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TECHNOLOGICAL CHANGE DURING THE ENERGY TRANSITION

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The energy transition from fossil fuels to alternative energy sources has important consequences for technological change and resource extraction. We examine these consequences by incorporating a nonrenewable resource and an alternative energy source in a market economy model of endogenous growth through expanding varieties. During the energy transition, technological progress is nonmonotonic over time: It declines initially, starts increasing when the economy approaches the regime shift, and jumps down once the resource stock is exhausted. A moment of peak-oil does no longer necessarily occur, and simultaneous use of the resource and the alternative energy source will take place if the return to innovation becomes too low. Subsidies to research and development (R&D) and to renewables production speed up the energy transition, whereas a tax on fossil fuels postpones the switch to renewable energy.

Keywords: Alternative Energy Sources, Endogenous Growth, Energy Transition, Nonrenewable Resources, Technological Change

1. INTRODUCTION

Economic growth and natural resource use have been intrinsically linked throughout history. While in the Malthusian era land improvement and expansions allowed for population increases, in the modern economy era coal and later oil made the steady growth of manufactured output per capita possible. Because fossil resources have seemed so abundant for most of the time since the industrial revolution, our theories of growth could safely ignore the role of resources and focus on capital investment and technological change. However, fossil resources are nonrenewable

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and at some point resource scarcity will be likely to restrict growth. The limited availability of our main current sources of energy gives rise to two possible scenarios: Either we need to gradually reduce energy use and prevent sudden declines in energy supply, or substitutes for fossil energy need to be introduced. Both scenarios involve costs and the natural question is to what extent growth will be influenced. In particular, the question is how the engine of growth in our modern economies, namely investment and innovation, will be affected.

To answer this question, we propose a model in which growth is driven by research and development (R&D) and that integrates the use of energy from potentially two sources: Nonrenewable (fossil) resources that can be extracted without cost from the earth's crust and a form of energy that is produced by using renewable resources. Nordhaus (1973) was the first one to introduce such a substitute technology that is not constrained by exhaustibility, which he called a "backstop technology." Examples of already available backstop technologies for natural resources are nuclear energy, solar energy, and wind energy. We contribute to the literature by studying the effects of the availability of a backstop technology on the rate of technological progress and on the resource extraction path in an analytically tractable, general equilibrium model.

Intuition suggests that technological progress as the engine of growth might falter in the long run, because incentives for developing labor- and capital-augmenting technology become smaller as resource stocks dwindle and the increasing resource income share puts downward pressure on the income shares of capital and labor. Taking the existence of a substitute for fossil fuels into account, however, we find the opposite result: Technological progress prospers instead of falters when resource stocks dwindle during the energy transition. Underlying the surge in innovation is a consumption smoothing motive: Agents convert part of the resource stock into knowledge, thereby transferring resource wealth to the backstop technology era. Moreover, we show that, if the backstop technology is expensive, a large increase in R&D investment is required for a smooth transition. As a result, the marginal return to innovation falls sharply and may even become equal to the return to conserving fossil when the backstop technology is already used. In this case, part of the consumption smoothing will take place through a regime of simultaneous use of the resource and the backstop technology. We also find that due to the availability of the backstop technology, the time profile of resource extraction may remain upward-sloping until the stock is depleted. Finally, we show that a research subsidy and a backstop technology production subsidy both speed up the energy transition, whereas a tax on resource use postpones the switch to renewables.

The first building block of our analysis is the so-called Dasgupta–Heal–Solow–Stiglitz (DHSS) model. The DHSS model integrates nonrenewable resources into the neoclassical exogenous growth framework.[1] Although the DHSS model does not focus on the energy transition toward backstop technologies, some of the early studies do take the existence of substitutes for the nonrenewable resource into account. Dasgupta and Heal (1974) and Dasgupta and Stiglitz (1981) allow for the invention of a backstop technology, which occurs each period with an exogenously given probability. Kamien and Schwartz (1978) introduce the possibility of undertaking R&D to affect the probability of invention. In partial equilibrium settings, Hoel (1978) and Stiglitz and Dasgupta (1982) assume that a backstop technology already exists. They show that the relative price of the resource compared to the backstop technology increases over time and the backstop is adopted once prices are equalized.

Tsur and Zemel (2011) study the transition from fossil fuels to renewable energy in general equilibrium, assuming that both types of energy are perfect substitutes, as in our model. In their model, the distinctive feature of renewable energy is that it requires a specific type of capital. Once a stock of capital in the renewable energy sector has been built up, energy can be generated up to the corresponding capacity at zero marginal costs. As a result, firms in the renewables sector need to be forward-looking when they decide whether and how much to invest in capital that can only be used to generate energy. In contrast, energy generation in the fossil sector comes with constant marginal costs and does not require any capital. In this paper, we abstract from investment in the renewables sector. The crucial distinction in our model between fossil and renewable energy is in terms of scarcity: Fossil energy is derived from a finite resource stock, whereas renewables have an infinite resource base. Strikingly, regarding the energy transition, we find a result similar to Tsur and Zemel (2011). Provided that the price of fossil energy is above a certain threshold, they also obtain an initial fossil phase, followed by a regime of simultaneous use and eventually a regime during which only renewables are used. Like in our case, the existence of an intermediate regime of simultaneous use is driven by households' desire to smooth consumption over time. Hence, our study is complementary to that of Tsur and Zemel (2011): While they abstract from scarcity in the fossil sector and we abstract from investment in the renewable sector, we get a similar three-phase pattern of energy generation over time, which also provides an indication of the robustness of our results.

In the models discussed so far, gradual technological progress was either absent or exogenous. Barbier (1999) was one of the first to study the role of endogenous technological change in alleviating resource scarcity. Scholz and Ziemes (1999) investigate the effect of monopolistic competition on steady-state growth in a model with a necessary nonrenewable resource.[2] More recently, Bretschger and Smulders (2012) explore the consequences of poor input substitution possibilities and induced structural change for long-run growth prospects in a multisector economy. These three endogenous growth models, however, ignore the existence of a backstop technology for the natural resource. Tsur and Zemel (2003) fill this gap in the literature, by introducing R&D directed at a backstop technology. In their model, accumulation of knowledge gradually decreases the per unit cost of the backstop technology. Alternatively, Chakravorty et al. (2012) assume that per unit costs of the backstop technology decrease over time through learning-by-doing. Both studies, however, are casted in a partial equilibrium framework.

Accordingly, the existing literature on nonrenewable resources in which technological progress is explained endogenously appears to suffer from a dichotomy: Either backstop technologies or general equilibrium effects are being ignored. A synthesis of both strands of the literature is, however, desirable and likely to generate new insights [cf. Valente (2011)]. After all, contrary to the presumption in the partial equilibrium literature that imposes a fixed resource demand function, output growth and biased technological change both affect the demand for the resource. Moreover, changes in the rate of interest induced by the energy transition should be taken into account, because they affect the level of investment and innovation, and the extraction path through Hotelling's rule.

There are a few notable exceptions that are not subject to the dichotomy criticism. First, Tsur and Zemel (2005) develop a general equilibrium model, where the unit costs of the backstop technology decrease as a result of R&D. However, R&D is only possible in the backstop sector, so that effects on aggregate technological progress cannot be addressed. Second, Tahvonen and Salo (2001) study the transition between renewable and nonrenewable resource in general equilibrium. In their model, though, technological change results from learning-by-doing and does not come from intentional investments. Moreover, they resort to a Cobb–Douglas specification for final output, thereby ignoring poor substitution between resources and man-made inputs. Third, Valente (2011) constructs a general equilibrium model in which the social planner optimally chooses whether and when to abandon the traditional resource-based technology in favor of the backstop technology. The differences with our analysis are that Valente abstracts from poor input substitution by imposing Cobb–Douglas production, assumes a costless endowment of the backstop technology, and derives the social optimum instead of the decentralized market equilibrium. Moreover, his focus on the optimal timing of backstop technology adoption and on the optimal jumps in output and consumption at the regime switching instant is different from ours.

The remainder of the paper is structured as follows. Section 2 presents the structure of the model. Section 3 discusses the energy transition and regime shifts. Section 4 provides a numerical illustration. Finally, Section 5 concludes.

2. THE MODEL

We model an economy in which final output is produced with intermediate goods and energy. The production of intermediate goods requires labor. Energy is derived from a nonrenewable natural resource that can be extracted at zero costs, or generated by a backstop technology that uses labor. The elasticity of substitution between energy and intermediate goods is assumed to be smaller than unity, in line with the empirical evidence in Koetse et al. (2008) and Van der Werf (2008). Technological progress in the model is driven by labor allocated to R&D directed at the invention of new intermediate goods. The remainder of this section describes the structure of the model in more detail.

2.1. Production

Final output Y is produced with energy E and an intermediate input M, according to

$$Y = A\left[\bar{\theta}E^{\frac{\sigma-1}{\sigma}} + (1-\bar{\theta})M^{\frac{\sigma-1}{\sigma}}\right]^{\sigma/(\sigma-1)}, \tag{1}$$

where A is a productivity parameter, $0 < \bar{\theta} < 1$ and $\sigma \in (0, 1)$ denotes the elasticity of substitution between energy and the intermediate input.[3]

The intermediate input is a CES aggregate of intermediate goods k_j with an elasticity of substitution between varieties of $1/(1-\beta) > 1$. At time t, there exists a mass of $N(t)$ different intermediate goods. When intermediate goods producers are identical, the equilibrium quantity of variety j is the same for all varieties, so that $k_j = k, \forall j$. By defining aggregate intermediate goods as $K \equiv Nk$, the intermediate input can be written as

$$M = \left(\int_0^N k_j^\beta dj\right)^{1/\beta} = N^\phi K, \tag{2}$$

where $\phi \equiv (1-\beta)/\beta$ measures the gains from specialization: While keeping aggregate intermediate goods K constant, the intermediate input M rises with the number of varieties N through increased specialization possibilities in the use of intermediate goods [cf. Ethier (1982), Romer (1987, 1990)].

Energy is generated by the nonrenewable resource R and a backstop technology H:

$$E = R + A_H H, \tag{3}$$

where A_H is a productivity index.

Final goods producers maximize profits in a competitive market. They take their output price p_Y, the prices of intermediate goods p_{K_j}, the resource price p_R, and the price of the backstop technology p_H as given. Because R and H are perfect substitutes, final good producers will only use the energy source with the lowest relative price per unit of energy and they are indifferent between the two if their prices are equal. Relative demand for intermediate goods and energy is therefore given by[4]

$$\begin{aligned}
K/R &= \left(\frac{p_R}{p_K}\right)^\sigma \left(\frac{1-\bar{\theta}}{\bar{\theta}}\right)^\sigma N^{-\phi(1-\sigma)} \quad \text{and} \quad H = 0 \text{ if } \quad p_H/A_H > p_R;\\
K/H &= \left(\frac{p_H}{p_K}\right)^\sigma \left(\frac{1-\bar{\theta}}{\bar{\theta}}\right)^\sigma (A_H N^{-\phi})^{1-\sigma} \quad \text{and} \quad R = 0 \text{ if } \quad p_H/A_H < p_R;\\
K/E &= \left(\frac{p_E}{p_K}\right)^\sigma \left(\frac{1-\bar{\theta}}{\bar{\theta}}\right)^\sigma N^{-\phi(1-\sigma)} \quad \text{if} \quad p_H/A_H = p_R,
\end{aligned} \tag{4}$$

where p_E denotes the price of energy.

