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Modelling policing strategies for departments with limited resources

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Crime prevention is a major goal of law-enforcement agencies. Often, these agencies have limited resources and officers available for patrolling and responding to calls. However, patrolling and police visibility can influence individuals to not perform criminal acts. Therefore, it is necessary for the police to optimize their patrolling strategies to deter the most crime. Previous studies have created agent-based models to simulate criminal and police agents interacting in a city, indicating a "cops on the dots" strategy as a viable method to mitigate large amounts of crime. Unfortunately, police departments cannot allocate all of the patrolling officers to seek out these hotspots, particularly since they are not immediately known. In large cities, it is often necessary to keep a few officers in different areas of the city, frequently divided up into beats. Officers need to respond to calls, possibly not of a criminal nature. Therefore, we modify models for policing to account for these factors. Through testing the policing strategies for various hotspot types and number of police agents, we found that the methods that performed the best varied greatly according to these factors.

Key words: Agent Based Methods; Crime Models; Hot Spots; Policing Strategies

1 Introduction

Crime has been decreasing since the 1990s [16], nevertheless it still occurs. Law-enforcement agencies facing budget freezes and cuts require them to manage their resources more effectively [7,19]. We aim to optimize policing practices given resource constraints by simulating various strategies. Although criminal offenses vary, we will focus our analysis on crimes that often cluster in time and space, such as residential burglaries [21].

Crime is not uniformly distributed throughout a city. Certain neighbourhoods exhibit more criminal acts than others [13]. This behaviour can be explained by the criminal opportunity theory [11]. This theory summarizes the likeliness of an individual committing a criminal act, which depends on the individual's self-interest, the network of the individual, and the welcoming of criminal behaviour [11]. A person's network can have a significant impact on influencing other members of one's network, such as family and friends, to commit criminal acts. A person's self-interest sparks when there is a tempting opportunity that entices one into criminal action. The theory also states that offenders are likely to commit a crime if the risk of being caught is relatively low in comparison to the reward.

Another component in the criminal opportunity theory is target selection [11]. Criminologists view target selection as a multi-level process in which an offender first seeks a general target, such as neighbourhood, followed by a specific target or residence [20]. An offender often decides to select targets through everyday movement [6], tending to commit criminal acts in familiar settings, particularly if previous offenses in the region were successful. In regions with high crime rates, certain neighbourhoods exhibit a sense of lawlessness, vulnerability, and a notion of crime tolerance [7, 19].

In the case of residential burglaries, the same location and its surrounding neighbours are more likely to be burglarized again within a short time immediately following the event. This behaviour is known as near-repeat victimization [13–15, 20, 27, 31]. The area surrounding a recently burglarized home attracts more crimes, creating a hotspot, a spatial region with a relatively higher crime rate [5, 10]. According to the "broken-window" theory, regions of disorder and lack of maintenance signal to offenders that criminal acts would likely go undetected, especially when compared to regions of order and cleanliness [30]. Many researchers have sought to identify different types of hotspots, both spatially and temporally, to assist police in identifying problematic areas [3, 8, 24]. Others have formulated theories to determine why hotspots form [6].

According to the criminal opportunity theory, the presence of a police officer has some influence on whether a criminal offender commits a crime [11]. Thus, law-enforcement agencies have police officers patrol a city in efforts to mitigate crime. Since it is unrealistic to have a police officer at each location in a city, effective patrolling strategies are required. Law-enforcement agencies have experimented with policing strategies. For example, police officers in Minneapolis randomly patrolled 55 different hotspots during high-risk times, not answering calls for service at times [26]. In 2015, the Los Angeles Police Department used helicopters to patrol over the city. When there were more weekly helicopter patrols, the number of reported crimes decreased compared to when there were less patrols [17]. Although more studies need to be done to conclude that there is a connection between the helicopter patrols and reported crime numbers, the results of this experiment are very promising. However, the presence of the police officers does not always help mitigate crime. In Kansas City, police agencies experimented with unbiased patrolling routes and did not yield a significant decrease in crime [29]. Simulated results verified how random patrolling was ineffective in mitigating crime [15], confirming the observed case study in Kansas City [29]. In another study where hotspot policing was experimented, 80% of the test regions reported significant crime reduction, suggesting more targeted policing strategies are more effective [4].

Computer simulations are effective tools in bridging theorized explanations and depictions of crime patterns [2, 9, 23]. Simulations allow for the opportunity to experiment with different environmental conditions. Several methods use agent-based models (ABM), a method that consists of agent entities that interact and make decisions [15, 27, 31]. In crime models, agents can represent police officers, criminals, victims, or other entities [9, 12]. Agents may make decisions on movement direction and actions. Furthermore, geographic factors can be introduced into the model [12]. Simulations allow policing strategies to be tested without having to use policing resources.

Our work focuses on creating an ABM that incorporates realistic policing strategies, including typical features of a patrolling police officer's day. When police officers patrol, they often have an assigned patrolling region called a beat [18]. Further, throughout the day an officer will have to respond to calls or has interactions with individuals or criminals that prevent further patrolling. These are features we will include in our model. We also test the strategy of having multiple patrolling types simultaneously. We will introduce the baseline models in Section 2 and our proposed methods in Section 3. We simulate each of the patrolling strategies for varying numbers of police agents, testing how the methods perform for cities with a limited number of police officers available for patrolling. The results are presented in Section 4 with figures and tables demonstrating how each method performed on crimes with different hotspot characteristics [27].

2 Agent-based methods for residential burglaries

We add more realistic features to existing ABM for residential burglaries [15, 21, 27, 31]. Short *et al.* developed a model to help explain how hotspots of criminal activities form by having criminal agents move through a city, committing residential burglaries and increasing the attractiveness of the region [27]. We discuss this model in detail in Section 2.1. Police agents have been added to the Short *et al.* model in multiple models [21, 27, 31]. We will modify the Jones *et al.* model, described in Section 2.2, through the addition of features often seen during a patrolling police officer's day. Our proposed model is explained in Section 3.

2.1 Agent-based method of hotspot formation

The goal of [27] was to examine how macroscopic behaviour of residential burglaries could be explained by behaviours of individuals at a microscopic level. Short *et al.* created an ABM of criminal agents with biased movement and the ability to perform criminal acts. They also derived the continuum limit and performed linear stability analysis to further understand the behaviour of the model. This allowed them to determine different parameter regimes that produce different hotspot types.

The criminal agents within the model move with bias towards attractive sites. At each location, the agent has a choice of burglarizing, going home, or moving to a neighbouring site. If an agent commits a crime or decides to return home, the agent is removed from the grid. New agents are added to the system at a pre-determined rate, Γ.

The decision of where to move and whether to burglarize a residence at site s depends on the attractiveness of the site, $A_s(t)$. This attractiveness can be calculated by

$$A_s(t) = A_s^0 + B_s(t), \tag{2.1}$$

where A_s^0 is the baseline attractiveness of the site and $B_s(t)$ is the attractiveness incorporating near-repeat victimization. This $B_s(t)$ attractiveness varies over time and depends on the historical criminal activity during the course the simulation. If a criminal burglarizes a site, the attractiveness of that site increases immediately, and then decays over time.

This gives the update for $B_s(t)$ as

$$B_s(t+\delta t) = B_s(t)(1-\omega\delta t) + \theta E_s(t).$$

Here, δt is the time step, ω gives the time frame for the increased likelihood of a repeated offense, and θ gives the amount of increase in attractiveness caused by a single burglary. Additionally, $E_s(t)$ gives the number of events that occurred at site s during the timestep.

The neighbouring sites are also influenced by the criminal act since neighbouring residences have a higher likelihood of being burglarized for a short time following the event. This is incorporated by updating the attractiveness of the neighbouring sites for the next time step according to

$$B_s(t+\delta t) = \left[(1-\eta)B_s(t) + \frac{\eta}{z}\sum_{s'\sim s} B_{s'}(t)\right](1-\omega\delta t) + \theta E_s(t). \tag{2.2}$$

Here, η determines how influential are the neighbouring effects from the near-repeat victimization, and z gives the number of neighbouring sites s' of site s. The notation $s' \sim s$ refers to all sites s' such that s' is a neighbour of s.

