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THE COST CHANNEL OF MONETARY POLICY AND INDETERMINACY

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We study the conditions that guarantee equilibrium determinacy in a standard sticky price model augmented with a cost channel. A central bank that assigns some positive weight to the output gap in its reaction function makes the economy *more prone* to multiple equilibria relative to the standard case. The value of the threshold on the interest rate response to inflation is *above one* and depends on the fraction of firms that need to borrow their bills payment.

1. INTRODUCTION

The sticky price New-Keynesian model is a popular framework for analyzing the transmission of monetary policy. Movements in the interest rate influence the demand side of the economy and have an impact on inflation only to the extent that they affect the output gap. If firms, however, need to pay a fraction of their bills before production a change in the nominal interest rate will also affect the cost of borrowing and, in turn, the aggregate inflation rate.

The dependence of marginal costs on the nominal interest rate has been recently investigated in a number of empirical contributions. Barth and Ramey (2001) find a significant cost channel effect on U.S. data at industry level. Christiano, Eichenbaum, and Evans (2005) estimate a DSGE model of the U.S. economy and find that monetary policy operates also through the supply side. Ravenna and Walsh (2006), Chowdhury, Hoffmann, and Schabert (2006), and Tillmann (2006) present single equation estimates for the G7 countries and they reject the hypothesis that the nominal interest rate has no direct effect on inflation through marginal costs.[1]

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Despite the growing empirical literature, the consequences of introducing a cost channel for the solution of an otherwise standard sticky price model coupled with a simple interest rate rule have not yet been investigated. We show that the sensitivity of marginal costs to the nominal interest rate influences the equilibrium dynamics in a number of important dimensions.

First, an interest rate rule that also responds to the output gap is *more likely* to generate indeterminacy relative to the standard model. The indeterminacy region enlarges because under the cost channel a rise in the policy rate has two direct consequences: a negative impact on aggregate demand and a positive impact on aggregate supply. The *threshold* for ruling out multiple equilibria hence requires a value of the interest rate response to inflation *above one*, and depends on the interaction between the coefficient of central bank reaction to the output gap and the fraction of firms borrowing their bills payment. This finding is in contrast to the standard sticky price model in which a positive response to the output gap makes the economy *less prone* to indeterminacy.

Second, the cost channel introduces an upper boundary to the interest rate response to current inflation that guarantees a unique equilibrium. Third, in the case of a forward-looking policy rule the reaction to *expected* inflation is associated with an upper limit that is higher than that implied by a sticky price model without cost channel.

This work is closely related to a recent literature that studies equilibrium determinacy in models in which money enters production and hence the level of the nominal interest rate determines the marginal costs [see, for instance, Benhabib, Schmitt-Grohé, and Uribe (2001)]. The literature on money in the production function, however, focuses mainly on interest rate rules that respond to inflation only. We show here that, in the presence of the cost channel, the monetary policy response to the output gap is a far more important source of equilibrium indeterminacy.

Section 2 briefly describes the model. Section 3 presents results for a forward-looking sticky price model using current-, forward-, and backward-looking interest rate rules. Section 4 concludes, and the proofs of the conditions for a unique equilibrium are in the Appendix.

2. THE MODEL

This section describes a log-linearized, microfounded New Keynesian sticky price model of the business cycle augmented with a cost channel. Details of the derivations can be found in Christiano, Eichenbaum, and Evans (2005), Ravenna and Walsh (2006), and Rabanal (2006). This model consists of the following three aggregate relationships:

$$x_t = E_t x_{t+1} - \tau (R_t - E_t \pi_{t+1}) + \varepsilon_t^x \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\, (x_t + \theta R_t) + \varepsilon_t^\pi \tag{2}$$

$$R_t = \rho R_{t-1} + (1-\rho)(\psi_\pi \pi_t + \psi_x x_t) + \varepsilon_t^R \tag{3}$$

$$\text{with } \tau \equiv \frac{1}{\sigma} \geq 0\,, \kappa \equiv \lambda\,(\sigma+\eta)\ , \lambda \equiv \frac{(1-\omega)\,(1-\omega\beta)}{\omega}\,,$$
$$\theta \equiv \frac{\gamma}{\sigma+\eta} \text{ and } \gamma \in [0, 1], \tag{4}$$

where x_t is defined as the gap between output and the level consistent with flexible prices, π_t represents inflation, and R_t is the nominal interest rate. Inflation and the interest rate are expressed as percentage deviations from their steady state values. The terms ε_t^x, ε_t^π and ε_t^R are exogenous disturbances.[2]

