Insular Gravity and Oceanic Isostasy. By Sir JOSEPH LARMOR, Lucasian Professor.

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A century ago geodetic and gravitational universal surveys were mainly concerned with determining the effective (gravitational) ellipticity of the Earth, after due allowance had been made for local anomalies, with especial view to the exact purposes of physical astronomy. One of the chief of these anomalies was exhibited by a remark of Airy, after scrutiny of the available data in his treatise (1830) on "Figure of the Earth" in the *Encyclopedia Metropolitana*, that the observations show gravity to be abnormally in excess on island stations. It appeared, for instance, that this cause might make the mass of the Moon uncertain up to 2 per cent. A very refined explanation of this anomaly of island stations (which will be seen presently to be only partially effective) was offered by Sir George Stokes, from whom this last remark is quoted, in the course of a memoir*, fundamental for theoretical geodesy, in which he demonstrated that no outside survey could lead to any certain knowledge of the distribution of mass inside the Earth, even in its outer crust, except as a matter of probability when backed up by geological knowledge.

It is explained there that the form of the sea-level must be locally depressed over a deep ocean, owing to defect of density; and in consequence on insular stations gravity at sea-level is measured abnormally nearer to the centre of the Earth as a whole, so that from this cause its value is greater than that belonging to the mean spheroidal surface. In fact, the form of the ocean is an equipotential surface, including therein the potential of the centrifugal force of rotation in the familiar manner: but the part of the potential arising from the local water is abnormally small on account of its low density, and this defect must, in absence of local compensation, be made up by a greater potential of the Earth as a whole, which demands depression of the local ocean surface towards the Earth's centre.

The opposite result would arise from excess matter of an adjacent mountain or island peak: that would raise the ocean level in its vicinity and thereby indirectly diminish gravity, measured at sea-level, as determined by levelling operations.

For example, at the centre of a circular oceanic basin of radius

* *Cambridge Transactions* (1849): reprinted in *Math. and Phys. Papers*, vol. II. Some idea of the great debt owed by the Indian and other gravitational surveys to the continuous amateur advice of Sir G. G. Stokes, spread over half a century of their development, may be gleaned from the collection of his *Scientific Correspondence* (Camb. Univ. Press), vol. II, pp. 253–325.

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b and uniform depth h, its defect of potential would be with sufficient accuracy $\int \gamma\rho' h 2\pi r\,dr/r$, where ρ' is the defect of density of the water below that of the average terrestrial crust; thus it is $2\pi\gamma\rho' bh$, where γ is the constant of gravitation given by $\gamma E/a^2 = g$. Here $E = \frac{4}{3}\pi a^3 \rho$, ρ being $\frac{11}{2}$, is the mass of the Earth of radius a. As the potential of the Earth as a whole is $V = \gamma E/r$, this change of local potential, say δV_0, would be compensated by change of sea-level δh, where $\delta V_0/V = -\delta h/r$. Thus in the present case the fall of level relative to depth of ocean is given by the expression

$$-\frac{\delta h}{h} = \frac{a 2\pi\rho' b}{E/a} = \frac{3}{2}\frac{\rho'}{\rho}\frac{b}{a} = \frac{9}{22}\frac{b}{a},$$

while

$$\frac{\delta g}{g} = -2\frac{\delta h}{a}.$$

If the radius b of the oceanic basin is 50 miles this fall would be the fraction $\frac{9}{22}\cdot\frac{50}{4000}$ or $\frac{1}{200}$ of its depth; if the radius were larger it would increase in direct proportion until it is a considerable fraction of the Earth's radius. A cup-shaped ocean could be similarly treated.

The steady sea-level would thus be depressed by $\frac{1}{10}$ of a mile owing to local causes, at the centre of a basin of 500 miles radius and 2 miles deep, in free communication with the other oceanic waters: and this approach to the Earth's centre would involve increase of g measured at ocean level, given by $\delta g/g = -2\delta h/a$, or here $\delta g = \cdot 05$ cm./sec.^{2}, where g is about 981, which is over one-third of the order of magnitude of the observed excesses at island stations.