The resulting model produced different types of behaviours, including no significant hotspots, stationary and dynamic hotspots, and hotspots of varying sizes [27]. In Section 2.2, we describe how Jones *et al.* added police into this model with a few notational changes [15]. They applied their model to the three different behaviours of no significant hotspots, small hotspots, and large hotspots.

2.2 Agent-based method of policing strategies

Jones *et al.* incorporate police agents into the Short *et al.* model [15, 27]. In order to accomplish this, the criminal agents need to have some behaviour modification due to police presence. The attractiveness of a site s is adjusted by decreasing $A_s(t)$ depending on the number of police agents at the site, $\kappa_s(t)$, given by

$$\tilde{A}_s(t) = e^{-\chi\kappa_s(t)}A_s(t). \tag{2.3}$$

The parameter χ indicates how much the police presence influences the attractiveness. Using this $\tilde{A}_s(t)$, the probability that a criminal agent will burglarize the residence at site s is given by

$$\tilde{p}_s(t) = \frac{\epsilon\tilde{A}_s(t)}{1+\epsilon\tilde{A}_s(t)}, \tag{2.4}$$

where the parameter ϵ determines the likelihood of a crime occurring.

Other adjustments to the model were made using $\tilde{p}_s(t)$ in the update formula for $B_s(t)$ with $\delta t = 1$, giving

$$B_s(t+1) = \left[B_s(t) + \frac{\eta}{4}\Delta B_s\right](1-\omega) + \theta\tilde{p}_s(t)n_s(t). \tag{2.5}$$

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Here, $n_s(t)$ gives the number of criminal agents located at site s, and

$$\Delta B_s = \sum_{s' \sim s} B_{s'}(t) - 4B_s(t)$$

is the discrete Laplacian applied to B_s on a 5-point stencil.

During the simulation, criminal agents move throughout the region. At each time step, the agent first decides whether or not to return home depending on the number of police agents at a given location. This probability is given by

$$\frac{J\kappa_s(t)}{1 + J\kappa_s(t)}, \tag{2.6}$$

where J is a parameter determining the influence of the police agents in the decision. If the agent decides not to return home, it then decides whether to commit a crime with probability $\tilde{p}_s(t)$. If a crime is not committed, the criminal agent at site s moves to a neighbouring location x with probability

$$\frac{\tilde{A}_x(t)}{\sum_{s' \sim s} \tilde{A}_s(t)}. \tag{2.7}$$

Similar to [27], criminal agents are added to the simulation at rate Γ.

Now that the behaviour rules for the criminal agents with police presence have been established, the police agents must decide how they will move throughout the region. This is where different patrolling strategies are implemented. The methodology proposed in [15] include random walks (RW-J), cops on the dots (CoD-J), and peripheral interdiction (PI-J), which will be described next.

2.2.1 *Random walk patrolling (RW-J)*

Random walk (RW) patrolling represents a policing strategy where there is no bias in movement. This type of movement is simulated by giving each police agent an equal probability of moving to any of its neighbouring sites. We will refer to this patrolling strategy with the Jones *et al.* implementation as RW-J.

2.2.2 *Cops on the dots patrolling (CoD-J)*

Cops on the dots (CoD) patrolling is essentially hotspot policing. In this strategy, police officers focus their time in hotspot regions where there is a higher level of criminal activity. In simulations, the police agents are biased in movement into regions with higher levels of attractiveness. The police agent at site s moves to a neighbouring location x with probability

$$\frac{\tilde{A}_x(t)}{\sum_{s' \sim s} \tilde{A}_s(t)}. \tag{2.8}$$

The movement is similar to that of the criminal agents. We will refer to this patrolling strategy with the Jones *et al.* implementation as CoD-J.

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2.2.3 *Peripheral interdiction patrolling (PI-J)*

Peripheral interdiction (PI) patrolling is a strategy where police officers patrol around a hotspot as opposed to entering a hotspot directly. It was theorized by Jones *et al.* that this strategy would have the best results with large hotspots [15]. They implemented the strategy by having police agents move with bias proportional to

$$e^{-|c_1 B_{s'} - c_2|}, \tag{2.9}$$

where c_1 and c_2 are chosen by the model parameters. These movement rules bias the police agents to a particular attractiveness level of the neighbouring sites.

2.3 Continuum PDE models of ABM policing strategies

To better understand agent-based methods, continuum models are often derived, giving other techniques to analyze the dynamics of the model for different parameter regions. Short *et al.* derived the continuum equations for equation (2.2), obtaining

$$\begin{aligned} \frac{\partial B}{\partial t} &= \eta \nabla^2 B - B + \rho A \\ \frac{\partial \rho}{\partial t} &= \nabla \cdot \left[\nabla \rho - \frac{2\rho}{A} \nabla A \right] - \rho A + \bar{B} \end{aligned}$$

after non-dimensionalizing [27]. Here, the $\bar{B}$ gives the spatially averaged attractiveness, and $\rho(\mathbf{x}, t)$ gives the density of criminals. Noting the similarity to the Keller–Segel aggregation model for chemotaxis, Rodriguez *et al.* verified local existence and uniqueness of solutions [25].

Incorporating the behaviour modification into the Short *et al.* model, Jones *et al.* derived the continuum equations for equation (2.5) [15],

$$\begin{aligned} \frac{\partial B}{\partial t} &= \frac{\eta}{4} \Delta B - \omega B + \theta \rho \epsilon \tilde{A} \\ \frac{\partial \rho}{\partial t} &= [-\epsilon A - \Lambda - J\kappa]\, \rho + \Gamma + \frac{1}{4} \Delta \rho - \frac{1}{2} \nabla \cdot [\rho \nabla \log A]\,. \end{aligned}$$

The number of police agents, $\kappa(\mathbf{x}, t)$, is determined by the strategy. For the random walk patrolling (RW-J), the continuum equation for κ is

$$\frac{\partial \kappa}{\partial t} = \frac{1}{4} \Delta \kappa.$$

For the cops on the dots patrolling (CoD-J), the continuum equation is

$$\frac{\partial \kappa}{\partial t} = \frac{1}{4} \Delta \kappa - \frac{1}{2} \nabla \cdot (\kappa \nabla \log A)\,.$$

As noted by Jones *et al.*, there are difficulties in deriving the continuum equations for the peripheral interdiction patrolling (PI-J) strategy. Since many of our methods use either a combination with a modified peripheral interdiction patrolling strategy or have

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nontrivial new behaviours, we will only include the agent-based formulations of our policing strategies.

2.4 Other ABM models with policing strategies

Other researchers have modified the Jones *et al.* approach or viewed different strategies [21, 28, 31]. Sutanto examined four different tactics, including an active response (police placed in high-attractiveness areas), stationary police (police are equally distributed through space on a grid), specified patrolling (police target areas between grid locations), and concentrated police (police are distributed in varying levels) [28]. Zipkin *et al.* modified the deterrence term to the cops on the dots (CoD) strategy for the continuum limit [31]. Nam implemented the strategies of Jones *et al.* and added three additional routines, including police agents moving as if they were criminal agents, police moving according to a previous state of the system, and finally by specified patterns like spirals [21].

3 Police beat ABM with response to calls

While previous extensions of the Jones *et al.* model focus on adding different routines, we modify the model by adding in features often seen in a typical patrol. In particular, we allow police agents to respond to calls. Further, they are allowed to return home or to the station, thus removing them from patrolling. We also restrict police agents' movements to particular beats. In addition to the police agents' presence deterring criminal agents at a location, we allow for neighbouring locations to have some deterring effect as well. The details of these changes are given in Sections 3.1–3.4.

We will test our ABM with the same strategies as [15] but with the modifications of police-agent behaviour. Further, we add a few more policing strategies, including mixed patrols. Sections 3.5–3.6 provide the details for these new routines.

3.1 Police beats

Cities are often partitioned into beats and districts for patrolling and reporting purposes. For our ABM, we assign each police agent a beat number, allowing movement only within this particular beat. The criminal agents are still free to travel anywhere in the region. One advantage to having beats is to ensure that all regions of the grid maintain a certain number of police agents. This will help to minimize any crime that is displaced due to policing hotspots. In our simulations, we chose four beats, similar to the four quadrants of a grid. All beats are squares of equal size.

