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Stabilization of a Tractor with *n* Trailers in the Presence of Wheel Slip Effects

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SUMMARY
The purpose of this paper is to design a stabilizing controller for a car with n connected trailers. The proposed control algorithm is constructed on the Lyapunov theory. In this paper, the purpose of navigating the system toward the desired point considering the slip phenomenon as a main source of uncertainty is analyzed. First mathematical models are presented. Then, a stabilizing control approach based on the Lyapunov theory is presented. Subsequently, an uncertainty estimator is taken into account to overcome the wheel slip effects. Obtained results show the convergence properties of the proposed control algorithm against the slip phenomenon.

KEYWORDS: Lyapunov stability; Slip; Nonholonomic constraint; Tractor-trailer; Stabilization.

1. Introduction

The applications of mobile robots and intelligent vehicles are growing in the industry and research today.[1–4] For some purposes in robotics, multi-trailer systems are applicable. They consist of several relatively similar units. Some units may be active while others may be passive. A multi-trailer structure allows a robotic system to accommodate different configurations for moving in the environment and increasing its capacity relative to a single-body structure.

There are examples of multi-trailer vehicles in agricultural and transport applications, but these systems are manned and in the future, these activities will be automated by mobile robots. Multi-trailer systems have significant transportation advantages. The cargo capacity is increased by adding trailers. Also, the ability to change the structure and extend the number of trailers is another positive point.[5] On the other hand, the cost of a multi-trailer vehicle is much less than multiple separate vehicles. Applications include automated trailer systems, carriage and passenger land transport, agricultural activities, transportation of goods to factories, airports, ports, and so on.[6]

Stability is the most important property for the control systems. Each control system, whether linear or nonlinear, will be involved with the stability issue, which should be carefully analyzed. Since physical systems are nonlinear, the Lyapunov linearization method is commonly used to justify the examination of linear control techniques in practical applications. Therefore, the linear stabilizer design guarantees the stability of the nonlinear systems around the equilibrium points. The Lyapunov linearization method is related to the local stability of the nonlinear systems. This approach stems from the notion that nonlinear systems have properties similar to the linearized nonlinear systems around their equilibrium points.[7,8] The most common and useful method for studying the stability of the nonlinear control systems is described by the Lyapunov stability in various forms.[9–11] This theory includes two approaches, that is, linearization and direct methods.[12] In the linearization method, the nonlinear system is linearized around the equilibrium points and using the stability measures of the linear systems, the stability is analyzed around the equilibrium points. In the direct method, the definition of a quasi-energy function is used for the system to examine the nonlinear system stability.[13,14]

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The Lyapunov theory is one of the most important techniques for stability analysis of the nonlinear systems in modern and classical control. An important point in Lyapunov stability is that all of the Lyapunov theorems determine sufficient conditions for analyzing stability. Therefore, investigating a Lyapunov function for a dynamic system may be much difficult in some cases. As stated above, in most of the control problems the Lyapunov theory is utilized to validate the stability based on the direct or linearization approaches for continuous systems. But, there exist fewer investigations for discontinuous systems. The variable gradient method based on Clarke's generalized gradient is also a useful method for constructing Lyapunov functions for discontinuous systems. This method, for some discontinuous systems, leads to the discovery of a Lyapunov function.[9,10]

Possible motion tasks for the tractor-trailer wheeled robots (TTWRs) result in two main control problems, that is, tracking control[15] and point stabilization. Stabilization around the desired configurations with kinematic control inputs as a closed-loop control problem is one of the important problems in Wheeled Mobile Robots (WMRs), which will be discussed in this paper. In the stabilization problem around a given desired configuration, the robot should converge a desired ultimate position from a desired initial situation.[16] It may seem that stabilization is a special case of the trajectory tracking problem, but it is not and requires different control strategies. The methods used for trajectory tracking problems do not solve the stabilization problem around a given desired configuration as a particular case.[17] A driver also feels the difference that parking a car precisely around a fixed steady-state configuration is different from tracking a trajectory. In references,[18,19] the literature review for the regulation of nonholonomic systems based on the kinematic models is presented. Some solutions are also presented for stabilizing the nonholonomic systems in refs. [20–22]. For nonholonomic systems, it is difficult to design a stabilizing feedback law.[23–25] Indeed, the problem is the stabilizing of a controllable multi-input-multi-output dynamical system. Based on the Brockett theorem, nonholonomic systems cannot be stabilized using time-invariant continuous control laws.[26] This is the most important limitation that has attracted much attention.

