

BOOK REVIEWS

HAMMER, J., *Unsolved Problems concerning Lattice Points* (Pitman Research Notes in Mathematics; no. 15, Pitman Publishing Ltd., 1977), 101 pp.

The author attempts to cover his subject comprehensively in a short book. To this end, he omits proofs and, indeed, any indication of the techniques involved. He gives references to the literature for all proofs. As a result, the reader who wishes to get the full flavour of the subject will need access to a well-stocked library.

The book is in three sections. The first two concern, respectively, geometrical and combinatorial problems for a single lattice. The third concerns problems on sets of lattices. In each section, there is a wide range of classical results, each with the subsequent refinements and generalisations, and with related open questions.

Given the nature of the presentation, the book is unsuitable as an introduction to the subject, but it should have some appeal to those who have a knowledge of the geometry of numbers.

W. W. STOTHERS

GIBSON, C. G., *Singular Points of Smooth Mappings* (Pitman, 1979), £8.00.

The recent upsurge of interest in singularity theory has led to the publication of a number of books on its various aspects. Nevertheless the present text is a valuable addition to the literature.

The layout is as follows. An introductory first chapter is followed by chapters on the basic ideas of transversality and unfolding respectively. The heart of the book is Chapter IV on singular points of smooth functions (mappings from smooth manifolds into the reals) and here we are given the list of singularities of codimension ≤ 5 . In the final chapter more general mappings are considered, giving the classification of stable germs by their local algebras. The author has understandably refrained from getting involved in catastrophe theory. Nevertheless the book has some overlap with that of Poston and Stewart ("Taylor expansions and catastrophes") in the same series. However the reviewer found it mathematically much more appealing than the latter. This is only to be expected as the book is directed specifically at a postgraduate (but not expert) mathematical audience. The approach is intuitive but rigorous, although the omission of some proofs tends to make it somewhat lightweight. Many excellent geometric examples are given. There are numerous misprints, particularly in the last two chapters, but almost all are of a trivial nature.

In conclusion this book is to be highly recommended as a clear and stimulating introduction to the subject.

A. G. ROBERTSON

BOLLOBÁS, BÉLA, *Extremal Graph Theory* (Academic Press, 1978), 488 pp., £19.50.

Extremal graph theory is usually considered to have its origins in a 1941 paper of the Hungarian mathematician Paul Turán. In that paper Turán found how many edges a graph G on n vertices can have if G contains no complete subgraph K_r on r vertices; further, he obtained all "extremal" graphs G with the maximal number of edges. This work is typical of the vast body of extremal graph theory which has grown up since then, "developed and loved by Hungarians", as Bollobás writes. A further quote from the preface explains more fully the nature of the subject. "In extremal graph theory one is interested in the relations between the various graph invariants, such as order, size, connectivity, minimum degree, maximum degree, chromatic number and diameter, and also in the values of those invariants which ensure that the graph has certain properties."

This encyclopaedic work, for which the author's article "Extremal problems in graph theory" in the first volume of the *Journal of Graph Theory* acts as a short introduction, is the first attempt to

get so much of the widespread theory into one volume. The book grew out of Part 3 courses given at Cambridge, and contains a great deal of material not otherwise available in book form. There are almost 450 pages of text, and 607 references of which 303 belong to the last ten years. The debt to Paul Erdős is clear from the 70 papers listed, and recent work is seen to be greatly influenced by the author's own contribution of 47 papers. The topics covered include connectivity, matchings, cycles, diameter, colourings, existence of complete subgraphs; the final chapter is on complexity and packing, where a lot of work of Bollobás and Eldridge is presented.

The flavour of the book can perhaps be appreciated from a brief sketch of one of the sections, that on fundamental matching theorems. The König-Hall theorem, earlier deduced from Menger's theorem, is proved in its generalised form due to Marshall Hall. It is remarked that transversal theory as such is outside the scope of the book, but it is pointed out that Philip Hall's result is contained in Dilworth's theorem on partially ordered sets, which is then proved. Here, as elsewhere, the author presents elegant simple proofs whenever possible (some of them new), thus attempting to unify the work. Tutte's theorem on 1-factors is then proved using Hall's theorem, and generalisations of Berge and Lovász are quickly deduced. But then follows a quick deduction of the Erdős-Gallai result that, if β denotes the maximal number of independent edges of G , where G has $n > 2\beta$ vertices, then the maximum number of edges possible in G is

$$\max \left\{ \binom{3\beta+1}{2}, \binom{\beta}{2} + \beta(n-\beta) \right\},$$

the bound being attained if and only if G is one of listed extremal graphs. The structure of graphs which in a sense "just" satisfy the conditions of Tutte's theorem is then studied (but the work of Sumner on minimal antifactor sets is not mentioned), and this leads to a study of the number $F(G)$ of 1-factors of G . The chapter then continues with f -factors, and recent work on matchings in graphs with given maximum and minimum degrees.

The book is a valuable contribution to the literature, collecting together in a coherent way a vast amount of work in a fast-growing subject, and bringing the reader right up to date. The exercises at the end of each chapter include a variety of recent research results (with references), conjectures and unresolved problems. My one criticism is that there are signs of haste in some irritating printing errors; within half a dozen pages on matchings there are several mathematical slips, two wrong references and at least three occasions when 1-factors are called 2-factors. But, such minor blemishes apart, this is a book which should be in every university library.

I. ANDERSON

BAKER, A. and MASSER, D. W. (editors), *Transcendence Theory: Advances and Applications* (Academic Press, 1977), £11.00.

Transcendence theory: advances and applications is the proceedings of a small technical conference held in Cambridge in 1976. Thus its primary function is the publication of recent work in transcendental number theory. As such it is surprisingly accessible to the student and the non-specialist.

The contributors to this volume have made, by editorial policy, a noteworthy attempt to expound the context of their contributions. Each article commences with the history and motivation of the problem and terminates in a substantial bibliography. It would be unreasonable to expect such a volume to be self-contained, and many fundamental lemmas are quoted from earlier publications. However the authors have made significant efforts to minimise this dependence by sketching background areas in preliminary sections and, on occasion, re-proving key results. Thus the reader who wishes to become acquainted with the current state of transcendence theory would be well-advised to turn first to this volume.

To the general reader the first four chapters will have most interest. These present the theory of linear forms in logarithms of algebraic numbers, and applications. The current state of the theory is summarised in two chapters, one on the latest form of Baker's estimate for such forms (which includes or surpasses almost all previous estimates) and the other on the analogous p -adic problem. There follows a history of the application of these tools to Diophantine equations, illustrated with