3.2 Response to calls

Part of a patrolling officer's job is to respond to calls and address suspicious activity [22]. In a study in Indianapolis and St. Petersburg, the time spent by a patrol officer was recorded and categorized for a typical shift [22]. The average number of encounters per 8 hours depended on the officer assignment and ranged from 9 to 11. These encounters ranged from brief to casual to full encounters, each requiring different lengths of time.

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An officer will not continuously be on patrol duty. Officers work according to shifts and also will have responsibilities at other locations, such as the police station [22]. Thus, we include removal of police agents from our simulation to represent cessation of patrolling. For our model, when a police agent and criminal agent are at the same location, we say that an interaction has occurred. In order to address the situation with the criminal agent, the police agent remains at that location for some period of time before returning to patrol. Removal from active patrol occurs after a number of interactions, sampled from a normal distribution where the mean is determined by the average number of encounters a typical police officer has on a given shift.

A police agent is added to the simulation when another police agent is removed from patrol duty. This new agent has the same assigned beat as the removed agent. Initial simulations had police agents added at the centre of the grid, connecting the four beats, as though there were a police station at this location. Results from these simulations produced a low level of criminal activity immediately surrounding the centre of the grid, but the remainder of the region had higher criminal activity. Having all agents patrol immediately around the station seemed unreasonable as police agents may decide to head to a location far from the station before initiating patrols, similar to a Lévy flight [7]. Thus, we have the police agents added to random locations within their designated beat.

3.3 Neighbouring deterrent effect

The Jones *et al.* model only included a deterrent effect at the location of the police agent [15]. However, if a criminal agent notices there is a police agent at the neighbouring site, then the criminal agent should be less likely to commit a crime than when there are no police agents present. Thus, we include a neighbouring deterrent effect for the 8 neighbouring locations: north, northeast, east, southeast, south, southwest, west, and northwest.

The neighbouring deterring effect $D_s(t)$ for a site s is given by

$$D_s(t) = \kappa_s(t) + \sum_{s' \sim s} \kappa_{s'}(t) e^{|s-s'|^2/v}, \tag{3.1}$$

where v determines the decay. Then, the attractiveness from Jones *et al.* is modified to

$$\tilde{A}_s(t) = e^{-\chi D_s(t)} A_s(t). \tag{3.2}$$

If there are no police agents at the neighbouring sites, then this is the original $\tilde{A}_s(t)$ from equation (2.3).

3.4 Other modifications

We made some slight modifications to the movement dynamics. We allow agents to move to any of the eight neighbouring nodes, as opposed to only the directions north, south, east, and west. By allowing the agents to move in more directions, we must change the discrete Laplacian to use a 9-point stencil rather than a 5-point stencil.

3.5 Patterned patrol (PAT)

In addition to the three strategies of Jones *et al.* [15], we implemented another policing strategy, patterned patrol (PAT). This patrolling method allows police agents to cover the entire grid to search for criminal activity. The police agents traverse the beat or grid in vertical lines, initially heading eastward until they reach the edge of the region and change direction to the west. Similarly, when the agents reach the western edge of the region, they change their direction back to the east.

Often, police departments have officers patrol via bicycles or by foot, and some experiments with bicycle patrols have shown decreases in certain criminal acts [1]. To further simulate more realistic patrolling, we enable police agents to have two different speeds. This would account for the possibility of patrolling via different means. Since patterned patrolling is the only strategy that we are examining that gives the police agents a specified route, this will be the only method we allow agents to have different speeds.

3.6 Mixed patrols

The four main strategies for patrolling under the new features of the ABM are random walk (RW), cops on the dots (CoD), peripheral interdiction (PI), and patterned patrol (PAT). The strategies from Jones *et al.* are implemented similarly but with the modifications outlined in the previous sections. We also performed mixed patrols where half of the police agents are patrolling according to one strategy, and the other half of the police agents are patrolling according to another. The different possible combinations are RW – CoD, RW – PI, RW – PAT, CoD – PI, CoD – PAT, and PI – PAT.

3.7 Implementation

To test the methods, we first compare the results to that of Jones *et al.* [15]. The parameters used in our implementation are listed in Table 1, which are the same values used in their simulations. The time step in the Short *et al.* method was $\delta t = 0.01$, which could be interpreted in units of days [27]. This time step was modified in Jones *et al.* to be $\delta t = 1$, which is to be interpreted as a unit of time in which agents are updated [15]. Since we are modifying the Jones *et al.* model, we have taken $\delta t = 1$ as well. We used a 100×100 grid with 64,000 iterations. Pseudo-code for the algorithm is presented in Algorithm 1. Depending on the method, the hotspot parameter regime, and the number of police agents, a single simulation could take a few minutes or a few hours.

We applied all methods to preexisting hotspots of three different hotspot parameter regimes, namely no significant hotspots, small hotspots, and large hotspots. The preexisting hotspots shown in Figure 1 were generated using our implementation of Jones *et al.* with no police agents.

4 Results

We compare the different methods that we implemented among each other as well as to Jones *et al.* [15]. By comparing to Jones *et al.*, we test for differences with the new model's features. For a reasonable comparison, we used the same number of police agents as their

Table 1. *This table provides the parameters and the values used in our simulations, taken from [15].*

Hotspot type	Parameters	Values
No significant hotspots	ω	0.0004
	γ	0.00025
	ϵ	0.02
	λ	0.000625
Small hotspots	ω	0.003
	γ	0.004
	ϵ	0.008
	λ	0.004
Large hotspots	ω	0.0001875
	γ	0.00025
	ϵ	0.0005
	λ	0.000625
All types	θ	1
	A_0	0.1
	η	0.05
	l	1
	χ	4.60517
	J	0.66
	c_1	1
	c_2	0.05
	μ_w	5
	σ_w	0.7
	μ_i	26
	σ_i	10
	ν	5.77

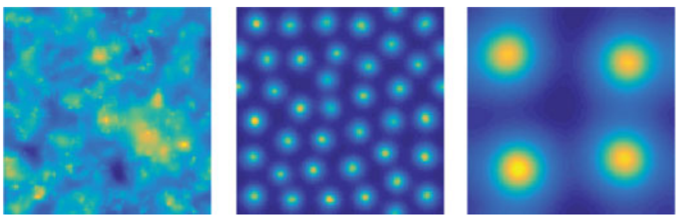

FIGURE 1. These plots give the pre-existing hotspots used for the three different parameter regimes that produce no significant hotspots, small hotspots, and large hotspots. The pre-existing hotspots were generated using the Jones *et al.* model with no police agents, which is equivalent to the Short *et al.* model [15,27]. (Left) No significant hotspots. (Middle) Small hotspots. (Right) Large hotspots.

simulations, 300. We also varied the number of police agents between 0 to 400 in running each method five times. Further, we ran all methods for 150 police agents 100 times in the small hotspot parameter regime to analyze the variability among runs. These results are presented in Table 4 in Section 5.5.

The B_{ave} value is the average value of $B_s(t)$ over the entire grid at the end of one simulation. This value indicates the level of criminal activity and is a good metric to

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compare the amount of crime between methods. We performed comparisons for all of the methods with and without beats to determine the effect police beats has on the overall level of criminal activity.