In recent years, different stabilization algorithms have been investigated for nonholonomic systems. The proposed approaches can be classified into time-varying algorithms,[27] discontinuous controllers,[28] and hybrid approaches.[29] In this paper, a discontinuous controller is proposed for stabilizing the last trailer of a multi-trailer system around a given desired configuration. Nonholonomic constraints are due to non-slip limitations assumed ideally for robot wheels; some references proposed tracking controllers in presence of wheel slips or self-collision such as refs. 30–32.

In this paper, the properties of the Lyapunov functions for the stabilization problem of WMRs are examined. First, some stability concepts of the nonlinear systems are described using the Lyapunov stability criterion. Next, the variable gradient method based on Clarke's generalized gradient is utilized to investigate a discontinuous Lyapunov-based controller for the stabilization problem. Also, a slip estimation method is presented to overcome the wheel slip phenomenon. Obtained results show the efficiency of the proposed algorithm in the stabilization of a multi-trailer system.

2. Problem Description

TTWR consists of a driving robot (called tractor) and one or more driven parts (called trailer) which are hinged behind the tractor. In this section, as shown in Fig. 1 the generalized coordinates for the TTWR are $q = (x_n, y_n, \theta_0, \theta_1, \ldots, \theta_n)^T$, the system velocities are (u_i, ω_i), $(i = 0, 1, \ldots, n)$, and $(P_1, P_2, \ldots P_n)$ are the inactive revolute joints and n is the number of trailers.

L_i is the spacing among the i-th trailer wheel axle and the connection joint with the (i-1)th unit, and L_{hi} is the spacing among the i-th trailer wheel axle and the connection joint with the (i+1)th unit. This type of connection causes the algebraic relationship between the control inputs, which can be accessed easily through the inputs. Here, the main purpose is controlling the last trailer with the generalized coordinates $q_n = (x_n, y_n, \theta_n)^T$.

3. Mathematical Model

In system modeling, it is assumed that the TTWR is made up of rigid bodies, the TTWR moves in a planar surface, and wheels are assumed as solid disks having a point contact with motion surface. The kinematic model of an n trailer WMR is as:

$$\dot{q}_i = G(q_i)v_i \qquad (i = 1 \rightarrow n) \tag{1}$$

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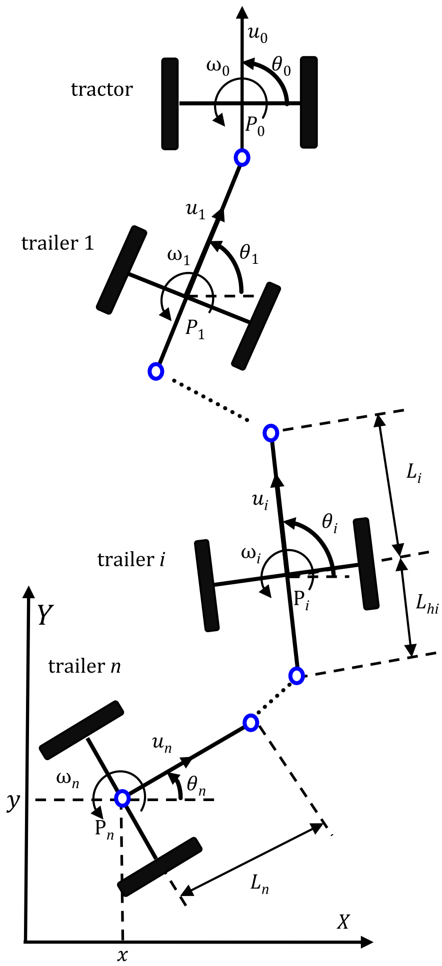

Fig. 1. The NTWMR and its configuration parameters.

where $v_i = \left[\,\omega_i \;\, u_i\,\right]^T$ and

$$G(q_i) = \begin{pmatrix} 1 & 0 \\ 0 & cos\theta_i \\ 0 & sin\theta_i \end{pmatrix} \tag{2}$$

The kinematic model for the n-th trailer robot assuming the slip disturbances parallel with the control inputs is as:

$$\dot{q}_n = G(q_n)v_n + G(q_n)\,\delta_n \tag{3}$$

where $\delta_n = \left[\,\delta_{n1} \;\, \delta_{n2}\,\right]^T$ express the slip effects parallel with the linear and angular velocity of the last trailer. A general formula can be used for transforming the kinematic inputs between the robot units as:

$$v_i = J_i v_{i-1} \tag{4}$$

where J_i is the conversion matrix as

$$J_i = \begin{bmatrix} -\dfrac{L_{hi}}{L_i}cos(\theta_{i-1} - \theta_i) & \dfrac{1}{L_i}sin(\theta_{i-1} - \theta_i) \\ L_{hi}sin(\theta_{i-1} - \theta_i) & cos(\theta_{i-1} - \theta_i) \end{bmatrix} \tag{5}$$