Firms in the intermediate goods sector need a patent to produce one specific variety according to the production function $k_j = l_{K_j}$, implying $K = L_K$, where l_{K_j} denotes labor demand by firm j and L_K is aggregate labor demand by the intermediate goods sector. Imperfect substitutability between varieties implies

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that the intermediate goods market is characterized by monopolistic competition [cf. Dixit and Stiglitz (1977)]. Each producer maximizes profits and faces a price elasticity of demand equal to $(1+\phi)/\phi$. As a result, all firms charge the same price

$$p_K = (1+\phi)w, \tag{5}$$

where w denotes the wage rate. Profits of intermediate goods producers are used to cover the costs of obtaining a patent. Combining (5) with the intermediate goods production function, we obtain an expression for individual profits:

$$\pi = p_K k - wk = \frac{\phi w K}{N}. \tag{6}$$

The resource market is characterized by perfect competition. Resource owners can extract their resource at a constant cost, which we normalize to zero. The initial resource stock is assumed to be of finite size: $0 < S_0 \ll \infty$. By imposing this structure on the resource market, we orient our analysis toward the exploitation of conventional, proved oil reserves, as opposed the more expensive and abundant unconventional types of oil reserves (e.g., oil sands and oil shales). One reason to do this is that, if effective climate policies will be implemented, a large share of unconventional oil reserves will never be extracted [cf. Van der Ploeg and Withagen (2012a,b)]. Another reason is that—even without climate policies—due to the rapidly declining costs of renewable energy [cf. International Energy Agency (2015b)], unconventional oil with its relatively high extraction costs probably will not be able to compete with renewable energy. The total world proved oil reserves were about 1,700 billion barrels at the end of 2014, equal to 52.5 years of global production [BP (2015)]. Extraction, or "lifting" costs of conventional oil varied from 4.32 to 11.90 dollars per barrel in 2008 [U.S. Energy Information Administration (2009)], whereas the average market price for West Texas Intermediate (WTI) and Brent Crude oil was about 98 dollars in 2008 [U.S. Energy Information Administration (2015b)]. Hence, when focusing on the conventional proved reserves, a setting with a finite stock and zero extraction costs seems appropriate.[5]

Firms in the perfectly competitive backstop technology sector use labor to produce energy according to the production function $H = \eta L_H$, where L_H denotes aggregate labor demand by the backstop technology sector. The price of one unit of the backstop equals its marginal cost[6]:

$$p_H = \frac{w}{\eta}. \tag{7}$$

2.2. Research and Development

R&D undertaken by firms in the research sector leads to the invention of new intermediate goods varieties. Following Romer (1990), we assume that the stock of public knowledge evolves in accordance with the number of invented intermediate goods. New varieties are created according to

$$\dot{N} = \frac{1}{a} L_R N, \tag{8}$$

where L_R denotes labor allocated to research and a is a cost parameter. The right-hand side of (8) features the stock of public knowledge, to capture the "standing on shoulders effect": Researchers are more productive if the available stock of public knowledge is larger [cf. Romer (1990)].[7] Moreover, we assume spillovers from the stock of public knowledge to the backstop technology sector by imposing $A_H = N^{\phi}$.[8] We define the innovation rate as $g \equiv \dot{N}/N$.

We abstract from technological change in the resource sector. In a related study, Van der Meijden and Smulders (2017) show that when allowing for resource-augmenting technical change, there may exist an initial regime in which both labor-augmenting and resource-augmenting technical change take place. However, resource-augmenting change vanishes already before the switch to the backstop technology has taken place. The reason is that resource-augmenting technologies become obsolete from the moment of the switch onward. By abstracting from resource-augmenting technical change, in this paper, we focus on the part of the energy transition during which the future switch to renewable energy has made investments in fossil-saving technologies unattractive.

Free entry of firms in the research sector implies that whenever the cost of inventing a new variety, aw/N, is lower than the market price of a patent, p_N, entry of firms in the research sector will take place until the difference is competed away. Therefore, free entry gives rise to the following condition:

$$aw/N \geq p_N \qquad \text{with equality (inequality) if } g > 0 \ (g = 0). \tag{9}$$

Throughout, we restrict our attention to the case of a positive innovation rate. In equilibrium, investors earn the market interest rate r on their investment in patents:

$$\pi + \dot{p_N} = r p_N, \tag{10}$$

By combining (5), (6), (9), and (10), we obtain an expression for the return to innovation:

$$r = \frac{\phi}{a} K + \hat{w} - g \text{ if } g > 0, \tag{11}$$

where hats denote growth rates. The return to innovation depends positively on K, because of a market size effect. The term $\hat{w} - g$ takes account of the change in the patent price over time. The parameter a has a negative effect on the return to innovation, because it is related negatively to the productivity of researchers. The parameter ϕ has a positive effect, because of its positive relationship with the markup on the price of intermediate goods.

2.3. Factor Markets

Equilibrium on the labor market requires that aggregate labor demand from the intermediate goods sector, the backstop technology sector, and the research sector equals the fixed labor supply $L_K + L_H + L_R = K + \frac{H}{\eta} + ag = L$. We define the income shares of energy and intermediate goods, and the expenditure shares of the backstop technology and the resource in total energy costs as follows:

$$\theta \equiv \frac{p_E E}{p_Y Y}, \qquad 1-\theta = \frac{p_K K}{p_Y Y}, \qquad \omega \equiv \frac{p_H H}{p_E E}, \qquad 1-\omega = \frac{p_R R}{p_E E}. \quad \textbf{(12)}$$

Using these definitions together with (5) and the backstop production function, labor market equilibrium implies

$$K = \frac{1-\theta}{(1+\phi)\omega\theta + 1 - \theta}(L - ag). \quad \textbf{(13)}$$

Resource extraction depletes the resource stock S according to

$$\dot{S}(t) = -R(t), \quad S(0) = S_0, \quad R(t) \geq 0, \quad S(t) \geq 0, \quad \textbf{(14)}$$

which implies that total extraction cannot exceed the initial resource stock.

2.4. Households

The representative household lives forever, derives utility from consumption of the final good, and inelastically supplies L units of labor at each moment. It owns the resource stock with value $p_R S$ and all equity in intermediate goods firms with value $V = p_N N$. The household maximizes lifetime utility $U(t) = \int_t^\infty \ln C(z)e^{-\rho(z-t)}dz$, subject to its flow budget constraint $\dot{V} = rV + p_R R + wL^S - p_Y C$, and the transversality conditions $\lim_{z\to\infty} \lambda_V(z)V(z)e^{-\rho z} = 0$ and $\lim_{z\to\infty} \lambda_S(z)S(z)e^{-\rho z} = 0$, where ρ denotes the pure rate of time preference, and λ_V and λ_S the shadow price of financial wealth and the resource stock S, respectively. Final output cannot be stored, so that consumption equals output, i.e., $C = Y$.

Straightforward manipulations of the standard first-order conditions for the optimization problem of the representative household yield two familiar rules[9]:

$$\hat{p}_Y + \hat{Y} = r - \rho, \quad \textbf{(15a)}$$

$$\hat{p}_R = r. \quad \textbf{(15b)}$$

The first one, (15a), is the Ramsey rule, which relates the growth rate of consumer expenditures to the difference between the nominal interest rate and the pure rate of time preference. Equation (15b) is the Hotelling rule, which ensures that owners of the resource stock are indifferent between (i) selling an additional unit of the resource and investing the revenue at the interest rate r, and (ii) conserving it and earn a capital gain at rate $\hat{p}_R$.

3. DYNAMICS OF THE MODEL

In this section, we discuss the dynamics of the model. Because the resource and the backstop technology are perfect substitutes, only the cheapest of the two will be used at a particular moment in time. If the two energy sources have equal prices, simultaneous use may occur. Therefore, three different regimes of energy use exist: a fossil regime, a simultaneous use regime, and a backstop regime. We proceed by first describing the dynamic behavior of the economy during each regime. Subsequently, we describe the energy transition by linking the regimes together.

3.1. The Fossil Regime

In the fossil regime, energy generation relies exclusively on the natural resource. The model described in Section 2 with $H = 0$ imposed can be condensed to a three-dimensional block-recursive system of differential equations in the energy income share θ, the innovation rate g, and the reserve-to-extraction rate $y \equiv S/R$. The system is block-recursive in the sense that the system of θ and g can be solved independently from y. Beyond simplifying the mathematical analysis, the reexpression of the model in terms of θ, y, and g also helps to clarify the economics behind our results. These variables, namely, have a clear interpretation as they are indicators of energy scarcity and the rate of technological progress. In this section, we analyze the (θ, g)-subsystem described in Lemma 1, and we postpone the solution of the differential equation for y until Section 3.6.

LEMMA 1. *Provided that $g(t) > 0$, the dynamics in the fossil regime are described by the following two-dimensional system of first-order differential equations in $\theta(t)$ and $g(t)$:*

$$\dot{\theta}(t) = \theta(t)[1 - \theta(t)](1 - \sigma)\,[r(t) - \hat{w}(t) + \phi g(t)]\,, \tag{16a}$$

$$\dot{g}(t) = \left[\frac{L}{a} - g(t)\right]\{\rho + \theta(t)(1 - \sigma)\phi g(t) - [1 - \theta(t)(1 - \sigma)]\,[r(t) - \hat{w}(t)]\}\,, \tag{16b}$$

where the term $r(t) - \hat{w}(t)$ is a function of $g(t)$:

$$r(t) - \hat{w}(t) = \phi\frac{L}{a} - (1 + \phi)g(t). \tag{16c}$$

Proof. See Appendix A.2.

Equation (16a) shows that the energy income share increases if the price per unit of energy increases relative to the price of intermediate goods, i.e., if $r - \hat{w} + \phi g > 0$. Expression (16b) is derived from the labor market equilibrium (13), which requires that the innovation rate declines if the input of labor in the intermediate sector $L_K = K$ increases.

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3.2. Simultaneous Use Regime

The simultaneous use regime is characterized by equal effective prices of the resource and the backstop technology. As a result, the energy income share will be constant and the innovation rate will be declining over time, as shown in Lemma 2.

