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Dynamic modelling of a 3-DOF parallel manipulator using recursive matrix relations

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SUMMARY

In this paper, a simple and convenient method – Recursive Matrix method – is proposed for kinematic and dynamic analysis of all types of complex manipulators. After addressing the principle of the method, an example – a 3-DOF parallel manipulator with prismatic actuators – is demonstrated for the efficiency of the method in solving kinematic and dynamic problems of complex manipulators. With the inverse kinematic solutions, the inverse dynamic problem is solved with the virtual powers method. Matrix relations and graphs of the acting forces and powers for all actuators are analysis and determined. It is shown that the proposed method is an effective mean for kinematic and dynamic modelling of parallel mechanisms.

KEYWORDS: Recursive matrix; Dynamic modelling; Complex manipulators; Parallel mechanisms.

I. INTRODUCTION

Parallel mechanisms generally comprise two platforms which are connected by joints or legs acting in parallel.[1] Over the past decades, parallel mechanisms have received more and more attention from researchers and industries. They can be found in several practical applications, such as aircraft simulators,[2] positional tracker,[3] telescopes[4] and micro-motion device.[5]

More recently, they have been used in the development of high precision machine tools[6–8] by many companies such as Giddings & Lewis, Ingersoll, Hexel, Geodetic and Toyoda, and others. The Hexapod machine tools are one of the successful applications.

Parallel mechanisms have significant advantages over serial mechanisms and in particular possess reduced moving mass and higher stiffness. Thus, parallel mechanisms can work at higher velocities, and yet maintain sufficient rigidity to deliver high levels of accuracy.

The analysis of parallel manipulators is usually implemented through analytical method in classical mechanics,[9–11] in which projection and resolution of vector equations on the reference axes are written in a considerable number of cumbersome, scalar equations and the solution are rendered by large scale computations together with time consuming computer codes.[11] In this paper, a new method – recursive matrix method – is introduced. It has been proved to reduce the number of equations and computation operations significantly by using a set of matrices for kinematics and dynamic modelling.[12,13]

A spatial 3-DOF parallel mechanism [8], which can be used in several applications, including machine tools is proposed in this paper. Existing 3-DOF parallel mechanisms can be classified in terms of the types of actuated joints, and the structures of the supporting frames. In regard to the types of actuated joints, they can be either revolute or prismatic. Since the prismatic joints can easily achieve high accuracy and heavy loads, the majority of the 3DOF parallel mechanisms in reality use actuated prismatic joints. A prismatic joint can have an extensible length or a fixed length. The 3DOF parallel mechanism discussed in this paper belongs to the type with extensible length and a passive leg located in the centre to improve the stiffness. In comparison with an approach for kinematic modelling of robot manipulator, a dynamic modelling approach could be systematized easily. Some systematic approaches have been developed for general-purpose parallel mechanisms analysis.[14–17] Meanwhile, quite a few of special approaches have been conducted for dynamic modelling of specific parallel mechanism configurations;[18–22] Kane and Levinson[23] obtained some vector recursive relations concerning the equilibrium of generalized forces that are applied to a serial manipulator. Sorli et al[24] conducted the dynamic modelling for Turin parallel manipulator, though the mechanism has three identical legs, it has 6-DOF. However, to the best of our knowledge, there is no efficient dynamic modelling approach available for parallel manipulator.

II. INVERSE KINEMATIC MODELLING

Let $Ox_0y_0z_0(T_0)$ be a fixed Cartesian frame. A spatial manipulator with three degrees of freedom is moving with respect to this frame; the mechanism consists of four kinematical chains, including three variable length legs with identical topology and one passive constraining leg, all connecting the fixed base to a moving platform (Fig. 1). In this 3-DOF parallel mechanism, a kinematical chain, associated with one of the three identical active legs, was introduced between the base and the moving platform. It consists of a fixed Hook joint, characterized by the angular