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The inverse transformation is also defined as

$$J_i^{-1} = \begin{bmatrix} -\frac{L_i}{L_{hi}} cos(\theta_{i-1} - \theta_i) & \frac{1}{L_{hi}} sin(\theta_{i-1} - \theta_i) \\ L_i sin(\theta_{i-1} - \theta_i) & cos(\theta_{i-1} - \theta_i) \end{bmatrix} \tag{6}$$

The determinant of J_i can be obtained as $det(J_i) = -\frac{L_{hi}}{L_i}$, where $L_i \neq 0$ is necessary for reversibility. Also the inverse of Eq. (4) can be obtained as

$$v_{i-1} = J_i^{-1} v_i \tag{7}$$

Equation (4) is a recursive equation and can be simplified as

$$v_i = \prod_{j=i}^{1} J_i v_0, \quad i = 1, \ldots, n \tag{8}$$

The inverse equation can be written as

$$v_{i-1} = \prod_{j=i}^{N} J_i^{-1} v_n, \quad i = 1, \ldots, n \tag{9}$$

4. Mathematical Background

Theorem 1 (Ref. 33)**.** *Consider the autonomous system $\dot{x} = f(x)$ in which f is continuous, and suppose $\boldsymbol{V}(x)$ is a continuous differentiable scalar function. Suppose:*

(a) *For some $l >\ 0$, the region $\boldsymbol{\Omega}_l$ defined by the condition $\boldsymbol{V}(x) < l$ is bounded.*
(b) *For all x inside $\boldsymbol{\Omega}_l$, $\dot{\boldsymbol{V}}(x) \leq 0$*

Suppose $\mathbb{R}$ is all points inside $\boldsymbol{\Omega}_l$ in which $\dot{\boldsymbol{V}}(x) = 0$ and $\boldsymbol{M}$ is the largest invariant set in $\mathfrak{R}$. Then every response $x(t)$ that starts at $\boldsymbol{\Omega}_l$ as $t \to \infty$ *tends to* $\boldsymbol{M}$.

Remark. It is not necessary to have a positive definite Lyapunov function.

Theorem 2 (Ref. 11)**.** *It is assumed that $\boldsymbol{\Omega}$ as a compact set, therefore the Fillipov solution of the system $\dot{x} = f(x)$, starting from $x(0) = x(t_0)$ in $\boldsymbol{\Omega}$ remains in $\boldsymbol{\Omega}$, $\forall\ t\ \geq 0$. Let $V : \boldsymbol{\Omega} \to \mathbb{R}$ be a regular time-independent function of time with $v\ \leq 0$, $\forall v\ \in \tilde{\dot{\boldsymbol{V}}}$, (if $\tilde{\dot{\boldsymbol{V}}} = 0$ has a series of empty and continuous solutions). Defining $\boldsymbol{S}\ =\ \{x \in \boldsymbol{\Omega} | 0 \in \tilde{\dot{V}}\}$. Then every trajectory in $\boldsymbol{\Omega}$ converges to the largest invariant set in $\boldsymbol{S}$.*

5. Discontinuous Control

Consider the following differential equation:

$$\dot{x} = f(x) \tag{10}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is a possibly discontinuous and bounded column vector function of n-dimensional state variables x. Special values of the state vector are called a "point" because it represents a point in the system state space. Also, n is system order. A solution for $x(t)$ of Eq. (1) shows a curve in the system state space in which the time t is from zero to infinity, which is called the state trajectory. $x(t)$ is a Filipov solution. It should be noted that Eq. (1) represents a closed-loop control system and there is not the effect of the control signal. The open-loop equation is as $\dot{x} = f(x, u)$, where the control input has the form $u = g(x)$ and therefore $\dot{x} = f(x, g(x)) = f(x)$.

$x(t)$ is absolutely continuous and $\dot{x} \in K[f](x)$, where $K[f](x)$ is the Filipov set defined as $K[\ f](x) = co\,\{\lim f(x_i)\ | x_i \to x, x_i \notin H\}$, where H indicates the set of points where f is discontinuous and co denotes the convex closure.

