern is to generate the desired hand motion on-line. The motion should be generated so, that the upward motion starts before the impact and after the impact the hand moves into its initial position. To generate the hand motion on-line based on the state of the yo-yo we propose a motion generator, as shown in Figure 13. This controller generates the hand motion according to the "pulse" input. If the input is "high" then hand is moved upward with acceleration $\ddot{h}_r$ and if input is "low" the hand is moved into its initial position. The principal motion pattern is given in Figure 14.

Let $t_{b,i}$ denote the time of the bottom impact and also the start of i-th cycle. The hand motion has two phases: the upward acceleration phase follow by the relaxation phase, when the hand is moved back to its initial position. The pulse P_i has to start before the bottom impact, $t_{s,i} < t_{b,i}$, and should end at $t = t_{b,i}$. We propose to select the time instant $t_{s,i}$ as the moment when the yo-yo reaches the height y^*_{trig}. To return

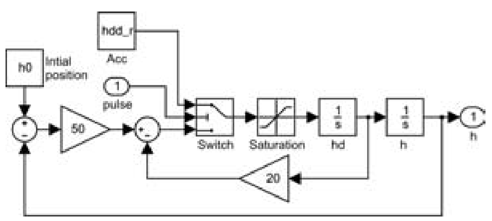


Fig. 13. Block scheme of hand motion generator.

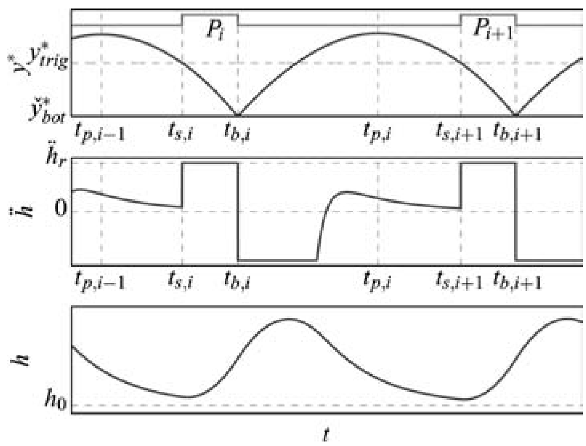


Fig. 14. Hand motion generation.

hand to its initial position after $t_{b,i}$, a simple PD controller is used. The saturation of the acceleration is used for safety to prevent that the hand accelerations exceed the limitations of a real system (downward acceleration is bounded to gravity acceleration).

As the pulse P_i is position triggered the duration of the pulse depends on the peak height in the previous cycle. Neglecting the string thickness, friction, and assuming no hand motion ($\ddot{h} = 0$), it is easy to obtain from Eqs. 9 and 11 the following relations

$$P_i = t_{b,i} - t_{s,i} = \sqrt{t_1^2 + \frac{4(y^*_{trig} - \check{y}^*_i)}{g(1-\zeta)}} - t_1$$

$$t_1 = t_{s,i} - t_{p,i-1} = \sqrt{\frac{4(\hat{y}^*_{i-1} - y^*_{trig})}{g(1-\zeta)}} \qquad (22)$$

where $\hat{y}^*_i$ and $\check{y}^*_i$ are the peak and bottom yo-yo positions, respectively. Eq. 22 shows that higher peak $\hat{y}^*_{i-1}$ gives shorter

https://doi.org/10.1017/S0263574705002043 Published online by Cambridge University Press

pulse P_i and *vice versa*. Considering all neglected yo-yo parameters, a similar but more complex relation between P_i and $\hat{y}^*_{i-1}$ could be found.

The amount of energy transferred to the yo-yo in the acceleration phase depends on the duration of the acceleration pulse P_i and its amplitude $\ddot{h}_r$. For stable yo-yo motion the energy loss has to be compensated by energy supplied by hand motion. From Eqs. 11 and 15 the following relation for the stationary motion can be obtained

$$\dot{\varphi}_o = \zeta\left(\dot{\varphi}_o + \frac{mr_o}{I + mr_o^2}\int_{t_{s,i}}^{t_{b,i}} \ddot{h}\,\mathrm{d}t\right) \tag{23}$$

Substituting $\dot{\varphi}_o$ from Eq. 20 yields after some calculations

$$\int_{t_{s,i}}^{t_{b,i}} \ddot{h}\,\mathrm{d}t = \ddot{h}_r P = \sqrt{2(1-\zeta)g\,\Delta y} \tag{24}$$

Now, assume that after achieving a stable yo-yo motion, the peak height decreases for some reason. Consequently, the pulse duration increases (see Eq. 22) and more energy is transferred to the yo-yo. Therefore, the peak height increases. Similar, if the peak height is too high, the pulse duration decreases and the height decreases. This property of the proposed control strategy is crucial for the stable operation of the yo-yo.

