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Kant on the Continuity of Alterations

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Abstract

The metaphysical “Law of Continuity of Alterations” (“LCA”) says that whenever an object alters from one state to another, it passes through a continuum of intermediate states. Kant treated LCA as a transcendental law of understanding. The primary purpose of the paper is to reconstruct and evaluate Kant’s three arguments for LCA. All three are found to be inadequate. However, a secondary goal of the paper is to show that LCA would have more naturally been construed as a regulative principle of reason (rather than a law of understanding). I conclude with some remarks about how this could work.

Keywords: Kant; continuity; alterations; time; causation; regulative principles

1. Introduction

From his earliest writings, Kant followed Leibniz¹ in defending what I’ll call the “Law of the Continuity of Alterations” (“LCA”). According to this metaphysical principle, any time an object alters from one state to another, it passes through a continuum of infinitely many intermediate states along the way.² This is an instance of the general principle that “*in mundo non datur saltus*” (A229/B281; cf. *Metaphysik Mrongovius*, 29:862; *Metaphysik Dohna*, 28:661),³ or roughly, “nature makes no leaps.” While he was content to make dogmatic use of the principle in his early works,⁴ it isn’t until the “Inaugural Dissertation” (from 1770) that Kant attempts to offer an argument for the principle. There, Kant suggests that the continuity of alterations follows as a consequence of the fact that time is continuous (i.e., that there is no smallest, indivisible unit of time [2:399]). The argument of the *Inaugural Dissertation* is repeated in metaphysics lectures in the 1780s and ’90s (see *Metaphysik Mrongovius* 29:920 and *Metaphysik Dohna* 28:662). Meanwhile, in *Critique of Pure*

¹Leibniz used the principle to argue against discontinuous change in motion (1996, 56), against absolutely solid atoms (1989, 131–32), and for the claim that all conscious states arise or diminish gradually (1996, 56). All of these anticipate Kant’s views.

²To put it in the terms I’ll be using throughout the paper, LCA says that: Any time an object *O* alters from state *a* at t_1 to *b* at t_2 , for any *c* between *a* and *b*, there is a t_3 between t_1 and t_2 such that *O* is *c* at t_3 . This statement of the principle formalizes the intuitive idea behind it, often expressed as the law that “nature makes no leaps.” On the one hand, it is clear that Kant takes LCA to rule out the possibility of discontinuities (“leaps”). For instance, in the discussion of the principle from the Second Analogy (which I’ll be discussing at length), he says that an altering object must pass “through *all* the infinite degrees of reality” (A209/B254; emphasis added). On the other, however, he sometimes characterizes continuity in terms that align more closely with what mathematicians nowadays call “density,” describing a series as continuous as long as between any two values in the series there is an intermediate one. Kant seems not to have realized that this in fact leaves open the possibility of discontinuities. This will turn out to be a problem for him in the first of the three arguments I reconstruct in the paper. (See note 17.)

³All quotations of Kant are taken from the *Cambridge Editions of the Works of Immanuel Kant*. Citations of *Critique of Pure Reason* are given with the standard A/B pagination. Citations of all other works by Kant are given by volume and page number of the *Akademie Ausgabe* editions.

⁴See *True Estimation of Living Forces* (1:37–38, 1:155–56) and *New Doctrine of Motion and Rest* (2:21–22).

Reason, Kant's interest in various forms of continuity is on full display. He discusses it in the Anticipations (A169/B211ff.), Analogies (A208/B253ff.), Postulates (A224/B271), Antinomies (A527/B555), and in the Appendix to the Dialectic (A658/B686ff.). Kant's most developed arguments for LCA occur towards the end of the Second Analogy, drawing on not just the metaphysics of time, but the metaphysics of causality and perceptual apprehension as well.

Kant makes it clear that he thinks that LCA occupies a privileged spot within his metaphysical system. In metaphysics lectures from the mid-1770s, he says that, "this law of continuity is no metaphysical whim, but rather a law that is spread through the whole of nature" (28:201), and that "this is the first law of nature, whose necessity can be comprehended a priori" (28:203). In the *Critique*, he claims to have established it as a "principle of transcendental origin" (A229/B282) and affirms that the law "rest[s] on pure transcendental and not empirical grounds" (A660/B688). I take these strong assertions to indicate that Kant was convinced that LCA is a transcendental law of understanding. That is, it is intended as an a priori law governing all possible objects of experience.⁵ The primary goal of this paper is to evaluate the basis of Kant's confidence that LCA can have such a lofty status.