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Let $V:\mathbb{R}^n \to \mathbb{R}$ be a Lipschitz and regular function which can be assumed as an attractive or repulsive potential field. Then, $V(x(t))$ is absolutely continuous, $\frac{d}{dt}V(x(t))$ exists almost everywhere, and $\frac{d}{dt}V\ (x(t)) \in^{a.e.} \dot{\tilde{V}}(x)$, where the set-valued map $\dot{\tilde{V}}(x)$ is the generalized time derivative $\dot{\tilde{V}}(x) = \cap_{\xi\in\partial V(x(t))}\xi^T K[\ f](x(t))$ and ∂V is Clarke's generalized gradient.

In other words for a dynamical system containing discontinuities, the time derivative of the Lyapunov function is not a single-valued function in a set obtained by the connection of all values from dot multiplication of the gradient of the Lyapunov function with a vector enclosing the Filippov set as a portion of its ingredients.

The generalized time derivative for the TTWR can be obtained as

$$\dot{\tilde{V}}(x) = \begin{bmatrix} \frac{\partial V}{\partial x_n} & \frac{\partial V}{\partial y_n} \end{bmatrix} K \begin{bmatrix} u_n cos\theta_n \\ u_n sin\theta_n \end{bmatrix} = K\ [u_n]\left(\frac{\partial V}{\partial x_n}cos\theta_n\ + \frac{\partial V}{\partial y_n}sin\theta_n\right) \tag{11}$$

Assuming the first control input as

$$u_n = -\alpha_u \left\{\frac{\partial V}{\partial x_n}cos\theta_n\ + \frac{\partial V}{\partial y_n}sin\theta_n\right\} \tag{12}$$

where α_u is a positive controller gain, results in the following negative semi-definite function:

$$\dot{\tilde{V}}(x) = -\alpha_u\left(\frac{\partial V}{\partial x_n}cos\theta_n\ + \frac{\partial V}{\partial y_n}sin\theta_n\right)^2 \tag{13}$$

It can be concluded that the robot tends to the biggest invariant subset of the set $\boldsymbol{S} \triangleq \left\{\frac{\partial V}{\partial x_n} = \frac{\partial V}{\partial y_n} = 0 \vee \frac{\partial V}{\partial x_n}cos\theta_n\ + \frac{\partial V}{\partial y_n}sin\theta_n = 0\right\}$. However, whenever $\frac{\partial V}{\partial x_n}cos\theta_n\ + \frac{\partial V}{\partial y_n}sin\theta_n = 0$, therefore, $|arctan2(\frac{\partial V}{\partial y_n}, \frac{\partial V}{\partial x_n}) - \theta_n| = \frac{\pi}{2}$, or $|\omega_n| = \frac{\pi}{2}$ which means that $\theta_n \neq 0$. Therefore, the choice of $\omega_n = \alpha_\omega\left\{tan2(\frac{\partial V}{\partial y_n}, \frac{\partial V}{\partial x_n}\ ) - \theta_n\right\}$ renders the surface $\frac{\partial V}{\partial x_n}cos\theta_n\ + \frac{\partial V}{\partial y_n}sin\theta_n = 0$ noninvariant whenever the robot is not located at $\frac{\partial V}{\partial x_n} = \frac{\partial V}{\partial y_n} = 0$. Therefore, the biggest invariant set in S is $S_0 \triangleq \left\{\frac{\partial V}{\partial x_n} = \frac{\partial V}{\partial y_n} = 0\right\}$. An attractive positive semi-definite Lyapunov potential function for V can be assumed as $V(x_n, y_n) = \exp\left(\frac{-\ sgn\ (x_n)x_n}{x_n^2+y_n^2}\right)$,[34] which has the minimum value at the origin, where the sign(.) function is defined as

$$sgn(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{14}$$

Based on the defined function V, the system configuration space can be partitioned into two connected sets $M^1 = \{.(x_n, y_n, \theta_n)|\ x_n \geq 0\}$ and $M^2 = \{.(x_n, y_n, \theta_n)|\ x_n < 0\}$ which are divided with the surface $S = \{.(x_n, y_n, \theta_n)|\ x_n = 0\}$. Therefore, negative semi-definite function

$$\dot{\tilde{V}}(x) = \frac{-sgn(x_n)\alpha_u exp\left(\frac{-\ sgn\ (x_n)x_n}{x_n^2+y_n^2}\right)}{\left(x_n^2+y_n^2\right)^2}\left[\left(x_n^2 - y_n^2\right)cos\theta_n\ + 2x_n y_n sin\theta_n\ \right]^2 \leq 0 \tag{15}$$