Algorithm 1 Police Patrolling Agent-Based Model Pseudo-Code

Input: ω, γ, ϵ, λ, θ, A^0, η, l, χ, J, μ_i, σ_i, μ_w, σ_w, v, *# of police agents, pre-existing hotspot map, policing strategy*
for `<each police agent>` **do**
  if *beats* **then**
    Randomly place police agents within their assigned beat.
  else
    Randomly place police agents.
  end if
  Sample y_{cop} from $Y \sim \mathrm{N}(\mu_i, \sigma_i)$.
  Set $T_w = 0$, police agent's waiting-time counter.
  Set $T_i = 0$, police agent's interaction counter.
end for
Set B_0 = *pre-existing hotspot map.*
Calculate $\kappa_s(t)$, number of police agents at site s.
Calculate $A_s(t)$, equation (2.1).
Calculate $D_s(t)$, equation (3.1).
Calculate $\tilde{A}_s(t)$, equation (3.2).
for `<each iteration>` **do**
  Introduce criminal agents at rate Γ.
  for `<each criminal agent>` **do**
    Calculate equation (2.6), P(going home|neighbouring police agents)
    Sample x_1 from Unif(0, 1).
    if $x_1 < P$(going home|neighbouring police agents) **then**
      Remove the criminal agent from the system.
    else Calculate equation (2.4), P(committing a crime).
      Sample x_2 from Unif(0, 1).
      if $x_2 < P$(committing a crime) **then**
        Remove the criminal agent from the system.
      else
        Calculate equation (2.7).
        Sample from Unif(0, 1) and move the criminal agent to site s' based on probability relative to equation (2.7).
      end if
    end if
  end for
  Calculate $n_s(t)$, number of criminal agents at site s.
  for `<each police agent>` **do**
    if $n_s(t) > 0$ **then**
      $T_i = T_i + n_s(t)$
      if $T_i \geqslant y_{cop}$ **then**
        if *beats* **then**
          Randomly place agent within assigned beat.
        else
          Randomly place police agent.
        end if
        Sample y_{cop} from $Y \sim \mathrm{N}(\mu_i, \sigma_i)$.
        Set $T_i = 0$.
      end if
    end if

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if $T_w == 0$ **then**
  if $n_s(t) > 0$ **then**
    Sample w_{cop} from $W \sim \mathrm{N}(\mu_w, \sigma_w)$.
    $T_w = T_w + 1$.
  else
    Move police agent according to *policing strategy*.
  end if
else if $T_w > 0$ && $T_w < w_{cop}$ **then**
  $T_w = T_w + 1$.
end if
if $T_w \geqslant w_{cop}$ **then**
  $T_w = 0$.
  Move police agent according to *policing strategy*.
end if
end for
Update $\kappa_s(t)$.
Update $B_s(t)$, equation (2.5).
Update $A_s(t)$, equation (2.1).
Update $D_s(t)$, equation (3.1).
Update $\tilde{A}_s(t)$, equation (3.2).
end for
Output: $B_s(t)$, $n_s(t)$

4.1 Criminal activity for 300 police agents

Our first comparison is to that of Jones *et al.* [15]. Their results indicate that the criminal activity does not improve with more police agents performing the random patrol strategy. However, for the CoD and PI strategies, more police agents indicate lower criminal activity.

In order to give a full comparison of all strategies with 300 police agents, we include the $B_s(t)$ plots for the final iteration of the simulation in Figure 2. Further, we include a table with all of the mean B_{ave} values for all simulations with 300 police agents in Table 2. The appendix contains additional figures with the results for our methods without beats and 300 police agents, allowing for a better comparison with Jones *et al.* and the impact of beats.

4.2 Varying number of police agents for small hotspots

One of our primary goals is to determine which methods are useful for agencies with limited resources. Therefore, we compare all of the methods by varying the number of police agents from 0 to 400. We only include the results for the small hotspots here in Figures 3 and 4; the remaining figures can be found in the appendix.

4.3 Criminal activity for 150 police agents

Noticing that there is a difference in the performance for the methods when there are fewer police agents, we do a further analysis of all methods on all hotspot types with only 150 police agents. The results are included in the Table 3.

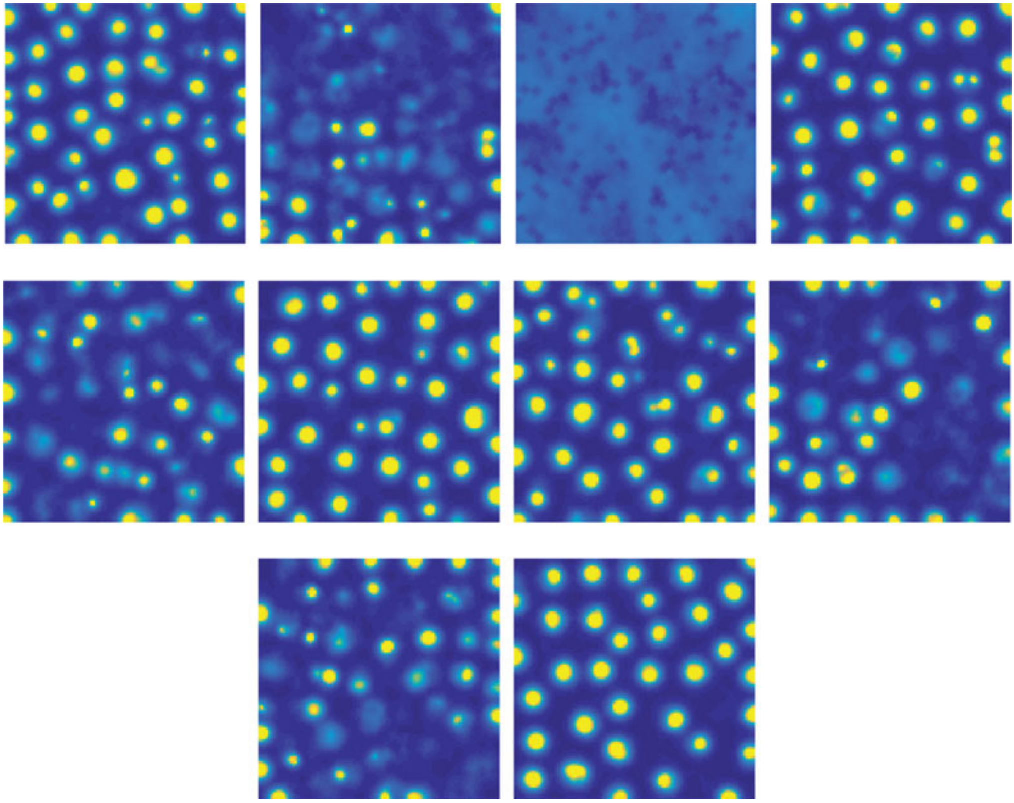

FIGURE 2. Each sub-plot gives the $B_s(t)$ values of the final iteration, indicating the level of criminal activity. Darker colours (blue) indicate less criminal activity, whereas light colours (yellow) indicate higher amounts of criminal activity. All simulations were run with 300 police agents using pre-existing small hotspots. These simulations included police beats. Figures without police beats are included in the appendix. From left to right, top row: (a) RW, (b) CoD, (c) PI, (d) PAT. Middle row: (e) RW-CoD, (f) RW-PI, (g) RW-PAT, (h) CoD-PI. Bottom row: (i) CoD-PAT, (j) PI-PAT.

5 Analysis

In this section, we will describe the results produced from the computer simulations done in Section 4. We notice a trend throughout all discussed methods. Less criminal activity is correlated with more criminals. This means that the police agents are effective in deterring crime, but not in reducing the number of criminal agents ready to commit criminal acts. This is similar to the results of Jones *et al.* [15].

5.1 Comparison to existing ABM patrolling methods

We next compare our results to the Jones *et al.* methods. When looking at the 300 police agent results in Table 2, we notice that the order of RW, CoD, and PI without beats has a similar ranking as Jones *et al.* in our implementations. Further, our strategies actually had lower criminal activity than in the Jones *et al.* methods, except for the CoD with

Table 2. *This table provides the mean B_{ave} value and the number of criminals for each method, averaged over five runs, using a total of 300 police agents. C_{ave} is the average number of criminals. The methods were applied to pre-existing hotspots of three different types: no significant hotspots, small hotspots, and large hotspots. The previous methods listed are taken from [15], labelled RW-J, CoD-J, and PI-J, representing the random walk, cops on dots, and peripheral interdiction, respectively. The remaining results were taken from our methods, which includes responding to calls and neighbouring deterrent effects; the methods are random walk (RW), cops on dots (CoD), peripheral interdiction (PI), patterned patrol (PAT), and a hyphen between two methods indicates a mixed method between the two.*