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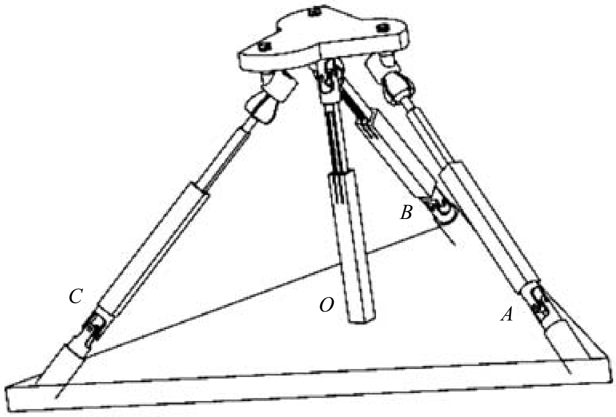

Fig. 1. Parallel manipulator with prismatic actuators.

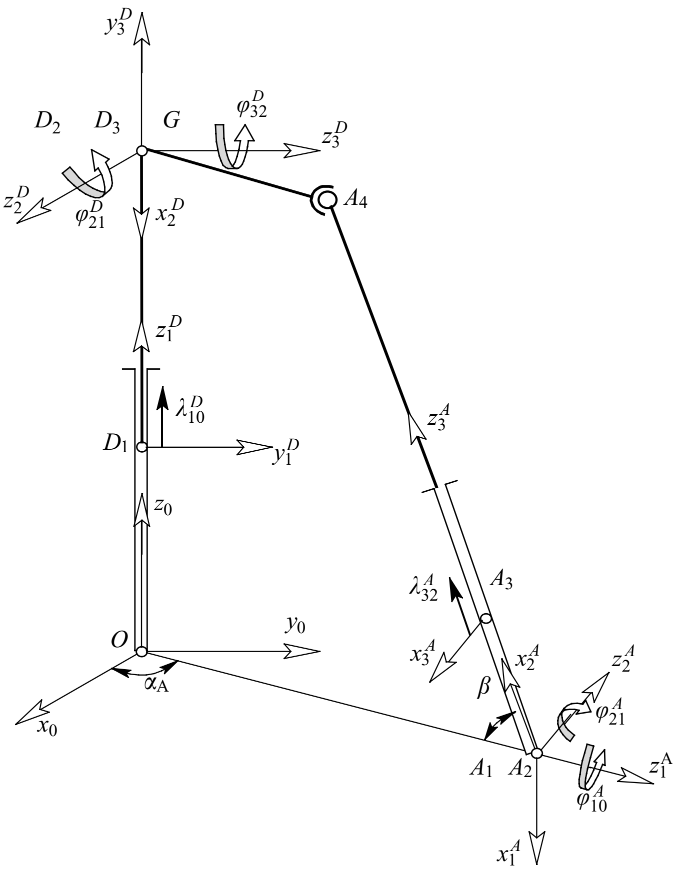

Fig. 2. Kinematical scheme of the manipulator.

velocity $\omega_{10}^A = \dot{\varphi}_{10}^A$ and the angular acceleration $\varepsilon_{10}^A = \ddot{\varphi}_{10}^A$, and a moving link of length l_2, mass m_2 and tensor of inertia $\hat{J}_2$, which has a relative rotation with the angle φ_{21}^A, so that $\omega_{21}^A = \dot{\varphi}_{21}^A$, $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$. An actuated prismatic joint is as well as a moving link to the $A_3 x_3^A y_3^A z_3^A (T_3^A)$ frame, having a relative displacement λ_{32}^A, velocity $v_{32}^A = \dot{\lambda}_{32}^A$ and acceleration $\gamma_{32}^A = \ddot{\lambda}_{32}^A$. It has the length l_3, mass m_3 and tensor of inertia $\hat{J}_3$. Finally, a ball-joint is attached to the platform.

The fourth constraining chain has a different architecture. It consists of a prismatic joint attached to the base and a moving link of length l_1 and mass m_1, having a purely vertical displacement λ_{10}^D, velocity $v_{10}^D = \dot{\lambda}_{10}^D$ and acceleration $\gamma_{10}^D = \ddot{\lambda}_{10}^D$. A Hook joint is attached to the moving platform. The moving platform is an equilateral triangle with edge l, mass m_4 and tensor of inertia $\hat{J}_4$.