LEMMA 2. *In the simultaneous use regime, the income share of energy* θ *remains constant and is equal to*

$$\theta_S = \left[(\eta + \eta\phi)^{1-\sigma}\left(\frac{1-\bar{\theta}}{\bar{\theta}}\right)^{\sigma} + 1\right]^{-1}. \tag{17a}$$

The innovation rate is decreasing over time, according to the following differential equation:

$$\dot{g} = -g(\phi g + \rho). \tag{17b}$$

Proof. See Appendix A.2.

Intuitively, as long as $\theta < \theta_S$, the resource is relatively cheaper than the backstop technology so that only the resource will be used for energy generation. If $\theta = \theta_S$, effective prices of the resource and the backstop technology are equal, which enables a regime of simultaneous use as long as θ remains constant. The declining innovation rate follows from the constant energy income share during the simultaneous use regime. A constant income share requires that the relative price of intermediate goods and energy remains unchanged: $r - \hat{w} + \phi g = 0$. As a result, K goes down over time, because the constant income share implies $\hat{K} = r - \hat{w} - \rho$. According to (11), g consequently needs to decline in order to ensure that $r - \hat{w} = -\phi g < 0$ remains satisfied: A decrease in g is needed to keep the return to innovation from dropping below the rate of interest as a result of the declining market size.

3.3. The Backstop Regime

The backstop regime is characterized by a constant energy income share and a constant innovation rate, as described in Lemma 3.

LEMMA 3. *In the backstop regime, the energy income share* θ *and the innovation rate* g *remain constant and are equal to, respectively,*

$$\theta_B = \theta_S, \tag{18a}$$

$$g_B = \left(\frac{L}{a} + \rho\right)\frac{\phi}{1+\phi}(1 - \theta_B) - \rho. \tag{18b}$$

Proof. See Appendix A.2.

Intuitively, Hicks-neutral technological change between the backstop and intermediate goods implies a constant energy income share. Given that the resource stock

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is depleted, innovation is the only remaining investment possibility. The constant income share of intermediate goods implies an unchanging return to innovation, resulting in a constant innovation rate over time.

3.4. The Energy Transition

Assuming that the initial stock is large enough to get $p_H(0)/A_H(0) > p_R(0)$, implying that $\theta(0) < \theta_S = \theta_B$, the economy will start in the fossil regime. Due to increasing scarcity and resource using technological change, the energy income share increases over time until this inequality is no longer satisfied. At this moment, the fossil regime will end. Depending on the productivity of the backstop technology and on characteristics of the innovation process, the switch to the backstop technology can either take place abruptly or gradually through an intermediate regime of simultaneous use. Both cases will be discussed in turn.

Abrupt shift. By imposing $\dot{\theta} = \dot{g} = 0$ in (16a) and (16b), we obtain the following steady-state loci in the fossil regime:

$$g|_{\dot{\theta}=0} = \phi\frac{L}{a}, \tag{19a}$$

$$g|_{\dot{g}=0} = \frac{\phi(L/a)\,[1-\theta(1-\sigma)] - \rho}{(1+\phi) - \theta(1-\sigma)} < g|_{\dot{\theta}=0}. \tag{19b}$$

Moreover, by combining (12) with (15a) and (15b), we get $\hat{R} = \hat{\theta} - \rho$, so that the resource extraction isocline can be written as

$$g|_{\hat{R}=0} = g|_{\dot{\theta}=0} - \frac{\rho}{(1-\sigma)(1-\theta)}, \tag{19c}$$

where we have used (16a) and (16c) to substitute for $\hat{\theta}$. Figure 1 shows the phase diagram for the fossil regime in the (θ, g)-plane. The growth rates of the effective prices of intermediate goods and energy are equal along the flat $\dot{\theta} = 0$ line, leading to constant income shares. At points below the income share locus, the effective price of energy relative to the intermediate goods increases, i.e., $r - \hat{w} + \phi g > 0$, so that the income share of energy rises over time and *vice versa*. The dynamic behavior of θ is illustrated by the horizontal arrows in the phase diagram. The real interest rate in the fossil regime can be found by combining $\hat{p}_Y = \theta r + (1-\theta)(\hat{w} - \phi g)$ and (16c), which gives

$$r - \hat{p}_Y = (1-\theta)\left(\phi\frac{L}{a} - g\right). \tag{20}$$

It follows from this expression that the real return to innovation (and hence the real rate of interest) depends negatively on the energy income share θ, because a higher income share of energy implies a lower income share of intermediate

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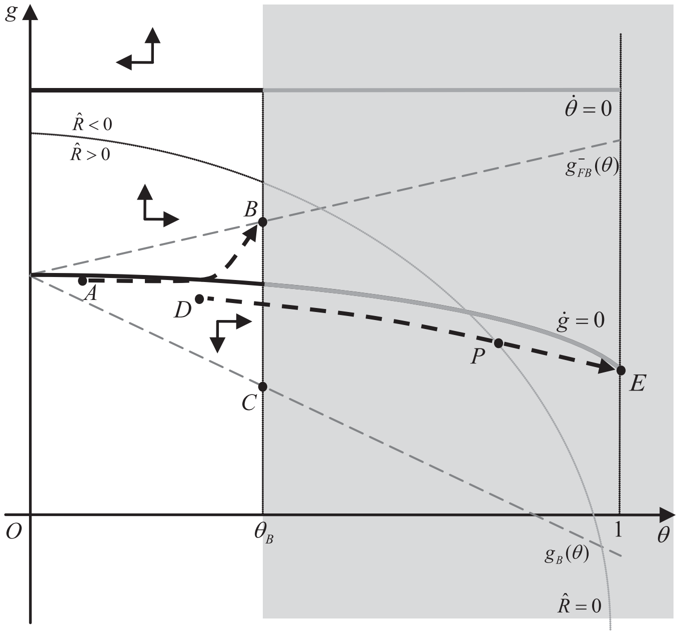

FIGURE 1. Phase diagram: fossil regime. The dashed arrow from point A to point B represents the equilibrium path when a backstop technology is available. The shaded area of the phase diagram is not relevant when a backstop technology is available. The dashed arrow from point D to point E represents the equilibrium path when no backstop technology is available.

goods and therefore a lower return to innovation. At points above the downward-sloping innovation locus, the real interest rate and output growth are lower than in steady-state equilibrium, so that $L_K = K$ declines and the innovation rate increases over time and *vice versa*. The dynamic behavior of g is illustrated by the vertical arrows in the phase diagram. The figure also contains the extraction isocline $\hat{R} = 0$, which slopes downward and has a vertical asymptote at $\theta = 1$. At points above the $\hat{R} = 0$ isocline, the real interest rate and therefore output growth are lower than required for constant extraction, so that extraction growth becomes negative and *vice versa*.

Without the existence of a backstop technology, the fossil regime lasts forever and the economy converges along the stable manifold from point D to point E in Figure 1.[10] This equilibrium path is characterized by an ever decreasing innovation rate and an energy income share that converges to unity. Peak-oil occurs at point P, where the equilibrium path crosses the extraction isocline. Because of the vertical asymptote of the extraction isocline at $\theta = 1$, resource use is necessarily declining in the long run. The occurrence of peak-oil, however, depends crucially on the elasticity of substitution between intermediate goods and energy. If this elasticity is high enough, the extraction isocline is located entirely below the equilibrium path. In that case, point P does not exist and extraction is declining throughout.[11]

When a backstop technology exists, however, the resource will not be used anymore if $\theta > \theta_B$, which is the case in the shaded area of the figure. In equilibrium, the resource will then be exhausted at the moment when θ hits θ_B and the economy will shift abruptly to the backstop technology. The negatively sloped dashed line in the figure represents (18b) and gives g_B for each possible θ_B. Hence, point C shows the steady-state equilibrium in the backstop regime, where the economy ends up immediately after the switch. The end point of the fossil regime can be found by using the Ramsey rule (15a), which implies that consumption should be continuous at each point in time as long as the interest rate is finite. Output in either regime can be written as

$$Y = N^{\phi} \left(\frac{1 - \bar{\theta}}{1 - \theta} \right)^{\sigma/(\sigma-1)} K. \qquad \textbf{(21)}$$

Hence, given that prices and therefore income shares are continuous, due to the required continuity of output, K needs to be continuous as well. Accordingly, labor market equilibrium (13) with $\omega = 0$ before the switch and $\omega = 1$ after the switch gives

$$L - a g_{FB}^{-} = \left(\frac{1 - \theta_B}{\theta_B (1 + \phi) + 1 - \theta_B} \right) (L - a g_B), \qquad \textbf{(22)}$$

where g_{FB}^{-} denotes the innovation rate just before the switch at time T_{FB} from the fossil to the backstop regime.[12] Substitution of (18b) into this expression yields the innovation rate at the end of the fossil regime:

$$g_{FB}^{-} = \frac{L}{a} - \frac{1 - \theta_B}{1 + \phi} \left(\frac{L}{a} + \rho \right). \qquad \textbf{(23)}$$

The positively sloped dashed line in Figure 1 gives g_{FB}^{-} for each possible value of θ_B. Point B denotes the end point (θ_B, g_{FB}^{-}) of the fossil regime. The equilibrium path that leads to the end point B is indicated by the dashed arrow starting at A. Along this path, the income share of energy is increasing over time and the innovation rate is higher than it would have been in an economy without the backstop technology available. The innovation rate is initially decreasing, but as soon as the economy crosses the innovation locus, the growth rate starts to increase until the moment of the regime switch. Intuitively, in order to prevent consumption from falling discontinuously when the resource stock is exhausted, the representative household now starts to increase savings when the regime switch comes near. So doing, the household effectively smooths consumption by converting part of the resource wealth into knowledge, thereby transferring consumption possibilities to the future regime in which the resource stock is depleted. In the figure, the extraction path is upward-sloping throughout the fossil regime, as the equilibrium path is entirely located below the extraction isocline. At time T_{FB}, the economy shifts from the fossil to the backstop regime and the innovation rate jumps down to free enough labor for the production of energy with the backstop technology while keeping output unaffected. Note that the end point (θ_B, g_{FB}^-) is located below the $\dot{\theta} = 0$ line, i.e., $\frac{\theta_B - \phi^2}{1-\theta_B}\frac{L}{a} < \rho \Leftrightarrow g_{FB}^- < \phi\frac{L}{a}$, which is a necessary condition for the abrupt shift from the fossil to the backstop regime to occur. Proposition 1 summarizes the findings of this section:

PROPOSITION 1. *If* $\frac{\theta_B - \phi^2}{1-\theta_B}\frac{L}{a} < \rho$, *the economy shifts abruptly from the fossil regime to the backstop regime and the innovation rate jumps down at* T_{FB}.