The question whether Kant has a successful argument for LCA is important for at least two reasons. First, answering this question would help delineate the scope of his a priori metaphysics of nature. Kant holds that his system entails synthetic a priori principles regarding extensive and intensive magnitudes, substantiality, causality, and modality (i.e., the "principles of pure understanding"). He also holds that this system can be extended to natural science through the addition of the empirical concept of matter (which is the project of *Metaphysical Foundations of Natural Science*, about which I'll say more in a moment). But are there any other synthetic principles governing all possible objects of experience which (a) go beyond the original or "official" principles of pure understanding, but which (b) remain completely a priori? That is, can we know anything more about the necessary structure and behavior of objects than that they're causally efficacious substances with determinate extensive and intensive magnitudes? Or is Kant's pure metaphysics of nature restricted to this bare-bones structure? Kant's endorsement of LCA within the context of the *Critique's* Analytic of Principles indicates that he acknowledges such an expanded scope for his metaphysical system. Thus if Kant can successfully justify his confidence that LCA is a transcendental law of understanding, it would provide a case study in how his metaphysics can be extended. If not, it might be taken to provide some evidence that pure metaphysics must remain rather austere. But then again, it might also leave open the possibility that LCA could occupy a different place within Kant's system, for instance in reason's system of regulative principles and ideas, rather than understanding's system of laws. On this last option (considered in the final section of the paper), LCA would govern not objects themselves, but rather understanding's systematic investigation of objects and, in this sense, would retain a privileged place in Kant's philosophy of science. Moreover, if LCA did find a place within reason's system of principles, it would provide a hint that

⁵Some have taken a passage from the Anticipations of Perception to imply that LCA was never intended as an a priori principle in the first place. Kant writes, "The proposition that all alteration (transition of a thing from one state to another) is also continuous could be proved here easily and with mathematical self-evidence, if the causality of an alteration in general did not lie entirely beyond the boundaries of a transcendental philosophy and presuppose empirical principles. For the understanding gives no inkling a priori that a cause is possible which alters the state of things" (A171/B212–13). Guyer's take is that it is "the *Critique's* unqualified assertion that the continuity of change is only an empirical matter" (1987, 205; Kemp Smith [1918, 380] agrees). This reading is implausible though, since there are so many other places where Kant clearly assigns an a priori status to the principle. Further, it wouldn't be at all clear what Kant was getting at with the remark about the possible "mathematical self-evidence" of the principle. Thus I don't think the passage should be read as assigning an empirical status to LCA. Instead, what's empirical is the fact that there are alterations in nature at all (this reading is supported at A206–7/B252). Since LCA articulates a law governing all alterations, this law would be vacuous without the existence of real, empirical objects exerting causal powers on each other. Nevertheless, the law itself remains a priori. (For an alternative charitable reading of the passage, see Watkins [2001, 81].)

an expansion of Kant's metaphysics beyond the core principles of pure understanding requires not just understanding's laws, but reason's as well.

Second, an analysis of LCA's place within Kant's system would help clarify the extent to which transcendental philosophy can provide a metaphysical basis for physical science. Kant thought that there could be a mutually beneficial relation between the general metaphysics of the first *Critique* and the metaphysically grounded physics of *Metaphysical Foundations of Natural Science (MFNS)*. Kant could offer physics a proper grounding in a priori metaphysics (4:469). In return, the successful applicability of Kant's general metaphysics to physics would flesh out the former by way of "instances *in concreto*" and thereby "give a mere form of thought sense and meaning" (4:478). In *MFNS*, Kant had shown that the laws of inertia, universal attraction, and conservation of mass all find grounding in his system. Given the centrality of laws describing continuous changes in the new physics (e.g., regarding change of position, direction, velocity, acceleration, force, etc.), a transcendental proof of LCA would make the grounding of physics in Kant's system even more robust.⁶ In fact, he says that since LCA "seems to amplify our cognition of nature so much," it is important to understand how it "is possible completely a priori" (A209/B254). The importance of LCA for natural science is due not least of all to the fact that continuity is a mathematical concept since "a doctrine of nature will contain only as much proper science as there is mathematics capable of application therein" (Kant, 2002b, 4:470).⁷