Rotations of the moving platform are defined by the angles φ_{21}^D and φ_{32}^D in the local coordinates (Fig. 2). The orientation of the joints A, B, C on the fixed platform (Fig. 1) is given by

$$\alpha_A = 0,\ \alpha_B = \frac{2\pi}{3}\ \alpha_C = -\frac{2\pi}{3}. \tag{1}$$

For convenience, the independent coordinates of the platform have been chosen as $z_0^G, \alpha_1, \alpha_2$, where z_0^G is the height of the platform and α_1, α_2 are the angles of the Hook joint.

Assuming the passive leg D on the $OD_1D_2D_3$ path, the passing matrices are derived:

$$d_{10} = \hat{e},\ d_{21} = d_{21}^{\varphi} a_1,\ d_{32} = d_{32}^{\varphi} a_2. \tag{2}$$

Now, considering the $OA_1A_2A_3A_4$ track of the limb A, the transfer matrices are given by

$$a_{10} = a_{10}^{\varphi} a_1 a_{\alpha}^A,\ a_{21} = a_{21}^{\varphi} a_{\beta} a_2,\ a_{32} = a_1, \tag{3}$$

Where:[25]

$$a_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},\quad a_2 = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$a_{\alpha}^A = \begin{bmatrix} \cos\alpha_A & \sin\alpha_A & 0 \\ -\sin\alpha_A & \cos\alpha_A & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a_{\beta} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$a_{k,k-1}^{\varphi} = \begin{bmatrix} \cos\varphi_{k,k-1}^A & \sin\varphi_{k,k-1}^A & 0 \\ -\sin\varphi_{k,k-1}^A & \cos\varphi_{k,k-1}^A & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$a_{k0} = \prod_{j=1}^{k} a_{k-j+1,\,k-j},\quad (k = 1, 2, 3).$$

The same equations can be written for the other two loops O-B and O-C of the mechanism (Fig. 2).

Suppose the absolute motion of the platform is given by

$$z_0^G = l_1 + l_6 + z_0^{G*}\left(1 - \cos\frac{2\pi}{3}t\right),$$
$$\alpha_i = \alpha_i^*\left(1 - \cos\frac{2\pi}{3}t\right)\quad (i = 1, 2). \tag{5}$$

To solve the inverse kinematic problem, the passive leg can be taken as a serial 3-DOF mechanism with the coordinates determined by the conditions

$$d_{30}^{\circ T} d_{30} = a,\quad \vec{u}_3^T\{\vec{r}_{10}^D + \lambda_{10}^D \vec{u}_3 + d_{10}^T \vec{r}_{21}^D\} = z_0^G, \tag{6}$$

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where $a = a_2 a_1$ is a rotation matrix from the frame $Ox_0y_0z_0$ to $Gx_Gy_Gz_G$, and $\vec{r}_{10}^{D} = l_6\vec{u}_3$, $\vec{r}_{21}^{D} = l_1\vec{u}_3$

$$d_{30}^{\circ T} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \tilde{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$$a_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}, \quad a_2 = \begin{bmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{bmatrix}$$

It results

$$\lambda_{10}^{D} = z_0^G - l_1 - l_6, \quad \varphi_{21}^{D} = \alpha_1, \quad \varphi_{32}^{D} = \alpha_2. \tag{8}$$

Once the solution of the inverse kinematic of this 3-DOF serial chain is found, the complete position and orientation of the limbs A, B, C can be determined by the following geometric conditions

$$\begin{aligned} &\vec{r}_{10}^{A} + \sum_{k=1}^{3} a_{k0}^{T}\vec{r}_{k+1,k}^{A} - d_{30}^{T}\vec{r}_{3}^{A_4} \\ &= \vec{r}_{10}^{B} + \sum_{k=1}^{3} b_{k0}^{T}\vec{r}_{k+1,k}^{B} - d_{30}^{T}\vec{r}_{3}^{B_4} \\ &= \vec{r}_{10}^{C} + \sum_{k=0}^{3} c_{k0}^{T}\vec{r}_{k+1,k}^{C} - d_{30}^{T}\vec{r}_{3}^{C_4} \\ &= z_0^G\vec{u}_3, \end{aligned} \tag{9}$$