Proof. The case in which the economy relies upon the resource forever without switching to the backstop technology can be excluded, because eventually $\theta > \theta_B$ would hold, implying that the backstop technology is cheaper than the resource. Hence, there exists a time T_{FB} at which the fossil regime ends. Moreover, due to the inequality the end point of the fossil regime is located below the income share locus so that the price of the resource relative to the backstop keeps on rising until the stock is depleted, which implies that simultaneous use cannot take place, so that the economy shifts from the fossil to the backstop regime at T_{FB}. The downward jump in the innovation rate follows immediately when subtracting (18b) from (23), yielding

$$g_{FB}^- - g_B = \theta_B\left(\frac{L}{a} + \rho\right) > 0. \tag{24}$$

■

Gradual transition. If the inequality in Proposition 1 is violated, the economy will not experience an abrupt shift from the fossil to the backstop regime. In this case, the shift in energy usage occurs more gradually, through a regime in which both energy sources are used simultaneously.

PROPOSITION 2. *If* $\frac{\theta_B - \phi^2}{1-\theta_B}\frac{L}{a} > \rho$, *the economy shifts from the fossil regime to an intermediate regime of simultaneous use at* T_{FS}.[13] *Subsequently, the economy shifts from the simultaneous use to the backstop regime at* $T_{SB} > T_{FS}$. *The innovation rate is continuous and equal to*

$$g_{FS}^- = g_{FS}^+ = \phi\frac{L}{a}, \tag{25}$$

at T_{FS}. *The innovation rate decreases during the simultaneous use regime and jumps down from*

$$g_{SB}^- = \frac{\phi(1-\theta_S)\left(\frac{L}{a} + \rho\right)}{(1+\phi)(1-\phi)} > 0, \tag{26}$$

to g_B at T_{SB}. During the simultaneous use regime, the real interest rate equals zero, the backstop expenditure share ω increases, while resource extraction declines gradually over time.

Proof. See Appendix A.2.

The negatively sloped dashed–dotted line in panel (a) of Figure 2 represents (26) and gives g_{SB}^{-} for each possible θ_B. The end point of the fossil regime is indicated by B. The economy moves along the equilibrium path from point A to point B during the fossil regime. The income share of energy increases over time, while the innovation rate again exhibits a nonmonotonic time profile: It decreases initially but starts to increase once the economy has passed the innovation locus. If the equilibrium path starts out below the extraction isocline, resource extraction peaks at point P and decreases afterward. Once point B is reached, both the levels and the growth rates of the effective resource and backstop price are equal, and the economy shifts to the simultaneous use regime. The evolution of the innovation rate from g_{FS}^{+} to g_{SB}^{-} during this regime of simultaneous use of the resource and the backstop is indicated by the dotted arrow from point B to point C in panel (b). Once point C is reached, the economy shifts to the backstop technology and jumps to point D in panel (b) of the figure to free labor for the backstop production while leaving output unchanged.

The existence of simultaneous use can be explained from the desire to smooth consumption between the different regimes of energy generation. Because resource conservation and investment in innovation each provide a channel for households to smooth consumption, the existence of a simultaneous use regime depends on the profitability of innovation (i.e., on ϕ) and on the costs of the backstop technology (i.e., on θ_B through η). If innovation revenues would be zero (i.e, if $\phi = 0$), there would be no investment in R&D at all. Without investment in R&D, there necessarily exists a regime of simultaneous use. The reason is that if $g = 0$, $L = L_K + L_H$, so that any jump in L_H will imply a jump in L_K and hence in consumption and marginal utility. Therefore, L_H must gradually increase from zero to its long-run value. In a market equilibrium with positive R&D (when $\phi > 0$), labor market equilibrium reads $L = L_K + L_H + ag$ so that households have an additional way to smooth consumption: By reducing innovation at the time of the regime shift, labor becomes available for energy generation with the backstop technology without a need to reduce consumption. In scenarios with profitable innovation possibilities and a relatively cheap backstop technology, consumption smoothing may completely take place through this new channel: simultaneous use will not occur. If, however, innovation is less profitable, or the backstop technology is relatively expensive so that it will absorb a substantial amount of the labor supply after the regime switch, part of the consumption smoothing still takes place through a temporary regime of simultaneous use with a zero real interest rate, during which the production of the backstop technology starts from zero at the beginning of this regime and gradually increases, until it jumps up to its constant long-run level at time T_{SB}.

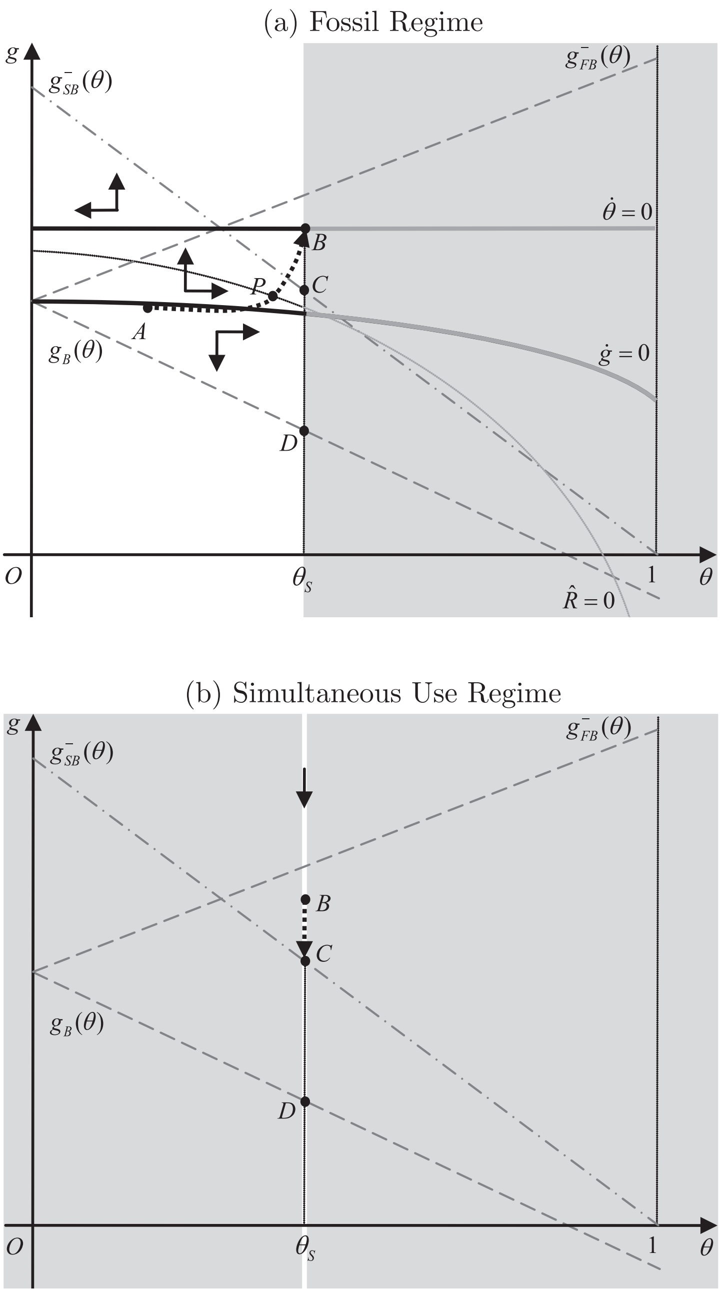

FIGURE 2. Phase diagrams: (a) fossil regime and (b) simultaneous use regime. In panel (a), the dotted arrow represents the equilibrium path of the fossil regime. In panel (b), the dotted arrow shows the equilibrium path of the simultaneous use regime. In both panels, the irrelevant parts of the phase diagrams are shaded in gray.

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3.5. Consumption: Level and Growth

So far, we have characterized the energy transition in terms of the energy income share and the innovation rate. In order to clearly see the implications of the transitional dynamics for consumption, Lemma 4 lists the level and the growth rate of consumption in each of the three regimes that we have described.

LEMMA 4. *The level and the growth rate of consumption (which equals output) in the three different regimes are given in the expressions below.*

(i) *In the fossil regime*

$$C(t) = N(t)^{\phi}\left(\frac{1-\theta(t)}{1-\bar{\theta}}\right)^{\sigma/(1-\sigma)}(L - ag(t)), \quad \hat{C}(t) = (1-\theta(t))\left(\phi\frac{L}{a} - g(t)\right) - \rho. \tag{27a}$$

(ii) *In the simultaneous use regime*

$$C(t) = N(t)^{\phi}\left(\frac{1-\theta_B}{1-\bar{\theta}}\right)^{\sigma/(1-\sigma)} a\frac{\phi}{1-\phi}g(t), \quad \hat{C}(t) = -\rho. \tag{27b}$$

(iii) *In the backstop regime*

$$C(t) = N(t)^{\phi}\left(\frac{1-\theta_B}{1-\bar{\theta}}\right)^{\sigma/(1-\sigma)}(L - ag_B)\frac{1-\theta_B}{1+\phi\theta_B}, \quad \hat{C}(t) = \phi g_B. \tag{27c}$$

Proof. Substitution of (13) with $\omega = 0$ in (21) gives the first expression in (27a). The second one is obtained by combining (15a) and (20) while noting that $C = Y$. To obtain (27b), note that $r - \hat{w} = -\phi g$ during a regime of simultaneous use. Combining this expression with (11) and (13), and with $\hat{p}_Y = \theta r + (1-\theta)(\hat{w} - \phi g)$ gives the first and second part of (27b), respectively. Substitution of (13) with $\omega = 1$ into (21) gives the first part of (27c). Finally, the second part is obtained by taking the growth rate of the first part and using (18a) and (18b). ■

The expressions for the level of consumption clearly show that a high-energy income share depresses consumption. Intuitively, during the fossil regime a high-energy income share is an indicator of absolute resource scarcity and during the simultaneous use and backstop regimes, the energy share is positively related to the production costs of the backstop technology. In the former case, a low level of resource input lowers output and consumption, whereas in the latter case consumption possibilities are depressed because energy generation uses up a lot of resources (i.e., labor).