Equation (1) is a log-linearized IS curve derived from the household's intertemporal problem. The parameters σ and η denote the inverse of the consumption and labour supply elasticities. Equation (2) captures the staggered feature of a Calvo-type world in which each firm adjusts its price with a constant probability, $1-\omega$, in any given period, and independently from the time elapsed from the last adjustment. The parameter $0 < \beta < 1$ is the agents' discount factor.

The important difference relative to the standard New Keynesian model is that under a cost channel the marginal costs are not only a function of the output gap but also depend on the nominal interest rate. The idea is that firms have to pay their wage bills before production takes place and thus, at the beginning of each period, they need to borrow the relevant amount at the interest rate R_t. Following Rabanal (2006), we define the parameter γ as the fraction of firms subject to a cost channel.[3]

Equation (3) characterizes the behavior of the monetary authorities. This is an interest rate rule according to which the central bank adjusts the policy rate in response to inflation and the output gap. These adjustments are implemented smoothly, with ρ measuring the degree of interest rate smoothing.

3. THE TAYLOR PRINCIPLE REVISITED

In this section, we investigate the properties of the equilibrium of the sticky price model presented above. First, we derive the analytical conditions that prevent indeterminacy when the structure of the economy is purely forward-looking and the interest rate responds to current inflation and current output gap. We then analyze equilibrium dynamics using forward- and backward-looking policy rules as well as a hybrid version of the New Keynesian model.

3.1. Current- and Forward-Looking Policy Rules

The conditions for a unique equilibrium in the model (1)–(3) are summarized by the following propositions.

PROPOSITION 1. *Let $\tau \geq 2\theta$. Then, under a current-looking policy rule equilibrium determinacy obtains if and only if*

$$max\left\{1+\theta\psi_x-\left(\frac{1-\beta}{\kappa}\right)\psi_x,\theta\psi_x\right\}<\psi_\pi.$$

Proof. See Appendix. ■

The assumption $\tau \geq 2\theta$ corresponds to a situation in which the weight of the demand channel of monetary policy transmission is relatively larger than the weight of the cost channel. To see this, notice that the partial equilibrium effect of a unitary interest rate increase on inflation is $\kappa\tau$ for the demand channel and $\kappa\theta$ for the cost channel. In the special case $\gamma = \theta = 0$, which corresponds to the absence of a supply-side transmission mechanism, the lower bound reduces to the condition derived by Woodford (2003) for the standard New Keynesian model. In the special case $\gamma = 1$, the condition $\tau \geq 2\theta$ reduces to $\sigma \leq \eta$. We will return to this after Proposition 2.

The cost channel affects the condition for a unique equilibrium through the interest rate response to the output gap. When $\psi_x = 0$, in contrast, the minimum response to inflation that rules out indeterminacy is independent from the cost channel. To gain intuition for this result, it is useful to iterate equation (2) forward:

$$\pi_t = \kappa\sum_{k=0}^{\infty}\beta^k E_t\left(x_{t+k}+\theta R_{t+k}+\varepsilon^{\pi}_{t+k}\right) \tag{5}$$

To the extent that inflation is monotonic with respect to the output gap and the nominal interest rate, and the output gap is monotonic with respect to the real interest rate, the sign of the terms $(x_{t+k}+\theta R_{t+k})$ is crucial to determine whether inflation expectations become self-fulfilling.

Consider an interest rate increase that, following a supply or a demand shock, makes the real interest rate positive. A negative output gap is a necessary condition to offset the inflationary consequences of the shocks, and a positive real rate delivers a negative output gap.[4] Under the cost channel, however, inflation is also directly related to the nominal interest rate, and therefore a negative output gap is no longer a sufficient condition for a non-positive inflation rate.