		300 police agents					
		No significant hotspots		Small hotspots		Large hotspots	
	Method	B_{ave}	C_{ave}	B_{ave}	C_{ave}	B_{ave}	C_{ave}
Previous methods	RW-J	0.3104	156	0.9814	1068	0.0081	279
	CoD-J	0.2545	151	0.2888	1756	0.0071	245
	PI-J	0.3900	99	0.4904	2473	0.0680	239
Without beats	RW	0.1896	139	0.6676	1108	0.0055	216
	CoD	0.1613	121	0.3506	1768	0.0052	181
	PI	0.3079	77	0.3602	1750	0.0495	185
	PAT	0.1424	110	0.5504	1112	0.0045	166
	RW-CoD	0.1713	125	0.4355	1531	0.0050	179
	RW-PI	0.1785	133	0.6796	1050	0.0051	177
	RW-PAT	0.1723	114	0.6483	1114	0.0049	164
	CoD-PI	0.1647	120	0.4514	1397	0.0050	177
	CoD-PAT	0.1693	128	0.3827	1579	0.0048	174
	PI-PAT	0.1690	120	0.7365	1003	0.0048	185
With beats	RW	0.1973	131	0.6511	1131	0.0058	216
	CoD	0.1576	123	0.3537	1712	0.0051	178
	PI	0.3128	76	0.3658	1740	0.0506	169
	PAT	0.1361	115	0.6121	1087	0.0042	148
	RW-CoD	0.1652	131	0.4741	1471	0.0053	198
	RW-PI	0.1700	121	0.6913	1097	0.0052	193
	RW-PAT	0.1810	127	0.6470	1091	0.0049	182
	CoD-PI	0.1711	125	0.4588	1379	0.0050	180
	CoD-PAT	0.1758	131	0.4343	1484	0.0052	211
	PI-PAT	0.1924	120	0.7438	993	0.0047	167

small hotspots. The neighbouring deterrent effect seems to lower the amount of criminal activity. However, the removal of police agents from small hotspots seems to increase the criminal activity when using the CoD strategy. The temporary removal and replacement of police agents would allow the criminal agents to act while the police agents travel to the hotspots.

In further comparisons with the Jones *et al.* methods for 150 police agents, we notice there is a marked difference between the rankings for the three strategies. For Jones *et al.* runs, the CoD-J strategy always had the best performance. With our implementation

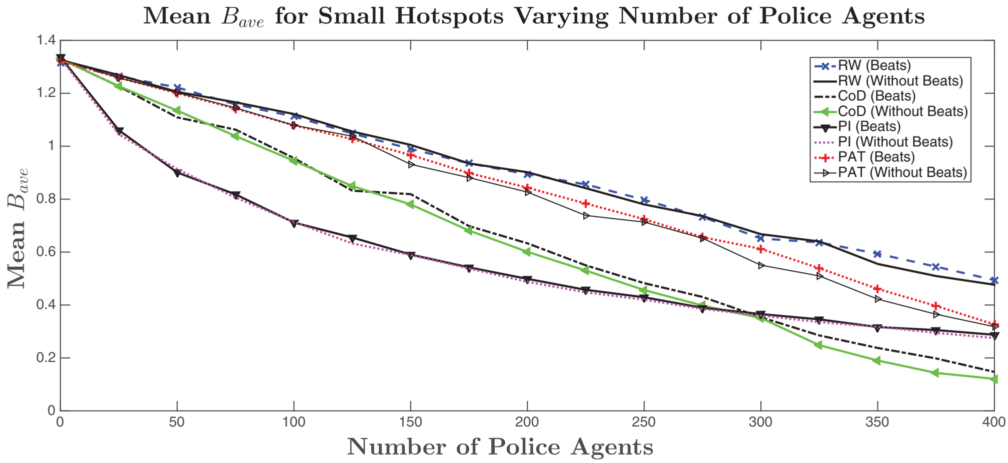

FIGURE 3. Single strategies for small hotspots. We evaluate our single patrolling strategies for small hotspots. This figure gives the mean values for B_{ave} for five runs for varying numbers of police agents. This plot gives the following strategies with beats and without beats: cops on the dots (CoD), peripheral interdiction (PI), random walk (RW), and patterned patrol (PAT). The remaining hotspot cases are presented in the appendix.

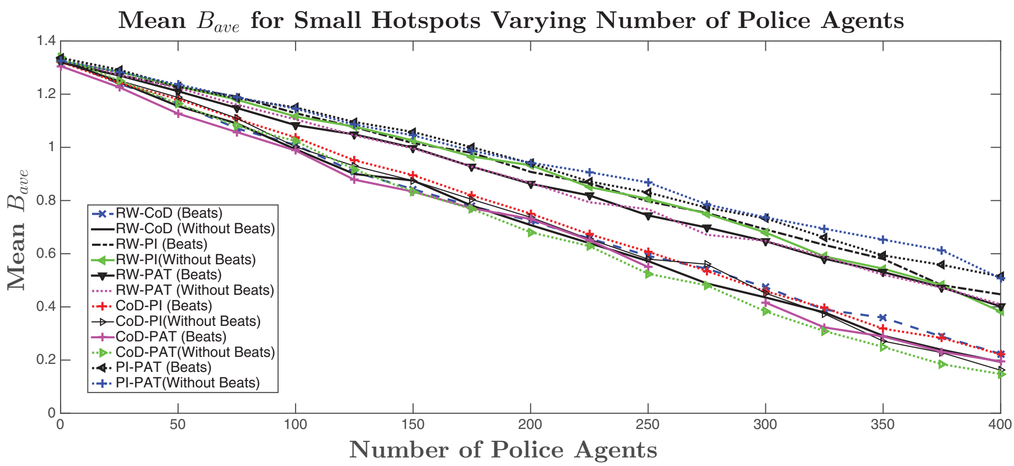

FIGURE 4. Mixed strategies for small hotspots. We evaluate our mixed patrolling strategies for small hotspots. This figure gives the mean values for B_{ave} for five runs for varying numbers of police agents. This plot gives the following mixed strategies with beats and without beats: RW–CoD, RW–PI, RW–PAT, CoD–PI, CoD–PAT, PI–PAT. The remaining hotspot cases are presented in the appendix.

without beats for CoD, RW, and PI, the RW method performed the best when there were no significant hotspots and for large hotspots, and the PI method did the best for the small hotspots. This indicates that for a smaller number of police agents, the CoD strategy is not ideal.

Table 3. *This table provides the mean B_{ave} value and the number of criminals for each method, averaged over five runs, using a total of 150 police agents. C_{ave} is the average number of criminals. The methods were applied to pre-existing hotspots of three different types: no significant hotspots, small hotspots, and large hotspots (shown in Figure 1). The previous methods listed are taken from [15], labelled RW-J, CoD-J, and PI-J, representing the random walk, cops on dots, and peripheral interdiction, respectively. The remaining results were taken from our methods, which includes responding to calls and neighbouring deterrent effects; the methods are random walk (RW), cops on dots (CoD), peripheral interdiction (PI), patterned patrol (PAT), and a hyphen between two methods indicates a mixed method between the two.*

150 police agents

	Method	No significant hotspots		Small hotspots		Large hotspots	
		B_{ave}	C_{ave}	B_{ave}	C_{ave}	B_{ave}	C_{ave}
Previous methods	RW-J	0.4759	166	1.1476	1125	0.4755	162
	CoD-J	0.4169	157	0.6587	1375	0.0158	501
	PI-J	0.4701	133	0.6807	1366	0.1268	465
Without beats	RW	0.3513	149	1.0050	1039	0.0137	446
	CoD	0.3757	165	0.7800	1440	0.0144	448
	PI	0.4350	116	0.5889	3325	0.1165	437
	PAT	0.3245	144	0.9315	1041	0.0102	345
	RW-CoD	0.3613	153	0.8751	1286	0.0142	450
	RW-PI	0.3491	161	1.0259	1037	0.0125	424
	RW-PAT	0.3684	167	0.9960	1051	0.0112	404
	CoD-PI	0.3736	156	0.8745	1206	0.0153	490
	CoD-PAT	0.3806	158	0.8337	1262	0.0124	411
	PI-PAT	0.3745	154	1.0451	1005	0.0124	402
With beats	RW	0.3590	159	0.9900	1084	0.0139	461
	CoD	0.3598	155	0.8189	1421	0.0145	479
	PI	0.4207	107	0.5907	3415	0.1178	437
	PAT	0.3129	144	0.9669	1079	0.0103	352
	RW-CoD	0.3414	153	0.8425	1301	0.0135	448
	RW-PI	0.3485	155	1.0160	1031	0.0141	466
	RW-PAT	0.3597	160	0.9988	1058	0.0115	388
	CoD-PI	0.3718	157	0.8953	1184	0.0140	463
	CoD-PAT	0.3756	159	0.8441	1272	0.0130	433
	PI-PAT	0.3911	152	1.0579	1039	0.0117	393

5.2 Single strategies

The PAT without beats performed the best in our implementation of the four single strategies with no significant hotspots and large hotspots for both 150 and 300 police agents. However, it was not as successful for the small hotspots in both cases.