During the backstop regime, consumption growth is entirely driven by innovation, implying that consumption growth is positive as long as innovation takes place. In the simultaneous use regime, consumption growth is negative and constant because the real interest rate equals zero. Intuitively, the effect of growing productivity due to innovation is exactly offset by declining resource input and by the backstop sector, which uses more and more labor over time. During the fossil regime, the growth rate is ambiguous: Innovation tends to increase consumption

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over time, but declining inputs of resources and intermediate goods might more than offset this effect.[14]

3.6. Initial Condition

To determine the initial value for the energy income share θ, i.e., to find the location of point A in Figures 1 and 2, we exploit the fact that total resource extraction over time should be equal to the initial resource stock. From the demand function (4), we derive a relationship between the income share and the reserve-to-extraction rate $y \equiv S/R$ at the beginning of the fossil regime:

$$\frac{\theta(0)}{1-\theta(0)} = \frac{\bar{\theta}}{1-\bar{\theta}} \left(\frac{y(0)\,[L - af(\theta(0))]\, N_0^{\phi}}{S_0} \right)^{(1-\sigma)/\sigma}, \tag{28}$$

where the function $g = f(\theta)$ is defined by the equilibrium path in the (θ, g)-plane. A second relationship between $\theta(0)$ and $y(0)$ is obtained by deriving from (4) a differential equation for y:

$$\dot{y} = -1 + [\rho - (1-\theta)(1-\sigma)\,(r - \hat{w} + \phi g)]\, y. \tag{29}$$

In the simultaneous use regime, (11)–(13) imply $r - \hat{w} = -\phi g$, so that differential equation (29) can be expressed in terms of y only. In the fossil regime, substitution of (16c) yields an expression in terms of y, θ, and g. The end condition for y is given by $y(T_{FB}^{-}) = 0$ in the scenario without simultaneous use and by $y(T_{SB}^{-}) = 0$ in the scenario with simultaneous use. Using the already determined paths for θ, g, and ω, the differential equation for y gives a unique equilibrium path in the (θ, y)-plane. The initial point $(\theta(0), y(0))$ is given by the intersection of this equilibrium path with (28) in the (θ, y)-plane.

4. NUMERICAL ILLUSTRATION

In this section, we perform numerical simulations of the model to quantify the transitional dynamics of the model.[15] Moreover, we will explore the effects of taxes and subsidies on the energy transition. We focus on a scenario in which an intermediate regime of simultaneous use exists. As a robustness check, we also provide simulation results for a formulation of our model in which the resource and the backstop technology are good instead of perfect substitutes, according to a CES function.[16] We first calibrate the model and then present the simulation results.

Van der Werf (2008) reports elasticities of substitution between a capital-labor aggregate and energy ranging from 0.17 to 0.61.[17] We take the average of these values to obtain $\sigma = 0.4$. By choosing $\phi = 0.25$, we get a markup of prices over marginal costs within the range of estimation results that Roeger (1995) reports for the US manufacturing sector. Recall $\phi \equiv (1-\beta)/\beta$, which implies that $\phi = 0.25$ corresponds to an elasticity of substitution between the different

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varieties $1/(1-\beta)$ of 5. We set the production function parameter $\bar{\theta}$ and the rate of pure time preference ρ to 0.1 and 0.01, respectively. Labor supply L and the initial knowledge stock N_0 are normalized to 1 and 0.1, respectively. The initial resource stock is chosen such that the initial share of resource expenditures in gross domestic product (GDP) $\theta(0)$ equals 7.5%, in line with the average US energy expenditure share in GDP over the period 1970–2009 [U.S. Energy Information Administration (2012)]. We use the research productivity parameter a to obtain an initial consumption growth rate $\hat{C}(0)$ of 1.7%, which is equal to the average yearly growth rate of GDP per capita in the United States over the period 1970–2010 [The Conference Board (2012)]. By imposing $A = 166$, we obtain an initial global GDP of 78 trillion US 2014 dollars, in line with the global GDP in 2014 [World Bank (2015)].

Our benchmark calibration implies an initial reserve-to-extraction rate of $y(0) = 52$, which lies within the range of the reserve-to-production ratios for oil, natural gas, and coal in 2008 of 44, 58, and 127, respectively [U.S. Energy Information Administration (2012)]. Initially, the ratio between the per unit energy price of the backstop technology and the resource amounts to 4.[18] This ratio is within the range of the projected levelized cost of electricity generated with renewable energy sources relative to the levelized cost of electricity generated by fossil fuels in the electricity sector in the United States in 2020, which varies from 0.77 to 5.43, according to Table 2 in U.S. Energy Information Administration (2015a).[19] In our calibrated model, the current era in which energy generation relies on fossil fuels ends in roughly 45 years. In reality, the current share of renewables in total energy supply is already positive: the global share amounted to 13.2% in 2012 [International Energy Agency (2015a)]. The zero initial share in our model is due to the assumption of perfect substitutability between fossil and renewable energy. However, in this section, we will also investigate the case of imperfect substitution as a robustness check in which renewable energy use is strictly positive from the beginning.

4.1. Transitional Dynamics

Panel (a) of Figure 3 shows the saddle path in the (θ, g)-plane generated by our simulation. Panels (b)–(f) depict the time profiles during the energy transition (i) in our benchmark model (solid black lines), (ii) in a world similar to the benchmark economy, but without the availability of a backstop technology (solid gray lines), and (iii) in a model in which the resource and the backstop technology are good, but imperfect substitutes (dashed lines).

In panel (a), the black dots depict the equilibrium path leading to the implementation of the backstop technology, whereas the gray dots show the saddle path toward the fossil steady state. In both cases, the economy moves from left to right in the diagram. The solid and dashed black lines in panel (b), representing the scenarios with perfect and imperfect substitution, almost coincide. They show that the energy income share gradually increases over time during the energy transition, until the level θ_B is reached, after which it will be constant. In the

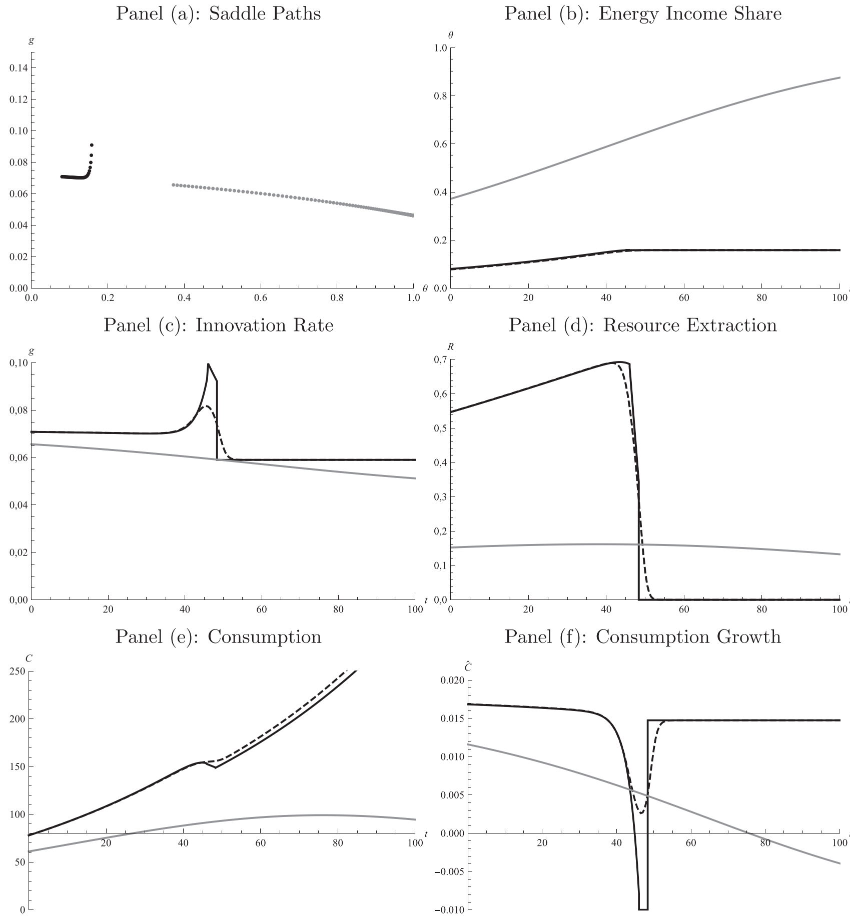

FIGURE 3. Saddle paths and time profiles. The solid lines represent the benchmark scenario in which a backstop technology that provides a perfect substitute for the resource is available. The gray line represents the scenario in which there is no backstop technology available. The dashed line represents the scenario in which a backstop technology provides a good, but imperfect substitute for the resource. In the latter scenario, η is adjusted to obtain $\theta_\infty = \theta_B$.

model without the backstop technology, the income share starts at a higher level and is monotonically increasing, asymptotically approaching unity in the long run.

The solid black line in panel (c) depicts the time profile of the innovation rate during the energy transition. The innovation rate first decreases slightly over time, but starts to increase after the stable manifold has crossed the innovation locus. During the simultaneous use regime, the innovation rate is declining. When the shift to the backstop technology takes place, the innovation rate jumps down to its constant long-run level. The gray line shows that, in contrast to the benchmark case, innovation in a world without the backstop technology decreases monotonically over time and starts out lower. As shown by the dashed lines, the imperfect substitutes model yields time paths that, though smoother, are quite similar to those generated by our simpler model in which the resource and the backstop technology are perfect substitutes.

Panel (d) shows that resource extraction is increasing initially, peaks just before the economy switches to the simultaneous use regime and then decreases rapidly until the resource stock is exhausted. Due to the finite exhaustion time, extraction starts out considerably higher than in the model without a backstop technology. According to panels (e) and (f), consumption is initially growing during the transition toward the backstop technology. When the implementation of the backstop technology is near, consumption growth rapidly declines and even turns negative at the end of the fossil regime and during the simultaneous use regime. At the moment of the switch to the backstop technology, consumption growth jumps up and remains positive and constant forever. In the model without the backstop, consumption is relatively lower. The time profile of consumption is initially increasing. Over time, consumption growth decreases and eventually turns negative.

4.2. Comparative Dynamics

So far, we have abstracted from government intervention. In this section, we extend our baseline model to include different policy instruments. We will focus on instruments that are typically associated with affecting the energy transition: an R&D subsidy, a renewables subsidy, and a carbon tax.[20] More specifically, we will introduce a constant ad valorem subsidy to production of energy with backstop technology, s_H, a constant ad valorem subsidy to profits from R&D, s_N, and a constant specific tax on resource use, τ_R. As a result, equations (6), (7), and (15b) now become, respectively,[21]

$$\pi = \frac{\phi w K}{N}(1 + s_N), \tag{30a}$$

$$p_H = \frac{w}{\eta}(1 - s_H), \tag{30b}$$

$$\hat{p}_R = r\frac{p_R - \tau_R}{p_R}. \tag{30c}$$

We investigate the introduction of ad valorem backstop and research subsidies of 20% (i.e., $s_H = s_N = 0.2$) and of the specific tax on fossil fuel use equal to 20% of the initial resource price (i.e., $\tau_R = 0.2 \times p_R(0) = 0.0122$). The solid lines in Figure 4 are the time profiles in the benchmark scenario without subsidies and taxes. The dashed lines represent the scenario with $s_H = 0.2$. Comparing the dashed lines with the solid lines in panel (a), we observe that the backstop subsidy increases initial fossil fuel use. The reason is that the subsidy effectively makes the substitute to fossil fuels cheaper. As a result, the initial fossil fuel price drops.

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This effect of an increase in initial fossil fuel use upon an increase in subsidies for renewable energy is known in the literature as the "Green Paradox" [Sinn (2008), Van der Ploeg and Withagen (2015)]. Panels (a) and (b) show that the backstop subsidy brings forward the switch to the backstop technology and increases the long-run energy supply. As shown in panel (c), the energy income share is lower throughout due to the backstop subsidy. In panel (d), we note from the dashed lines that the long-run innovation rate goes down upon the introduction of a backstop subsidy, due to a reallocation of labor from R&D to the generation of energy.

