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Low velocity ion slowing down in a de-mixing binary ionic mixture

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Abstract

We consider ion projectile slowing down at low velocity $V_p < V_{the}$, target thermal electron velocity, in a strongly coupled and de-mixing H-He ionic mixture. It is investigated in terms of quasi-static and critical charge-charge structure factors. Non-polarizable as well as polarizable partially degenerate electron backgrounds are given attention. The low velocity ion slowing down turns negative in the presence of long wavelength and low frequency hydromodes, signaling a critical de-mixing. This process documents an energy transfer from target ion plasma to the incoming ion projectile.

Keywords: Critical de-mixing; Binary ionic mixtures; Low velocity ion stopping

INTRODUCTION

A sustained and widespread interest is currently documented for low velocity ion slowing down (LVISD) in binary ionic mixtures (BIM) (Tashev *et al.*, 2008; Fromy *et al.*, 2010) fully neutralized by an electron fluid jellium (Leger & Deutsch, 1988*a*, 1988*b*, 1992).

We pay specific attention to the hydrogen-helium strongly coupled mixture of sustained astrophysical concern (Stevenson, 1975; Stevenson & Salpeter, 1977; Vorberger *et al.*, 2007; Lorenzen *et al.*, 2009). Up to now, most of these studies were conducted through, a weakly coupled BIM immersed into a classical high temperature electron background taken in a dielectric Fried-Conte description (Fried & Conte, 1961; Arista & Brandt, 1981). Specifying BIM ion coupling on species i ($i = 1, 2$) with charge Z_i one has

$$\Gamma_i = \frac{Z_i^2 e^2}{k_B T a_i}, \qquad (1)$$

where $a_i = ((4\pi/3)\ \mathrm{n_i})^{-1/3}$ with temperature $T = T_e = T_i$. Here, we stress strongly coupled BIM with $\Gamma_i \gg 1$, able to display a critical de-mixing process (2, 4).

Pertaining thermodynamic and hydrodynamic features have already been investigated at length (Leger & Deutsch, 1988*a*, 1988*b*, 1992). In this context, a critical de-mixing behavior is observed on the ion-ion structure factor $S_{\alpha\beta}(q)$ in the long-wavelength $q \to 0$ limit. As usual, LVISD amounts to evaluating low velocity ion stopping with neglected intrabeam scattering and in-flight correlation, in a low velocity regime $V_p \ll V_{the}$ target electron thermal velocity. On the other hand, the opposite $V_p > V_{the}$ situation is now rather well documented in the so-called standard stopping model (Deutsch, 1986; Deutsch *et al.*, 1989) where most of the ion projectile stopping arises from its interaction with target electron fluid. To the contrary, LVISD is essentially monitored by the projectile interaction with target ions. A foremost motivation for the present undertaking is a possible involment of LVISD in BIM in probing and testing the ion de-mixing process. In the past, some of us (Leger & Deutsch, 1988*a*, 1988*b*, 1992) have already investigated a critical BIM de-mixing signature on target electrical resistivity. Within a dielectric framework for target particles, the nonrelativistic ion stopping thus reads as (Arista & Brandt, 1981).

$$S = -\frac{dE}{dx} = -\frac{1}{V_p}\left(\frac{dE}{dt}\right)_0 = \frac{2}{\pi}\left(\frac{Z_p e}{V_p}\right)^2 \int_0^\infty \frac{dq}{q} \int_o^{qV_p} d\omega\omega\ \mathrm{Im}\left(-\frac{1}{\varepsilon(q,\omega)}\right), \qquad (2)$$

which can be straightforwardly re-expressed in terms of the ion charge-ion charge structure factor when switching to

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very low

$$V_p \leq \overline{V_{thi}} = C_1 V_{th1} + C_2 V_{th2},$$

with V_{thi}, thermal velocity and C_i, relative concentration of ion i.

Incoming ion projectile could then be able to probe every available ion fluid fluctuations in target by restricting the global dielectrics expression ε(q,ω) to its *i*-component. Such a picture highlights the BIM electron background following the ionic fluctuations, within a polarization concept.