We now examine the differences between the four single strategies with and without beats. The rankings of these four strategies are identical for each hotspot type for the 150 and 300 police agents. Further, there is no clear distinction of an improvement in the level of criminal activity with or without police beats. This is seen in Figure 3 where the methods are run for various numbers of police agents. The methods with beats perform very similarly to the methods without beats for all numbers of police agents.

The visualization of the four single strategies in the first row of Figure 2 demonstrate how the different methods with beats deter criminal activity. For the RW strategy, it appears that there is no significant reduction in the hotspot activity. The CoD strategy eliminated many of the hotspots. For the PI patrolling, all of the hotspots appear to have been eliminated, but the crime has been spread throughout the region. The PAT strategy appears to do only slightly better than the RW strategy partially eliminating a few hotspots.

5.3 Mixed strategies

For the mixed strategies, we see a similar relationship for the methods with and without beats in both the Table 2 and Figure A 4 in the appendix. When there were no significant hotspots, the best mixed strategies were RW-CoD and RW-PI for 150 police agents, both with and without beats. For the 300 police agents, the best three strategies without beats were CoD-PI, CoD-PAT, and PI-PAT. However, the best two strategies with beats were RW-CoD and RW-PI. Overall, the differences between the B_{ave} values were relatively small, indicating the best method was not overwhelmingly better. For the small hotspots, RW-PI and PI-PAT had the worst results for both 150 and 300 police agents. The mixed strategies with CoD performed the best here. This is quite the opposite from the large hotspots, which had the best results for RW-PAT and PI-PAT. The overall differences between the B_{ave} values were relatively small. In analyzing the mixed strategies from the plots in the second and third rows of Figure 2, we see that the strategies perform similarly to those of the single strategies.

5.4 Overall comparisons

Comparing all the methods, the patterned patrolling performed the best with and without beats when there were no significant hotspots for both 150 and 300 police agents. For small hotspots, the PI performed the best for 150 police agents, with or without beats, but the CoD performed the best for 300 police agents. Thus, having less police agents or different hotspot types would suggest the need for a different patrolling strategy.

5.5 Variation among runs

In order to better understand the variation among runs, we took 150 police agents and ran each method 100 times for only the small hotspot regime. The standard deviations for the B_{ave} of these runs are located in Table 4. For all of the runs, the standard deviations are less than 0.035. In our analysis for the small hotspot regime, the variation among runs is large enough to change the order of two closely performing methods, but not large enough to drastically change the order. For example, for 150 police agents, the mean B_{ave} difference between the top two strategies is 0.2, giving PI a strong lead. Overall, the variation among runs for the small hotspots does not greatly affect the rankings.

As a further comparison, we looked at 100 runs of the RW method with beats for the three different hotspot regimes. When there are no significant hotspots, the B_{ave} standard

Table 4. *This table provides the standard deviations of the B_{ave} value and the number of criminals for each method over 100 runs. The methods were applied to pre-existing small hotspots with 150 police agents. The methods are random walk (RW), cops on dots (CoD), peripheral interdiction (PI), patterned patrol (PAT), and a hyphen between two methods indicates a mixed method between the two. We also include RW-J, CoD-J, and PI-J for comparison.*

150 police agents with small hotspots				
	Without beats		With beats	
Method	B_{ave}	Criminals	B_{ave}	Criminals
RW-J	0.0124	33.818	—	—
CoD-J	0.0253	45.182	—	—
PI-J	0.00530	57.279	—	—
RW	0.02023	42.503	0.01614	36.397
CoD	0.02977	60.072	0.03109	58.412
PI	0.01363	141.325	0.01377	149.182
PAT	0.01575	35.059	0.02075	39.709
RW-CoD	0.0308	66.488	0.03412	73.590
RW-PI	0.01922	34.872	0.01786	32.622
RW-PAT	0.0224	45.905	0.02523	46.531
CoD-PI	0.03062	49.451	0.03328	52.943
CoD-PAT	0.03322	63.609	0.01843	37.124
PI-PAT	0.01723	36.074	0.01762	36.710

deviations for 150 and 300 police agents were 0.0155 and 0.0118, respectively. For the small hotspots, the standard deviations are 0.0161 and 0.0224 for 150 and 300 police agents, respectively. For the large hotspots, the standard deviations for 150 and 300 police agents were 0.000982 and 0.000336, respectively. Given the proximity of the mean B_{ave} values for the large hotspots for both 150 and 300 police agents, the ranking of these methods might change only slightly since the standard deviations are also quite small. Further, the gap in the mean B_{ave} values for the best and worst methods for both 150 and 300 police agents are large in comparison to the standard deviations. Therefore, we do not expect the stochasticity in the model to produce a largely different ranking of the methods.

6 Conclusion

In this study, we extended an existing ABM for police patrolling strategies in order to include more realistic patrolling features [15]. The features that we introduced were police beats, responding to calls, and the neighbouring deterring effect. We tested our methods for varying numbers of police agents to determine the best strategies for agencies with limited resources. We used Jones *et al.* as a baseline to compare our models to see the differences between the new features. The results were mostly consistent, except for simulations with 150 police agents in the small hotspot regime. Here, the PI strategy performed better than CoD, which was the best method from Jones *et al.* simulation.

We also implemented a new strategy for patterned patrols (PAT). Additionally, we tested mixed strategies with half of the police agents moving according to one strategy

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and the other half another strategy. Overall, the methods that performed the best varied depending on the type of hotspots and the number of police agents. The mixed strategies did not perform the best, but were often better than many of their single strategies. Further, mixed strategies often performed well when one of their single strategies run alone performed poorly.

The best policing strategy varied depending on the number of police agents and the type of hotspots. From the simulations, we found that for the case of no significant hotspots, patterned patrols were the best, followed by cops on the dots and mixed cops on the dots strategies. The worst strategy was peripheral interdiction. In the case of small hotspots, peripheral interdiction was the best strategy when there were fewer than 300 police agents, and cops on the dots was the best strategy when there were at least 300 police agents. Mixed strategies with cops on the dots also performed well, but the random walk and the patterned patrols performed poorly, even when mixed with the peripheral interdiction strategy. For the large hotspots, patterned patrolling as a single or mixed strategy performed the best. The peripheral interdiction strategy performed the worst, which is interesting since Jones *et al.* theorized that it would be the best strategy for large hotspots. Overall, when there are small concentrated regions of crime, placing agents on or near those regions have the most impact on the criminal activity. When there are large spread out hotspots or no significant hotspots, a more thorough search of the region (such as patterned patrol) will have the larger impact on the criminal activity.

This model can be improved by varying A_0 in order to account for the baseline attractiveness that varies through space. We could further incorporate transportation hubs to facilitate movement of agents, similar to automobiles, buses, and trains that transport individuals farther distances. Further, we could include other types of agents, such as victims or individuals that have calls needing police assistance. We would like to apply these models to cities to see if we can observe the same levels of criminal activity.

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References

[1] Barclay, P., Buckley, J., Brantingham, P. J., Brantingham, P. L. & Whinn-Yates, T. (1996) Preventing auto theft in suburban Vancouver commuter lots: Effects of a bike patrol. In: R.V. Clark (Ed.), Preventing Mass Transit Crime, 133–161. Monsey, NY: Criminal Justice Press.

[2] Birks, D., Townsley, M. & Stewart, A. (2012) Generative explanations of crime: Using simulation to test criminological theory. *Criminology* **50**(1), 221.

[3] Block, R. L. & Block, C. R. (1995) Space, place and crime: Hot spot areas and hot places of liquor-related crime. *Crime Place* **4**(2), 145–184.

[4] Braga, A. A. (2001) The effects of hot spots policing on crime. *Ann. Am. Acad. Political Soci. Sci.* **578**(1), 104–125.

[5] Brantingham, P. J. & Brantingham, P. L. (1984) *Patterns in Crime*, Macmillan, New York.