To reach the asymptotic stability from $\dot{\tilde{V}} = 0$, it can be concluded that $S \triangleq \{.\ (x_n, y_n, \theta_n)|\ x_n((y_n^2 - x_n^2)cos\theta_n\ - 2x_n y_n sin\theta_n\ ) = 0\}$. In any point where $(y_n^2 - x_n^2)cos\theta_n\ - 2x_n y_n sin\theta_n = 0$, one has $|arctan2(2x_n y_n, x_n^2 - y_n^2) - \theta_n| = \frac{\pi}{2}$, which means that $\theta_n \neq 0$ and only $x_n = y_n = 0$; therefore, from $\omega_n = \alpha_\omega\ \{arctan2(2x_n y_n,\ x_n^2 - y_n^2) - \theta_n\}$ it can be concluded that $\omega_n \neq 0$. Also $\dot{\tilde{V}} = 0$ is satisfied for $(x, y) = (0, 0)$ and for all θ_n. For any invariant set $(0\ ,\ 0\ , \theta_n)$ from $\omega_n = 0$ one obtains $\theta_n = 0$

Therefore, the stabilizing control input is proposed as follows for the last trailer:

$$\begin{aligned} u_n &= \alpha_u\ sgn(x)\left[\left(y_n^2 - x_n^2\right)cos\theta_n\ - 2x_n y_n sin\theta_n\ \right] \\ \omega_n &= \alpha_\omega\left\{arctan2\ \left(2x_n y_n, x_n^2 - y_n^2\right) - \theta_n\right\} \end{aligned} \tag{16}$$

where α_u and α_ω are positive controller gains.

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6. Slip Estimation

To estimate the slips as the main source of uncertainties in WMRs, the system state variables $q = (x \;\; y \;\; \theta_n \;\; \theta_{n-1} \ldots \;\; \theta_0)^T \in \mathbb{R}^{n+3}$ are measured with specific sampling time, using sensors. The performance of the system is compared with the desired states and accordingly, the decision is made to adapt the system. The correction is applied to the system to overcome the uncertainties and reach the desired states. Therefore, several stages are required for identifying, deciding, and correcting a perturbed system. Achieving performance specifications in control systems is required when the dynamic characteristics of the controlled process are largely uncertain and these characteristics change over and over again during system operation. The idea is to design a controller that can be adapted to the variations of the process dynamics and disturbance characteristics. The first step is to identify unknown phenomena, such as slips imposed on the system. The next step is the decision making and the correction of the uncertainties of the system or using the input signal. The designed method uses an adaptation law to eliminate the error and converge the trajectories to the reference values. The simplest method for estimating the slip values is using the data from the previous time step. In this regard, using the system kinematic model, the slip vector $\hat{\delta} = (\hat{\delta}_1 \; \hat{\delta}_2)^T$ can be estimated. In other words, the estimate of the slip vector can be obtained using the data from the (k-1)th time step as

$$\hat{\delta} = G^{\#}(q_n(k))\{\dot{q}_n(k) - G(q_n(k-1))\, u_n(k-1)\} \tag{17}$$

where $G^{\#}$ is the pseudo-inverse of the matrix G which is defined as $G^{\#} = \{G^T(q_n(k))G(q_n(k))\}^{-1}G^T(q_n(k))$.

For obtaining the slip vector estimate, using Eq. (1) it can be concluded that

$$\dot{q}_n(k) - G(q_n(k-1))\, u_n(k-1) = \begin{pmatrix} \dot{x}_n(k) - u_n(k-1)cos\theta_n(k-1) \\ \dot{y}_n(k) - u_n(k-1)cos\theta_n(k-1) \\ \dot{\omega}_n(k) - \omega_n(k-1) \end{pmatrix} \tag{18}$$

Then Eq. (3) for the last trailer yields

$$\hat{\delta} = G^{\#}\{\dot{q}_n(k) - G(q_n(k-1))\, u_n(k-1)\} \tag{19}$$

Therefore, estimates of the system slip vector can be used to eliminate the slip effects in the feedback control algorithm.

7. Obtained Results

It is assumed that $n = 3$ and the aim is to obtain the tractor's linear and angular velocities as control inputs to stabilize the last trailer. The relation among the kinematic inputs of the 3rd trailer and the tractor can be obtained using the equations presented for the off-axle linking among the robot units. The kinematic inputs vector for the last trailer is as

$$\upsilon_3 = \begin{bmatrix} \omega_3 \\ u_3 \end{bmatrix} \tag{20}$$

Based on the designed control method (16), one has

$$\upsilon_3 = \begin{bmatrix} \omega_3 \\ u_3 \end{bmatrix} = \begin{bmatrix} \alpha_\omega \left\{arctan2\left(2x_3y_3,\, x_3^2 - y_3^2\right) - \theta_3\right\} - \hat{\delta}_{31} \\ \alpha_u\, sgn(x) \left[\left(y_3^2 - x_3^2\right) cos\theta_3 - 2x_3y_3 sin\theta_3\right] - \hat{\delta}_{32} \end{bmatrix} \tag{21}$$