Energy exchange between ion projectile and thermalized medium is expressed in terms of emission and absorption processes, according to the protocol (Pines, 1964)

$$\left(\frac{dE}{dt}\right)_0 = \int_{\omega>0} d^3\vec{q} N(\omega) f(\vec{q},\omega) - \int_{\omega>0} d^3\vec{q}(N(\omega)+1) f(\vec{q},\omega)$$

yielding, up to the first order in the interaction, a quarter involving only spontaneous emission. The present approach stands at variance with the so-called *T*-matrix one (Gericke *et al.*, 1996; Gericke & Schlanges, 1999) advocating a strong binary interaction between incoming ion projectile and one target electron thus probing mostly short wavelength modes with $q \to \infty$. Then,

$$S_{zz}(\vec{q},\omega) = \frac{\hbar q^2}{4\pi^2 e^2} N(\omega) \mathrm{Im}\left(-\frac{1}{\varepsilon(\mathrm{q},\omega)}\right), \tag{3}$$

where $N(\omega)^{-1} = e^{\beta\hbar\omega} - 1$, $\beta = (k_B T)^{-1}$ allow to put Eq. (2) under the form

$$S = \frac{2}{\pi}\left(\frac{Z_p e}{V_p}\right)^2 \int_0^\infty \frac{dq}{q} \int_o^{qV_p} d\omega \frac{4\pi^2 e^2 \omega}{\hbar q^2} \times S_{zz}(\vec{q},\omega)\left(e^{\beta\hbar\omega} - 1\right), \tag{4}$$

with the usual electron ε(q,ω) now extended to the ion components building up the BIM, and the charge-charge structure factor

$$S_{zz}(q) = \sum_{\alpha,\beta=1}^{2} (c_\alpha c_\beta)^{1/2} Z_\alpha Z_\beta S_{\alpha\beta}(q), \tag{5}$$

where $C_1 + C_2 = 1$. C_i refers to concentration of species i within a BIM built on target ion charges Z_1 and Z_2.

Focusing attention on the slow and long wavelength hydromodes $\omega \to 0$, $q \to 0$ monitoring BIM de-mixing, one can safely restrict Eq. (4) to its static limit $\omega \to 0$

$$S = \frac{8\pi}{3}(Z_p e^2)^2 \beta V_p \int_0^\infty dq S_{zz}(q) \tag{6}$$

in-terms of $S_{zz}(q) = \int_0^\infty d\omega S_{zz}(q,\omega)$ Eq. (6) also highlights the expected LVISD linear $V_p -$ dependence $\leq \overline{V_{thi}}$ Eq. (6) implies an average over every ω-fluctuation, available to $S_{zz}(\vec{q},\omega)$.

CRITICAL $S_{ZZ}(q)$

Mean field classical description of BIM de-mixing could be rather straightforwardly explained with the static charge-charge structure factor (De Gennes & Friedel, 1958; Fisher & Langer, 1968),

$$S_{ZZ} = \frac{\Sigma |t|^{-\gamma}(qa_i)^2}{3\bar{\Gamma}\bar{Z}^2\left((q\xi)^2+1\right)}, \tag{7}$$

where $t = (T - T_c/T_c)$, T_c = critical temperature, $\gamma = 1$, and ξ = ion-ion correlation length featuring $\lim_{|t|\to 0} \xi \to \infty$. $\bar{Z} = C_1 Z_1 + C_2 Z_2$ and $\bar{\Gamma} = C_1\Gamma_1 + C_2\Gamma_2$. Σ denotes a constant normalizing factor accessed through the sum rule (q in $\bar{a}_i^{-1}$)(where $\bar{a}_i = C_1 a_1 + C_2 a_2$)

$$\int_0^\infty dq q^2 \left[S_{zz}(q) - \overline{z^2}\right] = -\frac{3\pi}{2}, \tag{8}$$

where $\overline{z^2} = 1 + C_1 C_2 (Z_1 - Z_2)^2/\bar{z}^2$.

Eq. (7) mostly emphasizes long distance hydromodes, of significance at critical de-mixing.

Paying attention first to non-polarizable BIM with a fixed and rigid electron background (Leger & Deutsch, 1988*a*, 1988*b*), the correlation length reads as

$$\left(\frac{\bar{a}_i}{\xi}\right)^2 = 3\bar{\Gamma}\,\bar{z}^2 \frac{D_I}{D_R}, \tag{9}$$

where D_I and D_R, respectively, denote the $q \to 0$ limit of

$$D_I(q) = \bar{Z}^2 - C_1 C_2\left[Z_1^2 \hat{C}_{22}^R(q) + Z_2^2 \hat{C}_{11}^R(q) - 2Z_1 Z_2 \hat{C}_{22}^R(q)\right], \tag{10}$$

and

$$D_R(q) = 1 - c_1 \hat{C}_{11}^R(q) - c_2 \hat{C}_{22}^R(q) - c_1 c_2 \det\left|\hat{C}_{\alpha\beta}^R(q)\right|, \tag{11}$$

in terms of

$$\hat{C}_{\alpha\beta}^R(q) = \hat{C}_{\alpha\beta}^R(q) + Z_\alpha Z_\beta \hat{\nu}(q), \tag{12}$$