While LCA's importance in Kant's system has long been appreciated, most of the literature discussing it focuses on its connection to other parts of Kant's system, without much consideration of the arguments Kant gives for the principle itself. For instance, Warren (2001a), Friedman (2013), and McNulty (2019) focus on its role in articulating and justifying Kant's "dynamical" theory of matter. Ewing (1924, 122–23), Paton (1936, 289–93), and Watkins (2005, 255–57) discuss it in connection to Kant's theory of causality. Cohen (1883 and 1885), Bennett (1966, 170–80), and Guyer (1987, 204–5) discuss it in relation to Kant's theory of intensive magnitudes. However, while a few commentators have considered aspects of Kant's arguments for LCA,⁸ there hasn't yet been a systematic attempt to reconstruct and evaluate all of Kant's arguments for the principle. This paper aims to fill that gap in the literature.

By my count, Kant offers three distinct arguments in support of LCA. This paper will reconstruct and evaluate all three. After some preliminary remarks in section 2, in section 3 I consider the argument from the "Inaugural Dissertation." According to that argument, LCA is a direct consequence of the fact that time is continuous. In section 4, I consider the first of two arguments in the *Critique's* Second Analogy of Experience. According to this argument, the continuity of alterations is supposed to follow from a metaphysical claim about causation: causes bring about their effects gradually, never all at once. In section 5, I consider a third argument for LCA, which appears at the very end of the Second Analogy. This argument (which may be intended as a stand-alone argument or may be intended as a continuation of, or supplement to, the previous one) attempts to support LCA by appeal to the continuity of our *perceptions* of alterations. All three of these arguments will turn out to be inadequate. While I'll be pushing several different lines of objection, the recurring mistake will be that Kant's arguments implicitly rest on hidden (question-begging) assumptions of

⁶For instance, LCA could naturally supplement Kant's claims about acceleration due to gravitation or repulsion, which are described by continuous functions (inverse-square and inverse-cube laws respectively; see 4:321 and 4:514–22). Friedman also notes this connection (2013, 387).

⁷Kant would hardly have been the first to notice the importance of continuity in the physical sciences of the eighteenth century. For instance, Johann Bernoulli (1727) and Boscovich (1754) followed Leibniz in using a law of continuity to argue against the existence of absolutely hard bodies. And in a biological context, Reimarus (1766) considers the possible continuity of biological forms, and even considers Leibniz's suggestion regarding the possibility of living species that are intermediate between plants and animals ("*thierartige Pflanzen*"). For an extensive overview of various laws of continuity in eighteenth-century science and metaphysics, see Tonelli (1963).

⁸See Ewing (1924, 122–23), Bennett (1966, 170–80), Watkins (2001, 80–81), and Warren (2001a, 101).

some version of LCA. The lesson provided by these analyses is that it will not frequently be possible to establish metaphysical principles as transcendental laws of understanding given that any such argument can rely on only the austere resources of the core of Kant's transcendental theory of experience (*viz.*, the categories and their corresponding principles, together with features of the form of intuition). Thus if LCA (and perhaps other principles like it) is to have any place in Kant's metaphysics, its basis must lie elsewhere.

Accordingly, while the primary goal of the paper is to show that Kant has no successful argument for LCA as a transcendental law of understanding (which is how he intended it), a secondary goal is to show that the principle might nevertheless be rehabilitated elsewhere in his system. I'll conclude (section 6) with some remarks in this direction. For even if LCA is not justifiable as a law of understanding, it may still (1) be empirically true, or (2) be justifiable a priori within the limited scope of Kant's dynamical theory of physics, or (most interestingly) (3) be properly construed as a regulative principle of reason. I'll devote the most attention to this third option and will make the case that there is good reason to think that Kant should have recognized that LCA is more at home in reason's system of regulative principles than understanding's system of constitutive principles.