The effects of subsidizing R&D are shown by the dotted lines. Comparing these with the solid lines, we observe that the R&D subsidy increases innovation throughout [panel (d)]. Furthermore, due to its enhancing effect on resource-using technical change, the subsidy to R&D brings forward the switch to the backstop and increases resource use before the switch [panel (a)]. Because the relative scarcity of fossil fuels compared to labor goes down in production, the subsidy lowers the initial resource income share [panel (c)]. Furthermore, the reallocation of workers toward the research sector lowers the long-run energy supply [panel (b)]. Finally, as shown by the dash–dotted lines, the fossil fuel (or: carbon) tax lowers initial resource use and delays the transition to the backstop technology, but does not have long-run consequences.

5. CONCLUSION

We have investigated the effects of the availability of a backstop technology on the time paths of resource extraction and the rate of technological progress, taking into account that natural resources and man-made inputs are poor substitutes and that generation of energy with the backstop technology is costly. To this end, we introduce a nonrenewable resource and a backstop technology in a simple endogenous growth model. The backstop technology can be used to produce a perfect substitute for the natural resource. Technological progress is driven by workers in R&D, who build upon previously generated knowledge. We solve the model analytically and develop a graphical apparatus to visualize its transitional dynamics and regime shifts. Moreover, we quantify the results by calibrating the model and performing a simulation analysis. The results are robust to relaxing the assumption of perfect substitutability between the resource and the backstop technology.

Our main findings can be divided into three categories: energy regimes, technological change, and resource extraction. Regarding the first category, we find that the economy experiences different regimes of energy generation. Initially, the economy relies exclusively on the natural resource. In the long run, the natural resource will be abandoned in favor of the backstop technology. In between these two regimes, there may exist an intermediate era during which the resource and the backstop technology are used simultaneously. This feature is noteworthy, because the model does not impose the convexities in resource extraction or backstop production costs that are normally required to obtain this result. The reason for the existence of a regime of simultaneous use is the desire to smooth consumption:

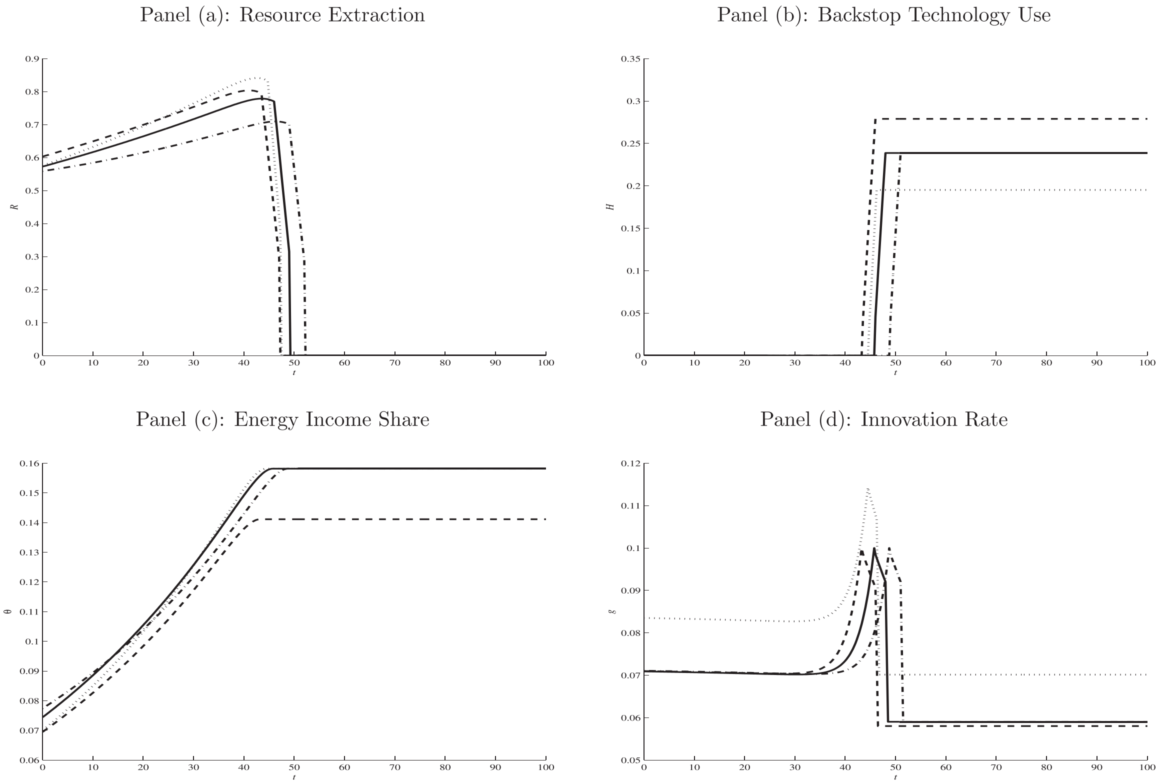

FIGURE 4. Effects of subsidies and taxes. The solid lines represent the benchmark scenario. The dashed and dotted lines give the cases with a subsidy to production with the backstop technology ($s_H = 0.2$) and a subsidy to research and development ($s_N = 0.2$), respectively. The dash–dotted lines represent the case with a specific tax on resource use ($\tau_R = 0.0122$, corresponding to 20% of the initial resource price).

By introducing the backstop technology gradually during the simultaneous use regime, households effectively shift part of the resource wealth to the future. We show that a subsidy for R&D and a subsidy for the generation of energy by the backstop technology both speed up the energy transition, whereas a tax on fossil fuel use postpones the switch to renewables.

Second, the introduction of a backstop technology in the model crucially affects the shape of the time path of technological progress, measured by the rate of innovation. Instead of monotonically decreasing as it would be without the backstop technology, the rate of innovation exhibits a nonmonotonic development over time: It first decreases gradually, but during the run-up to the backstop technology it starts to increase. The reason for the surge in innovation is again consumption smoothing: By investing in innovation, households effectively convert resource wealth into knowledge, thereby shifting consumption possibilities to the future in which energy generation is costly. If the return to investment in innovation remains high enough, consumption smoothing entirely takes place through investment in innovation so that the simultaneous use regime disappears. Once the economy enters the backstop regime, the rate of innovation jumps down to its long-run value to release resources for production in the backstop sector. During the entire transition towards the backstop regime, the rate of innovation is strictly higher than it would have been without the availability of a backstop technology.

Third, the introduction of the backstop technology has notable implications for the development of resource extraction over time. The resource extraction path does no longer have to become downward-sloping eventually. Depending crucially on the elasticity of substitution between energy and man-made inputs, the extraction path can be monotonically upward-sloping or downward-sloping until exhaustion, or exhibit an internal maximum, known as "peak-oil."

The most important direction for further research is the introduction of stock-dependent extraction costs and pollution from combustion of the natural resource. In combination with the backstop technology, these features make it interesting to compare the decentralized outcome to the social optimum, in order to shed light on optimal environmental policy. Another useful extension of the current analysis would be the introduction R&D activities in the resource and backstop sector, so that the direction of technological progress becomes endogenous.

NOTES

1. The DHSS model consists of Dasgupta and Heal (1974), Solow (1974a,b), and Stiglitz (1974a,b). Recently, Benchekroun and Withagen (2011) have developed a technique to calculate a closed-form solution.

2. Following Dixit and Stiglitz (1977), a natural resource is defined to be "necessary" if production is zero without input of the resource.

3. Time arguments are omitted if there is no possibility of confusion.

4. Appendix A.1 derives the relative factor demand by solving the profit maximization problem of final good producers.

5. In Section 4.2, we discuss the introduction of a specific tax on oil extraction, which has similar effects as a positive constant extraction costs.

6. We abstract from investment in the backstop technology sector, implying that firms in the backstop sector do not need to be forward-looking. The case with investment and forward-looking firms in the backstop technology sector is studied in Tsur and Zemel (2011). They, however, abstract from scarcity of fossil resources to keep their model tractable. In terms of the energy transition, both approaches result in similar outcomes.

7. Ang and Madsen (2015) provide estimates of the "ideas production function" using data over the past 140 years. They find empirical evidence for strong knowledge spillovers, with a coefficient for the knowledge stock consistently close to one, as we have implicitly assumed in (8).

8. The assumption of $A_H = N^{\phi}$ implies Hicks-neutral technological change between intermediate goods and the backstop technology. Technically, the assumption ensures that the energy income share is constant in the backstop regime, as discussed in Section 3.3. Making this assumption is equivalent to assuming that the backstop is produced by using final output instead of labor.

9. Appendix A.1 derives the solution to the utility maximization problem of the representative household.

10. Appendix A.3 shows that point E in Figure 1 is the only attainable steady state of the model without a backstop technology that satisfies the transversality condition.

11. Note from (19c) that $\frac{\partial g|_{\hat{R}=0}}{\partial \sigma} < 0$ and $\lim_{\sigma \to 1} g|_{\hat{R}=0} = -\infty$, so that extraction would be declining throughout with Cobb–Douglas production.

12. We use the conventional shortcut notation $x_{ij}^{+} \equiv \lim_{t \downarrow T_{ij}} x(t)$ and $x_{ij}^{-} \equiv \lim_{t \uparrow T_{ij}} x(t)$.

13. If $\frac{\theta_B - \phi^2}{1-\theta_B} \frac{L}{a} = \rho$, the simultaneous use regime is degenerate with $T_{FS} = T_{SB}$.

14. The growth part of equation (27a) could be used to plot a consumption growth isocline $g|_{\hat{C}=0} = g|_{\dot{\theta}=0} - \frac{\rho}{1-\theta}$ in the phase diagrams of Figure 1 and panel (a) of Figure 2, which divides the phase diagrams in two regions: one with positive and one with negative consumption growth. This isocline has a vertical intercept between the extraction and income share locus and tends to minus infinity if the energy income share tends to unity.

15. For the simulation, we use the relaxation algorithm reported in Trimborn et al. (2008).

16. The specification of the CES function is $E = \left[\bar{\omega}(A_H H)^{(\gamma-1)/\gamma} + (1-\bar{\omega})R^{(\gamma-1)/\gamma}\right]^{\gamma/(\gamma-1)}$ with $\bar{\omega} = 0.9$ and $\gamma = 50$.

17. Chen (2017) develops a new empirical strategy to estimate the elasticity of substitution in the presence of biased technical change. He, however, applies his method to capital-labor instead of capital-labor-energy substitution.

18. Using (4), this ratio is given by $\beta/\eta[\bar{\theta}/(1-\bar{\theta})]^{\sigma/(1-\sigma)}[(1-\theta(0))/\theta(0)]^{1/(1-\sigma)}$.