Finally, from Eq. 24 it follows that the amplitude of the yo-yo motion can be selected by changing $\ddot{h}_r$ or y^*_{trig} (note that $P = P(y^*_{trig})$. When selecting these two parameters we have to consider limitations of the robot system. We have already mentioned the acceleration bounds. Additionally, it is necessary to consider also the robot workspace bounds. The robot can operate a yo-yo only if the required tip positions are in the workspace, and too high $\ddot{h}_r$ combined with too long P_i may yield too large vertical motion of the hand. Therefore, a special attention is needed when the yo-yo peak height is lower than the goal value. Although, the robot could operate the yo-yo at goal value, the pulse P_i for lower heights could be too long. In such cases we suggest to decrease the trigger height temporarily and then stepwise increase it as the peak height approaches the goal value. In fact, it could be easily verified that the hand motion height is lower when pulses are shorter and accelerations higher, for the same amount of transferred energy, of course.

V. EXPERIMENTS

To illustrate the capabilities of the proposed control strategy based on on-line hand motion generation we have used the Mitsubishi PA10 robot arm. The experimental setup is shown in Figure 15. The yo-yo has been tied to the tip of the robot. To measure the position of the yo-yo a vision system has been used. In the current implementation the vision system is using a simple USB WebCam 16. The proposed control strategy relies on good timing and synchronization. Using only the video information it is very hard to determine the moment of the bottom impact precisely. To improve the performance, a force sensor, which measures forces in the string, has been used to detect the bottom impact (Figure 16).

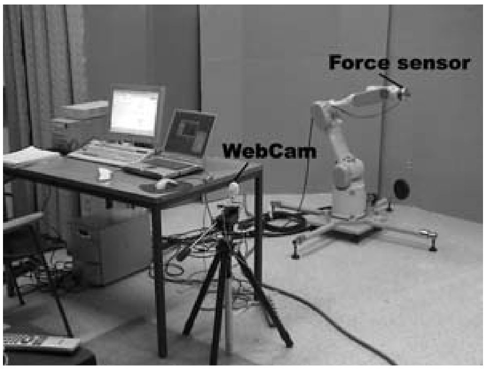


Fig. 15. Experimental setup.

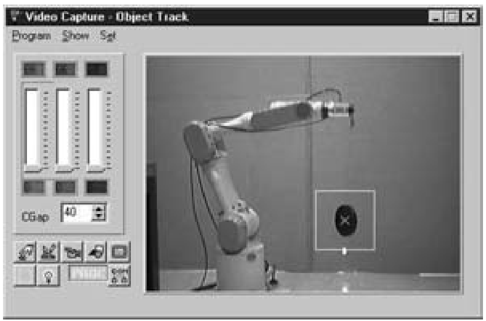


Fig. 16. Capturing and identification of the yo-yo position with WebCam.

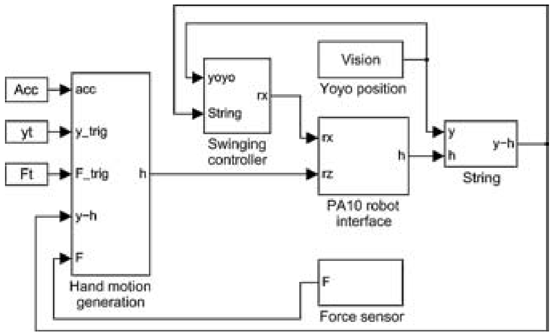


Fig. 17. Block scheme of the controller implemented in Simulink.

The control has been implemented on PC's in MATLAB/SIMULINK environment. For controlling the robot tip position the kinematic control implemented in PA10 motion control board has been used. Special Simulink drivers for interfacing the PA10 robot control board, vision system and force sensors have been developed. The main controller generates the up and down motion of the tip of the robot. The sampling rate of the controller has been 100 Hz, except for the video frame rate which has been 25 Hz. The control scheme is presented in Figure 17.