2. Extensive and intensive continuities

Stated generally, the Law of Continuity asserts that "*in mundo non datur saltus*" (A229/B281), or roughly, "nature makes no leaps." It turns out though, that there are just as many particular laws of continuity in Kant's metaphysics as there are sorts of leaps that nature (supposedly) refuses to make. For instance, in *MFNS*, Kant describes the continuity of mechanical change, which pertains to changes in momentum in moving bodies (4:552–53).⁹ In the Antinomies, Kant argues that matter is continuous qua infinitely divisible (A523/B551ff.). In the Appendix to the Dialectic, he argues that between any two "species" (by which he means something like "possible natural kinds") there are infinitely more intermediate species (A659/B687ff.). And he even sometimes describes a "logical" law of continuity, which has to do with the applicability of a concept to objects across a continuous range of cases (see *Metaphysik Dohna*, 28:662). The primary focus of this paper is the Law of Continuity of Alterations (LCA) (*i.e.*, the claim that every alteration from one state to another must be continuous). In this section, I want to focus on two other laws of continuity which are required for a full articulation of LCA.

Extensive Magnitudes. Kant argues that space and time as extensive magnitudes are "*quanta continua*" (A169/B211) (call this the "Law of Continuity of Extensive Magnitudes," or "LCEM"). An extensive magnitude is a quantity "in which the representation of the parts makes possible the representation of the whole" (A162/B203). The discrete (*i.e.*, discontinuous) mathematics of counting and addition (synthesizing parts into wholes) is grounded in extensive magnitudes. Nevertheless, Kant argues that extensive magnitudes are also continuous insofar as these extended wholes can be divided into arbitrarily small parts: "no part of [space or time] can be given except as enclosed between boundaries (points and instants), thus only in such a way that this part is again a space and time" (A169/B211). For Kant, the continuity of space and time is equivalent to the fact that no part of either is a smallest unit.

Intensive Magnitudes. An intensive magnitude is one "which can only be apprehended as a unity, and in which multiplicity can only be represented through approximation to negation =0" (A168/B210). Kant holds that between any two degrees of intensity, there will always be infinitely more (call this the "Law of Continuity of Intensive Magnitudes," or "LCIM"). Here Kant is *defining* intensive magnitudes as continuous. However, the metaphysical import of LCIM consists not in the claim that all intensive magnitudes are continuous, but in the claim that all "realities" are intensive

⁹This "mechanical" law is arguably just a specific case of the general (or "metaphysical") LCA. I'll return to this issue in section 6.

magnitudes.¹⁰ In general, all of the material determinations that we can discern in objects are realities in Kant's sense of the term. Kant lists fundamental physical properties (e.g., heat and weight) as realities (A169/B211), as well as the attractive and repulsive forces that constitute material substance itself (*MFNS* 4:499, 4:523). Psychological entities, especially sensations, are realities (A143/B182, B207), and even consciousness itself may be a type of reality capable of continuous variation in intensity (B413ff.). When Kant articulates LCA in terms of the continuous alteration of the "state" of an object, the kinds of states he has in mind are these continuous, intensive "realities."

Here it is worth mentioning that LCA is supposed to hold *only* for alterations in realities, or the "real determinations" in objects. There are many other ways that we could conceive of an object to change that wouldn't track alterations in realities and wouldn't obey LCA. For instance, when one "alters" from being unmarried to being married, there are no intermediate states to pass through. Changes like this are no threat to LCA, however, because this sort of "state" is not a reality in Kant's sense, and so not the sort of property that LCA is supposed to govern.¹¹ Furthermore, I also take it that alterations are always "within" a single type of reality. Say r_1 and r_2 are different types of realities. If an object alters from some degree of r_1 (with no r_2) to some degree of r_2 (with no r_1), the change is properly described as two alterations: a diminution of r_1 to nothing, and a simultaneous growth of r_2 from nothing. A corollary of this is that every alteration consists in nothing but an increase or decrease of the intensity of some reality.

