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A detail shape grammar. Using Alberti's column system rules to evaluate the longitudinal elevation of the nave of Sant'Andrea church generation

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Abstract

This paper shows the way Leon Battista Alberti treatise *De Re Aedificatoria* column systematization shape grammar was built, and how the longitudinal elevation of the nave of the Sant'Andrea church was designed, evidencing some steps involved in the grammar construction process, its derivation, and the partial outputs gained from the grammar evaluation. The goal of this grammar is to infer Alberti's treatise rules through a computational approach evidencing its algorithmic nature, illustrating it, and understanding the *De Re Aedificatoria* rules that may be applied in the generation of some parts of buildings such as the Sant'Andrea church, showing a complementary method to archaeological, bibliographical, historical ones, among others, that have been used to understand the built environment. The rules provided by the grammar may be applied to an initial insertion point to produce a complete column and to a pre-existing plan or section element of a building generating its facade. It was proved, evaluating the grammar with a simple linear regression method, to what extent the treatise influenced the design of the facade in the study.

Introduction

The shape grammar presented in this paper may be considered as a detailed grammar. Its construction started from a doric base element described in Alberti's *De Re Aedificatoria* column system. The conjugation of different elements, mostly molds, with other elements leads the grammar user to the generation of parts of larger architectonic elements, such as facades. This process is based on the application of a grammar called *Alberti's Column Systematization Shape Grammar*, that is applied to generate part of a specific building – the longitudinal elevation of the nave of the Sant'Andrea church in Mantua, Italy.

The column systematization grammar allowed us to understand a set of possible combinations of elements, starting from a sequential manipulation of molds, revealing a certain number of possible conjugations of those elements, testing the generative characteristics of *De Re Aedificatoria* (the original treatise did not include any drawings), as Alberti specifies in Book IX, Chapter VI, pp. 592, that is, the reader should use the treatise descriptions at his discretion and not be "absolutely" constrained by them (Alberti, 2011). The algorithmic nature of the treatise is related to these premises: it describes ways of articulating and manipulating the molds and the column system elements (as desired by the user), not a combinatorial specification of them.

The research presented in this paper was related to the Alberti Digital research project (Kruger et al., 2011), whose goal was to determinate the extension of Alberti's influence on Portuguese architecture during the counter-reform period.

This paper is centered in the decoding process of the *De Re Aedificatoria* by inferring the corresponding shape grammar using the computational framework provided by description grammars (Stiny, 1981), shape grammars (Stiny & Gips, 1972) (Li, 2001), and the transformations in languages of designs (Knight, 1994). The grammar in the treatise was compared with the grammar of existing buildings designed by Alberti as well as with Portuguese buildings from the same period.

Alberti's *De Re Aedificatoria* consists of ten books: Book I about the Lineaments; Book II on materials, Book III on construction; Book IV on public works; Book V on buildings for private purposes, and Book VI on the ornament. Book VII is dedicated to the ornament of sacred buildings; Book VIII on the ornament of secular buildings; Book IX on the ornament of private buildings and, finally, Book X is dedicated to the restoration of buildings.

The treatise is based on the three fundamental dimensions that architecture must obey: *necessitas, commoditas*, and *voluptas* (necessity, convenience, and pleasure) differing from those set by Vitruvius *De Architectura* as *firmitas*, *utilitas*, and *venustas* (firmness, utility, and beauty), and essentially applicable to design processes (Alberti, 2011, p. 23). The method used by Alberti in the construction of his treatise is Aristotlelic, as he relates the whole to the parts, starting from an abstract idea towards a concrete example. From design (project) to performance (build). But in the treatment of ornaments, he inverts the hierarchical pyramid starting from a detail element towards the whole, from the description of the temple ending with the private home (Frommel, 2007, p. 696).

The column is the main ornamental element being distinguished as part of the wall, and the entablement as part of the roof. Both elements constitute a single syntactic element, that is, the column system (formed by the pedestal, base, shaft or column, architrave, frieze, and cornice). These elements are presented in detail in Book VII, chapter 7, dedicated to ionic and doric bases; chapter 8 is dedicated to the doric, ionic, corinthian, and composite capitals, and chapter 9 to the doric, ionic, and corinthian entablatures (architrave, frieze, and cornice).

The abstraction of the capital and other elements represents an albertian invention. The function of the capital is not dependent on the function of the building, or even of the column. In chapter 8 of book VII, Alberti explains that the column and the capital do not form a single entity, and are rather autonomous units with their own laws. Pure beauty (represented by numbers) and its relationship with the ornament and the column system is evoked in Book IX. By applying them it is possible to generate a temple that is, according to Alberti, the formal building "entity" that leads the generation of all kinds of buildings.

This paper shows the work of constructing a shape grammar using the Alberti treatise descriptions and real buildings' ornamental elements, and is organized as follows: (1) Strategies for the interpretation and formalization of Alberti's rules; (2) Column system shape grammar rules; (3) longitudinal elevation of the nave of the *Sant'Andrea of Mantua* church generation; (4) Evaluating the degree of influence of Alberti's treatise in the design of the longitudinal elevation of the nave of the *Sant'Andrea* church.

Strategies for the interpretation and formalization of Alberti's rules

The treatise contains two types of descriptions that were taken in account in the decoding work: a type in which the author clearly provides very detailed instructions on how to perform a given element type, as in the passage describing the doric base in Book VII, Chapter VII, pp. 203 (Alberti, 1988), "… made the height of the base half the diameter. The width of the die in either dimension would be no more than one and a half and no less than one and third times that of the diameter. The scotia consists of a hollow channel and two thin fillets running around the edges of the channel. Each fillet takes up a seventh of the thickness; the remainder is hollowed out."; and another, with less detailed descriptions, presented in Book IX, Chapter IV pp. 301 (Alberti, 1988), "The pediment to a private house should not emulate the majesty of the temple in any way. …. The remainder of the wall may be crowned on either side with the slighter projection of acroteria."

In the first approach, there is clearly an effort to quantify the elements represented.

In the second, the approach is more qualitative, leaving the proportional values of building elements open. Although this type is more generative, therefore offering a more open and less oriented interpretation, the first type provides a more objective formulation of the rule set to be used in the shape grammar.

The column system elements and their applications, presented in the text (columns, independent columns, and intercolumn), were therefore generated from the specifications present in Books VI, VII, and VIII. These descriptions were coded in grammar rules by dividing the text into parts of paragraphs containing specific instructions about how to draw a specific element or part of it. Diagram 1 has the function of guiding the production of the respective elements following the sequence of text.

One problem encountered during the decoding process was related to some ambiguities, contradictions or lacking parts of the text. These aspects led us to develop a strategy called *fill in the gaps*, which compiles a set of alternative information such as the LCS system, innate numbers, proportions relating musical consonances, and other information extracted from real buildings, as seen in Diagram 2.

The LCS system is a minimal vocabulary defined by Alberti in Book VII, Chapter VII, composed of the letters L, C, and S along with their variations, C and S reversed. With these letters Alberti defines the localization where to apply the molds of specific ornamentation: a *Fascia* (or *Fillet*) is an L but with a larger *fillet*; the *Recess* is a very prominent L; one *Ovolo* is the combination of an L with a C; the *Cord* is a parametric variation of the last; the *Channel* is the combination of the inverted L with C; the L and S make a *Gullet,* and finally the *Wave* consists of a reversed L with an S. It is on those elements that are applied the latest level of detail, particularly: the *Fascia* may contain shells, scrolls, and inscriptions, the *Recess* denticles; the *Ovolo* eggs and leaves; the *Cord* is connected with wires; the *Gullet* and the *Wave* are populated with leaves. The *fillet* is the only element that does not contain any ornaments. Parts of certain elements (such as a doric base, a plinth, a torus) may be further decomposed, comprising a set of elements defined with the LCS system, and these combinations may be generated by addition operations. A set of molds of the LCS are present in Figure 1 (Coutinho et al., 2011, pp. 788–798).