19. The levelized costs of energy are defined as "the per-kilowatthour cost (in real dollars) of building and operating a generating plant over an assumed financial life and duty cycle" [U.S. Energy Information Administration (2015a, p. 1)].

20. Our model features two different market failures: imperfect competition in the market for intermediate goods and intertemporal knowledge spillovers in the research sector. As a result, implementing the social optimum in a decentralized equilibrium requires a subsidy to the production of intermediate goods and a research subsidy.

21. The introduction of s_H, s_N, and τ_R will affect the equations (13), (16a)–(17a), (18b), (19a), (19b)–(20), (22), (23)–(26), (28), and (29) as well. Details are available from the authors upon request.

22. The simultaneous regime can only exist if the g_{FB}^{-} line intersects the income share locus of the fossil regime, which requires $\phi < 1$.

REFERENCES

Ang, J. B. and J. B. Madsen (2015) What drives ideas production across the world? *Macroeconomic Dynamics* 19, 79–115.

Barbier, E. (1999) Endogenous growth and natural resource scarcity. *Environmental & Resource Economics* 14(1), 51–74.

Benchekroun, H. and C. Withagen (2011) The optimal depletion of exhaustible resources: A complete characterization. *Resource and Energy Economics* 33(3), 612–636.

BP (2015) *BP Statistical Review of World Energy*. London: BP p.l.c.

Bretschger, L. and S. Smulders (2012) Sustainability and substitution of exhaustible natural resources: How structural change affects long-term R&D-investments. *Journal of Economic Dynamics and Control* 36(4), 536–549.

Chakravorty, U., A. Leach, and M. Moreaux (2012) Cycles in nonrenewable resource prices with pollution and learning-by-doing. *Journal of Economic Dynamics and Control* 36(10), 1448–1461.

Chen, X. (2017) Biased technical change, scale, and factor substitution in U.S. manufacturing industries. *Macroeconomic Dynamics* 21(2), 488–514.

Dasgupta, P. and G. Heal (1974) The optimal depletion of exhaustible resources. *Review of Economic Studies* 41, 3–28.

Dasgupta, P. and J. Stiglitz (1981) Resource depletion under technological uncertainty. *Econometrica* 49(1), 85–104.

Dixit, A. K. and J. E. Stiglitz (1977) Monopolistic competition and optimum product diversity. *American Economic Review* 67(3), 297–308.

Ethier, W. J. (1982) National and international returns to scale in the modern theory of international trade. *American Economic Review* 72(3), 389–405.

Hoel, M. (1978) Resource extraction, substitute production, and monopoly. *Journal of Economic Theory* 19(1), 28–37.

International Energy Agency (2015a) *Medium-Term Renewable Energy Report*. Paris: IEA Publications.

International Energy Agency (2015b) *Projected Costs of Generating Electricity*. Paris: IEA Publications.

Kamien, M. I. and N. L. Schwartz (1978) Optimal exhaustible resource depletion with endogenous technical change. *Review of Economic Studies* 45(1), 179–196.

Koetse, M. J., H. L. de Groot, and R. J. Florax (2008) Capital-energy substitution and shifts in factor demand: A meta-analysis. *Energy Economics* 30(5), 2236–2251.

Nordhaus, W. D. (1973) The allocation of energy resources. *Brookings Papers on Economic Activity* 4(3), 529–576.

Roeger, W. (1995) Can imperfect competition explain the difference between primal and dual productivity measures? Estimates for U.S. manufacturing. *Journal of Political Economy* 103(2), 316–30.

Romer, P. M. (1987) Growth based on increasing returns due to specialization. *American Economic Review* 77(2), 56–62.

Romer, P. M. (1990) Endogenous technological change. *Journal of Political Economy* 98(5), S71–102.

Scholz, C. and G. Ziemes (1999) Exhaustible resources, monopolistic competition, and endogenous growth. *Environmental & Resource Economics* 13(2), 169–185.

Sinn, H.-W. (2008) Public policies against global warming: A supply side approach. *International Tax and Public Finance* 15(4), 360–394.

Solow, R. M. (1974a) The economics of resources or the resources of Economics. *American Economic Review* 64(2), 1–14.

Solow, R. M. (1974b) Intergenerational equity and exhaustible resources. *Review of Economic Studies* 41, 29–45.

Stiglitz, J. (1974a) Growth with exhaustible natural resources: Efficient and optimal growth paths. *Review of Economic Studies* 41, 123–137.

Stiglitz, J. E. (1974b) Growth with exhaustible natural resources: The competitive economy. *Review of Economic Studies* 41, 139–152.

Stiglitz, J. E. and P. Dasgupta (1982) Market structure and resource depletion: A contribution to the theory of intertemporal monopolistic competition. *Journal of Economic Theory* 28(1), 128–164.

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Tahvonen, O. and S. Salo (2001) Economic growth and transitions between renewable and nonrenewable energy resources. *European Economic Review* 45(8), 1379–1398.

The Conference Board (2012) *Total Economy Database*. New York: The Conference Board, Inc.

Trimborn, T., K.-J. Koch, and T. M. Steger (2008) Multidimensional transitional dynamics: A simple numerical procedure. *Macroeconomic Dynamics* 12(03), 301–319.

Tsur, Y. and A. Zemel (2003) Optimal transition to backstop substitutes for nonrenewable resources. *Journal of Economic Dynamics and Control* 27(4), 551–572.

Tsur, Y. and A. Zemel (2005) Scarcity, growth and R&D. *Journal of Environmental Economics and Management* 49(3), 484–499.

Tsur, Y. and A. Zemel (2011) On the dynamics of competing energy sources. *Automatica* 47(7), 1357–1365.

U.S. Energy Information Administration (2009) *Performance Profiles of Major Energy Producers*. Washington, DC: U.S. Department of Energy.

U.S. Energy Information Administration (2012) *International Energy Statistics*. Washington, DC: U.S. Department of Energy.

U.S. Energy Information Administration (2015a) *Levelized Cost and Levelized Avoided Cost of New Generation Resources in the Annual Energy Outlook 2015*. Washington, DC: U.S. Department of Energy.

U.S. Energy Information Administration (2015b) *Petroleum & Other Liquids Spot Prices*. Washington, DC: U.S. Department of Energy.

Valente, S. (2011) Endogenous growth, backstop technology adoption, and optimal jumps. *Macroeconomic Dynamics* 15(03), 293–325.

Van der Meijden, G. and J. Smulders (2017) Carbon lock-in: The role of expectations. *International Economic Review*, forthcoming.

Van der Ploeg, F. and C. Withagen (2012a) Is there really a green paradox? *Journal of Environmental Economics and Management* 64(3), 342–363.

Van der Ploeg, F. and C. Withagen (2012b) Too much coal, too little oil. *Journal of Public Economics* 96(1), 62–77.

Van der Ploeg, F. and C. Withagen (2015) Global Warming and the green paradox: A review of adverse effects of climate policies. *Review of Environmental Economics and Policy* 90(2), 285–303.

Van der Werf, E. (2008) Production functions for climate policy modeling: An empirical analysis. *Energy Economics* 30(6), 2964–2979.

World Bank (2015) *World Development Indicators*. Washington, DC: World Bank.

APPENDIX

This appendix contains the derivations of the mathematical results in the main text.

A.1. HOUSEHOLD AND FIRM BEHAVIOR

The Lagrangian associated with the profit maximization problem of firms in the final output sector reads

$$\mathcal{L} = p_Y \left[\bar{\theta} E^{(\sigma-1)/\sigma} + (1-\bar{\theta}) \left(\int_0^N k_j^{\beta} dj \right)^{(\sigma-1)/\beta\sigma} \right]^{\sigma/(\sigma-1)} - \int_0^N p_{Kj} k_j dj - p_R R - p_H H + p_E \left(R + N^{\phi} H - E \right). \tag{A.1}$$

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The first-order conditions are as follows:

$$\frac{\partial \mathcal{L}}{\partial k_j} = p_Y A \left[\bar{\theta} E^{(\sigma-1)/\sigma} + (1-\bar{\theta}) \left(\int_0^N k_j^{\beta} dj \right)^{(\sigma-1)/\beta\sigma} \right]^{\sigma/(\sigma-1)-1} \times (1-\bar{\theta}) K^{-1/\sigma} N^{-\phi(1-\sigma)/\sigma} - p_K = 0, \quad \textbf{(A.2a)}$$

$$\frac{\partial \mathcal{L}}{\partial E} = p_Y A \left[\bar{\theta} E^{(\sigma-1)/\sigma} + (1-\bar{\theta}) \left(\int_0^N k_j^{\beta} dj \right)^{(\sigma-1)/\beta\sigma} \right]^{\sigma/(\sigma-1)-1} \bar{\theta} E^{-1/\sigma} - p_E = 0, \quad \textbf{(A.2b)}$$

$$\frac{\partial \mathcal{L}}{\partial R} = p_E - p_R \leq 0, \quad (p_E - p_R) R = 0, \quad \textbf{(A.2c)}$$

$$\frac{\partial \mathcal{L}}{\partial H} = p_E N^{\phi} - p_H \leq 0, \quad (p_E N^{\phi} - p_H) H = 0, \quad \textbf{(A.2d)}$$

where we have used $p_{Ki} = p_{Kj} \equiv p_K, \forall i, j$. Combining (A.2a)–(A.2d) with $H = 0$ ($R = 0$) gives the first (second) row in (4). The third row of (4) follows from combining (A.2a)–(A.2d) with $p_H N^{-\phi} = p_R$ imposed.

The Hamiltonian associated with the utility maximization problem of the representative household reads

$$\mathcal{H} = \ln C + \lambda_V \left[rV + p_R R + wL^S - p_Y C \right] - \lambda_S R. \quad \textbf{(A.3)}$$

According to the Maximum Principle, the necessary first-order conditions for an optimum are

$$\frac{\partial \mathcal{H}}{\partial C} = 0 \Rightarrow \frac{1}{C} - \lambda_V p_Y = 0 \Rightarrow \hat{C} + \hat{p}_Y = -\hat{\lambda}_V, \quad \textbf{(A.4a)}$$

$$\frac{\partial \mathcal{H}}{\partial R} = 0 \Rightarrow \lambda_V p_R - \lambda_S = 0 \Rightarrow \hat{\lambda}_V + \hat{p}_R = \hat{\lambda}_S, \quad \textbf{(A.4b)}$$

$$\frac{\partial \mathcal{H}}{\partial S} = -\dot{\lambda}_S + \rho\lambda_S \Rightarrow \dot{\lambda}_S + \rho\lambda_S = 0 \Rightarrow \hat{\lambda}_S = \rho, \quad \textbf{(A.4c)}$$

$$\frac{\partial \mathcal{H}}{\partial V} = -\dot{\lambda}_V + \rho\lambda_V \Rightarrow \lambda_V r = -\dot{\lambda}_V + \rho\lambda_V \Rightarrow \hat{\lambda}_V = \rho - r. \quad \textbf{(A.4d)}$$

Combining (A.4a) and (A.4d) gives the Ramsey rule (15a). The first-order conditions (A.4b)–(A.4d) yield the Hotelling rule (15b).