Both LCEM and LCIM are necessary for fully articulating LCA. For while LCA is a stronger metaphysical principle than the mere conjunction of LCEM and LCIM, it does presuppose both. It *presupposes* both because an alteration of some object O from state a at t_1 to b at t_2 could not be continuous unless (i) between t_1 and t_2 there is a continuum of intermediate times (LCEM) and (ii) between a and b there is a continuum of possible intermediate degrees (LCIM). Yet, LCA is *stronger* than the conjunction of the two because while LCEM and LCIM state that there is a continuum of possible intermediate degrees that O *could* pass through during the continuum of intermediate times, it takes LCA itself to say that O *must* in fact pass through all of them.¹² The central question of this paper, to which I now turn, asks whether Kant can justify this stronger claim as a transcendental law of the understanding governing all possible objects of experience.

3. The argument from the continuity of time

With the above preliminaries in place, we are in a position to reconstruct and evaluate Kant's arguments for LCA. I begin with the argument from the "Inaugural Dissertation." Its conclusion is: "*All changes are continuous* or flow: that is to say, opposed states only succeed one another through an intermediate series of different states" (2:399). The argument runs as follows:

For two states are in different moments of time. But between two moments there will always be an intervening time, and, in the infinite series of the moments of that time, the substance is

¹⁰I won't address Kant's argument for this central claim from the Anticipations. Many scholars have been highly skeptical of this argument (see Bennett 1966, 170–180 and Warren 2001b, 16). For more sympathetic readings, see Guyer (1987, 200), Bird (2006, 429–36), Cohen (1885, 422–38), and Jankowiak (2013).

¹¹Relatedly, the mere change of position of a body will not necessarily count as an alteration of the relevant sort. For if a body's motion is inertial, then the "motive force" in the body (which is a kind of reality) will remain unaltered (see A207n./B252n.). Hence, LCA should be taken to apply not to motions per se, but rather to changes in motion (i.e., acceleration, deceleration, and change of direction).

¹²To elaborate slightly on this point, note that LCIM is meant only as a claim about the *possible* values that a type of reality can take. But LCIM says nothing about which of these possible values will *actually* be instantiated by any given altering object. To see this point more clearly, note that a violation of LCA would not thereby violate LCIM: If an accelerating body altered discontinuously from 10 m/s straight to 12 m/s, it wouldn't mean that other bodies couldn't move at 11 m/s.

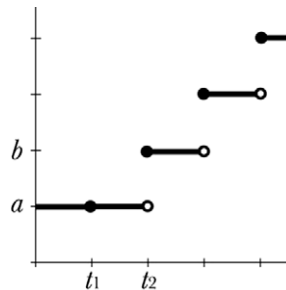


Figure 1. Step Function.

not in one of the given states, nor in the other, and yet it is not in no state either. It will be in different states, and so on to infinity. (2:399–400)

I'll refer to this as the Argument from the Continuity of Time, since the argument depends primarily on the claim that time is continuous (discussed above as LCEM).¹³

The reasoning can be reconstructed as follows:

1. Assume some O changes from a at t_1 to b at t_2 ($a \neq b$).¹⁴
2. Since O cannot be both a and b at the same time, t_1 and t_2 must be different.
3. Hence (given LCEM), there must be a finite duration between t_1 and t_2 . Call one of the instants in this duration t_3 .
4. At t_3 , O can be neither a nor b , but since it must have some value, there must be a c (between a and b) such that O is c at t_3 .
5. Since the same reasoning can be repeated *ad infinitum* (for shorter and shorter intermediate durations), it follows that the change is continuous.

There are two problems with this argument. The first is that it begs the question; the second is that it does not prove what it needs to prove. The first problem is in line 4: the claim that during the interval between t_1 and t_2 , O could be neither a nor b , but necessarily something in between. Kant begs the question in assuming that the alteration couldn't be described by something like the function shown in Figure 1. Perhaps O is in state a through the entire interval from t_1 right up until t_2 , and the first instant (after t_1) where O is not in state a is t_2 . Clearly, such an alteration would contain a discontinuity.¹⁵

¹³Kant continued to flirt with the idea that LCA is a direct consequence of LCEM throughout the critical period, indicating that he still found it persuasive even after the revolutionary insights of the 1770s. See *Metaphysik L1* (28:201–4), *Metaphysik Mrongovius* (29:920), and *Metaphysik Dohna* (28:662).