Inferring numbers, proportions, and parameters described in the treatise

Lionel March, in *Architectonics of Humanism: Essays on Number in Architecture* (March, 1998), relates the proportions of four orders described by Alberti with the ratio 30:5:3, that is, the proportions of the ark described in Genesis (5:6 .13–16). It is possible to deduce and use, with the fill in the gaps strategy, those numbers mentioned in Alberti's descriptions establishing the proportions of the following orders: Tuscan italic or 6:1; the composite of 10:1, which is the expression of Ionian 10:06 on their proportion 8:1, with an average between the Doric and ionic Tuscan, that is, 7:1, while the Corinthian has the same relationship between the Ionian and the composite at 9:1. Another significant value to derive the proportions of Noah's ark is 5:3. (March, 1998, p. 115)

In Book IX, Chapters V, VI, and VII, Alberti makes the description of all aspects listed in "all the genres of beauty and ornaments … that emanate from the universal principle of beauty." (Alberti, 2011, p. 591) The distinction is made between odd numbers such as 3, 5, 7, and 9, and even numbers such as 4, a divine number, and 6 as a perfect number that comprises the sum of all its integer dividers $1 + 2 + 3 = 6$. The numbers 8 and 10, are perfect since their squares are formed by the sum of the first 4^3, that is $10^2 = 1^3 + 2^3 + 3^3 + 4^3$. Alberti restricts the

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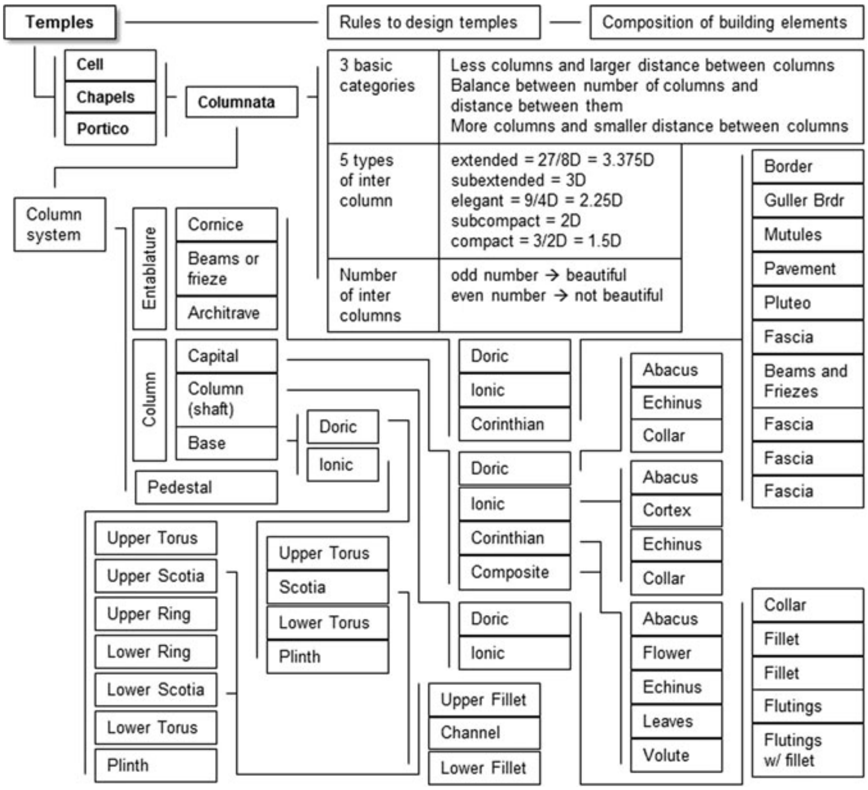

Diagram 1. Column system diagram structured according to the passage of the text.

number 10 as the maximum number of windows to be applied and the number 9 as the limit of intercolumns in the case of temples (Alberti, 2011, p. 597).

The *delimitation* consists of lines comprehending the length, width, and height. The musical consonances are: diapente or sesquiáltera – ratio of 2.3; diatesseron or sesquitércia – ratio of 3:4; diapason or double consonance – 1:2 ratio; diapason – diapente or triple consonance – 1:3 ratio; sesquioitava – ratio of 8:9 (difference between a fifth and a fourth 2:3 3:4, or 2:3/3:4 = 8:9); disdiapason or quadruple (or two octaves) – 1:4 ratio. These ratios are used as length and width in two dimensions, matching a third (height) to the space prescribed by the architect.

The innate proportions are a system determined by roots and powers such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{8}$, $\sqrt{12}$, and $\sqrt{16} = 4$, with the goal of

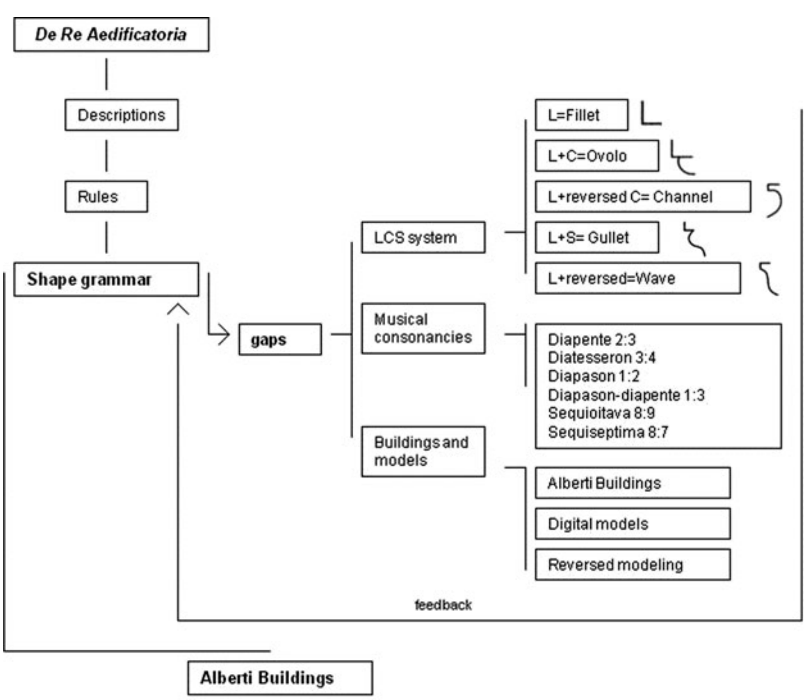

Diagram 2. Fill in the gaps system diagram.

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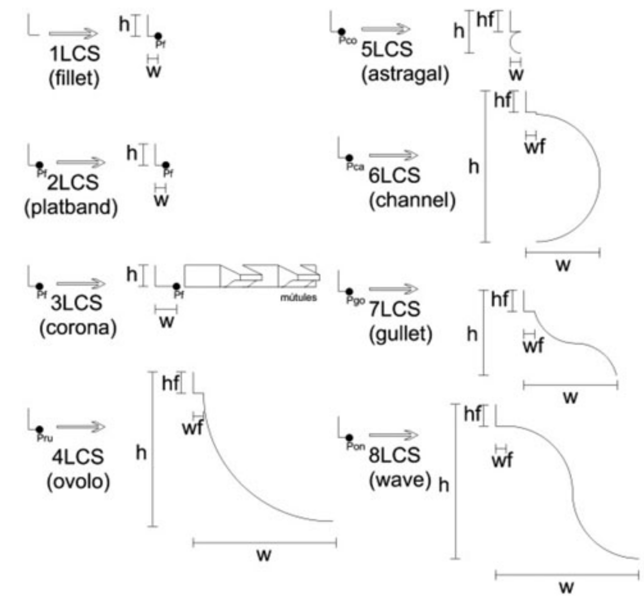

Fig. 1. LCS system rules 1–8.

assigning the shortest measure to the width of the area, the longest to the length, and the medium to the height. However, Alberti advises that this system should be used with flexibility because "sometimes will swap – up, taking into account the convenience of the buildings." (Alberti, 2011, p. 606).