From Eq. (7), one can write the following equation to obtain υ_2:

$$\begin{bmatrix} \omega_2 \\ u_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{L_3}{L_{h3}} cos\,(\theta_2 - \theta_3) & \dfrac{1}{L_{h3}} sin\,(\theta_2 - \theta_3) \\ L_3 sin\,(\theta_2 - \theta_3) & cos\,(\theta_2 - \theta_3) \end{bmatrix} \begin{bmatrix} \omega_3 \\ u_3 \end{bmatrix} \tag{22}$$

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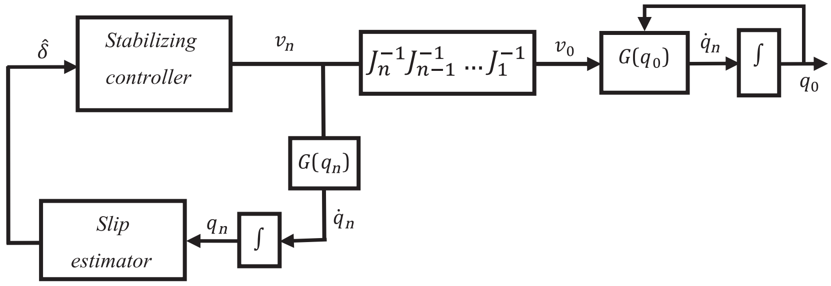

Fig. 2. Proposed algorithm block diagram.

Similarly, v_1 can be obtained as

$$\begin{bmatrix} \omega_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} -\frac{L_2}{L_{h2}} cos\,(\theta_1 - \theta_2) & \frac{1}{L_{h2}} sin\,(\theta_1 - \theta_2) \\ L_2 sin\,(\theta_1 - \theta_2) & cos\,(\theta_1 - \theta_2) \end{bmatrix} \begin{bmatrix} \omega_2 \\ u_2 \end{bmatrix} \tag{23}$$

Also to achieve v_0 one has

$$\begin{bmatrix} \omega_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} -\frac{L_1}{L_{h1}} cos\,(\theta_0 - \theta_1) & \frac{1}{L_{h1}} sin\,(\theta_0 - \theta_1) \\ L_1 sin\,(\theta_0 - \theta_1) & cos\,(\theta_0 - \theta_1) \end{bmatrix} \begin{bmatrix} \omega_1 \\ u_1 \end{bmatrix} \tag{24}$$

In summary, from Eqs. (22)–(24) the following equation can be obtained:

$$v_3 = J_1\ J_2\ J_3\ v_0 \tag{25}$$

In the simulations, the controller gains are chosen as $k_u = 10$, $k_\omega = 3$, $k_z = 10$, and the system geometrical parameters are $L_h = L_{h1} = L_{h2} = L_{h3} = 0.17$, $L = L_1 = L_2 = L_3 = 0.05$.

To have the performance and reasonable control inputs, the control gains have been chosen using the trial and error technique and the simultaneous checking of the closed-loop system performance and the control inputs. Because of the stability of the closed-loop system (according to Eqs. (13) and (15) and related clarifications), from different initial configurations in a limited time, the robot tracking errors converge to zero and the transient responses are eliminated, and the system tracks the desired trajectory appropriately.

The slip effects are introduced as

$$\delta_{31} = \begin{cases} 1 & 0.05 \le t \le 0.2 \\ 0 & cte \end{cases}$$
$$\delta_{32} = \begin{cases} 1 & 0.05 \le t \le 0.2 \\ 0 & cte \end{cases} \tag{26}$$

In Fig. 2, the overall closed-loop control algorithm is shown.

Stabilization of a 3-trailer TTWR from different initial conditions is depicted in Fig. 3. Time-history of stabilization errors in x and y directions is shown in Figs. 4 and 5. Kinematic control inputs are demonstrated in Fig. 6. The Lyapunov function candidate as a function of the time is also depicted in Fig. 7. Stabilization of a 3-trailer TTWR starting from different initial conditions and different initial orientations is depicted in Fig. 8. Stabilization of a 3-trailer TTWR starting from different initial conditions in the existence of wheel slippages using the slip estimator and without using the slip estimator is demonstrated in Figs. 9 and 10.