¹⁴Here and throughout the rest of the paper, the states (a , b , etc.) mentioned in discussions of alterations should all be assumed to be values (i.e., determinate intensive magnitudes) of one single type of reality. Thus when O alters from a to b , the change consists only in the degree to which a particular type of reality is present in O , not the type of reality instantiated. Also, a and b should always be assumed to differ by some determinate amount.

¹⁵Watkins is more sympathetic to this argument. While aware that Kant's inference might appear question-begging, he defends Kant by appealing to the example of an object moving through space, pointing out that a discontinuity in its motion would require the object to pop in and out of existence as it leaped past some space; but "[o]bviously, bodies do not move in such a fashion" (2001, 80). However, the fact that motion (qua change of place) is not discontinuous would not license an inductive inference to the claim that *all* alterations are continuous, especially considering that mere change of place is not even the sort of change of state described by LCA (see note ¹¹). And note that a body undergoing discontinuous changes in velocity would still traverse all intervening spaces and thus not "pop in and out of existence."

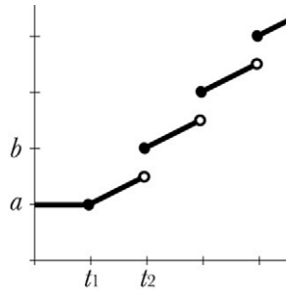


Figure 2. Slanted Step Function.

Now one might reasonably object that this is not a fair counterexample on the grounds that, in this example, the alteration didn't really begin at t_1 , since O remains in state a after t_1 . If the object hasn't yet changed, the alteration hasn't yet begun. This would be a fair complaint, but note that it would entail that Kant had in fact begged the question even earlier in the argument. Kant began the argument by inferring from $a \neq b$ to $t_1 \neq t_2$. If this is taken to imply (as the objection would have it) that t_1 is the last instant O was a and (by parity of reasoning) t_2 is the first instant O is b , then Kant would be assuming that (noninfinitesimal) changes in degree cannot take place in an infinitesimally¹⁶ brief instant (i.e., discontinuously). The LCA skeptic would retort that O might have become b infinitesimally soon after it ceased being a , and thus that there was no first instant that O was b (which is all consistent with Kant's observation that O cannot be a and b at the same time).

Thus Kant needs to justify the claim that there is a finite duration between the instant O stops being a and the instant it arrives at b . In the *Metaphysik L1* version of the argument, Kant seems to acknowledge this need when he adds the following premise: "If a body transfers from one state into another, then there must be a moment in which it goes out of the preceding state, and a moment in which it comes into the following state" (28:203). This claim could be read to mean the following:

3.5. In any alteration of an object from a to b , there will be a last instant at which it is a and a first instant at which it is b .

Kant's thought is presumably that, if 3.5 were true, then since these "last a " and "first b " instants would be different, there would be an interval between them during which O was neither a nor b , but rather something in between.

Kant offers no explanation as to why he would be entitled to assume 3.5 (and the LCA skeptic would surely balk at it), but even if we charitably granted the additional premise to him, the argument still would not prove what it needed to. This is because there are functions that obey 3.5 and are nevertheless discontinuous. The alteration represented in Figure 2 provides such a counterexample: since the value is always increasing, O will be in any one state for just one instant, hence there'd always be a first and last instant for any one value (they'd be the same instant), which satisfies the rule laid down in 3.5. Nevertheless, such a function clearly "leaps" over values.

Thus even if we grant that the argument is sound, the continuity described in the conclusion is not the real thing. For the argument proves only that: as O changes from a at t_1 to b at t_2 , for every t_3 (between t_1 and t_2) there is a c (between a and b) such that O is c at t_3 . The problem is that even though there might always be an intermediate state between any two instants, "leaps" over ranges of states remain a possibility. To fix this, the quantifiers would need to trade their variables. That is,

¹⁶Here and throughout the paper, I understand an infinitesimal quantity to be one which is greater than zero, but smaller than any specifiable real number. (Contemporary definitions of infinitesimals in terms of hyperreals—infinite sequences of real numbers—were a twentieth-century development.)