The *means* are defined among three numbers as being an extremity (higher end) greater than the inverse end (lower end) and a third, the average term extremes. With an arithmetic division, having $a > b > c$, being $a = 4$; $c = 8$; $c = a + x$, $a + x/2 = b$ is possible to achieve 6 as a *mean* of 4:8. With a geometric division, having $c = a^*x$; $b = a^*\sqrt{x} = (\sqrt{a^2})\ {}^*\sqrt{x} = \sqrt{(a^{2*}x)} = \sqrt{(a^*a^*x)} = \sqrt{(a^*c)}$. Taking $a = 4$; $c = 9$ we have $4^*9 = 36$, $\sqrt{36} = 6$; then 6 is the *mean* of 4:9. Finally, in the musical *mean*, having the condition $a > b > c$; $a = 1/2c$; $c - a = x$; $x/3 = k$ and $a + k = b$. If $a = 30$; $c = 60$, taking $c - a = x$, then $x = 30$. If $x/3 = k$, then $K = 10$. If $k = a + b$, then $b = 40$, confirming 30:40:60 as suggested by Alberti (Alberti, 2011, p. 606).

Application and calculation of column system elements combinations

There is no description in *De Re Aedificatoria* that establishes the specific kind of column system elements combinations that are permitted or not. Therefore, there are no restrictions on their use. This shows the generative power of the treatise, providing a series of possible grammar applications (Knight, 2003). If there are no combinations of "fully" pre-defined elements by the treatise, the predictable combinations of different elements were calculated and are present in Diagram 3, of circa 900 variations of column elements. However, the number could rise exponentially

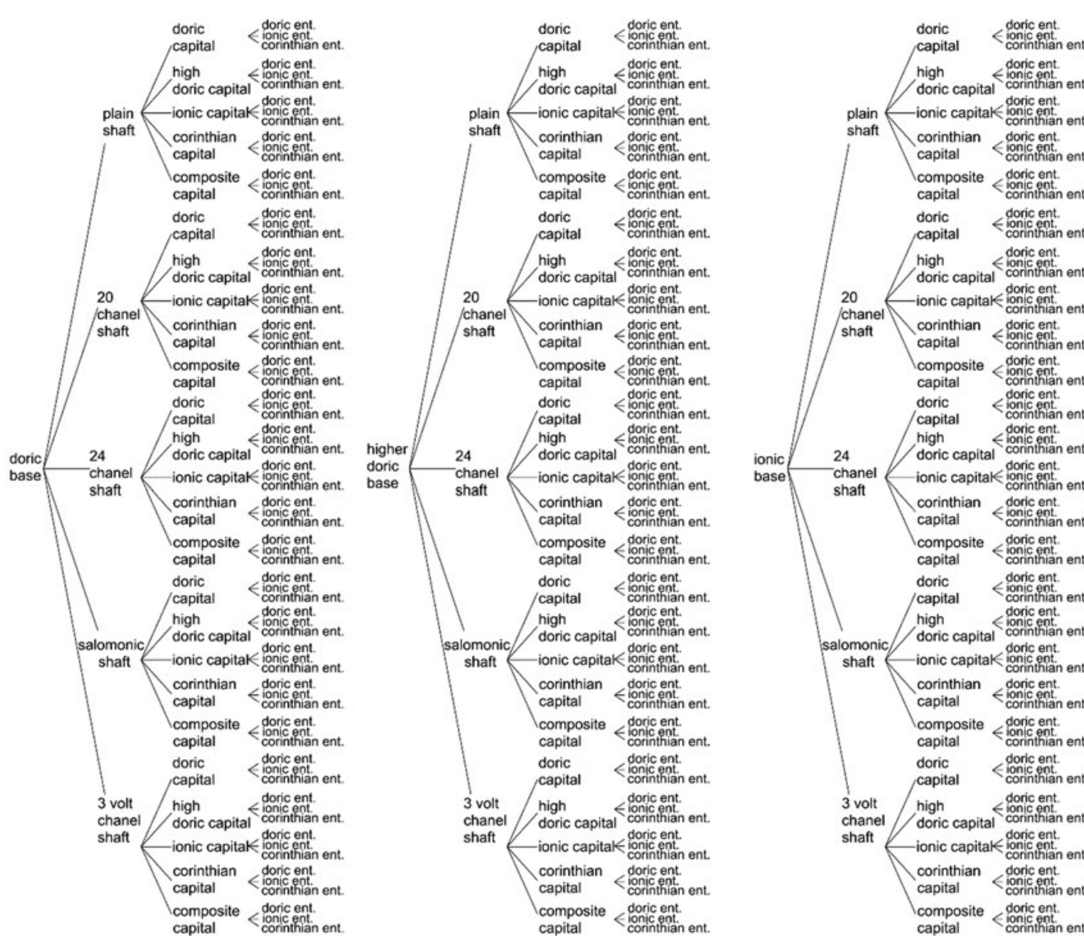

Diagram 3. Derivation trees of column system elements.

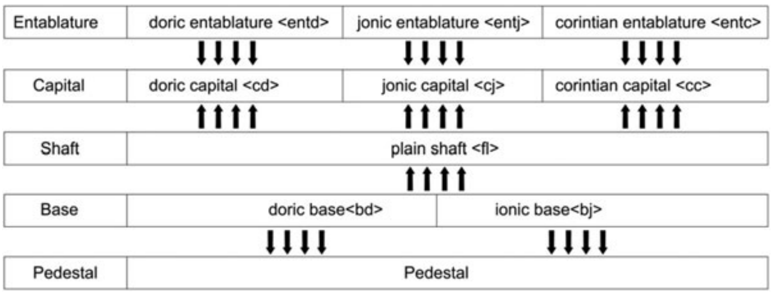

Diagram 4. Different grammars application fluxus.

if we consider the nature of topologies and maximal lines of different elements' variations. Some possible derivations are shown in Diagram 3.

Along with the grammar rules of different elements of the column a set of ordered classifications and combinations, that will be employed in the derivation of the grammar rules, is also provided as seen in Diagram 4.

The elements will be combined to generate a column from a specific facade such as the longitudinal elevation of the nave of Sant'Andrea: – pedestal<ped>, doric base<bd>, plain shaft <fl>, corinthian capital <CC>, doric entablature <entd>. These labels (for example <entd>, meaning doric entablature) contain insertion elements that should match the appropriate labels.

Combinatorial mode function

The derivation trees show in their initial vertices a Doric base, a Doric base 1 (variation), and an ionic base. To these bases, the shafts, the capitals, and the entablatures are added. The pedestal also has two variations. Finally, the elements produced with these combinations can be duplicated according to the diameter of the column – module D – or in *apophyge* or *listelo*, that is, the base of the column.