Experimental results have shown that when the robot is playing yo-yo, swinging of the yo-yo occurs (Assumption 1 violated). This swinging can even make the primary up-down motion of the yo-yo in vertical direction impossible. Therefore, to reduce the disturbing swinging we have implemented an additional controller which compensates the yo-yo swinging by moving the robot tip in horizontal direction.

https://doi.org/10.1017/S0263574705002043 Published online by Cambridge University Press

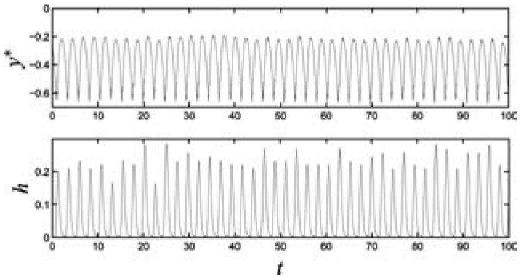


Fig. 18. Yo-yo height and robot tip position in the first 100 s of the experiment (Yo-yo A).

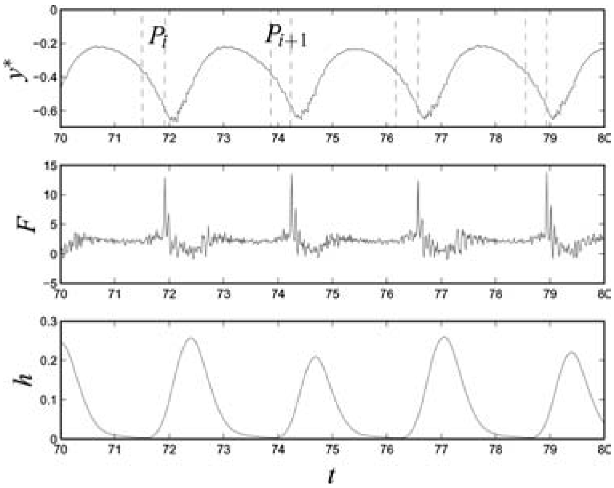


Fig. 19. Detailed view of yo-yo height, string force and robot tip position (Yo-yo A).

In the experiment we have used yo-yo A. Considering the acceleration bounds of the PA10 robot and the delay in the video information, the hand upward motion started when yo-yo has been 0.25 m above the bottom position ($\hat{y}^*_{trig} = y^*_{bot} + 0.25\,\mathrm{m}$) with the acceleration of $\ddot{h}_r = 1.6\,\mathrm{ms}^{-2}$. The upward acceleration stopped when the force in the string exceeded $F_{trig} = 4\,\mathrm{N}$.

The results are shown in Figures 18 and 19. Figure 18 shows the relative height of the yo-yo y^* and the robot tip position h in vertical direction in the first 100 s of the experiment. We can see that with the proposed control strategy the yo-yo motion is stable and that the peak height is preserved. Figure 19 shows a detailed view of 10 s motion. The small variations of the yo-yo peak height are due to the disturbances caused by swinging of the yo-yo. Although the controller parameters y^*_{trig} and $\ddot{h}_r$ have been constant during this experiment, we can see that the amplitude of the robot tip motion h changes. Namely, when the yo-yo peak height decreases the interval P increases and more energy is transferred to the yo-yo (E.g., Figure 19 in $P_i > P_{i+1}$). As previously explained, this stabilizing action is one of the key features of the proposed control strategy. One could conclude from Figure 19 that the acceleration interval P stops before yo-yo reaches the bottom position. However, this difference is due to the delay in vision system. Note that because the force sensor is very fast, the force signal gives better information about the time of bottom impact.

VI. CONCLUSION

This paper deals with the modelling of a yo-yo and selection of a control strategy for playing a yo-yo with a robot. Although, playing a yo-yo is an easy task for a human, it is an exciting piece of work for a robot. First of all, not all of the yo-yo states are measurable and secondly, the motion of the yo-yo can be controlled only by moving the free end of the string. To understand the system we have analysed the yo-yo motion and then we have developed a model. The proposed one DOF model captures all important features of the yo-yo important for the control design. When selecting a control strategy two things are important for robotic yo-yo: to select a suitable nominal hand motion strategy and to synchronize the robot motion with the yo-yo. Experiments with yo-yo have shown that visual feedback is essential for playing the yo-yo, because the hand motion upward should be started before bottom impact. "Feeling" the bottom impact can improve the operation but it is not required. We have compared two cases when the hand motion is generated using a predefined pattern and when the motion is generated on-line.

