

THE  
JOURNAL  
OF  
SYMBOLIC LOGIC

EDITED BY

MAX BLACK

ALONZO CHURCH

LEON HENKIN

S. C. KLEENE

*Managing Editor:* ROBERT E. LUCE

*Consulting Editors:*

W. ACKERMANN

FREDERIC B. FITCH

RÓZSA PÉTER

C. A. BAYLIS

CARL G. HEMPEL

W. V. QUINE

PAUL BERNAYS

LÁSZLÓ KALMÁR

BARKLEY ROSSER

G. D. W. BERRY

JOHN G. KEMENY

THORALF SKOLEM

EVERT BETH

J. C. C. MCKINSEY

A. R. TURQUETTE

ROBERT FEYS

ANDRZEJ MOSTOWSKI

H. E. VAUGHAN

VOLUME 15

1950

PUBLISHED QUARTERLY BY THE ASSOCIATION FOR SYMBOLIC LOGIC, INC.

WITH THE AID OF SUBVENTIONS FROM

HARVARD UNIVERSITY

PRINCETON UNIVERSITY

INSTITUTE FOR ADVANCED STUDY

SMITH COLLEGE

UNIVERSITY OF PENNSYLVANIA



THE  
JOURNAL  
OF  
SYMBOLIC LOGIC

EDITED BY

MAX BLACK

ALONZO CHURCH

LEON HENKIN

S. C. KLEENE

*Managing Editor:* ROBERT E. LUCE

*Consulting Editors:*

W. ACKERMANN

FREDERIC B. FITCH

RÓZSA PÉTER

C. A. BAYLIS

CARL G. HEMPEL

W. V. QUINE

PAUL BERNAYS

LÁSZLÓ KALMÁR

BARKLEY ROSSER

G. D. W. BERRY

JOHN G. KEMENY

THORALF SKOLEM

EVERT BETH

J. C. C. MCKINSEY

A. R. TURQUETTE

ROBERT FEYS

ANDRZEJ MOSTOWSKI

H. E. VAUGHAN

VOLUME 15

1950

PUBLISHED QUARTERLY BY THE ASSOCIATION FOR SYMBOLIC LOGIC, INC.

WITH THE AID OF SUBVENTIONS FROM

HARVARD UNIVERSITY

PRINCETON UNIVERSITY

INSTITUTE FOR ADVANCED STUDY

SMITH COLLEGE

UNIVERSITY OF PENNSYLVANIA

**The four numbers of Volume 15 were issued at the following dates:**

**Number 1, pages 1-80, March 29, 1950.**

**Number 2, pages 81-160, July 25, 1950.**

**Number 3, pages 161-240, September 30, 1950.**

**Number 4, pages 241-312, January, 1951.**

**All numbers of this volume are copyrighted by the Association for Symbolic Logic, Inc. Reproduction of copyrighted numbers of the Journal by photostat, photo-print, microfilm, or like process is forbidden, except by written permission of the Managing Editor.**

## TABLE OF CONTENTS

On syntactical categories. By YEHOSHUA BAR-HILLEL.....	1
A demonstrably consistent mathematics—Part I. By FREDERIC B. FITCH.....	17
A formal system of logic. By HAO WANG.....	25
Widerspruchsfreier Aufbau der Logik I. By WILHELM ACKERMANN.....	33
Reviews.....	58
Twelfth meeting of the Association for Symbolic Logic. By MORTON G. WHITE.....	76
Abstracts of papers.....	76
Elections.....	80
Institutional contributing subscribers to the JOURNAL.....	80
Completeness in the theory of types. By LEON HENKIN.....	81
The elimination of contextually defined predicates in a modal system. By RUTH BARCAN MARCUS.....	92
On natural deduction. By W. V. QUINE.....	93
Existence of classes and value specification of variables. By HAO WANG.....	103
Non-standard models for formal logics. By J. BARKLEY ROSSER and HAO WANG.....	113
A reduction in the number of primitive ideas of arithmetic. By JOHN R. MYHILL.....	130
On virtual classes and real numbers. By R. M. MARTIN.....	131
Reviews.....	135
On the reduction of the decision problem. Third paper. Peppis prefix, a single binary predicate. By LÁSZLÓ KALMÁR and JÁNOS SURÁNYI.....	161
Note on truth-tables. By JAN KALICKI.....	174
A test for the existence of tautologies according to many-valued truth-tables. By JAN KALICKI.....	182
A complete theory of natural, rational, and real numbers. By JOHN R. MYHILL.....	185
On complete atomic proper relation algebras. By FRANK HARARY.....	197
Reviews.....	199
Set-theoretical basis for real numbers. By HAO WANG.....	241
Zusammenhang der mehrfachen und transfiniten Rekursionen. By RÓZSA PÉTER.....	248
Über eine Art von Unvollständigkeit des Prädikatenkalküls der ersten Stufe. By GISBERT HASENJAEGER.....	273
A remark concerning decidability of complete theories. By ANTONI JANICZAK.....	277
Reviews.....	280
List of officers and members of the Association for Symbolic Logic.....	286
Index of reviews by subjects.....	292

### ERRATA

#### VOLUME 12

Page 63, line 10 from the bottom. For « p. 86 », read « pp. 83–86 ».

Page 63, line 5 from the bottom. Add at the end of the line: « [This note appears in German, and without title; the title which is here given was supplied by the author.] ».

#### VOLUME 13

Page 112, line 16. For « 1946 », read « 1947 ». Page 115, line 25. For « a », read « *a* ».