A combinatorial mode function aims to systematize the application of different elements through a mathematical function that generates different combinations of elements of columns. Not being the universal case (comprising all and not only possible combinations), an application to three columns with three different heights is presented, that is: column with $h = 7$, column with $h = 8$, column with $h = 9$. Each of these heights comprises a set of ordered pairs as:

$$se\, h = 7 \subset \begin{cases} (bd, cd) \\ (bd, cj) \\ (bd, cc) \\ (bd, cm) \end{cases}, se\, h = 8 \subset \begin{cases} (bd, cj) \\ (bd, cc) \\ (bd, cm) \\ (bj, cj) \\ (bj, cc) \\ (bj, cm) \end{cases}, se\, h = 9$$

$$\subset \begin{cases} (bd, cc) \\ (bd, cm) \\ (bj, cc) \\ (bj, cm) \end{cases}$$

We have the function

$$f(h) = \begin{cases} K\{h\}\ se\, h = 7 \\ L\{h\}\ se\, h = 8 \\ N\{h\}\ se\, h = 9 \end{cases}$$

as:

$$\begin{aligned} g(h) &= [\{(bd, cj)\} + l(h)] \\ m(h) &= [\{(bj, cc), (bj, cm)\}] \\ l(h) &= [\{(bd, cc), (bd, cm)\}] \end{aligned}$$

and

$$\begin{aligned} K(h) &= [\{(bd, cd)\} + g(h)] \\ L(h) &= [\{(bj, cj)\} + g(h) + m(h)] \\ N(h) &= [l(h) + m(h)] \end{aligned}$$

where $f(h)$ represents the function that allows one to combine and articulate different types of bases and capitals to a given shaft, to make different columns.

Column system shape grammar rules

There is a set of grammar rules for the shaft; the doric base with its two variations; the ionic base; the doric capital with its two variations; the ionic capital; the corinthian capital; the doric entablature; for the ionic entablature with its variations, and for the corinthian entablature. Each of these grammar rules contains three views: plan, elevation, and axonometric. Their algebras are represented by the Cartesian product, in particular: the algebras U_i, j representative of those forms (U) with the index i representing geometry (point, line, plane, and solid) and the index j representing the dimension of representation (dimensions 0, 1, 2, 3) respectively. Algebras V represent the markers at indices i and j.

Along with the designed rules, it is presented: the portion of the text of the treatise that originated the respective shape grammar rule; a set of descriptions of the variables highlighted in the text (such as h = height, w = width, L = length, D = diameter of imoscape); the parameters related to the variables or constraints (such as the parametric variation of the doric base plinth above mentioned as $1/2 \leq x \leq 1/3$, $x = h$ plinth) present in the treatise, which are used in the drawing's parameterization.

A set of functions and a set of equations explaining the mathematical expression referring to the set of variables present in a given rule are also presented. Finally, a description is provided, when necessary, explaining the changes between the left part and the right part of the rule. The grammar rules may be applied if an insertion point and a pre-existing plan or section element of a building are previously provided. A set of rules from the doric base, the shaft, and the corinthian capital may be seen in Figures 2–4.

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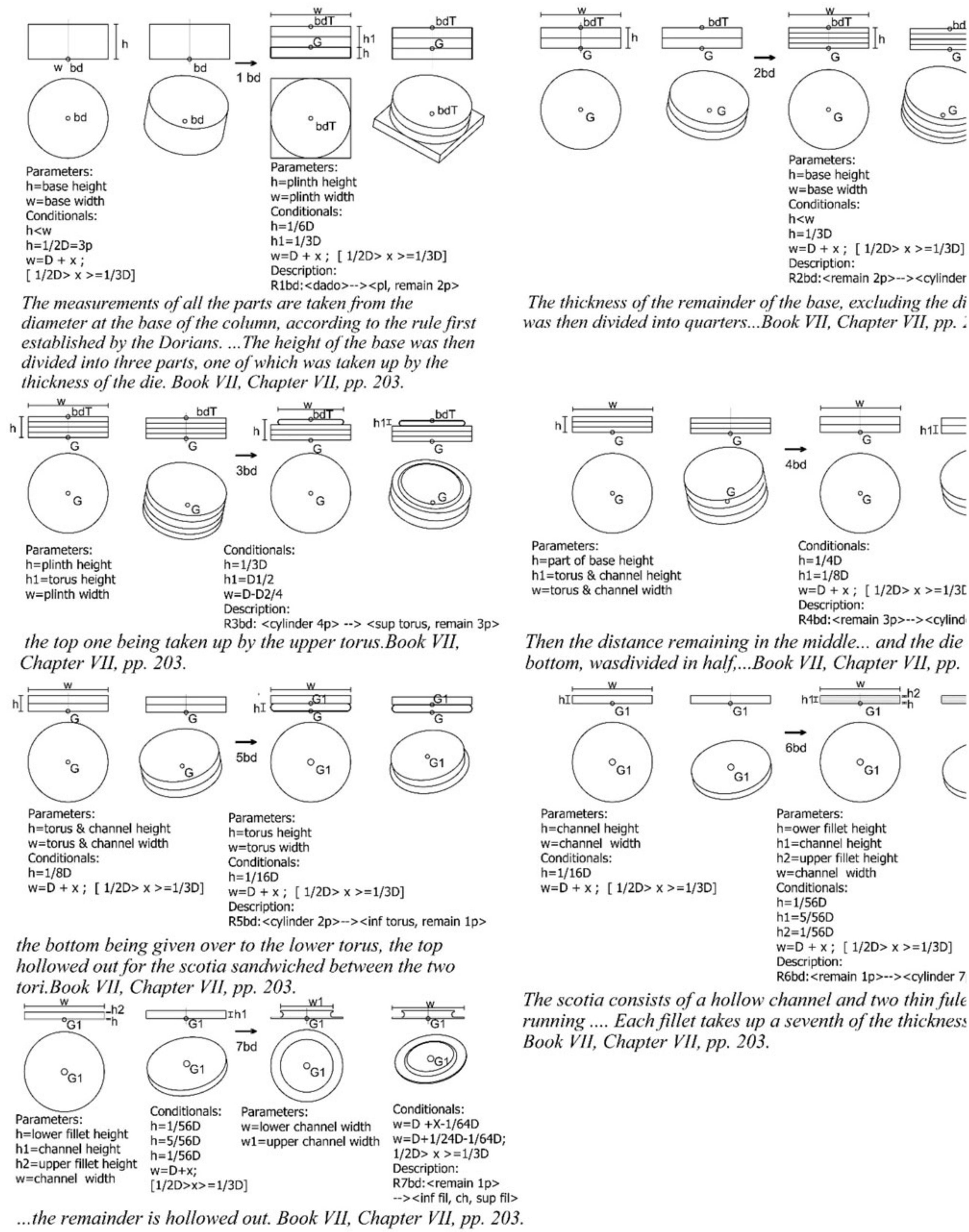

Fig. 2. Doric base shape grammar rules 1–7.