where

$$\vec{r}_{10}^{A} = l_0 a_\alpha^{AT}\vec{u}_1, \quad \vec{r}_{21}^{A} = \vec{0},$$
$$\vec{r}_{32}^{A} = l_5\vec{u}_1 + \lambda_{32}^{A} a_{32}^{T}\vec{u}_3, \quad \vec{r}_{43}^{A} = l_3\vec{u}_3.$$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{r}_3^{A_4} = \begin{bmatrix} -l_4\cos\alpha_A \\ 0 \\ l_4\sin\alpha_A \end{bmatrix} \tag{10}$$

III. Velocities and accelerations

The motion of all links is characterized by the following skew symmetric matrices [23]

$$\tilde{\omega}_{k0}^{A} = a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T} + \omega_{k,k-1}^{A}\tilde{u}_3. \tag{11}$$

These matrices are associated with the absolute angular velocities, given by the recurrence relations

$$\vec{\omega}_{k0}^{A} = a_{k,k-1}\vec{\omega}_{k-1,0}^{A} + \omega_{k,k-1}^{A}\vec{u}_3. \tag{12}$$

The velocity $\vec{v}_{k0}^{A}$ of the joint A_k can be obtained as

$$\begin{aligned} &\vec{v}_{k0}^{A} = a_{k,k-1}\{\vec{v}_{k-1,0}^{A} + \tilde{\omega}_{k-1,0}^{A}\vec{r}_{k,k-1}^{A}\} + v_{k,k-1}^{A}\vec{u}_3, \\ &\vec{v}_{\sigma,\sigma-1}^{A} = \vec{0} \quad (\sigma = 1, 2, 4). \end{aligned} \tag{13}$$

The following *matrix conditions of connectivity* constitute the inverse kinematical model

$$\begin{aligned} &\omega_{10}^{A}\vec{u}_i^{T}a_{10}^{T}\tilde{u}_3 a_{21}^{T}\{\vec{r}_{32}^{A} + a_{32}^{T}\vec{r}_{43}^{A}\} + \omega_{21}^{A}\vec{u}_i^{T}a_{20}^{T}\tilde{u}_3\{\vec{r}_{32}^{A} + a_{32}^{T}\vec{r}_{43}^{A}\} \\ &+ v_{32}^{A}\vec{u}_i^{T}a_{30}^{T}\vec{u}_3 = v_{10}^{D}\vec{u}_i^{T}\vec{u}_3 + \vec{u}_i^{T}d_{30}^{T}\tilde{\omega}_{30}^{D}\vec{r}_3^{A_4}. \end{aligned} \tag{14}$$

These relations generate Jacobian matrix of the mechanism, defining the robot workspace. From equation (14), relative velocities $\omega_{10}^{A}, \omega_{21}^{A}, v_{32}^{A}$ result as functions of platform's characteristic velocities

$$v_{10}^{D} = \dot{z}_0^G, \quad \omega_{21}^{D} = \dot{\alpha}_1, \quad \omega_{32}^{D} = \dot{\alpha}_2 \tag{15}$$

Assume that the robot has a virtual motion determined by the angular velocities $v_{32a}^{Av} = 1$, $v_{32a}^{Bv} = 0$, $v_{32a}^{Cv} = 0$. The characteristic virtual velocities, expressed as function of manipulator's position, are given by new connectivity conditions of relative velocities

$$\vec{u}_i^{T}a_{30}^{T}\{\vec{v}_{30a}^{Av} + \tilde{\omega}_{30a}^{Av}\vec{r}_{43}^{A}\} = \vec{u}_i^{T}d_{30}^{T}\{\vec{v}_{30a}^{Dv} + \tilde{\omega}_{30a}^{Dv}\vec{r}_3^{A_4}\}. \tag{16}$$