But this explanation fails because there is a predominant offset. The vertical attraction of the local ocean regarded as an extensive flat slab of water is abnormally small by $2\pi\gamma\rho' h$, where $g = \gamma E/a^2$, that is by $g\rho' 2\pi a^2 h/E$ or $\frac{3}{2}\frac{\rho'}{\rho}\frac{h}{a}g$; thus this direct defect in g may be much the greater, being $\frac{1}{2}a/b$ times the indirect excess. There is however some effect in the other direction due to excess density of the local land, which is usually a substantial correction. This preponderance destroys and even reverses the Stokes explanation of the oceanic anomaly. Indeed closer examination shows that, as based by him*, rather confusedly as it seems, it depends on a potential equation used by Laplace which can, in limited manner,

* *Math. and Phys. Papers*, vol. II, p. 153. Stokes did not make any correction in this reprint in 1883: but Dr Bowie states (*loc. cit. infra*) that there is no generally accepted explanation other than compensating excess of density beneath the ocean.

This analysis of Stokes in fact establishes as a general proposition that the effect of *distant* irregularities of surface mass consists of a direct vertical attraction, say g'', together with an indirect part due to change of level, equal to $-4g''$, thus countervailing four times: this influence, of wide range and presumably actually small, is superposed on the *local* effect here considered.

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apply only to a locally infinitely thin spherical layer. The principle of depressed level became familiar, simple examples being worked out, *ab initio* and so correctly, by way of illustration in Chap. IV of Colonel A. R. Clarke's standard treatise on Geodesy (1880), from the point of view however only of levelling operations, not of gravity.

But soon the discussion of the data of the Indian geodetic survey, by Archdeacon Pratt in India, revealed new features*, by showing strong residual defect of gravity on the Himalayas, such as could only be accounted for by a large defect of density underneath the mountains. Airy's idea that the mountains might be buoyed up by extensive roots floating in a denser magma, existing beneath a *thin* crust, could not of course now be maintained, at any rate in that form, in view of the high rigidity of the Earth as a whole. But there was much to be said, on various counts, for a thinner and deeper viscid stratum, lying between the crustal material and the solid core, in which in the tendency towards equilibrium the pressure due to the weight of the crust must in course of ages have become equalized laterally, at any rate partially, and the load upon it thus made uniform to that degree everywhere. It is implied that there are no local abnormalities of density in the core, which is reasonable as the core is probably metallic. This is the hypothesis of isostasy, propounded as a universal principle by Dutton and worked out systematically by Hayford and his colleagues of the American Survey, who found that it gave a fair account of the usually slighter anomalies (mainly of levelling) revealed in that great undertaking†.

Circumspection is, however, suggested in applying these ideas to the anomalies at oceanic stations; for the Stokes explanation already claimed to be an effective *vera causa*, without aid from compensation of density underneath. It happens that the subject is amenable in a general way to simple elucidation: and as the essential circumstances for submarine mountains and landscapes can perhaps be more directly estimated, it seems indeed to provide in some respects a closer test. On an ideal very narrow island-peak of negligible mass, in a wide ocean of uniform depth, with adjustment as a whole to general isostasy by denser horizontal strata underneath, there would be but slight resultant abnormality of the local part of the attraction. For the totality of the strata could almost be regarded as an extensive thin flat sheet, while local defect of potential on which change of sea-level depends would be still more closely compensated by the extra mass below‡. Hence,

* In 1855–9: cf. A. R. Clarke, *Geodesy*, pp. 96–8.

† Cf. the chapter in H. Jeffreys' recent treatise *The Earth*.

‡ In the case illustrated above, with radius of ocean about 500 miles and depth of compensation 100 miles, about 10 per cent. of the anomaly both of attraction and of potential would remain after compensation of the ocean.

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in contrast to the Stokes uncompensated case above, under isostatic conditions gravity and level ought both to be regular over a wide ocean of nearly uniform depth with strata nearly horizontal underneath.

Thus in considering gravity-data, say over the Pacific Ocean, we may on the isostatic hypothesis consider only this excess density, over that of water, of the local land distributions in an ocean with an ideal conveniently assigned flat bottom: and any gravity anomaly must be due to these excesses alone, together with isostatic compensations of opposite amount underneath to maintain the average total load for the crustal strata. The disturbance of ocean level due to them would be slight if they are merely local peaks. If the station is on a straight ledge even of a narrow island it will be considerable*.

This note, with its limitations as regards scope that are imposed by imperfect information, has been prompted by an important recent Report on "Isostasy in the Southern Pacific" by Dr W. Bowie† in which, from systematic calculation for the five insular stations that were examined, local isostasy is found to account for about three-fourths of the observed local excesses in g, which are there of the order of over 0·1 in 981. Since the Stokes effect of change of ocean level is found inadequate, the greater part of the excess of gravity ought to be due solely to direct attraction of the local excess of land density measured from an averaged flat ocean floor with a cancelling compensation underneath, if oceanic compensation is to hold good. A set of gravity determinations over the wide ocean, combined with soundings, would thus throw interesting light.