When $\psi_\pi > 1$ and $\psi_x = 0$, the negative output gap outweighs the positive direct effect of the interest rate on inflation.[5] But, if the central bank reacts also to the output gap, then the increase in the nominal interest rate will be smaller than the increase associated with $\psi_x = 0$. The overall effect of a smaller interest rate rise when $\psi_x > 0$ is a smaller output gap in absolute value, which might be insufficient then to generate a negative inflation rate. For given $\psi_\pi > 1$ and sufficiently large values of ψ_x, the increase in θR_t dominates the decline in x_t and the expectations of higher inflation become self-fulfilling.

PROPOSITION 2. *Let $\tau < 2\theta$. Then, under a current-looking policy rule equilibrium determinacy obtains if and only if*

$$max\left\{1+\theta\psi_x-\left(\frac{1-\beta}{\kappa}\right)\psi_x, \theta\psi_x\right\}$$
$$<\psi_\pi<\frac{\tau\psi_x\left[1+\beta+\kappa\theta\right]}{\kappa\left[2\theta-\tau\right]}+\frac{(1+\rho)\,(2+2\beta+\tau\kappa)}{\kappa\,(1-\rho)\left[2\theta-\tau\right]}.$$

Proof. See Appendix. ■

If the relative importance of the supply-side channel of monetary policy transmission is larger than in Proposition 1 so that $\tau < 2\theta$, then the interest rate response to inflation that rules out equilibrium indeterminacy is also constrained from above. To develop intuition for this result, it is useful to consider the special case of full cost channel, $\gamma = 1$, according to which the condition $\tau < 2\theta$ simplifies to $\sigma > \eta$.

Under the cost channel, an increase in the nominal interest rate can move inflation in different directions depending on two factors: the degree of policy activism and the labor supply elasticity. For a given consumption elasticity, an increase in the real interest rate boosts labor supply and it reduces real wages. Higher values of the labor supply elasticity (i.e., lower η) imply a smaller decline in real wages for a given change in the nominal interest rate. If the response of the nominal rate to inflation is too aggressive, then the cost of higher lending rates outweighs the benefit of lower real wages and firms will raise prices.[6] If, in contrast, labor supply is sufficiently inelastic (i.e., higher η), such as in Proposition 1, then the real wage effect always dominate the borrowing cost effect and the upper bound never materializes.

Similar results, not reported but available on request, are obtained in the case of a forward-looking policy rule according to which the nominal interest rate responds to expected inflation, $E\pi_{t+1}$, rather than to current inflation.[7]

3.2. Backward-Looking Policy Rules

In the standard New Keynesian model, backward-looking policy rules are less prone to deliver equilibrium indeterminacy [see Woodford (2003)]. In this section, we investigate whether the finding is robust to the presence of a cost channel of monetary policy transmission. Specifically, we consider the following backward-looking policy rule:

$$R_t=\rho R_{t-1}+(1-\rho)(\psi_\pi\pi_{t-1}+\psi_x x_{t-1})+\varepsilon_t^R. \quad \textbf{(6)}$$

The parameters of the cost channel-augmented New Keynesian model are calibrated using the values in Ravenna and Walsh (2006) and they read $\sigma = 1.5$, $\beta = 0.99$, $\eta = 1$ and $\lambda = 0.15$. To make more transparent the comparison between current- and backward-looking interest rate rules we set $\rho = 0$. The results presented here are robust to alternative values of the interest rate smoothing parameter