$\hat{C}_{\alpha\beta}^R(q)$ denotes the partial direct correlation function, viewed in the Ornstein-Zernikc (OZ) equations

$$\hat{h}_{\alpha\beta}(k) = \hat{C}_{\alpha\beta}(q) + \sum_{\nu=1}^{2} c_\nu \hat{h}_{a\nu}(q)\hat{C}_{\nu\beta}(q). \tag{13}$$

The right-hand side of Eq. (12) also features the dimensionless and screened Coulomb potential

$$\hat{\nu}(q) = \frac{4\pi\beta e^2 \bar{n}_i}{k^2 \varepsilon(q)} \equiv \frac{3\bar{\Gamma}}{\bar{a}_i^2}\frac{1}{k^2\varepsilon(q)}, \tag{14}$$

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with the dimensionless static electron fluid dielectric function ε(q). Close to criticality, one expects a characteristic diverging behavior of the correlation length, so that

$$\xi = \xi_0^{\pm} |t|^{-\nu}, t \to 0 \qquad (15)$$

with $\nu = 0.5$, in a standard mean field OZ approximation.

SUPERELASTIC LVISD

The introduction of Eq. (7) into Eq. (6), thus yields for $q\,\bar{a}_i \leq 1$

$$S = \frac{8\pi}{3}(Z_p e^2)^2 \cdot \frac{\beta V_p}{\bar{a}_i} \cdot \frac{\Sigma |t|^{-1}}{\bar{\Gamma}\bar{Z}^2}\left(\frac{\xi}{\bar{a}_i}\right)^2\left[1 - Tan^{-1}\left(\frac{\xi}{\bar{a}_i}\right)\right], \qquad (16)$$

demonstrating that ξ-diverging behavior (Eq. (15)) is compensated by that in Eq. (7).

Now we pay attention to correlation length (Eq. (9)) estimates by solving simultaneously (Leger & Deutsch, 1988*a*, 1988*b*) OZ Eq. (13) with the Hypernetted Chain (HNC) equations

$$g_{\alpha\beta}(r) = \exp\left[h_{\alpha\beta}(r) - (C^R_{\alpha\beta}(r)\right], \qquad (17)$$

valid for any Γ_i values.

Lindhard screening (Lindhard, 1954) involves (cf. Fig. 1)

$$\varepsilon(k) = \frac{k^2 + k_{TF}^2\, g_L(x)}{k^2}, \qquad (18)$$

where $k_{TF} = (6\pi n_e e^2/E_F)^{1/2}$ denotes the Thomas-Fermi wave vector and the function $g_L(x)$ depends only on the dimensionless variable $x = k/2k_F$, with

$$g_L(x) = \frac{1}{2} + \frac{1-x^2}{4x}\ell n\left|\frac{1+x}{1-x}\right|, \qquad (19)$$

and $k_F = (3\pi^2 n_e)^{1/3}$, Fermi wave number in terms of electron density n_e, while $E_F = \hbar_F^2 k_F^2/2m_e$. Moreover, Hubbard screening (Hubbard, 1966) with

$$g(x) = \frac{g_L(x)}{1 - \frac{k_{TF}^2}{k^2} g_L(x) G(x)}, \qquad (20)$$

where $G(x) = \frac{x^2}{2x^2+1/2}$, includes exchange-correlation contributions into the jellium background.

Present critical conditions are now significantly different. In Figure 2, critical de-mixing occurs at $C_2 = 0.75$ in lieu of $C_2 = 0.34$ in Figure 1 for the same $\Gamma = 60$. More importantly, ξ^2 is now allowed to increase positive on the largest r_s range side.

The resulting correlation length is now rising strongly when $|t| \to 0$, up to the standard means field behavior $(\xi/a_1)^2 \gg 1$.

The given LVISD is now negative featuring a super elastic interaction between the low velocity incoming ion projectile and the PBIMB target.