Page 122, line 12 from the bottom. For « pp. 261–266 », read « pp. 271–276 ».

Page 124, line 12. Add: « [As stated in the appendix, the work originally came out in parts: pp. 1–188 in 1941, pp. 189–328, 329–380 in 1942, pp. 381–492 in 1943, and the rest in 1945.] ».

Page 125, line 6 from the bottom. For « XII 55 », read « XII 52 ».

Page 149, line 19 from the bottom. The first word in the line should be « lead ».

Page 166, line 37 from the bottom. For *u*, read *U*.

Page 167, line 19 from the bottom. For « XIII 000 », read « XIII 166(1) ».

#### VOLUME 14

Page ii, line 5. For « January, 1950 », read « February 2, 1950 ».

Page 57, line 5 from the bottom; page 58, line 3 from the bottom; and page 279, first column, line 24. For « YRJÖÖ », read « YRJÖ ».

Page 61, line 23. For « pp. 248–260 », read « pp. 258–270 »; and last line, for « pp. 311–346 », read « pp. 331–346 ».

- Page 87, line 6. For  $u$ , read  $U$ .  
 Page 104, line 4. Add  $\wedge$  at the end of the line.  
 Page 109. In the last displayed formula, add the condition  $\sim Y = 0$ , so that the part of the formula in brackets becomes:

$$[\text{Int}(X) \wedge \sim X = 0 \wedge \sim Y = 0 \wedge (A = X^2 \vee X^2 = 1 + AY^2)].$$

- Page 111, line 6. For « A6 », read « A3 ».  
 Page 114. In [8], for « *Vollständigkeit* », read « *Vollständigkeit* ».  
 Page 141, line 14. For « *notwendig* », read « *nicht notwendig* ».  
 Page 143, line 3. The parentheses are part of the original title of the article, hence should appear as italic parentheses.  
 Page 193, line 21. For « vol. 222 », read « vol. 223 ».  
 Page 195, line 11. For « Amsterdam », read « Bussum ».  
 Page 211, line 11. For « o-transform », read « o-transform ».  
 Page 233, line 2. For the main implication sign in the formula, read a sign of disjunction,  $\vee$ .  
 Page 243, line 16. For  $e$ , read  $\acute{e}$ .  
 Page 244, line 9. For « o », read « of ».  
 Page 249, line 18. For « Kaufman », read « Kaufmann ».  
 Page 257, line 24. Insert a comma before « Bd. ».  
 Page 268, first column, line 26 from the bottom. For « Alfred », read « Alfred Jules ».  
 Page 269, second column, line 15. For « de », read « des ».

#### VOLUME 15

- Page 58, line 15. For « *Scientarum* », read « *Scientiarum* ».  
 Page 62, line 29. After « again », insert « (in connection with the definition of symmetry of a relation) ».  
 Page 64, line 39. For « *Pryczynek* », read « *Przyczynek* ».  
 Page 71, line 20. For « *existence* », read « *existence* ».  
 Page 73, line 30. For the second « ' » , read « ' » .  
 Page 77, line 10. After  $Y$ , insert a parenthesis,  $)$ .  
 Page 96, line 24. For « UI and EG », read « UG and EI ».  
 Page 120, line 21. For  $F_n^n$ , read  $F_n^n$ .  
 Page 121, line 5. After the first « type », insert « 1 ».  
 Page 123, line 23. For  $L''$ , read  $L'''$ .  
 Page 129, line 24. For « *philosophical* », read « *philosophica* ».  
 Page 133, line 9 from the bottom. For  $O_r$ , read  $O_r$ ; and for  $1_r$ , read  $1_r$ .  
 Page 133, line 8 from the bottom. For  $0_r$ , read  $0_r$ .  
 Page 143, line 19 from bottom. For « CARRUCIO », read « CARRUCCIO ».  
 Page 153, line 10. After the colon, insert « ' ».  
 Page 159, line 6. The parentheses are part of the original title of the article, hence should appear as italic parentheses.  
 Page 177, line 15 and line 16. For  $\equiv$ , read  $\sim$  (twice).  
 Page 180. In the next to last truth-table, for «  $\mathfrak{B.C.}$  », read «  $\mathfrak{B} \cdot \mathfrak{C}$  ».  
 Page 184. In the upper left corner, instead of « 84 », the page number should be « 184 ».  
 Page 188, line 6. For  $\equiv$ , read  $\neq$ .  
 Page 192, line 23. For  $(x; y)$  and  $(w; 0)$ , read  $(x; y; z)$  and  $(w; 0; z)$ , respectively.  
 Page 193, footnote 46. For « (1935) », read « (1936) ».  
 Page 198. In the first line of the footnote, for « remarks », read « remarks ».  
 Page 202, line 4 from the bottom. Insert « & » at the end of the line.  
 Page 218, line 21. For the first « of », read « on ».  
 Page 223, line 12. For « HELEN », read « HELENA ».  
 Page 226, line 3 from the bottom. For « vol. 14 no. 4 », read « vol. 14 no. 11 ».  
 Page 228, line 9. In the fifth formula in the line, supply a bar over  $\alpha$ .  
 Page 228. In the displayed formula, for  $\Lambda_x$ , read  $\Lambda_x$ .  
 Page 234, line 12 from the bottom. The parentheses are part of the original title of the article, hence should appear as italic parentheses.  
 Page 239, line 26. For « *Lattices* », read « *lattices* ».  
 Page 240, line 13. For « XV 156(1) », read « XV 156(2) ».