As can be seen, the designed algorithm can effectively navigate the TTWR around the origin starting from different initial configurations. In the existence of slip effects, although the designed method can stabilize the system around the origin, using the slip estimator slip effects can be compensated efficiently. Time-history of the Lyapunov function candidate from different initial conditions represents the stability of the closed-loop algorithm. As can be seen from the time-history of the tracking

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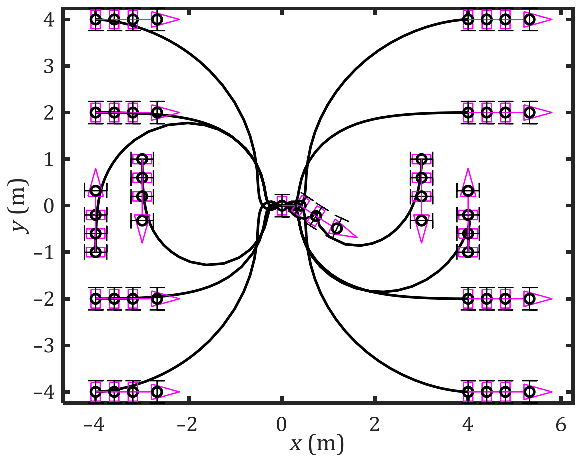

Fig. 3. Stabilization of a 3-trailer TTWR from different initial conditions.

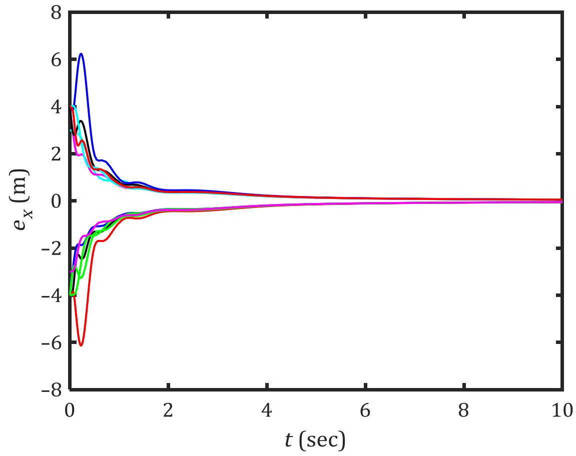

Fig. 4. Time-history of stabilization errors in the x direction for a 3-trailer TTWR from different starting configurations.

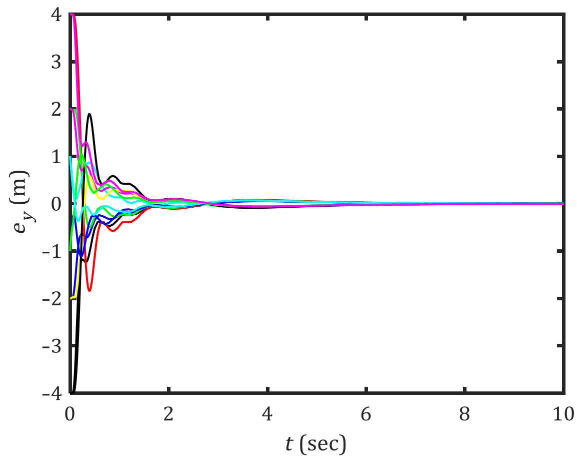

Fig. 5. Time-history of stabilization errors in the y direction for a 3-trailer TTWR from different starting configurations.

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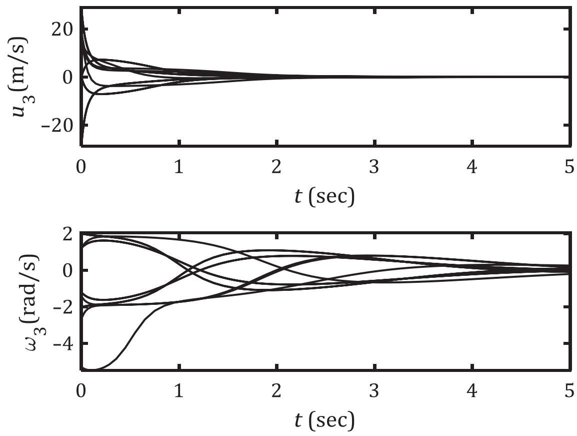

Fig. 6. Control inputs in the stabilization of a 3-trailer TTWR from different initial conditions.

