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REVIEWS

Probability: The Science of Uncertainty. BY MICHAEL A. BEAN (Wadsworth (Brooks/Cole), 2001)

There must be hundreds of books on elementary probability. They usually have titles like *Probability Theory with Applications in X*, and strive to show that probability is not just an arcane theory, but absolutely indispensable if you work in X. The present work is just such a book, with X equal to *Investments, Insurance and Engineering*. Actually, the great majority of the applications are in the area of investments and insurance, and the book is really written almost completely from this perspective.

The author makes a brave attempt to motivate almost everything he does with examples from finance. The fifth chapter, for example, on discrete distributions, gives an idea of his approach. The list of distributions is familiar (binomial, Poisson, negative binomial, geometric), but the motivating examples are unusual. The binomial distribution is introduced as a model for stock prices as follows: suppose a stock has price S at the start of a trading day; the price at the end of the trading day is $Su, u > 1$, with probability p and price S/u with probability $1 - p$. If successive trading days are independent, then, after n days, the number of upward movements is given by the binomial distribution, and the probabilities of the possible values of the stock are binomial probabilities. There is then a nice discussion of what independence means in relation to price movements.

The discussion of the binomial distribution continues with a methodical elaboration of its main properties, much like any other elementary text: we are taken through the probability mass function, the moments and the moment generating function, tricks for calculating probabilities (these really do strike me as out of date now; most students will have access to a statistics package which will calculate cumulative probabilities and probability masses at the touch of a few keys); and how the binomial is related to both the Poisson and the normal distribution (although the book has many diagrams, there are no graphical comparisons of the binomial with its Poisson or normal approximations). The section on the binomial distribution finishes with four worked examples: the number of claims received by a life insurance company; the number of policyholders making at least one claim on their auto insurance; the price of a stock after four days trading; and finally, an engineering example. There are exercises (with selected answers) on the binomial distribution at the end of the chapter, and again the author has taken considerable trouble to put the exercises in a financial setting.

As well as the emphasis on finance in the choice of examples and exercises, there are other features of the book which distinguish it from the usual elementary probability text. Some of the topics with important applications in finance to receive an extended coverage are:

(a) mixture distributions: the negative binomial distribution is seen, not only as a discrete waiting time distribution, but also as a mixture of Poisson distributions with a gamma mixing distribution;
(b) compound distributions and, in particular, the compound Poisson distribution;
(c) mixed distributions: the treatment here is very thorough, with good use of diagrams to bring out the partly continuous and partly discrete nature of the distribution; again the connection with insurance caps and deductibles is well made; and
(d) geometric means as an appropriate summary for the return on a single investment over a number of independent periods.

This is a long book (over 400 pages), and there were places where I was uncomfortable with the author's treatment. I did not like the discussion of the Poisson distribution. The probability mass function is derived as the limit of a binomial distribution as $n \to \infty$ with $np = \lambda$. Unfortunately, the derivation (p196), as presented, appears to be a piece of mathematical trickery, and is not connected to events occurring in time. The author has made the decision that statements like:

$$\Pr[\text{Claim occurs in } (t, t + \delta t] = \lambda\delta t + o(\delta t)$$

are out of place in an elementary book. However, there is no reason why he should not consider the number of events occurring in a time interval $[0, t]$ which is divided into n subintervals. This leads plausibly to the binomial distribution, and the Poisson limit arises naturally. This approach has the advantage of placing events occurring in time at the centre of the derivation of the distribution. It is very difficult to understand statements like: "Calls to a telephone service are made at random and independently at an expected rate of two per minute" (p198), without some elaboration of what "at random" means in this context.

I was even less happy with the introduction to the geometric mean (p44). The expectation of a random variable is defined on page 42. On page 44 he considers the value of a security with initial value of 1. The value of the security after n days is:

$$(1 + x_1)(1 + x_2) \ldots (1 + x_n)$$

where $x_1, x_2, \ldots, x_n$ are the returns on days $1, 2, \ldots, n$. The X_i are assumed to be independent and identically distributed as:

$$X = \begin{cases} 0.5 \text{ with probability } 1/2 \\ -0.4 \text{ with probability } 1/2. \end{cases}$$

What "should we expect the accumulation after n days to be"? I should expect the definition