[6] Brantingham, P. L. & Brantingham, P. J. (1999) A theoretical model of crime hot spot generation. *Stud. Crime Crime Prevention*, 8(1), 7–26.

[7] Chaturapruek, S., Breslau, J., Yazdi, D., Kolokolnikov, T. & McCalla, S. G. (2013) Crime modeling with Lévy flights. *SIAM J. Appl. Math.* **73**(4), 1703–1720.

[8] Eck, J., Chainey, S., Cameron, J. & Wilson, R. (2005) Mapping crime: Understanding hotspots. National Institute of Justice.

[9] Eck, J. E. & Liu, L. (2008) Contrasting simulated and empirical experiments in crime prevention. *J. Exp. Criminology* **4**(3), 195–213.

[10] Eck, J. E. & Weisburd, D. (1995) Crime places in crime theory. *Crime Place Prevention Stud.* **4**(1), 1–33.

[11] Felson, M. & Clarke, R. V. (1998) Opportunity makes the thief. *Police Research Series*. Paper 98. London: Home Office, 1–36.

[12] Hegemann, R. A., Smith, L. M., Barbaro, A. B. T., Bertozzi, A. L., Reid, S. & Tita, G. E. (2011) Geographical influences of an emerging network of gang rivalries. *Phys. Stat. Mech. Appl.* **390**(21).

[13] Johnson, S. D., Bernasco, W., Bowers, K., Elffers, H., Ratcliffe, J., Rengert, G. & Townsley, M. (2007) Space–time patterns of risk: A cross national assessment of residential burglary victimization. *J. Quant. Criminology*, **23**(3), 201–219.

[14] Johnson, S. D., Bowers, K. & Hirschfield, A. (1997) New insights into the spatial and temporal distribution of repeat victimization. *Br. J. Criminology* **37**(2), 224–241.

[15] Jones, P. A., Brantingham, P. J. & Chayes, L. R. (2010) Statistical models of criminal behavior: The effects of law enforcement actions. *Math. Models Methods Appl. Sci.* **20**(supp01), 1397–1423.

[16] Kearney, M. S., Harris, B. H., Jácome, E. & Parker, L. (2014) *Ten economic facts about crime and incarceration in the United States*. The Hamilton Project, Washington, DC.

[17] Mather, K. & Winton, R. (2015) LAPD uses its helicopters to stop crimes before they start. *Los Angeles Times*, March 7.

[18] Mitchell, P. S. (1972) Optimal selection of police patrol beats. *J. Criminal Law Criminology Police Sci.* **63**(4), 577–584.

[19] Mohler, G. O., Short, M. B., Brantingham, P. J., Schoenberg, F. P. & Tita, G. E. (2011) Self-exciting point process modeling of crime. *J. Am. Stat. Assoc.* **106**(493), 100–108.

[20] Moreto, W. D., Piza, E. L. & Caplan, J. M. (2014) "A plague on both your houses?": Risks, repeats and reconsiderations of urban residential burglary. *JQ: Justice Q.* **31**(6), 1102–1126.

[21] Nam, S. H. (2012) *Optimal Temporal-Spatial Deployment of Urban Law Enforcement Personnel: Theory, Analysis and Implementation*, PhD thesis, Princeton University.

[22] Parks, R. B., Mastrofski, S. D., Dejong, C., & Gray, M. K. (1999) How officers spend their time with the community. *Justice Q.* **16**(3), 483–518.

[23] Pitcher, A. B. (2010) Adding police to a mathematical model of burglary. *Eur. J. Appl. Math.* **21**(4–5), 401–419.

[24] Ratcliffe, J. H. (2004) The hotspot matrix: A framework for the spatio-temporal targeting of crime reduction. *Police Practice Res.* **5**(1), 5–23.

[25] Rodriguez, N. & Bertozzi, A. L. (2010) Local existence and uniqueness of solutions to a PDE model for criminal behavior. *Math. Models Methods Appl. Sci.* **20**(supp01), 1425–1457.

[26] Sherman, L. W. & Weisburd, D. (1995) General deterrent effects of police patrol in crime "hot spots": A randomized, controlled trial. *Justice Q.* **12**(4), 625–648.

[27] Short, M. B., D'Orsogna, M. R., Pasour, V. B., Tita, G. E., Brantingham, P. J., Bertozzi, A. L. & Chayes, L. B. (2008) A statistical model of criminal behavior. *Math. Models Methods Appl. Sci.* **18**(supp01), 1249–1267.

[28] Sutanto, A. (June 2011) *Optimal police patrol*, RWTH Aachen University Center for Computational Engineering Sciences – Undergraduate Research Opportunities Program.

[29] Taylor, B., Koper, C., & Woods, D. (2011) A randomized controlled trial of different policing strategies at hot spots of violent crime. *J. Exp. Criminology* **7**(2), 149–181.

[30] Wilson, J. Q. & Kelling, G. L. (1982) Broken windows. *Atlantic Mon.* **249**(3), 29–38.

[31] Zipkin, J. R., Short, M. B. & Bertozzi, A. L. (2014) Cops on the dots in a mathematical model of urban crime and police response. *Discrete Continuous Dyn. Syst.* **34**(2014), 1479–1506.

Appendix A

This appendix contains the images of our agent-based methods without beats for 300 police agents and small hotspots. We include the $B_s(t)$ plots for the final iteration of the simulation in Figure A 1. This appendix also contains the plots of the different methods as the number of police agents are varied from 0 to 400, located in Figures A 2–A 5. The results here are for the hotspot parameter regimes that give no significant hotspots and large hotspots. Small hotspot results are located in Figures 3 and 4.

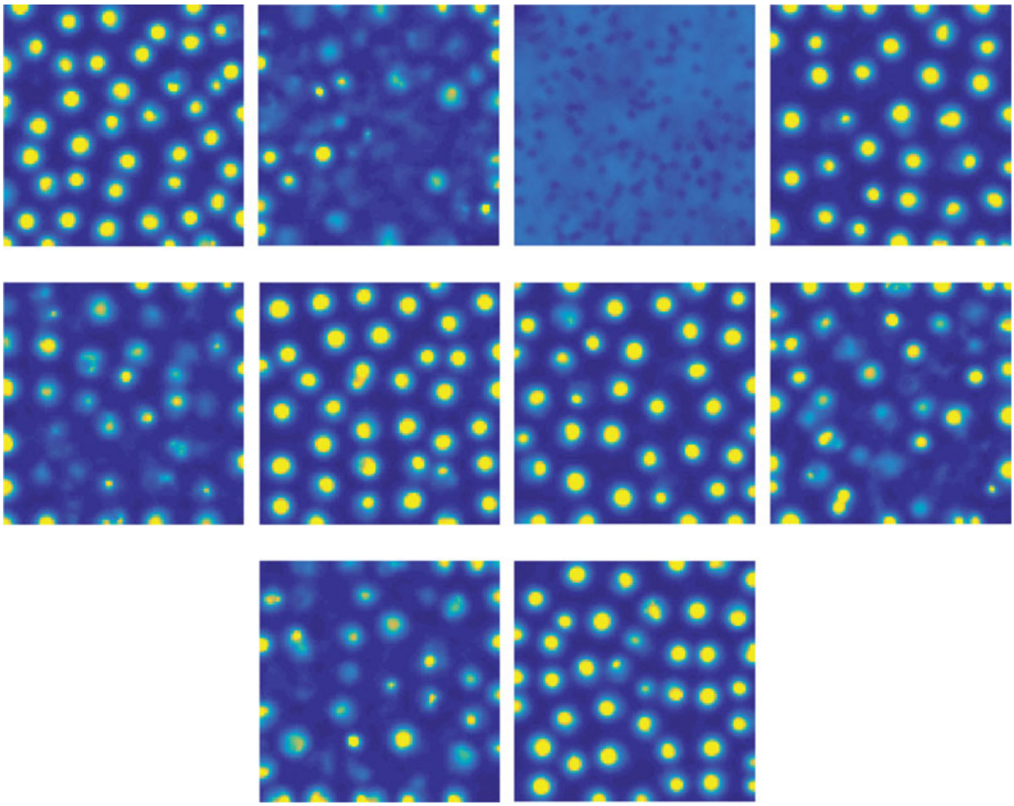

FIGURE A 1. Each sub-plot gives the $B_s(t)$ values of the final iteration, indicating the level of criminal activity. Darker colours (blue) indicate less criminal activity, whereas light colours (yellow) indicate higher amounts of criminal activity. All simulations were run with 300 police agents using pre-existing small hotspots. These simulations do not have police beats. Figures with police beats are included in Figure 2. From left to right, top row: (a) RW, (b) CoD, (c) PI, (d) PAT. Middle row: (e) RW-CoD, (f) RW-PI, (g) RW-PAT, (h) CoD-PI. Bottom row: (i) CoD-PAT, (j) PI-PAT.