To conclude, the main result is that for playing a yo-yo it is important to start the upward motion before the yo-yo reaches the bottom position and the acceleration has to be reversed after the bottom impact. The peak height of the yo-yo motion depends on the duration of the upward acceleration and on the acceleration amplitude. From the viewpoint of the robot workspace bounds it is better to use shorter acceleration pulses with greater amplitude as the hand position amplitude is lower in this case. However, shorter pulses require more precise timing. Therefore, we propose a control strategy where the robot tip motion is generated on-line depending on the yo-yo state: at a certain height before the bottom impact the upward motion of the robot is initiated and after the impact, which is detected by the force sensor, the robot moves to its initial position. This control strategy assures the stable yo-yo motion and by changing the controller parameters, the peak height of the yo-yo motion can be selected. The proposed control has been verified by experiments with the Mitsubishi PA10 robot. To keep the yo-yo running for a longer time, it has been necessary to implement an additional controller that suppresses the swinging of the yo-yo.

References

1. A. A. Rizi and D. E. Koditschek, "Progress in Spatial Robot Juggling," *Proc. Int. Conf on Robotics and Automation*, Nice, France (1992) pp. 775–780.
2. M. Buehler, D. E. Koditschek and P. J. Kindlmann, "Planning and Control of Robotic Juggling and Catching Tasks," *Int. J. of Robotic Research* **6**(1): 3–14 (1994).
3. S. Schaal and C. G. Atkeson, "Open Loop Stable Control Strategies for Robot Juggling," *Proc. IEEE Conf. Robotics and Automation*, Atlanta, Georgia (1993) pp. 913–918.
4. M. M. Williamson, "Rhythmic Robot Arm Control Using Oscilators," *Proc. 1998 IEEE/RSJ Int. Conf. On Intelligent Robots and Systems*, Victoria, Canada (1998) pp. 77–83.
5. A. J. Ijspeert, J. Nakanishi and S. Shaal, "Learning Rhythmic Movements by Demonstration using Nonlinear Oscilators," *Proc. of the 2002 IEEE/RSJ Int. Conf. On Intelligent Robots and Systems*, Lausanne, Switzerland (2002) pp. 958–963.

https://doi.org/10.1017/S0263574705002043 Published online by Cambridge University Press

6. K. Hashimoto and T. Noritsugu, "Modeling and Control of Robotic Yoyo with Visual Feedback," *IEEE Int. Conf. on Robotics and Automation*, Minneapolis, Minnesota (1996) pp. 2650–2655.
7. H.-L. Jin and M. Zackenhouse, "Yo-yo dynamics: Sequence of Collisions Captured ba a Restitution Effect," *Trans. of ASME J. of Dynamic Systems, Measurement and Control* **124**(3), 390–397 (2002).
8. L. Žlajpah, "Modelling and Control Strategy of Robotic Yo-yo," *Proceedings of 12th Int. Workshop on Robotics in Alpe–Adria–Danube Region* (*CD*), Cassino, Italy (2003).
9. L. Žlajpah and B. Nemec, "Control Strategy for Robotic Yo-Yo," *Proc. of the 2003 IEEE/RSJ Int. Conf. On Intelligent Robots and Systems*, Las Vegas, Nevada (2002) pp. 767–772.
10. L. Žlajpah and A. Bardorfer, "Playing virtual yo-yo with haptic interface," *Proceedings of the 5th EUROSIM Congress on Modeling and Simulation* (*CD*), Marne la Valle, France (2004).
11. H.-L. Jin and M. Zackenhouse, "Necessary Condition for Simple Oscillatory Neural Control of Robotic Yo-yo," *Proc. Int. Joint Conf. on Neural Nerowrks-World Congress on Computational Intelligence* (*IJCNN-WCCI'02*), Honolulu, HI (2002) pp. 1427–1432.
12. H.-L. Jin and M. Zackenhouse, "Robotic Yo-yo Playing With Visual Feedback," *IEEE Trans. on Robotics and Automation* **20**(4), 736–744 (2004).

Appendix

Yo-yo parameters used in experiments and simulation:

		A	*B*	*C*
r_o	[m]	$8.5 \cdot 10^{-3}$	$5.0 \cdot 10^{-3}$	$5.5 \cdot 10^{-3}$
k_r	[mrd^{-1}]	$8 \cdot 10^{-5}$	$5 \cdot 10^{-4}$	$1.6 \cdot 10^{-4}$
m	[kg]	0.273	0.052	0.050
I	[kgm^2]	$5.92 \cdot 10^{-4}$	$1.96 \cdot 10^{-5}$	$2.6 \cdot 10^{-5}$
B	[Nm^{-1}s]	$6 \cdot 10^{-6}$	$2 \cdot 10^{-6}$	$1 \cdot 10^{-6}$

https://doi.org/10.1017/S0263574705002043 Published online by Cambridge University Press