Other two compatibility relations of the loops O-B and O-C can be obtained, if one considers $v_{32b}^{Bv} = 1$ and $v_{32c}^{Cv} = 1$. Some new connectivity relations of the angular accelerations $\varepsilon_{10}^{A}, \varepsilon_{21}^{A}, \gamma_{32}^{A}$ of the links can be obtained

$$\begin{aligned} &\varepsilon_{10}^{A}\vec{u}_i^{T}a_{10}^{T}\tilde{u}_3 a_{21}^{T}\{\vec{r}_{32}^{A} + a_{32}^{T}\vec{r}_{43}^{A}\} + \varepsilon_{21}^{A}\vec{u}_i^{T}a_{20}^{T}\tilde{u}_3\{\vec{r}_{32}^{A} + a_{32}^{T}\vec{r}_{43}^{A}\} \\ &+ \gamma_{32}^{A}\vec{u}_i^{T}a_{30}^{T}\vec{u}_3 = \gamma_{10}^{D}\vec{u}_i^{T}\vec{u}_3 + \vec{u}_i^{T}d_{30}^{T}\{\tilde{\omega}_{30}^{D}\tilde{\omega}_{30}^{D} + \tilde{\varepsilon}_{30}^{D}\}\vec{r}_3^{A_4} \\ &- \omega_{10}^{A}\omega_{10}^{A}\vec{u}_i^{T}a_{10}^{T}\tilde{u}_3\tilde{u}_3 a_{21}^{T}\{\vec{r}_{32}^{A} + a_{32}^{T}\vec{r}_{43}^{A}\} \\ &- \omega_{21}^{A}\omega_{21}^{A}\vec{u}_i^{T}a_{20}^{T}\tilde{u}_3\tilde{u}_3\{\vec{r}_{32}^{A} + a_{32}^{T}\vec{r}_{43}^{A}\} \\ &- 2\omega_{10}^{A}\omega_{21}^{A}\vec{u}_i^{T}a_{10}^{T}\tilde{u}_3 a_{21}^{T}\tilde{u}_3\{\vec{r}_{21}^{A} + a_{32}^{T}\vec{r}_{43}^{A}\} \\ &- 2\omega_{10}^{A}v_{32}^{A}\vec{u}_i^{T}a_{10}^{T}\tilde{u}_3 a_{21}^{T}a_{32}^{T}\vec{u}_3 - 2\omega_{21}^{A}v_{32}^{A}\vec{u}_i^{T}a_{20}^{T}\tilde{u}_3 a_{32}^{T}\vec{u}_3. \end{aligned} \tag{17}$$

When the other two kinematical chains are pursued, the similar relations can be easily obtained.

The following relations are for the angular accelerations $\vec{\varepsilon}_{k0}^{A}$ and the linear accelerations $\vec{\gamma}_{k0}^{A}$ of the joints

$$\begin{aligned} \vec{\varepsilon}_{k0}^{A} &= a_{k,k-1}\vec{\varepsilon}_{k-1,0}^{A} + \varepsilon_{k,k-1}^{A}\vec{u}_3 \\ &\quad + \omega_{k,k-1}^{A}a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\vec{u}_3\tilde{\omega}_{k0}^{A}\tilde{\omega}_{k0}^{A} + \tilde{\varepsilon}_{k0}^{A} \\ &= a_{k,k-1}\{\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A} + \tilde{\varepsilon}_{k-1,0}^{A}\}a_{k,k-1}^{T} + \omega_{k,k-1}^{A}\omega_{k,k-1}^{A}\tilde{u}_3\tilde{u}_3 \\ &\quad + \varepsilon_{k,k-1}^{A}\tilde{u}_3 + 2\omega_{k,k-1}^{A}a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\tilde{u}_3\vec{\gamma}_{k0}^{A} \\ &= a_{k,k-1}\vec{\gamma}_{k-1,0}^{A} + a_{k,k-1}\{\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A} + \tilde{\varepsilon}_{k-1,0}^{A}\}\vec{r}_{k,k-1}^{A}, \\ &\quad \vec{\gamma}_{\sigma,\sigma-1}^{A} = \vec{0} \quad (\sigma = 1, 2, 4). \end{aligned} \tag{18}$$

The equations (14), (17) are the *inverse kinematical model* of the manipulator.