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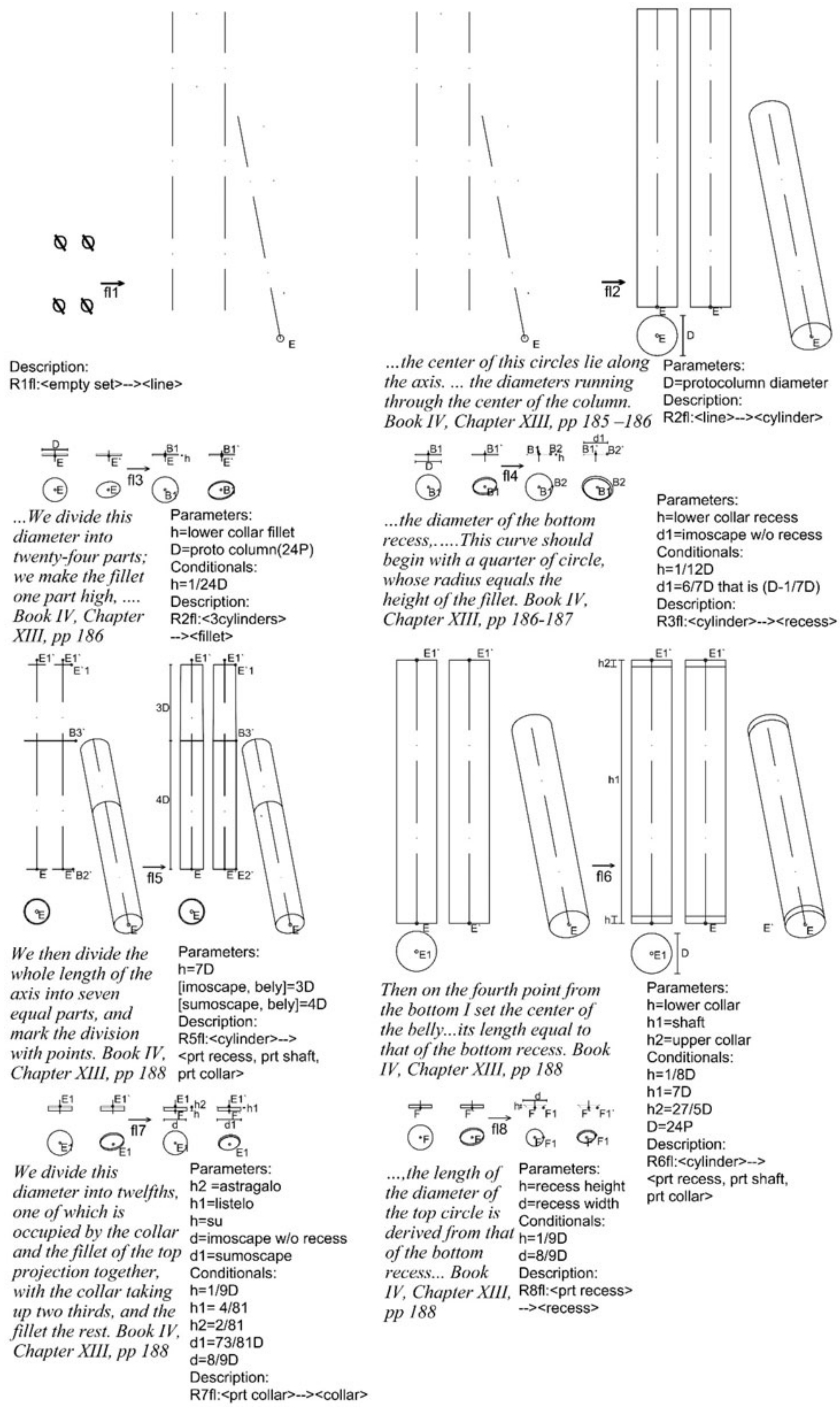

Fig. 3. Shaft shape grammar rules 1–8.

Automation rules program

The application of the right side of the grammar using the visual programming software Grasshopper (GH), allowed for the automation of the grammar's systematization reaching different formal solutions through the manipulation of pre-defined parameters. This created a set of medium scale (focus on the elements of the column system) and small scale (constituent parts of the elements of the column system) evaluation. Figure 5 shows the rules being automated with a GH code, generating a column with Corinthian capital (Coutinho et al., 2013*b*, p. 655–663)

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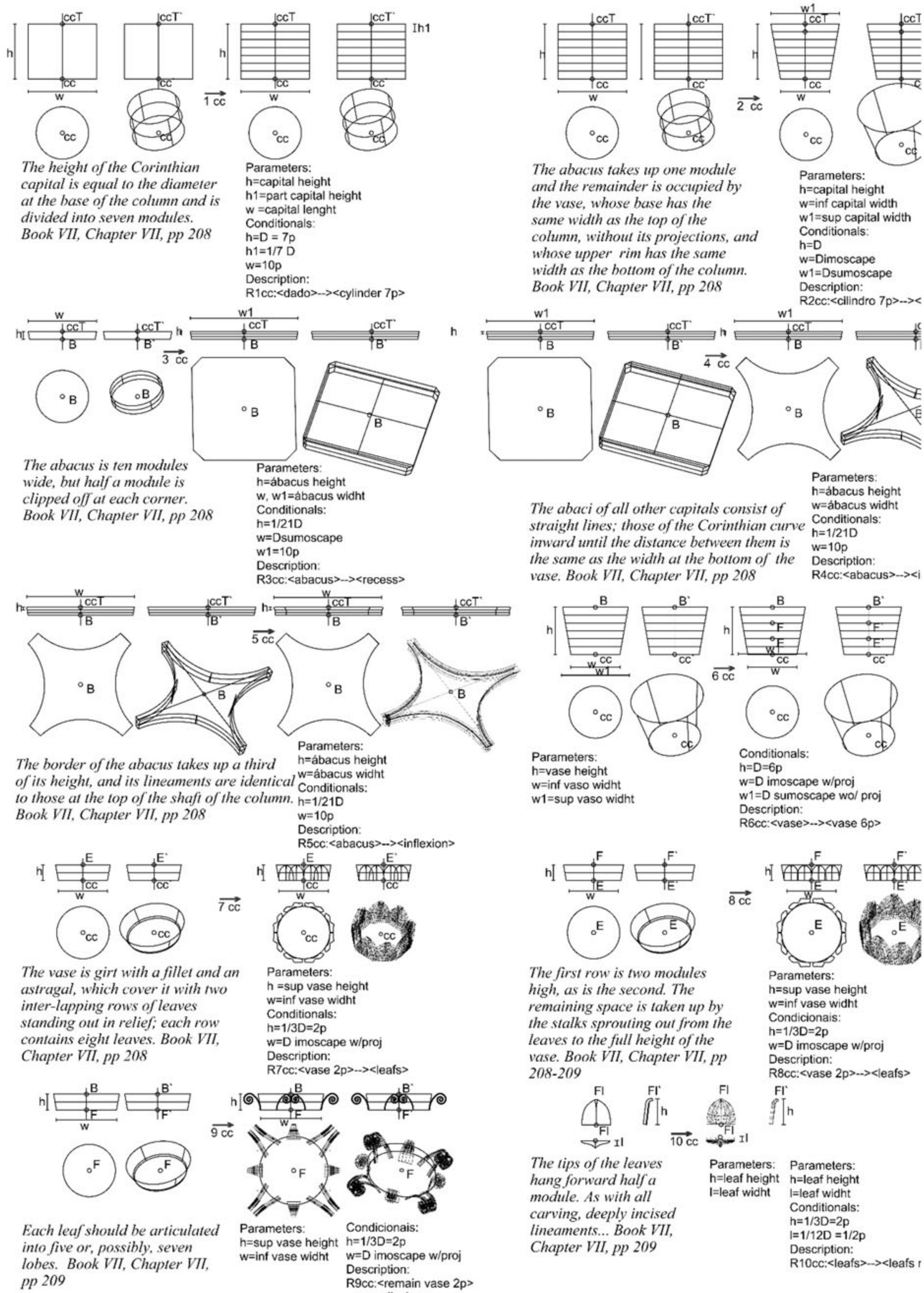

Fig. 4. Corinthian capital shape grammar rules 1–10.