A.2. PROOFS OF LEMMATA AND PROPOSITIONS

Proof of Lemma 1. By substituting the labor market equilibrium (13) with $\omega = 0$ imposed into (11), we find expression (16c) for the return to innovation in the fossil regime. We use the expenditure share definitions in (12) to rewrite the first line of the relative demand function (4):

$$\frac{\theta}{1-\theta} = \left(\frac{p_R}{p_K} \right)^{1-\sigma} \left(\frac{\bar{\theta}}{1-\bar{\theta}} \right)^{\sigma} N^{\phi(1-\sigma)} \Rightarrow \hat{\theta} = (1-\theta)(1-\sigma)\left(r - \hat{w} + \phi g \right). \quad \textbf{(A.5)}$$

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This completes the derivation of expression (16a) in Proposition 1. To obtain the second expression in the proposition, we first differentiate the labor market equilibrium condition (13) to get

$$\hat{K} = -\frac{\dot{g}}{(L/a) - g}. \tag{A.6}$$

By converting the energy income share definition (12) into growth rates while using the intermediate goods price (5) and the Ramsey rule (15a), we obtain

$$\hat{\theta} = -\frac{1-\theta}{\theta}\left[\hat{w} + \hat{K} - (r - \rho)\right]. \tag{A.7}$$

Combining (A.5), (A.6), and (A.7), we find (16b) in Lemma 1. ■

Proof of Lemma 2. In the simultaneous use regime, the effective prices of the resource and the backstop technology must be equal, as in the third line of (4):

$$p_H N^{-\phi} = p_R. \tag{A.8}$$

Substitution of $p_E = p_H N^{-\phi}$ and (A.8) into the third line of the relative demand function (4) and by using $p_K/p_H = \eta(1+\phi)$ from (5) and (7) gives

$$\frac{\theta}{1-\theta} = [(1+\phi)\eta]^{\sigma-1}\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^{\sigma}, \tag{A.9}$$

which implies $\theta = \theta_S$ and therefore proves the first part of the lemma. To proof the second part, we convert (A.8) into growth rates:

$$\hat{p}_H - \phi g = \hat{p}_R \Rightarrow r - \hat{w} + \phi g = 0, \tag{A.10}$$

where the latter expression uses (7) and (15b). Substituting (A.10) into (11), we find

$$-\phi g = \phi\frac{K}{a} - g. \tag{A.11}$$

Using (5), (12), (15a), (15b), and $\hat{\theta} = 0$ together with (A.11), we obtain

$$\hat{g} = \hat{K} = -\phi g - \rho, \tag{A.12}$$

which gives rise to the differential equation in Lemma 2. ■

Proof of Lemma 3. Using $p_K/p_H = \eta(1+\phi)$ from (5) and (7), the relative factor demand function (4) gives

$$\left(\frac{\theta}{1-\theta}\right) = [\eta(1+\phi)]^{\sigma-1}\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^{\sigma}, \tag{A.13}$$

which can be solved for θ to obtain θ_B. Combining the innovation return (11), the income share definition (12), labor market equilibrium (13), the Ramsey rule (15a), and the relative demand function (A.13), we find a differential equation for the innovation rate:

$$\dot{g} = -\left(\frac{L}{a} - g\right)\left[\phi\left(\frac{1-\theta_B}{\theta_B(1+\phi) + 1 - \theta_B}\right)\left(\frac{L}{a} - g\right) - g - \rho\right]. \tag{A.14}$$

Because this differential equation is unstable in g, the innovation rate immediately settles down at its steady-state value given by the second expression in Lemma 3. ∎

Proof of Proposition 2. The case in which the economy relies upon the resource forever without switching to the backstop technology can be excluded, because eventually $\theta > \theta_B$ would hold, implying that the backstop technology is cheaper than the resource. Hence, there exists a time at which the fossil regime ends. If there would not exist a regime of simultaneous use, continuity of consumption requires that the end point of the fossil regime would be given by (θ_B, g_{FB}^-). However, the inequality in the proposition implies $g_{FB}^- > \phi \frac{L}{a}$. Therefore, the dynamic path in the fossil regime that leads to (θ_B, g_{FB}^-) would necessarily intersect the vertical θ_B line before the fossil regime has ended. This would imply that only the resource is being used while the backstop technology is relatively cheaper, which violates optimality of the behavior of final good producers. As a result, there exists a time T_{FS} at which the economy shifts from the fossil to the simultaneous use regime. The simultaneous use regime cannot last forever, because the innovation rate is decreasing throughout a regime of simultaneous use, according to (17b), while (13) and (11) together with $r - \hat{w} = -\phi g$ imply a strictly positive lower bound on g due to $\omega \leq 1$. Therefore, there exists a time $T_{SB} \geq T_{FS}$ at which the economy shifts from the simultaneous use to the backstop regime.

We continue by showing that the innovation rate is continuous at T_{FS}. Together with the labor market equilibrium (13) with $\omega_{FS}^- = 0$, the continuity of output requires

$$L - ag_{FS}^- = \frac{1-\theta_B}{\omega_{FS}^+\theta_B(1+\phi) + 1 - \theta_B}\left(L - ag_{FS}^+\right). \tag{A.15}$$

Substituting the labor market equilibrium (13) into the innovation return equation (11) and noting that $r - \hat{w} = -\phi g$, we get

$$\omega = \frac{\phi L - ag}{ag(1+\phi)(1-\phi)} \frac{1-\theta_S}{\theta_S}. \tag{A.16}$$

Using this relationship to substitute for ω_{FS}^+ in (A.15), the matching condition reduces to

$$L - ag_{FS}^- = \frac{a}{\phi}(1-\phi)g_{FS}^+. \tag{A.17}$$

We have already argued that the innovation rate g_{FS}^- cannot exceed $\phi L/a$. Moreover, given that $\omega \geq 0$, it follows from (A.16) that the innovation rate g_{FS}^+ cannot exceed $\phi L/a$ either. Consequently, the only solution to (A.17) reads $g_{FS}^- = g_{FS}^+ = \phi L/a$.

To prove the downward jump of the innovation rate at T_{SB}, first note that, as a result of the required continuity of output, the labor market equilibrium (13) with $\omega_B = 1$ implies

$$\frac{1-\theta_S}{\omega_{SB}^-\theta_S(1+\phi) + 1 - \theta_S}\left(L - ag_{SB}^-\right) = \frac{1-\theta_B}{\theta_B(1+\phi) + 1 - \theta_B}\left(L - ag_B\right).$$

Substitution of (A.16) for ω_{SB}^- on the left-hand side and (18b) for g_B on the right-hand side, gives equation (26), where $g_{SB}^- > 0$ follows from $\phi < 1$, which is required for the simultaneous use regime to exist.[22] Subtracting (18b) from (26), we find

$$g_{SB}^- - g_B = \phi g_{SB}^- + \rho > 0,$$

implying that the innovation rate jumps down at T_{SB}.

Finally, we show that the real interest rate equals zero, that the backstop expenditure share increases, and that resource extraction decreases over time during the simultaneous use regime. Using $\hat{p}_Y = \theta r + (1-\theta)(\hat{w} - \phi g)$, the real interest rate can be written as

$$r - \hat{p}_Y = -\theta_S (r - \hat{w} + \phi g) = 0, \tag{A.18}$$

where the second equality follows from (A.10). Taking the time derivative of (A.16), we find

$$\hat{\omega} = \frac{L/a}{L/a - g} \frac{\beta(1-\theta_S) + \omega\theta_S}{\omega\theta_S}(\phi g + \rho) > 0. \tag{A.19}$$

By using the expenditure share definition (12), the Hotelling rule (15b), the backstop price (7), and $\hat{E} = \hat{K} = -\phi g - \rho$, we obtain

$$\hat{R} = -\frac{\omega}{1-\omega}\hat{\omega} - \rho = -\frac{\phi^2(1-\theta_S)L + a\left[(1-\phi)(1+\phi) + \phi^2(1-\theta_S)\right]\rho}{a\theta_S(1-\phi)(1-\omega)(1+\phi)} < 0, \tag{A.20}$$

where the last equality uses (A.16) and the labor market equilibrium (13). ■

A.3. STEADY STATES

Here, we show that point E in Figure 1 is the only attainable steady state of the model without a backstop technology that satisfies the transversality condition.

PROPOSITION 3. *The only attainable internal steady state of the model without a backstop technology that satisfies the transversality conditions is given by point E in Figure 1.*

Proof. Using asterisks (*) to denote steady-state values of this model, the other three steady states of the model satisfy

$$g^* = \frac{L}{a}, \quad \theta^* = 1, \tag{A.21a}$$

$$g^* = \frac{L}{a}, \quad \theta^* = 0, \tag{A.21b}$$

$$g^* = \frac{\phi}{1+\phi}\frac{L}{a} - \beta\rho, \quad \theta^* = 0. \tag{A.21c}$$

The first two steady states (A.21a) and (A.21b) do not satisfy the transversality condition, because substitution of $K^* = L - ag^* = 0$ into (16c) implies $(r - \hat{w})^* = -g^* < 0$ and the transversality condition in growth rates requires

$$\lim_{t\to\infty} = \hat{p}_N(t) + \hat{N}(t) - r(t) \leq 0 \Rightarrow \lim_{t\to\infty} r(t) - \hat{w}(t) \geq 0, \tag{A.22}$$

where the second inequality uses (9). Hence, the two steady states with $(r-\hat{w})^* = -g^* < 0$ do not satisfy the transversality condition. Steady state (A.21c) is located at the intersection of the innovation locus with the $\theta = 0$ line, and below the income share locus in the (θ, g)-plane. It is immediately clear from the dynamics around this point in Figure 1 ($\dot{\theta} > 0$) that this steady state cannot be attained. The economy can only be situated here if there is an infinite amount of oil available from the beginning (so that $\theta^* = 0$), which is impossible. Point E in Figure 1 satisfies the transversality condition, as $(r - \hat{w})^* = \rho > 0$ in this equilibrium. ■

A.4. INITIAL CONDITION

By using (11)–(13), $r - \hat{w} = -\phi g$ in the simultaneous use regime, and (16c) in the fossil regime, differential equation (29) can be expressed as

$$\begin{aligned} \dot{y} &= -y(1-\theta)(1-\sigma)\left(\phi \tfrac{L}{a} - g\right) + y\rho - 1, && \text{if } t < T_{FS}, \\ \dot{y} &= -1 + y\rho, && \text{if } T_{FS} \leq t \leq T_{SB}. \end{aligned}$$

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