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IV. Motion simulation

For the inverse dynamic problem, the virtual power method is applied. Graphs and recursive matrix relations for the forces and powers of the three actuators are obtained.

The three actuators forces are

$$\vec{f}_{32}^{A} = f_{32}^{A}\vec{u}_3, \quad \vec{f}_{32}^{B} = f_{32}^{B}\vec{u}_3, \quad \vec{f}_{32}^{C} = f_{32}^{C}\vec{u}_3. \tag{19}$$

The force of inertia and the resultant moment of the forces of inertia of the rigid body T_k are determined in respect to the centre of the joint A_k. On the other hand the characteristic vectors $\vec{f}_k^*$ and $\vec{m}_k^*$ evaluate the influence of the weight action $m_k\vec{g}$ and all other external and internal forces applied to the same T_k manipulator link.

Given the absolute motion of the platform by equation (5), the position, velocity and acceleration of each joint can be determined, and the wrench about A_k can also be determined. Assuming that friction forces at the joints are negligible, we can then calculate the actuating forces.

In the virtual-velocities method, the dynamic equilibrium condition of the mechanism requires that the virtual power of all external, internal and inertia forces, which is developed during a general virtual displacement, to be set null. Applying the *fundamental equations of the parallel robots dynamics*,[13] the following matrix equation is obtained

$$f_{32}^{A} = \vec{u}_3^{T}\{\vec{F}_3^{A} + \omega_{21a}^{Av}\vec{M}_2^{A} + \omega_{21a}^{Bv}\vec{M}_2^{B} + \omega_{21a}^{Cv}\vec{M}_2^{D} + v_{10a}^{Dv}\vec{F}_1^{D} + \omega_{32a}^{Dv}\vec{M}_3^{D}\}, \tag{20}$$

where

$$\begin{aligned}
\vec{F}_{k0}^{A} &= m_k^A\vec{\gamma}_{k0}^{A} + m_k^A\{\tilde{\omega}_{k0}^{A}\tilde{\omega}_{k0}^{A} + \tilde{\varepsilon}_{k0}^{A}\}\vec{r}_k^{CA} + 9.81 m_k^A a_{k0}\vec{u}_3\\
\vec{M}_{k0}^{A} &= m_k^A\tilde{r}_k^{CA}\vec{\gamma}_{k0}^{A} + \hat{J}_k^A\vec{\varepsilon}_{k0}^{A} + \tilde{\omega}_{k0}^{A}\hat{J}_k^A\vec{\omega}_{k0}^{A} + 9.81 m_k^A\tilde{r}_k^{CA}a_{k0}\vec{u}_3\\
\vec{F}_k^{A} &= \vec{F}_{k0}^{A} + a_{k+1,k}^{T}\vec{F}_{k+1}^{A},\\
\vec{M}_k^{A} &= \vec{M}_{k0}^{A} + a_{k+1,k}^{T}\vec{M}_{k+1}^{A} + \tilde{r}_{k+1,k}^{A}a_{k+1,k}^{T}\vec{F}_{k+1}^{A},
\end{aligned}$$
$$(k = 1, 2, 3). \tag{21}$$

The equations (20) and (21) represent the *inverse dynamic model* of the-3-DOF parallel manipulator with prismatic actuators.

An example is given with the following parameters

$$\alpha_1^* = \pi/12, \quad \alpha_2^* = \pi/18, \quad z_0^{G*} = 0.1\,\text{m}$$
$$l = 0.75\,\text{m}, \quad D_1D_2 = l_1 = 0.65\,\text{m},$$
$$l_2 = 1.2\,\text{m}, \quad A_3A_4 = l_3 = 1.25\,\text{m}$$
$$GA_4 = l/\sqrt{3}, \quad A_2A_3 = l_5 = 0.25\,\text{m}, \quad OD_1 = l_6 = 0.1\,\text{m}$$
$$\sin\beta = (l_1 + l_6)/(l_3 + l_5), \quad l_0 = (l_3 + l_5)\cos\beta + l_4$$
$$m_1 = 1\,\text{kg}, \quad m_2 = 2.5\,\text{kg}, \quad m_3 = 1.5\,\text{kg}, \quad m_4 = 5\,\text{kg}.$$

Finally, one obtains the graphs of the forces f_{32}^{A} (Fig. 3), f_{32}^{B} (Fig. 4), f_{32}^{C} (Fig. 5) and powers p_{32}^{A} (Fig. 6), p_{32}^{B} (Fig. 7), p_{32}^{C} (Fig. 8) of the three prismatic actuators.