For a land survey, as in the Himalayas, heights determined by levelling operations are reckoned from an ideal ocean level which would be affected in the same way as the actual ocean level near island masses, that is only by local excess masses at the surface above a mean compensated distribution when the local compensations extend very deep.

There is a statement near the end of Dr Bowie's Report that, in the cases examined, if the densities of the island "pedestals" were 10 or 15 per cent. (say 12) above the normal value $2\frac{1}{2}$, about one-eighth of the remaining non-isostatic excess of gravity would be accounted for. From this we may perhaps infer that the excess density over that of water, namely $1\frac{1}{2}$, along with a cancelling diminution below on account of the now complete compensation, would account for $\frac{100}{12}\cdot\frac{3}{5}\cdot\frac{1}{8}$, or say $\frac{3}{5}$ of the whole local excess of gravity. This fraction is not discordant with the estimates (around

* Cf. A. R. Clarke, *Geodesy*, p. 93; or Thomson and Tait, *Nat. Phil.* (1867); also *infra* for conical forms—and the remark added at the end.

† *Proc. Washington Academy of Sciences* (Dec. 4, 1925).

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$\frac{3}{4}$) in the Report, and is in so far confirmation. As the Stokes effect of change of ocean level is now itself obliterated by compensation, that over-compensation by about one-third could be accounted for, as Dr Bowie suggests, by an increase of mean excess density of crustal strata which need only be from $1\frac{1}{2}$ to about 2.

In illustration, for a conical island of height h and angle 2α the potential at its vertex is $\pi\gamma\rho' h^2 (\sec\alpha - 1)$ and the component attraction along its axis is $\pi\gamma\rho' h (1 - \cos\alpha)$. For a sector of the cone of angle β with its sharp edge along the axis, these are merely affected by a factor $\beta/2\pi$. The extra gravity at the summit of such a sharp promontory is due mainly to its direct attraction and so differs from that for a flat plate by the factor $\beta/2\pi \,.\, (1 - \cos\alpha)$. Due to this uncompensated plate of ocean by itself of depth h and defect of density ρ' ($= \frac{3}{2}$) the direct defect of gravity is (as *supra*) $\frac{3}{2}\frac{\rho'}{\rho}\frac{h}{a}g$, which is 0·1 in 981 per mile of depth. A conical peak of density ρ' and semi-angle 30° thus would produce an excess half this, diminished however by the compensation below, which is of the order of the fraction $\frac{1 + \cos\alpha}{3}\frac{h}{H}$ of it, so negligible if H, the depth of compensation, is 100 miles. In fact it is only the compensation for sideway excess mass, as it attracts more vertically, that is of sensible direct influence on g. Its sideway attraction deflects the plumb line, and so contributes to the oceanic rise of level, but in a way which must be already included in the potential effect, as *supra*. In a sector of a cylinder also, or for a sloping ocean floor, results may readily be calculated in general illustration. It is, of course, the transverse component of local attraction that affects levelling, while pendulums are affected by its vertical component. Theoretically the two are interconnected through the potential, as above indicated in general terms, in a manner illustrated by precise formulae for the case of a set of parallel mountain chains first (1867) in Thomson and Tait's *Natural Philosophy*, or more readily in simpler cases.

Generally, the conclusion is that the depression of sea-level at island stations is not adequate by itself to counteract the direct diminution in gravity due to the low density of the attracting water, much less to produce the observed excess. But if the oceanic defect of density is deeply compensated underneath, down to a uniform standard depth of ocean, any local anomaly of gravity should be due simply to the excess mass over that of water up to an ideal level surface of the local land standing in this ocean.

An illustration involving the potential is more recently reported*. It appears that astronomical observation and direct

* *Proc. Washington Acad.* (Jan. 16).

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measurement show a difference of as much as 1 mile in the distance between two stations on the north and south coasts of Porto Rico about 35 miles apart. Thus over the island the radius of a level surface is less than the Earth's radius R by 1 part in 35. Thus the expression for the potential locally is

$$V = -g\,(z - x^2/2 \,.\, \tfrac{34}{35}R + pxz + qz^2 + \ldots),$$

where p and q are to be determined from observed local gradient of g, as to which levelling alone does not give information. Discussions of this kind go back to Bouguer for the Andes, and Cavendish and Maskelyne for Schiehallion*.

* Cf. Cavendish, *Scientific Papers*, vol. II, pp. 402–7.

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