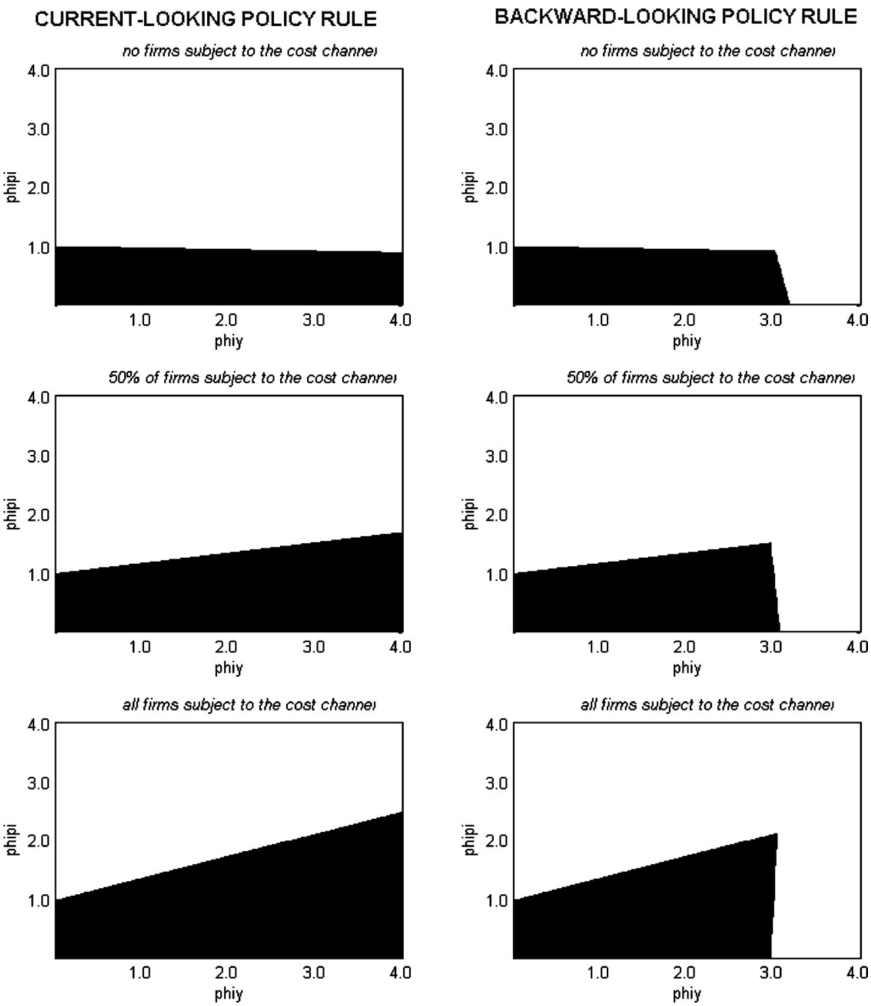

FIGURE 1. The cost channel of monetary policy transmission and indeterminacy. Dark region: indeterminacy; white region: uniqueness.

as well as to using a hybrid model in which the backward-looking weights in the aggregate demand and supply curves are equal to 0.5.

Figure 1 presents the findings for the current-looking policy rule in the left column and the backward-looking policy rule in the right column. The rows refer to three different constraints: when no firms, 50% of the firms or all firms are subject to the cost channel. The results are displayed as a function of the policy responses to inflation, ψ_π, and the output gap, ψ_x. We report results up to a value of 4 for both coefficients. The upper bound on ψ_π becomes binding only above 30.

The top panels correspond to $\gamma = 0$ and thus describes the properties of the standard New Keynesian model in which there is no cost channel. As for the

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current-looking rule, the Taylor Principle holds and *one* is the relevant threshold of the inflation response (vertical axis) above which uniqueness is guaranteed. The indeterminacy region shrinks as the response to the output gap (horizontal axis) increases. Under a backward-looking policy rule, the indeterminacy region is smaller and, in contrast to the current-looking case, sufficiently large values of ψ_x imply that the coefficient on the interest rate response to inflation, ψ_π, can become arbitrarily close to zero without delivering indeterminacy.

The middle panels show that when 50% of firms are subject to the cost channel, the region of multiplicity becomes larger. A positive response to the *current* output gap in the left column squeezes the uniqueness region. In the special case $\psi_x = 0$, the Taylor Principle of *one* is again the relevant threshold. As for the backward-looking rule, the number of parameter configurations associated with indeterminacy is now larger than in the model with no cost channel. Sufficiently large values of ψ_x still guarantee a unique equilibrium independently of the parameter ψ_π. Under the cost channel, however, for given values of ψ_x below a certain threshold, the interest rate response to inflation that guarantees a unique equilibrium is larger than in the absence of the cost channel.

The results in the middle panels are reinforced when all firms are subject to the cost channel (bottom panels). Using a hybrid model of aggregate demand and supply, the main message of a widening of the indeterminacy region is unchanged. A policy reaction to the output gap still makes the economy more prone to indeterminacy relative to the model with no cost channel, even when the interest rate responds to *past* inflation and *past* output gap. Our conclusions are robust to alternative parameterisations of the model: high values of the parameters σ, η, and ρ move further northeast of the upper right corner of the shaded area, and they make higher the threshold of the coefficient on *past* output gap above which equilibrium uniqueness is independent of ψ_π. Results are available on request.