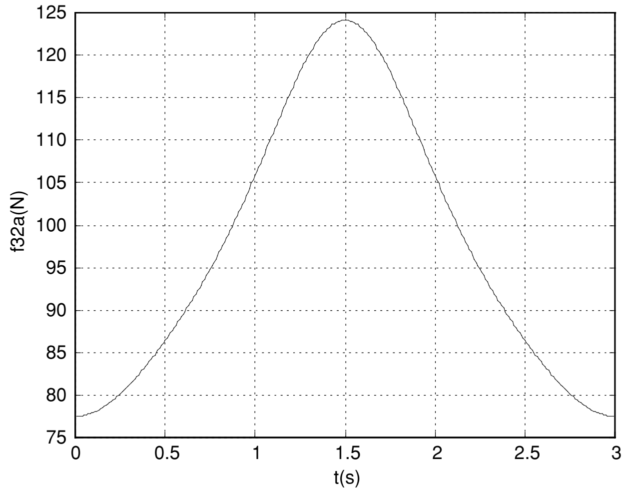

Fig. 3. Force f_{32}^{A} of the first actuator.

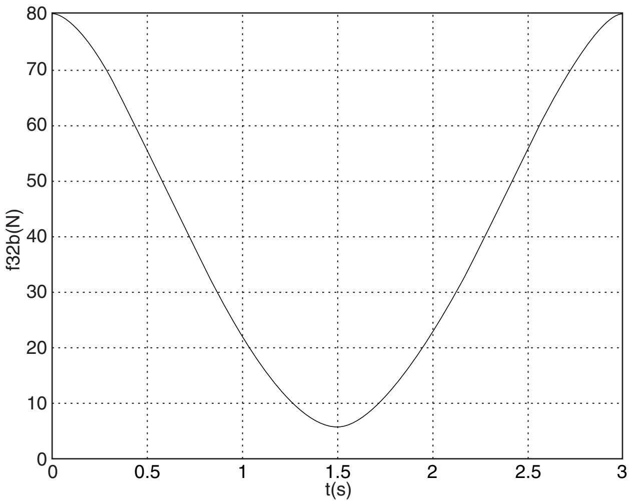

Fig. 4. Force f_{32}^{B} of the second actuator.

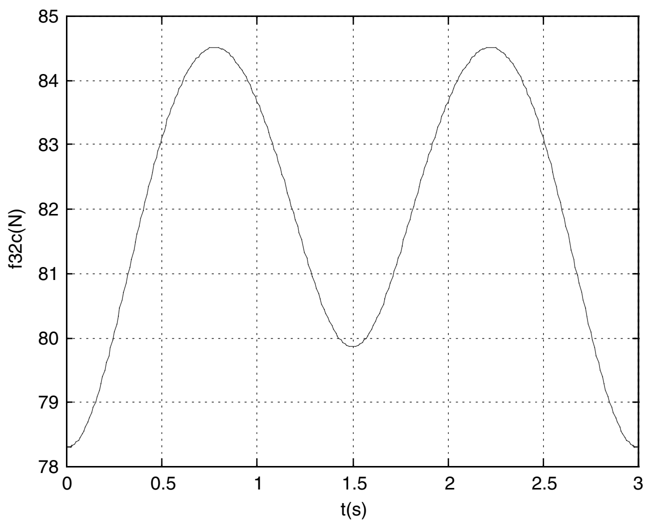

Fig. 5. Force f_{32}^{C} of the third actuator.