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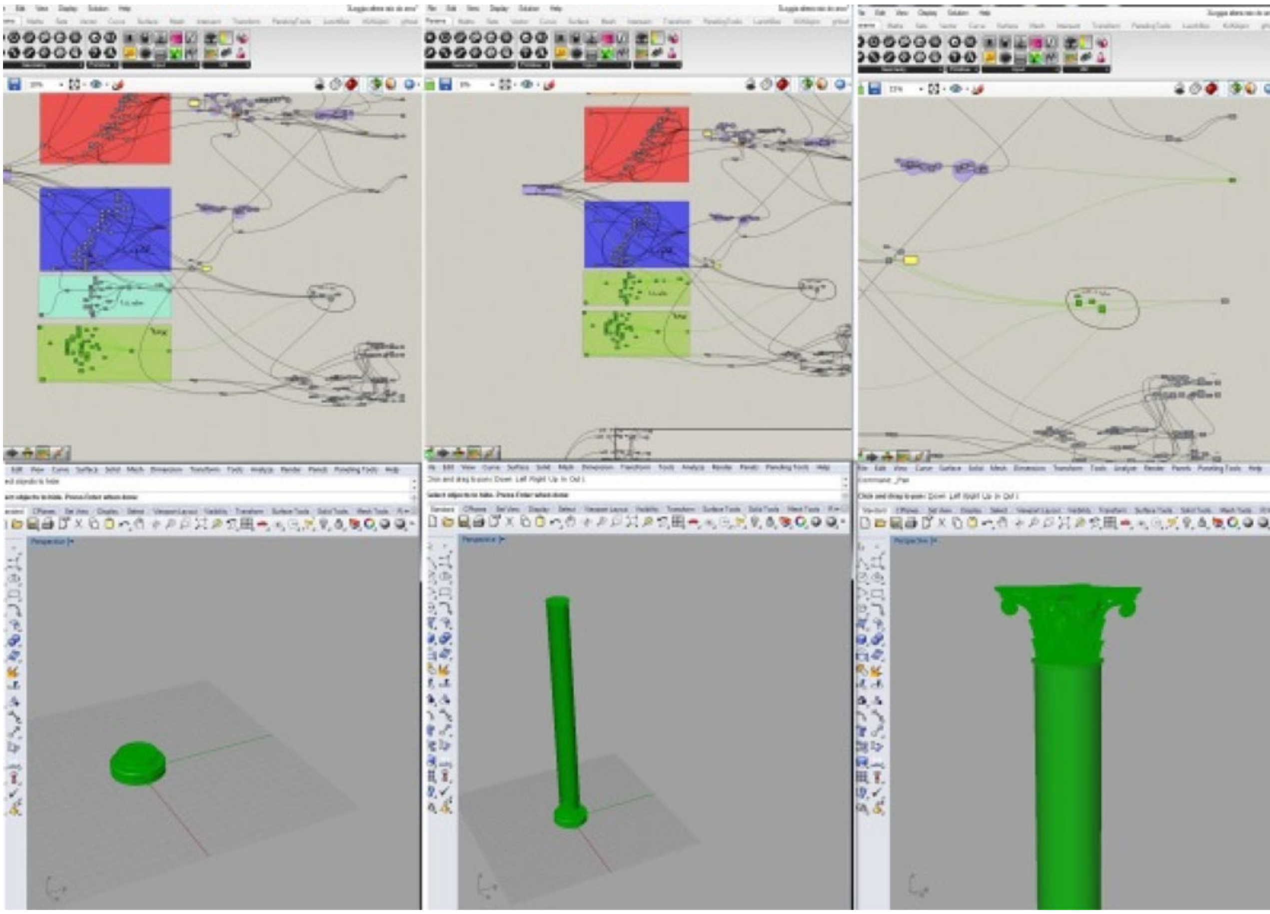

Fig. 5. Rules automated generating a column with a Corinthian capital.

The longitudinal elevation of the nave of Sant'Andrea church generation

The choice of this building for the study was based on its beauty, its complexity, and its relevance in Alberti's posthumous architectonic work.

Ludovico Gonzaga commissioned the design of Sant'Andrea Church in Mantua to Alberti. The project, assisted by Luca Fancelli, begun in circa 1470, and there is correspondence about this project from 1471 (note that Alberti died in 1472). It is estimated that the works began around 1473. In 1494, there were already 12 carried chapels built, where Andrea Mantegna did a series of *frescos*. (Frommel, 2007, pp. 695–725). The model suggested by Alberti's draft of Sant'Andrea church was the Etruscan temple, serving as a formal reference to the proportions of the *Basilica* of *Maxentius* in Rome.

Alberti provided a model, but also the measurements, the work and cost estimate, and estimated 3 years to carry out the work of construction on site, as well as two million bricks in Mantuan measure for the construction. According to Tavernor (1998), this type of information, along with its survey and comparison of building volume measurements versus estimated bricks, coincide with the Etruscan temple proposed by Alberti – consisting of a single nave temple with three chapels on each side and a main chapel without cruise – contrary to what exists, that is, a single nave church with three chapels on each side, a transept, and a main chapel. (Tavernor, 1998, pp. 160–163) Alberti prescribes that the internal horizontal scale should have a length of six parts and a width of five parts. The length is divided into two parts, one for the cell and the other for the column. The width, divided into ten parts, results in three parts to the left and the right, symmetrically, for the smaller cells. The remaining four parts are attributed to the main cell.

Rules and derivations stages

A specific structure of the stages and, within these stages, some rules, have been adapted to better meet the proposed objectives of generating the longitudinal elevation of the nave of *Sant'Andrea* church. In Diagram 5, the different stages are shown.

Stage 1 – The recognition grammar comprises a set of rules that recognize parts of a plant and a section. It is a parallel shape grammar, so it will be applied simultaneously in two planes: horizontal and vertical. A set of rules recognizes some shapes given from parts of a plan and a section of a church – a longitudinal elevation of the nave – generating floor and wall axes to be used in further stages. It comprehends five distinct rules, as shown in Figure 6.

Stage 2 – A meta-structure is applied, comprising a set of axes and insertion points of other elements (e.g. window and column system). The insertion axes of the ornament are independent of the internal structure of the building. A set of rules insert axes and labels of pilasters and column elements. These rules are

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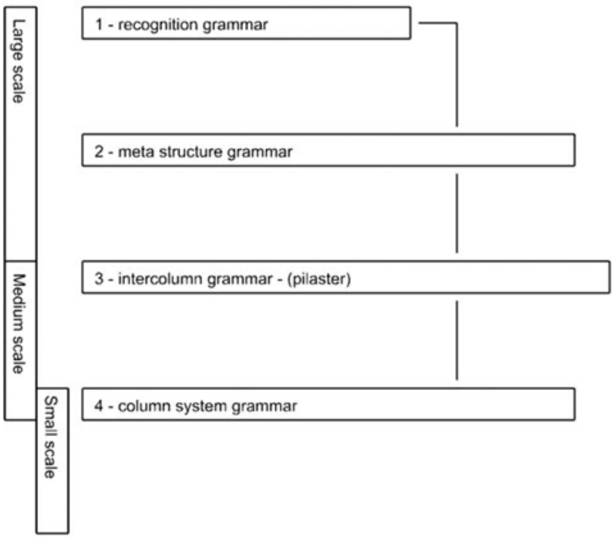

Diagram 5. Stages of derivation.

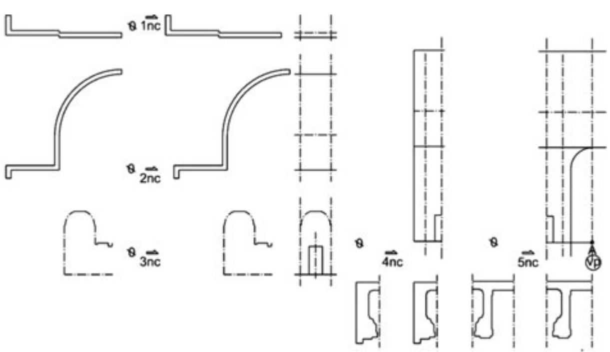

Fig. 6. Set of recognizing rules from stage 1.

parametric and behave according to certain measurement relations. A set of parameters and conditions are used to manipulate labels as seen at Figure 7.