3.3. Related Literature

In an important contribution, Benhabib, Schmitt-Grohé, and Uribe (2001) show that the conditions under which interest rate rules induce multiple equilibria depend on the way money is introduced in a model. Although the framework in Section 2 does not model money explicitly, other approaches, such as a cash-in-advance constraint and money in the production function, can also generate a cost channel of monetary policy transmission. It is therefore useful to relate the contribution of this work to previous literature.

Carlstrom and Fuerst (2000) and Schmitt-Grohé and Uribe (2006) study the conditions that guarantee equilibrium indeterminacy in a model with capital accumulation and a cash-in-advance constraint. Schmitt-Grohé and Uribe (2006) show that introducing output as additional argument in a Taylor-type feedback rule carries two unfortunate consequences. First, it can lead to significant welfare losses. Second, a positive response of the nominal interest rate to output enlarges the indeterminacy region relative to a model without the cost channel (compare

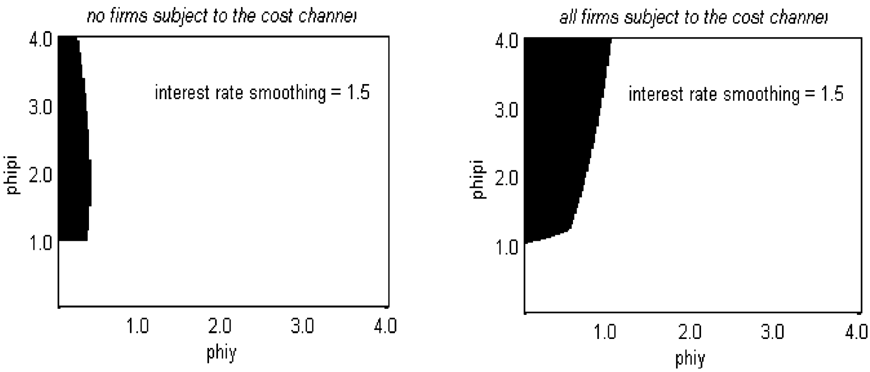

FIGURE 2. Superinertial backward-looking interest rate rules.

the panels (b) of Figures 1 and 2 in their paper). Their result that the interest rate response to output makes the economy more prone to multiple equilibria suggests that the findings obtained in this paper under a particular characterization of the cost channel are robust to using a cash-in-advance constraint.

A more general approach to introducing a cost channel is taken by Benhabib, Schmitt-Grohé, and Uribe (2003). The authors conduct a global analysis of equilibrium indeterminacy in a framework in which labor and real money balances may have different degrees of substitutability ranging from perfect substitutes to perfect complements. The main result is that a backward-looking feedback rule ensures global stability, provided that the coefficient on lagged interest rate is above unity.

Although a global analysis is beyond the scope of this paper, Figure 2 shows the regions of local determinacy for a superinertial backward-looking feedback rule with a coefficient on lagged interest rates, ρ, of 1.5. When all firms are subject to the cost channel (right panel), and labor and money are complements, responding to output shrinks the determinacy region, provided that ψ_x is below a certain threshold. A similar finding emerges for $\rho = 0$ in the right column of Figure 1. The left panel of Figure 2 reports results when no firms have borrowing constraints.

In line with other approaches to modeling the cost channel, under a superinertial backward-looking rule, the region of determinacy is larger that under a backward-looking rule with no interest rate smoothing. Furthermore, values of the smoothing coefficient above 2.5 are associated with a unique equilibrium. Results are available on request.

4. CONCLUSIONS

We have studied the interaction between the cost channel and monetary policy rules in an otherwise standard sticky price model of the business cycle. An interest

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rate response to inflation *above one* can still generate multiple equilibria if the central bank reacts also to movements in the output gap. Results are robust to using forward-, current-, and backward-looking policy rules as well as to using a hybrid model of aggregate supply and demand augmented with a cost channel. Our findings suggest that, when the cost channel is empirically important, trying to limit cyclical swings in real activity may result in undesired volatility of inflation and output.

NOTES

1. Rabanal (2007) investigates further the implications of the cost channel following a monetary policy shock and concludes that it is not possible to generate a positive response of inflation to such a shock in a DSGE model of the U.S. economy augmented with a cost channel [see Castelnuovo and Surico (2006) for an alternative explanation of the "price puzzle"].