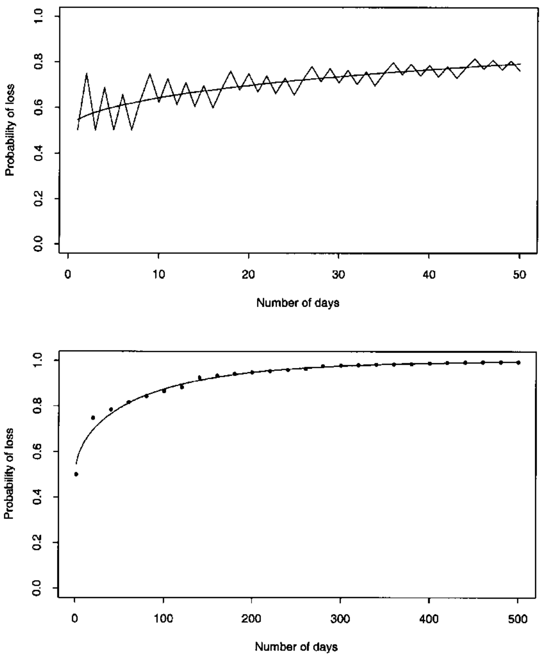

Figure 1. Probability of a loss with normal approximation for days: (top) $1, 2, \ldots, 50$; and (bottom) $1, 20, 40, 60, \ldots, 500$

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of expectation two pages earlier to be applied to give the answer 1.05^n. I should then expect a mighty alarm bell to be rung on the dangers of applying a linear operator to a product. Instead, the author gives a spurious argument which leads to an expected value of $0.9^{n/2}$. The following table is salutary, and shows that short-term behaviour is very surprising:

Number of days held	1	2	3	4	5	6	7
E(Value)	1.05	1.10	1.16	1.22	1.28	1.34	1.41
Pr(Profit)	0.5	0.25	0.5	0.31	0.5	0.34	0.5

The author writes: "we should expect our investment to be less than the initial amount after 2 days!"; he overlooks that, after 3, 5 and 7 days, we would have a 50% chance of making a healthy profit. A more satisfactory approach is to consider the log returns. Figure 1 shows the probability of a loss against the number of days held. The exact probabilities have a saw-tooth behaviour. The normal approximation using log returns is excellent. The conclusions are clear: the expected return tends to infinity, but the expected log return tends to minus infinity; the probability of making a profit tends to zero!

In conclusion, this book has much to offer the would-be actuary who wants to learn probability theory with a financial flavour. The financial motivation of probabilistic ideas is good, there are large numbers of worked examples and exercises, both with a largely financial setting, and much of the discussion has a nice economic insight. A useful addition to *Probability Theory with Applications in X*.

Iain Currie

An Introduction to Actuarial Studies. By M. E. Atkinson and D. C. M. Dickson (Edward Elgar Publishing, 2000)

This text is designed to be used in a general introductory first course in actuarial science. It is the basis for a 'taster' or 'attractor' course, to engage students' interest before they become more engrossed in the detail of actuarial theory.

Every academic actuary is aware that the students who select actuarial science have no real knowledge of what they have committed to until it is almost too late to escape! We start by teaching interest theory; not strictly actuarial, except in our love of notation. Not until they get into life contingencies do students really encounter the peculiarly actuarial blend of finance and uncertainty. This book is the basis of an introductory course which is designed to tackle this problem; it is three or four courses in actuarial science condensed into a single introductory course. Obviously a lot of detail must go; the students for whom this text is designed are new undergraduates, and relatively unsophisticated mathematically.

The first topic covered in the book is compound interest. Here the book goes into surprising detail. Coverage includes effective and nominal rates of interest; annuities immediate, due, $1/m$-thly; bond and commercial loan calculations. While this seems a lot — it is about 1/3 of a full compound interest course, it is also hard to see what could be omitted, given the later requirements in the life contingencies section. My main disagreement with the authors is that it would have been possible to teach the ideas without going into details of the $1/m$-thly annuity; a change of time unit achieves the same result without the confusing additional notation. Still, perhaps it would not be an honest introduction to actuarial science without a pile of notation to learn.

The second topic is demography — a bit of a surprise to some, as demography has been rather downplayed in actuarial syllabuses, both in the United Kingdom and North America. Here the student is assumed to have some training in continuous probability distributions, required for understanding the force of mortality — again, this maybe takes an introductory course further than one would expect.