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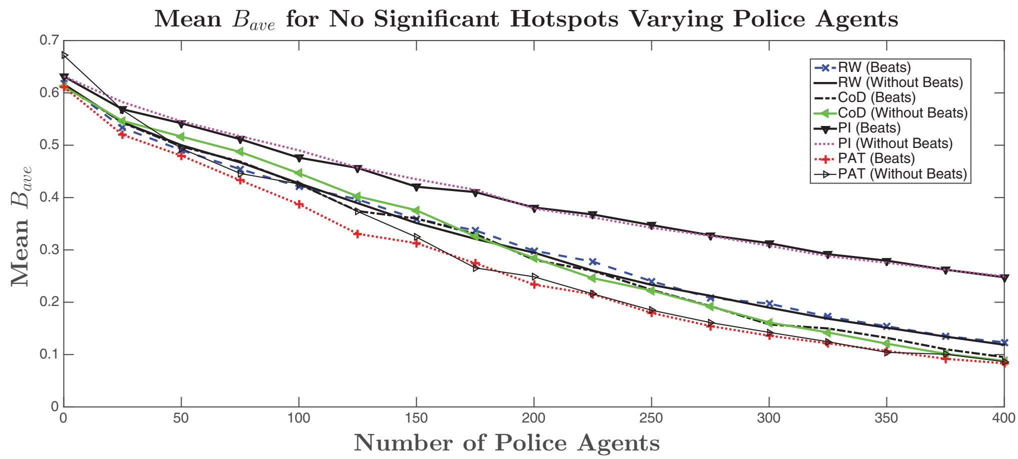

FIGURE A 2. Single strategies for no significant hotspots. We evaluate our single patrolling strategies for no significant hotspots. This figure gives the mean values for B_{ave} for 5 runs for varying numbers of police agents. This plot gives the following strategies with beats and without beats: cops on the dots (CoD), peripheral interdiction (PI), random walk (RW), and patterned patrol (PAT).

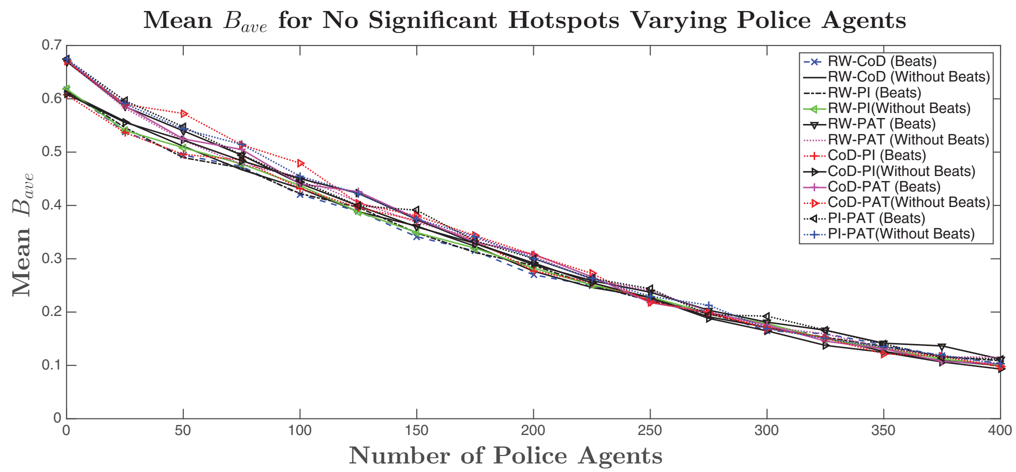

FIGURE A 3. Mixed strategies for no significant hotspots. We evaluate our mixed patrolling strategies for no significant hotspots. This figure gives the mean values for B_{ave} for five runs for varying numbers of police agents. This plot gives the following mixed strategies with beats and without beats: RW–CoD, RW–PI, RW–PAT, CoD–PI, CoD–PAT, PI–PAT.

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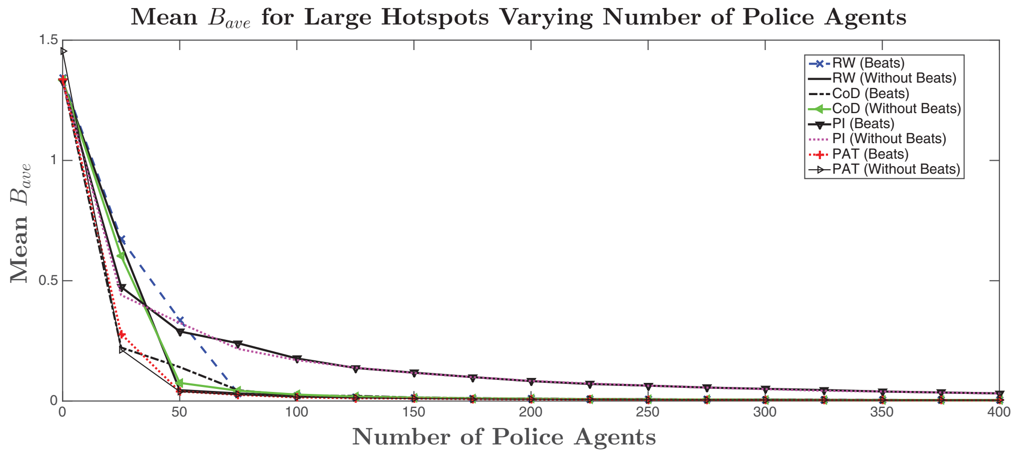

FIGURE A 4. Single strategies for large hotspots. We evaluate our single patrolling strategies for large hotspots. This figure gives the mean values for B_{ave} for five runs for varying numbers of police agents. This plot gives the following strategies with beats and without beats: cops on the dots (CoD), peripheral interdiction (PI), random walk (RW), and patterned patrol (PAT).

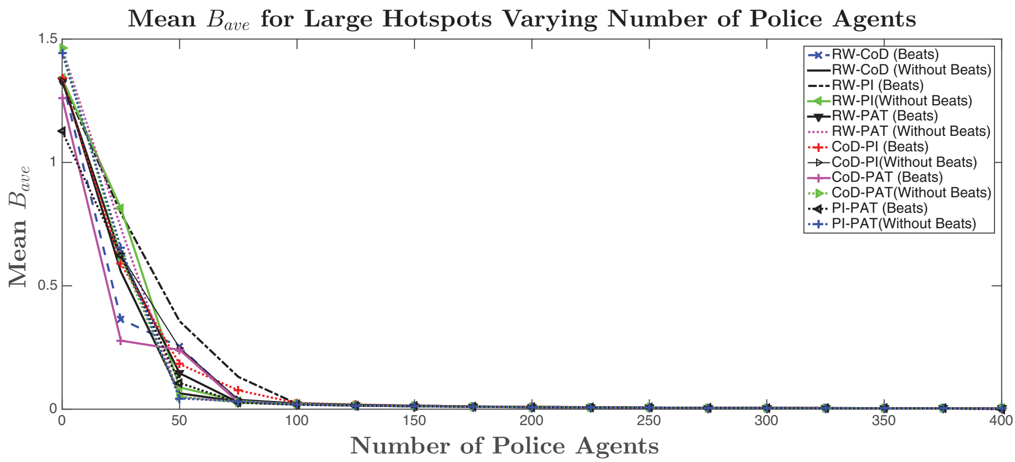

FIGURE A 5. Mixed strategies for large hotspots. We evaluate our mixed patrolling strategies for large hotspots. This figure gives the mean values for B_{ave} for five runs for varying numbers of police agents. This plot gives the following mixed strategies with beats and without beats: RW–CoD, RW–PI, RW–PAT, CoD–PI, CoD–PAT, PI–PAT.