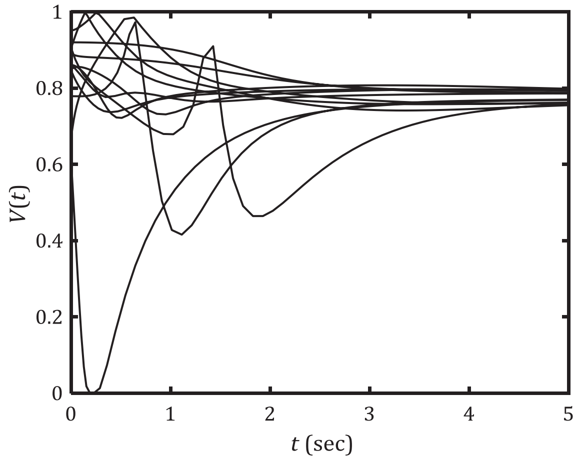

Fig. 7. Time-history of Lyapunov function candidate in the stabilization of a 3-trailer TTWR from different initial conditions.

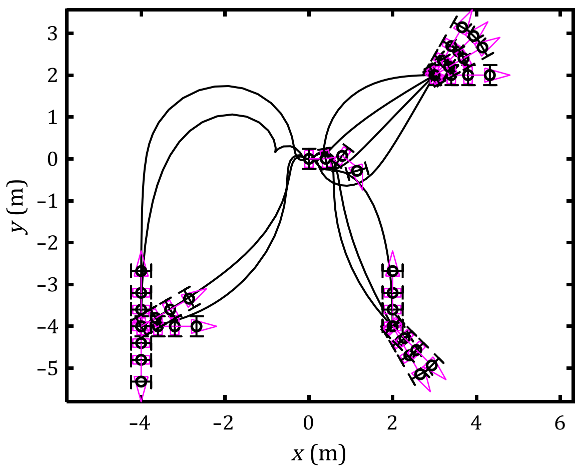

Fig. 8. Stabilization of a 3-trailer TTWR starting from different initial conditions and different initial orientations.

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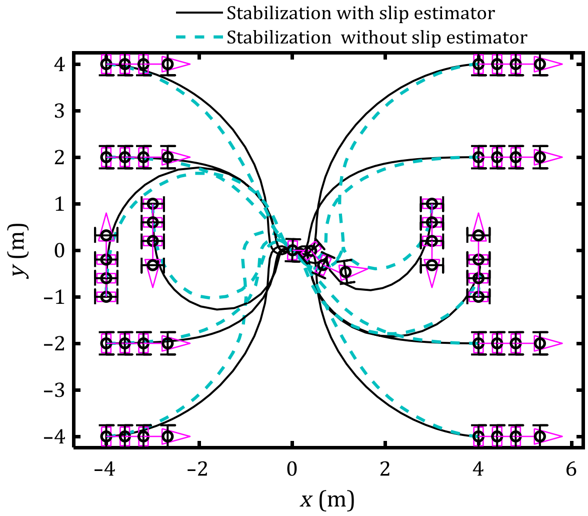

Fig. 9. Stabilization of a 3-trailer TTWR starting from different initial conditions in the existence of slip effects using slip estimator and without using slip estimator.

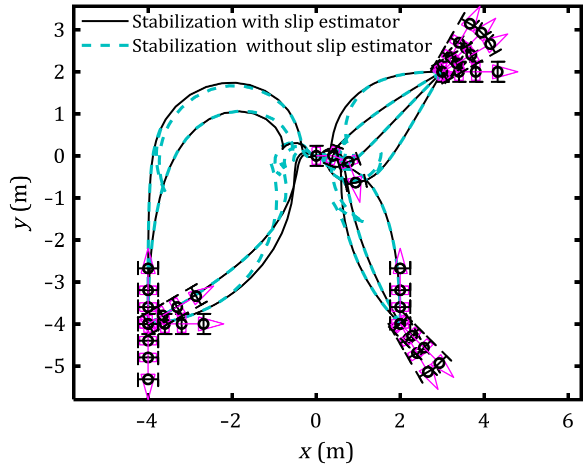

Fig. 10. Stabilization of a 3-trailer TTWR from different starting configurations in the existence of wheel slip effects using slip estimator and without using slip estimator.

errors, as was expected from the system stability analysis it can be concluded that the error signals are converging from different initial configurations.

8. Conclusion

A new discontinuous controller for the stabilization of a nonholonomic WMR towing n trailers in the presence of wheel slip effects was investigated. The proposed algorithm was a Lyapunov-based method accompanied by Clarke's generalized gradient method. The aim of the proposed method was the stabilization of the last trailer around the origin. The numerical comparative results demonstrate the efficiency of the presented algorithm to stabilize the TTWR around the origin that can be hardly achieved with the restricting nonholonomic constraints. It should be insisted that the robot base point was chosen on the last trailer, which is practically important and revealed the advantages of the recommended controller.

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