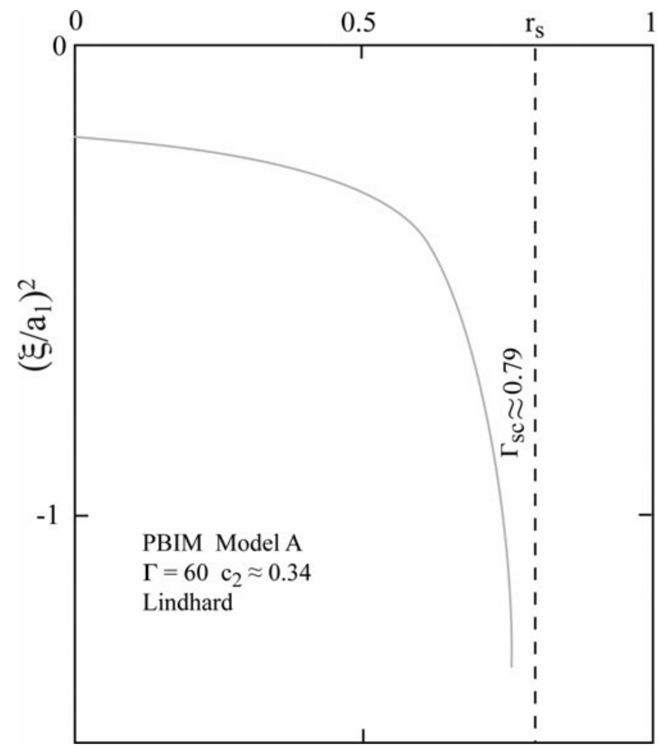

Fig. 1. Plot of the reduced squared correlation length $(\xi/a_1)^2$ in terms of r_s along a critical and vertical line (34% He, $\Gamma = 60$) with Lindhard screening. There ξ^2 remains negative.

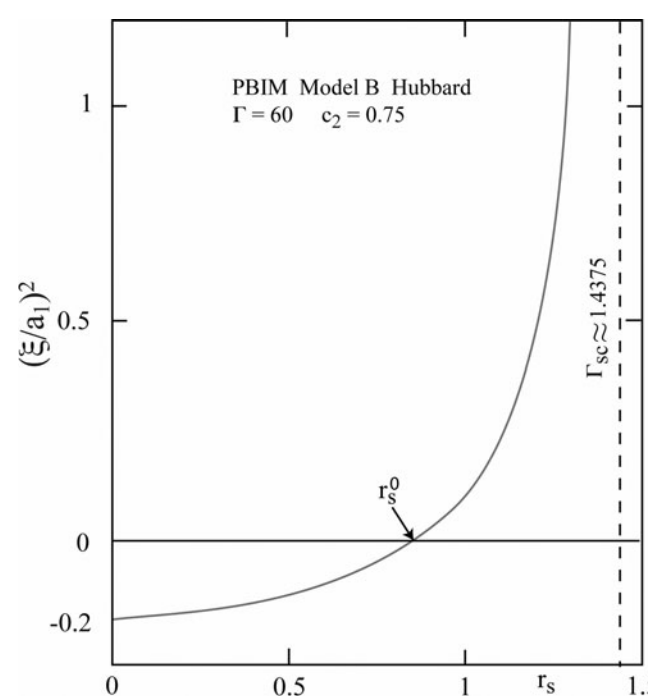

Fig. 2. PBIM model B with Hubbard screening. Plot of the reduced squared correlation length $(\xi/a_1)^2$ as a function of r_s along a critical and vertical line (75% He at $\Gamma = 60$) ξ^2 is now allowed to change sign for $r_s \geq r_s^0$.

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It can also be appreciated that before turning negative, LVISD (16) vanishes for $(\xi/a_1)^2 \sim 1.65$. The prefactor Σ in Eq. (16) is then straightforwardly derived from the sum rule (8) under the alternative forms,

$$\Sigma = \left(-\frac{3\pi}{2} + \overline{z^2}\right)^3 \frac{D_R|t|}{D_I} \tag{21}$$

in terms of BIM thermodynamics and also (cf. Eq. (15))

$$\Sigma = q\bar{\Gamma}^2\overline{z^2}\left(-\frac{3\pi}{2} + \overline{z^2}\right)^3 \left(\frac{\xi_0^{\pm}}{\bar{a}_i}\right) \tag{22}$$

FINAL REMARKS

The present developments highlight for the first time the intriguing interplay of a first order de-mixing process in a strongly coupled and binary ionic mixture with a low velocity incoming ion beam. The latter may be envisioned for diagnostics purposes or target conditioning in the subfields of ICF and warm dense matter, for instance.

