$$L(a,b) = 2\int_0^{\frac{\pi}{2}} \left(\sqrt{a^2\cos^2 t + b^2\sin^2 t} + \sqrt{a^2\sin^2 t + b^2\cos^2 t}\right)dt \geqslant \pi(a+b)$$

with equality if, and only if, $b = a$. Thus

$$\pi(a + b) \leqslant L(a, b) \leqslant 2\pi\sqrt{\frac{a^2 + b^2}{2}},$$

with equality if, and only if, $b = a$ as required.

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Cauchy-Schwarz via collisions

Consider a line of n railway trucks with masses $m_1, m_2, \ldots, m_n$ moving on a smooth straight track with velocities $v_1 > v_2 > \ldots > v_n$ (with negative velocities allowed) and spaced so that they successively couple together in the order Truck $n - 1$ to Truck n, Truck $n - 2$ to Trucks $n - 1$ and n, Truck $n - 3$ to Trucks $n - 2$ and $n - 1$ and n, etc. If, when all n trucks are coupled together, their common velocity is V, conservation of momentum shows that $V = \dfrac{\sum m_i v_i}{\sum m_i}$. But kinetic energy cannot be gained in the collisions, so

$$\frac{1}{2}\sum m_i v_i^2 \geqslant \frac{1}{2}\left(\sum m_i\right)V^2 = \frac{1}{2}\frac{\left(\sum m_i v_i\right)^2}{\sum m_i}$$

and thus

$$\left(\sum m_i v_i\right)^2 \leqslant \left(\sum m_i\right)\left(\sum m_i v_i^2\right). \qquad (*)$$

Moreover, physical intuition suggests that no kinetic energy will be lost only in the special case in which $v_1 = v_2 = \ldots = v_n$ where there are no collisions and the total kinetic energy is the same whether the trucks are viewed individually or *en masse*.

For two arbitrary sets of non-zero real numbers, $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$, substitute $m_i = a_i^2$, $v_i = \frac{b_i}{a_i}$ (so that v_i might be negative) and re-order so that $v_1 > v_2 > \ldots > v_n$. Then (*) rewrites as the Cauchy-Schwarz inequality,

$$\left(\sum a_i b_i\right)^2 \leqslant \left(\sum a_i^2\right)\left(\sum b_i^2\right), \qquad (**)$$

with equality when $v_1 = v_2 = \ldots = v_n$ so that the two sets of numbers are proportional. For two sets of real numbers where some of a_i or b_i might be zero, excluding the pairs (a_i, b_i) with $a_i b_i = 0$ leaves the LHS of (**) unchanged, whereas the RHS cannot decrease when these pairs are put back in so that Cauchy-Schwarz holds in this case as well.

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The constant acceleration formulae imply constant acceleration!

In this note we show that each of the five formulae for motion under constant acceleration, interpreted as differential equations describing the motion, imply that the acceleration is constant.

1. For example, $v = u + at$ rewrites as the general differential equation of motion $\dot{x} = \dot{x}_0 + \ddot{x}t$. Differentiating this gives $\ddot{x} = \ddot{x} + \dddot{x}\,t$ from which $\dddot{x} = 0$ and $\ddot{x}$ is constant.
2. $v^2 = u^2 + 2as$ gives the differential equation $\dot{x}^2 = \dot{x}_0^2 + 2\ddot{x}x$. Differentiating this gives $2\dot{x}\ddot{x} = 2\ddot{x}\dot{x} + 2\,\dddot{x}\,x$ from which $x\dddot{x} = 0$ so $\dddot{x} = 0$ whenever $x \neq 0$ and $\ddot{x}$ is constant.
3. $s = \frac{1}{2}(u + v)t$ gives the differential equation $x = \frac{1}{2}(\dot{x}_0 + \dot{x})t$. Differentiating this twice then gives $\dot{x} = \frac{1}{2}(\dot{x}_0 + \dot{x}) + \frac{1}{2}\ddot{x}t$ and $\ddot{x} = \frac{1}{2}\ddot{x} + \frac{1}{2}\ddot{x} + \frac{1}{2}\dddot{x}\,t$ from which $\dddot{x} = 0$ and $\ddot{x}$ is constant.
4. $s = vt - \frac{1}{2}at^2$ gives the differential equation $x = \dot{x}t - \frac{1}{2}\ddot{x}t^2$. Differentiating this gives $\dot{x} = \dot{x} + \ddot{x}t - \ddot{x}t - \frac{1}{2}\dddot{x}\,t^2$ from which $\dddot{x} = 0$ and $\ddot{x}$ is constant.
5. Now for the surprise! The fifth equation of motion $s = ut + \frac{1}{2}at^2$ gives the differential equation $x = \dot{x}_0 t + \frac{1}{2}\ddot{x}t^2$. Differentiating this twice gives $\dot{x} = \dot{x}_0 + \ddot{x}t + \frac{1}{2}\dddot{x}\,t^2$ and $\ddot{x} = \ddot{x} + \dddot{x}\,t + \dddot{x}\,t + \frac{1}{2}\ddddot{x}\,t^2$ so that $t^4\ddddot{x} + 4t^3\dddot{x} = 0$. Thus $t^4\dddot{x}$ is constant giving $\ddot{x} = A + \frac{B}{t^3}$ for constants A and B. $B = 0$ then avoids the singularity at $t = 0$ and implies constant acceleration.

This way of thinking about the constant acceleration formulae can be used to derive them from one another. For example, start with