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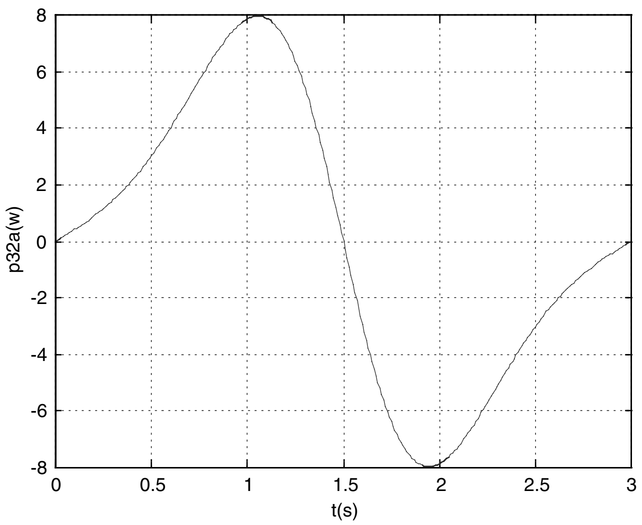

Fig. 6. Power p^A_{32} of the first actuator.

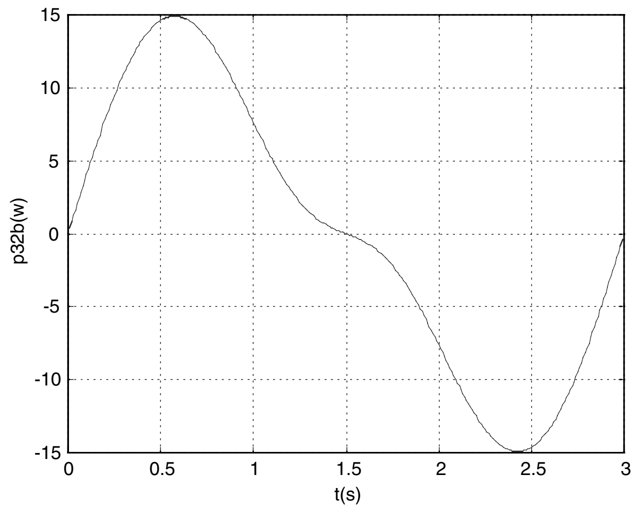

Fig. 7. Power p^B_{32} of the second actuator.

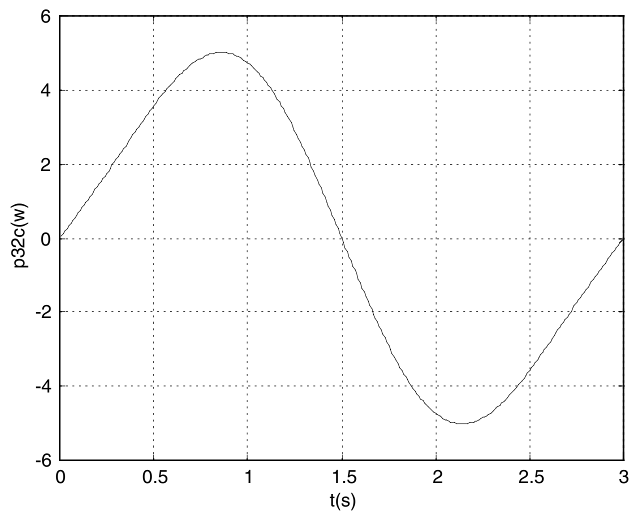

Fig. 8. Power p^C_{32} of the third actuator.

V. CONCLUSIONS

In the paper, the matrix relations for the real time computation of position, velocity and acceleration of each link of a 3-DOF parallel manipulator have been established. The analytical calculus involved in the Lagrange equation is very tedious, thus presenting an elevated risk for errors. Furthermore, the duration of numerical computations is getting longer when the number of components of the mechanism is increased. In this case, the new method showed its advantage over others. With the example of the 3-DOF parallel mechanism, the new method illustrated an efficient way to determine the real time variation of the forces and powers of all the actuators of a parallel manipulator. In a context of automatic control, the iterative matrix relations (20) and (21) given in this dynamic model can be easily transformed into a robust model for the computerised control of the most general parallel manipulators.

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