Within a fundamental statistical physics perspective, it should be appreciated that the above results document unambiguously the potentialities of probing collective very long wavelength phenomena occurring in a plasma target with low velocity ion beams via the evaluation of a transport coefficient, featured in the present context by a stopping power mechanism.

As far as a super elastic projectile-target interaction is witnessed at very low V_p, it should be noticed that it also can appear in the T-matrix approach (Gericke *et al.*, 1996).

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REFERENCES

Arista, N.R. & Brandt, W. (1981). Energy-loss and straggling of charged-particles in plasmas of all degeneracies. *Phys. Rev. A* **23**, 1898–1905.

De Gennes, P.G. & Friedel, J. (1958). Anomalie de resistivité dans certains metaux magnétiques (Resistivity anomalies in magnetic metals). *J. Phys. Chem. Solids* **4**, 71–77.

Deutsch, C. & Leger, D. (1993). Electron transport with ion demixing. *Contrib. Plasma Phys*. **33**, 409–420.

Deutsch, C. (1986). Inertial confinement fusion driven by intense ion beams. *Ann. Phys. Paris* **11**, 1–111.

Deutsch, C., Maynard, G., Bimbot, R., Gardes, D., DellaNegra, S.,Dumail, M., Kubica, B., Richard, A., Rivet, M.F., Servajean, A., Fleurier, C., Sanba, A., Hoffmann, D.H.H., Weyrich, K. & Wahl, H. (1989). Ion stopping in dense plasmas. A standard model approach. *Nucl. Instrum. Meth. A* **278**, 38–43.

Fisher, M.E. & Langer, J.S. (1968). Resistive anomalies at magnetic critical points. *Phys. Rev. Lett*. **20**, 665–668.

Fromy, P., Tashev, B. & Deutsch, C. (2010). Very low velocity ion slowing down in binary ionic mixtures: Charge-and mass-asymmetry effects. *Phys. Rev. Special Topics Accelerators and Beams* **13**, 1013C2–8.

Fried, B.D. & Conte, S.D. (1961). *The Plasma Dispersion Function*. New York: Academic Press.

Gericke, D.O., Schlanges, M. & Kraeft, W.D. (1996). Stopping power of a quantum plasma T-matrix approximation and dynamical screening. *Phys. Lett. A* **222**, 241–245.

Gericke, D.O. and Schlanges, M. (1999). Beam-plasma coupling effect on the stopping power of dense plasmas. *Phys. Rev. E* **60**, 904–909.

Hubbard, W.B. (1966). Studies in stellar evolution. V. Transport coefficients of degenerate stellar matter. *Astrophys. J*. **146**, 858–869.

Leger, D. & Deutsch, C. (1988). Critical demixing in strongly coupled ionic mixtures . I. Weak electron-ion coupling and stability requirements. *Phys. Rev. A* **37**, 4916–4929.

Leger, D. & Deutsch, C. (1988). Critical demixing in strongly coupled ionic mixtures. II. Hydrodynamic correlation functions. *Phys. Rev. A* **37**, 4930–4942.

Leger, D. & Deutsch, C. (1992). Linear and electronic transport in strongly coupled ionic H-He mixtures. *Phys. Fluids B* **4**, 3162–3184.

Lindhard, J. (1954). Electron dielectric function. *Mat. Fys. Medd. K. Dan. Vidensk. Selsk*. **28**, 1.

Pines, D. (1964). *Elementary Excitations in Solids*. New York: Benjamin.

Lorenzen, W., Holst, B. & Redmer, R. (2009). Demixing of hydrogen and helium at Megabars pressure. *Phys. Rev. Lett*. **102**, 115701/4.

Stevenson, D.J. (1975). The thermodynamics and phase separation of dense fully ionized H-He fluid mixtures. *Phys. Rev. B* **12**, 3999–4007.

Stevenson, D.J. & Salpeter, E.E. (1977). Phase-diagram and transport properties for H-He fluid planets. *Astrophys. J. Suppl*. **35**, 221–235.

Tashev, B., Baimbetov, F., Deutsch, C. & Fromy, P. (2008). Low ion velocity slowing down in dense binary ionic mixtures. *Phys. Plasmas* **15**, 10270.

Vorberger, J., Tamblyn, I., Militzer, B. & Bonev, S.A. (2007). H-He mixtures in the interior of giant planets. *Phys. Rev. B* **75**, 024206.