2. The model can also be written in term of the deviation of output from a long-run trend, y_t. The Phillips curve would then be expressed as $\pi_t = \beta E_t \pi_{t+1} + \kappa[(y_t - z_t) + \theta R_t]$ and the aggregate demand as $y_t = E_t y_{t+1} - \tau(R_t - E_t \pi_{t+1}) + \varepsilon_t^y$, where z_t represents an exogenous shift in the marginal cost of production [see Lubik and Schorfheide (2004, p. 193)]. For the sake of comparison with earlier contributions [see, for example, Woodford (2003)], we write the model in terms of the model-consistent definition of the output gap. Such modeling choice implies that ε_t^R also embodies mismeasurements of the natural level of output.

3. Alternatively, γ can be interpreted as the fraction of wage bills paid in advance by each firm.

4. In the standard New Keynesian model, demand shocks are not associated with a negative output gap as they can be fully offset by an appropriate increase in the nominal interest rate. Under the cost channel, in contrast, demand shocks generate a trade-off between output and inflation stabilization.

5. To see this, notice that for $\gamma = 1$, $\tau > \theta$.

6. It should be noted, however, that for empirically plausible values of the parameters of the model the upper bound only becomes binding at values of ψ_π as large as 30.

7. The only difference relative to the current-looking reaction function regards the upper limit of ψ_π. The condition $\tau \geq 2\theta$, which in the previous case did not impose any constraint on ψ_π from above, is now associated with the upper bound $\tau\psi_y(1 + \beta + \kappa\theta)/\kappa(\tau - 2\theta) + (1 + \rho)(2 + 2\beta + \kappa\tau)/\kappa(1 - \rho)(\tau - 2\theta)$. In contrast, the condition $\tau < 2\theta$ does not impose now any upper limit on ψ_π.

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APPENDIX

The forward-looking sticky price model analyzed in this paper is characterized by one predetermined variable, R_t, and two jump variables, π_t and y_t, whose associated characteristic, third-order polynomial, reads

$$\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 = 0.$$

Woodford (2003) shows that the necessary and sufficient conditions for two roots to lie outside and one root to lie within the unit circle are as follows:

- Case I:

$$1 + A_2 + A_1 + A_0 < 0, \tag{A.1}$$

$$-1 + A_2 - A_1 + A_0 > 0, \tag{A.2}$$

or

- Case II:

$$1 + A_2 + A_1 + A_0 > 0, \tag{A.3}$$

$$-1 + A_2 - A_1 + A_0 < 0, \tag{A.4}$$

$$A_0^2 - A_0A_2 + A_1 - 1 > 0, \tag{A.5}$$

or

- Case III:

$$1 + A_2 + A_1 + A_0 > 0, \tag{A.3}$$

$$-1 + A_2 - A_1 + A_0 < 0, \tag{A.4}$$

$$A_0^2 - A_0A_2 + A_1 - 1 < 0 \tag{A.6}$$

$$|A_2| > 3. \tag{A.7}$$

PROOF OF PROPOSITION 1

The model (1)–(3) has the following canonical form representation [see Lubik and Schorfheide (2003, 2004), for details]:

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + \Pi \eta_t,$$

where

$$s_t = [x_t, \pi_t, R_t, E_t x_{t+1}, E_t \pi_{t+1}]', \quad \eta_t = [(x_t - E_{t-1}x_t), (\pi_t - E_{t-1}\pi_t)]'$$

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\tau & 1 & \tau \\ 0 & 0 & \kappa\theta & 0 & \beta \end{bmatrix}, \Gamma_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \rho & (1-\rho)\psi_x & (1-\rho_R)\psi_\pi \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\kappa & 1 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ (1-\rho)\psi_x & (1-\rho)\psi_\pi \\ 1 & 0 \\ -\kappa & 1 \end{bmatrix}.$$