These parameters have an expression such as $L = k1\text{D} + 2\text{wIch} + 3/2\text{wch}$; $K_1 = 4$ where L = facade module; D = column (Pilaster) diameter; wlch = chapel intercolumn and; wch = chapel bay in rule 1vp and $L1 = k2\text{D} + 2\text{wIch} + 3/2\text{wch}$; $k2 = 5$ for rule 2vp. Both rules have the sum

$$\left(\sum_{j=1}^{k1+k2} \mathbf{D}\right) + \mathbf{6wIch} + \mathbf{9/2wch};\ \forall\{\mathbf{wIch},\ \mathbf{wch}, \mathbf{D}\} \in \mathbf{R} \wedge \mathbf{k} \in \mathbf{N}$$

Stage 3 – Dedicated to the pilaster, uses a set of rules from the intercolumn shape grammar. A description from Book VI, Chapter 12, pp. 402 produces a transformation rule, such as *<bd>* → *S* *<bd>* transforming a doric base with its plan inscribed in a circle used to sweep the vertical section (profile) in an extrusion, into another base inscribed in a rectangle extruding its vertical section (profile) along with it.

Stage 4 – Dedicated to the column system, where different elements of the column will be inserted in points in the facade previously defined in stage two providing ornamentation. A proto-parallelepiped inserted at the initial shape label pd, starts the generation. The pedestal is generated by applying its specific grammar rules comprising a height of circa 2D + 1/6D. A base and a shaft are generated adding a cylinder at label fu, and a transformation of its shape is then operated to obtain a pilaster. The generated base has a height of 1/2D and the shaft has a height of 8D. Then, at label cc, a composite capital with 1D of height is generated. Finally, a doric entablature with 2D of height is generated from the label ent. Rule e erases all labels as presented in the derivation in Figure 8.

The final derivation generated a digital model. The rules of the grammar applied and automated with GH code generated a model that was then produced with a CNC Milling machine. The results are shown in Figure 9.

Transformations

With the generation of the longitudinal elevation of the nave of Sant'Andrea we aim to verify if its elements can be obtained from Alberti's rules or some transformed rules. There are at least four different ways of transforming a grammar, namely rule addition, rule subtraction, and rule changing, which can be designated by letters *A*, *S*, and *C*, respectively (Knight, 1994). A fourth transformation type *I* can be added if we consider that a rule can remain unchanged. This transformation *I* is important for our study because each time such a transformation is used, it may suggest

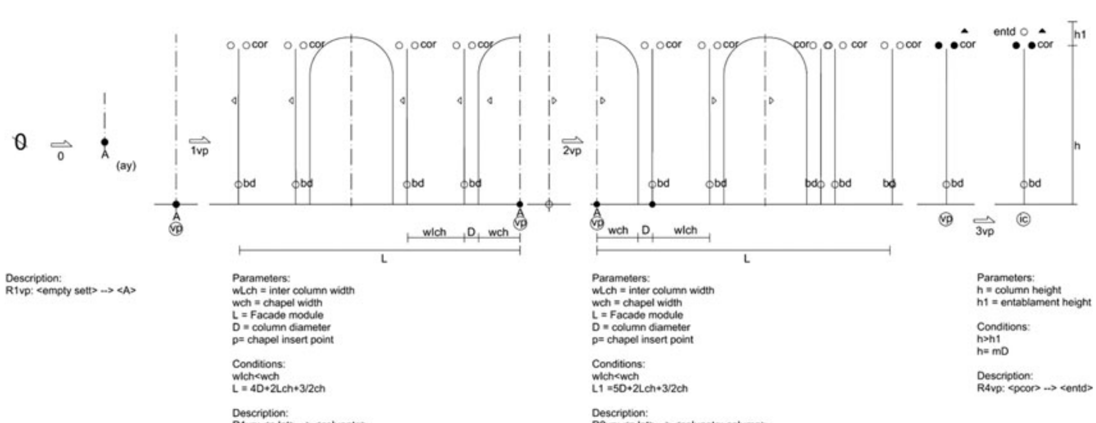

Fig. 7. Set of meta-structure rules from stage 2.

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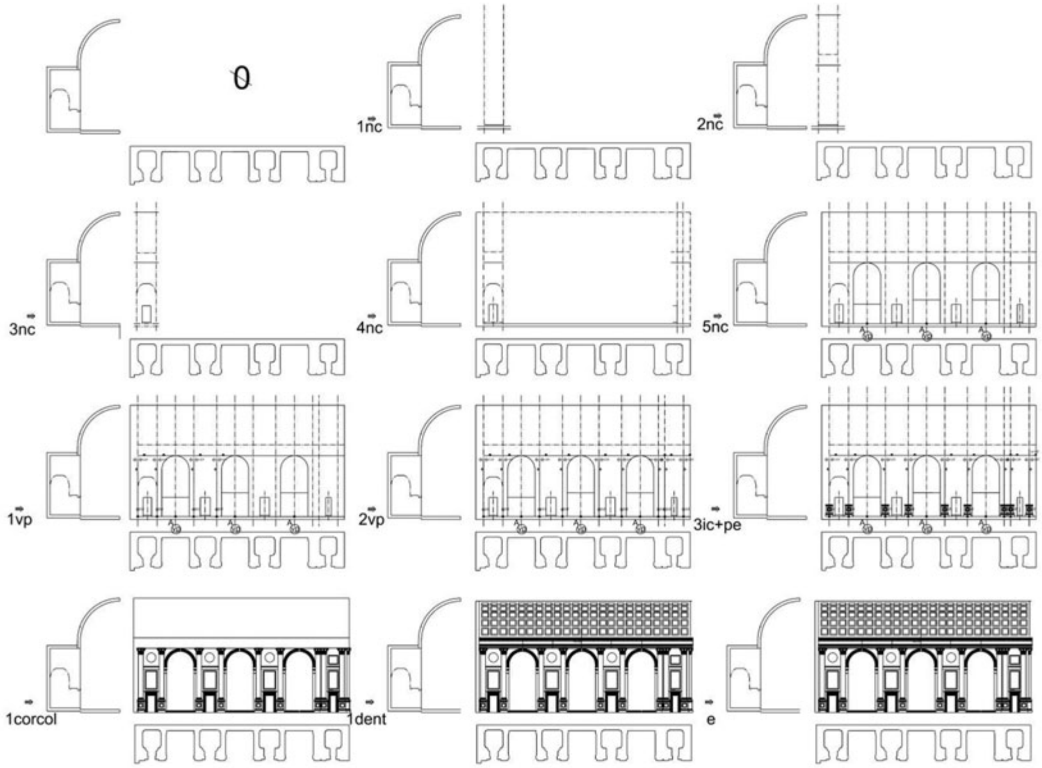

Fig. 8. Grammar derivation of the longitudinal elevation of the nave of Sant'Andrea church.

that there is strong evidence that the designer was knowledgeable of Alberti's rules (Coutinho et al., 2013*a*). At stage 2, transformations were verified in the intercolumn parameterization through $L = 4D + 2wIch + 3/2wch$; and $L = 5D + 2wIch + 3/2wch$.

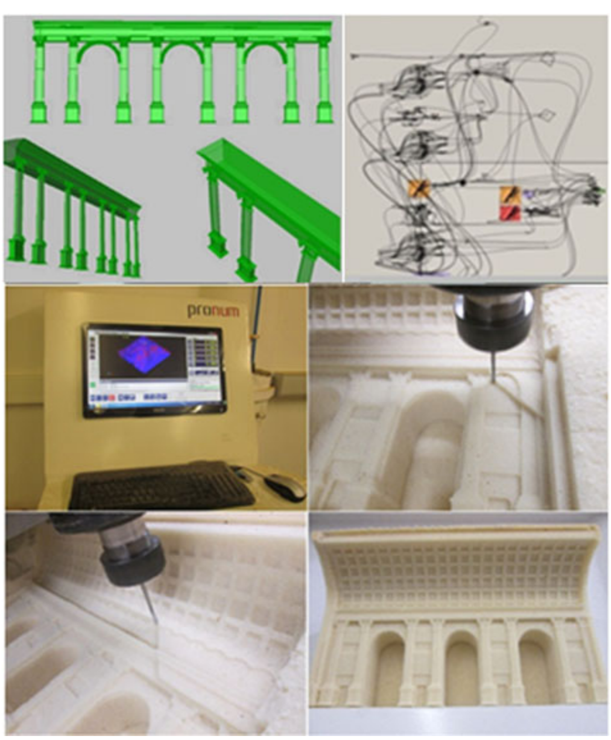

Fig. 9. Rules of the grammar applied and automated with GH code generating a model. Church model fabricated with a CNC Milling machine.