The properties of the matrix $\Xi = \left(\Gamma_0^{-1}\Gamma_1\right)$ determines the conditions for a unique equilibrium. In particular, the matrices of the characteristic polynomials are

$$A_2 = -\frac{1}{\beta}\left[\tau\beta\psi_x(1-\rho) - \kappa\theta\psi_\pi(1-\rho) + \tau\kappa\theta\psi_x(1-\rho) + \beta(1+\rho) + 1 + \tau\kappa\right]$$

$$A_1 = \frac{1}{\beta}\left[\tau\psi_x(1-\rho) - \kappa\theta\psi_\pi(1-\rho) + \tau\kappa\psi_\pi(1-\rho) + (1+\rho) + \rho\beta + \rho\tau\kappa\right]$$

$$A_0 = -\frac{\rho}{\beta}.$$

Consider Case II. Condition (A.3) corresponds to

$$\frac{1}{\beta}\left[(1-\beta)\tau\psi_x(1-\rho) + \kappa\tau\psi_\pi(1-\rho) - \tau\kappa\theta\psi_x(1-\rho) - \tau\kappa(1-\rho)\right] > 0,$$

which can be solved for the coefficient of the policy response to inflation as

$$\psi_\pi > 1 + \theta\psi_x - \left(\frac{1-\beta}{\kappa}\right)\psi_x. \quad \textbf{(A.8)}$$

Condition (A.4) implies

$$-1 - \frac{1}{\beta}\begin{bmatrix} (1+\beta)\tau\psi_x(1-\rho) - 2\kappa\theta\psi_\pi(1-\rho) + \tau\kappa\theta\psi_x(1-\rho) \\ +\tau\kappa\psi_\pi(1-\rho) + 2\rho\beta + \beta + 2(1+\rho) + \tau\kappa(1+\rho) \end{bmatrix} < 0,$$

which can be rewritten as

$$-1-\frac{1}{\beta}\left\{\begin{array}{c}\beta\left(1+2\rho\right)+\left(2+\tau\kappa\right)\left(1+\rho\right)\\ +\tau\psi_x\left(1-\rho\right)\left[\left(1+\beta\right)+\kappa\theta\right]+\kappa\psi_\pi\left(1-\rho\right)\left[\tau-2\theta\right]\end{array}\right\}<0. \qquad \textbf{(A.9)}$$

Assuming $\tau \geq 2\theta$, the inequality in (A.9) is always satisfied. This also contradicts (A.2), thereby eliminating Case I. Finally, condition (A.5) takes the following form:

$$\left(\frac{\rho^2+\rho\beta^2}{1-\rho}\right)+\beta\tau\psi_x\left(1-\rho\right)+\beta\left(1+\rho\right)+\beta\tau\kappa+\tau\kappa\rho\left(\psi_\pi-\theta\psi_x\right)+\beta\psi_\pi\left(\tau-\theta\right)>0. \qquad \textbf{(A.10)}$$

The last term of the left-hand side is always positive as the definitions in (4) imply that $\tau > \theta$. If $1-\left(\frac{1-\beta}{\kappa}\right)\psi_x > 0$, as it is likely to be the case empirically, then the inequality (A.8) implies that the last but second term in (A.10) is positive.

By contrast, if $1-\left(\frac{1-\beta}{\kappa}\right)\psi_x < 0$, the condition $(\psi_\pi-\theta\psi_x) > 0$ implies that the inequality (A.8) holds. Condition (A.5) is thus satisfied, whereas Condition (A.6) is violated, which implies that Case III is ruled out.

PROOF OF PROPOSITION 2

On the basis of the same algebra and reasoning developed in the proof of Proposition 1, Condition (A.8) and Condition (A.10) hold also for Proposition 2. The only difference between the two proofs concerns Condition (A.9) as now $\tau < 2\theta$. According to the latter assumption, the inequality (A.9) derived from Condition (A.4) can be rewritten as

$$\kappa\psi_\pi\left(1-\rho\right)\left[2\theta-\tau\right]<\tau\psi_x\left(1-\rho\right)\left[\left(1+\beta\right)+\kappa\theta\right]+2\left(1+\beta\right)\left(1+\rho\right)+\tau\kappa\left(1+\rho\right),$$

which implies

$$\psi_\pi<\frac{\tau\psi_x\left[1+\beta+\kappa\theta\right]}{\kappa\left[2\theta-\tau\right]}+\frac{\left(1+\rho\right)\left(2+2\beta+\tau\kappa\right)}{\kappa\left(1-\rho\right)\left[2\theta-\tau\right]}. \qquad \textbf{(A.11)}$$

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