At stage 3, transformations are related to the rule regarding pilaster change from a round section to a rectangular one. Finally, at stage 4, transformations using the application of *De Re Aedificatoria* are applied, that is: intercolumn width of 6D and 12D height, using the proportion of 1:2. The pedestal has a 2D height, the doric base has ½ D height, the shaft with 8D height, the Corinthian capital with 1D height, and the Corinthian entablature with 2D height. They largely observe a transformation *I*. See Table 1.

Evidencing that Alberti's treatise influenced the longitudinal elevation of the nave of Sant'Andrea of Mantua church design

The simple linear regression method (SLRM) was chosen to perform the analysis of the rules applied in the facade design, measuring the degree to which the treatise has influenced the design and construction of the buildings. To implement the

Table 1. Sant'Andrea of Mantua church central nave façade shape grammar proportions

	Doric	Ionic	Corinthian
Entablature		n. e	circa 2D
Capital		n. e	1D
Shaft		n. e	8D
Base	1/2D		
Pedestal	circa 2D + 1/6D		
Intercolumn	6D (with height of 13D + 2/3D)		

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SLRM the statistical software SPSS was used as observed by Bryman and Cramer (1992). For verification of the degree of influence two variables were used. The independent variable (IV), that is the values of the column system from the treatise, and the dependent variable (DV) consisting on the rules applied in the longitudinal elevation of the nave of Sant'Andrea church values. With this method, it is possible to describe the relationship between the two variables. This relationship may be seen through the line $Y_i = \beta0 + \beta_1 X_i + \varepsilon_i$; where X is the independent variable; Y is the dependent variable or predict; β_0 is the constant which represents the intersection of the straight line and the vertical axis; β_1 is a constant representing the slope of the line; and ε_i is the residual factor.

The objectives of SLRM are: to perform the measurement of how much one variable is explained by another, that is, how much DV is explained by IV; quantify the intensity and direction of the linear relationship between two variables; predict the DV from the IV; and infer that the model is adequate to explain the linear relationship between two variables. The dispersion model gives the quality measures of the model where the correlation coefficient (R) measures the intensity and direction of the linear relationship. R_2 is the coefficient of determination which measures the proportion or percentage of variation of the DV that is explained by the IV. This varies between [0, 1]. Where $R_2 = 0$, DV cannot be explained by IV. If $R_2 = 1$ it means that the DV can be 100% explained by the IV. The coefficient of determination then varies between 0% and 100%. It is assumed that 50% means that the DV can moderately be explained by IV. In this analysis, we focus on the readings R_2. To apply the SLRM it is necessary to assess five presuppositions (Carvalho, 2008): linearity of the studied phenomena; random variables with null value: $E\ (\varepsilon\ i) = 0$; constant variance of residual random variables: Var $(\varepsilon\ i) = \sigma\ 2$; independence of residual random variables: Cov $(i\ \varepsilon, v\ j) = 0$, $i \neq j$; normal distribution of residual random variables: $\varepsilon\ i \cap N\ (0, \sigma 2)$.

The treatise and transformed rules proportion parameters represented by h (height) of the column system element were selected to be analyzed. These proportions are dependent on a constant diameter D which represents the measure of the column imoscape projection. The 106 rules of the column system contain: eight rules from shaft; seven rules from doric base; seven rules from ionic base; 18 rules from doric capital; 18 rules from ionic capital; ten rules of the corinthian capital; two rules from composite capital; 13 rules of the doric entablature; 17 rules ionic entablature; one rule from corinthian entablature and five rules from the pedestal. The rules which contain more than one parameter were divided into sub-rules. For example, the rule of the shaft Rshaft 6 has $h = 1/8D$, $h1 = 7D$ and $h2 = 27/5D$, and was treated in sub-rules like Fuste6 $h = 1/8D$, Rfuste6A $h1 = 7D$, Rfuste6B $h2 = 27/5D$ to facilitate the database management.

The sample has 261×2 (different building elements) =522 parameters $=N$, that is the number of observation. To determine how much the rules from the treatise may have influenced the implemented rules at the longitudinal elevation of the nave of Sant'Andrea church, it was found 35.6% as the value of the coefficient of determination, proving an approximate median influence of the treatise in the generated facade. Figure 10 identifies the rules in the regression parameters. Most of the rules are in relation to the superior section of the line. It may mean that the rules corresponding to higher values of h (height) are the ones most similar, matching the rules of the treatise. There is a great homogeneity of the proportions of the rules applied to the column system in the longitudinal elevation of the nave of the *Sant'Andrea* church.

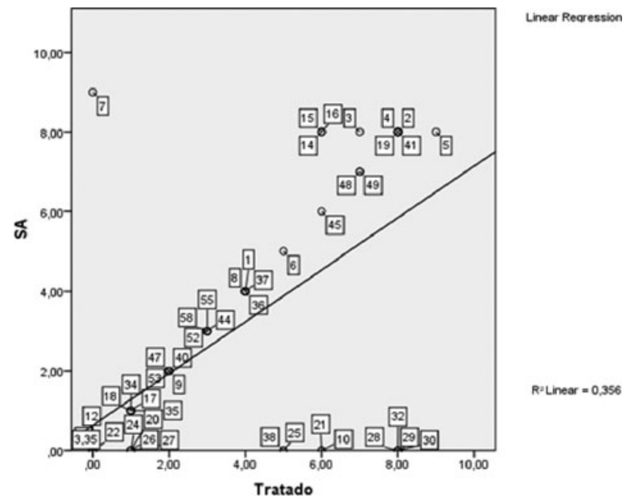

Fig. 10. Linear regression curve between the treatise and the longitudinal elevation of the nave Sant'Andrea church with the correspondent rule.

Conclusion

The column system elements were compiled in rules focusing the relation between the representation of the rules and the passages of the text, to ensure a faithful representation and coding.

The column systematization shape grammar, that may be considered as a detailed grammar, was used to verify which rules were needed to transform to generate the longitudinal elevation of the nave *Sant'Andrea* church. The evaluation of the grammar application showed some evidence of the use of Alberti's *De Re Aedificatoria* descriptions on the design of the longitudinal elevation of the nave of *Sant'Andrea*. The experiment presented in this paper is similar to an author's research that analyzed three other buildings, two of which by Alberti (Rucellai palace in Florence and Sant'Andrea church), and other Portuguese buildings such as São Vicente de Fora in Lisbon and the main elevation of Palácio Ducal in Vila Viçosa, using the same methodology but evidencing different results. The SLRM value observed in this experiment was the highest related to all other experiments, suggesting Alberti's authorship of the facade "design" or, at least, a great influence of his concepts in the construction works. Different methodologies such as: archaeological, bibliographical, historical, among others, may be empowered with this research methodology regarding the discovery of new insights in the built environment. Finally, the shape grammar proved to be efficient in providing and organizing the data to be evaluated with SLRM.

This paper also had the goal to demonstrate the process of construction of this detail grammar contributing to a better understanding of its specificities.

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