Kant needed to show instead that: as O changes from a at t_1 to b at t_2 , for every c (between a and b) there is a t_3 (between t_1 and t_2) such that O is c at t_3 . Only the latter would rule out discontinuities, but Kant hasn't demonstrated it yet.¹⁷

I think we should conclude that Kant was mistaken to think that LCA would follow directly from the continuity of time. The problem (as I hinted earlier) is that the LCEM and LCIM tell us only that there are infinitely many instants at which the object *could* instantiate any of the infinitely many possible intermediate degrees. We'd need an additional metaphysical principle to show that the object *must* pass through all of those degrees during the interval in question. The metaphysics of time alone is not sufficient to entail this stronger claim. As we'll see next, the arguments presented in the Second Analogy both incorporate additional metaphysical principles (in addition to the metaphysics of time) to strengthen the argument.

4. Argument from gradual causation

I turn now to the first of two arguments for LCA found at the end of the Second Analogy.¹⁸ I quote the core of it at length here:

Between two instants there is always a time, and between two states in those instants there is always a difference that has a magnitude (for all parts of appearances are always in turn magnitudes). Thus every transition from one state into another happens in a time that is contained between two instants ... *Now every alteration has a cause, which manifests its causality in the entire time during which the alteration proceeds. Thus this cause does not produce its alteration suddenly (all at once or in an instant), but rather in a time, so that as the time increases from the initial instant a to its completion in b , the magnitude of the reality ($b - a$) is also generated through all the smaller degrees that are contained between the first and the last. All alteration is therefore possible only through a continuous action of causality.* (A208/B253–54; emphasis added)

The main difference between this argument and the previous one is the appeal to the metaphysics of causation. Alterations don't just happen. Rather, they are brought about as the effects of causes (this is the central claim of the Second Analogy). Kant's crucial move here (the italicized portion of the text) is the claim that because causal efficacy is exerted *gradually* (instead of instantaneously), a cause whose effect is a change from a to b will bring the object through all intermediate states as it exerts its causal power throughout the alteration's duration.¹⁹ I'll call this the Argument from Gradual Causation.

¹⁷Arguably, Kant's mistake here is due to an insensitivity to the distinction between *density* and *continuity* (which would be forgivable since this distinction was not fully appreciated in Kant's time). A range of values is considered "dense" so long as between any two values there are further intermediate values. Kant sometimes hints that he takes density to be sufficient for continuity (see A169–70/B211–2, A209/B254, A659/B687, not to mention the text under consideration in this section), but the above counterexamples demonstrate otherwise. Since the nineteenth century, true continuity has been cashed out in terms of continuous functions, and this requires something stronger than mere density. According to a standard definition, a function $f(x)$ is continuous if and only if for any point $\langle x, f(x) \rangle$ on the curve, and for any $\varepsilon > 0$, there will be a $\delta > 0$ such that (for any x_1) if $|x - x_1| < \delta$, then $|f(x) - f(x_1)| < \varepsilon$. Such a function makes no "leaps." Had Kant been aware of the distinction between density and continuity, he probably would have recognized that he needed to demonstrate not just that alterations are dense, but that they are describable by continuous functions.

¹⁸A remark on the relation between this argument and the rest of the Second Analogy: I take it that Kant does not *need* to prove LCA in order for his main project of the Second Analogy to succeed (i.e., showing that all events have causes by appeal to conditions on the possibility of objective time-determination). That project is completed by about A202/B247. Rather, Kant wants to make the case here that LCA is a *consequence* of his account of causality.

¹⁹The appeal to gradual causation as justification for LCA is anticipated by the precritical "New Doctrine of Motion and Rest" (see 2:22).

I take it that what Kant is getting at with this additional premise is something like the following; I'll call it the Principle of Gradual Causation, or "PGC":

In any infinitesimal ("vanishingly small," as Kant puts it sometimes) quantity of time, a cause can only exert an infinitesimal²⁰ quantity of efficacy.

If Kant is entitled to PGC, then he has a successful argument for LCA. For PGC would entail that at any instant during an alteration, the changing object's state will have just arisen as the effect of its infinitesimally recent, infinitesimally efficacious cause, and will also be on its way to a new, infinitesimally different state an infinitesimally short time later. Accordingly, the change at any given instant will be infinitesimal, and so the object will never "leap" past some value in its alteration. In other words, all alterations are continuous because all alterations are driven by causes, and all causes happen continuously. The argument can be reconstructed more precisely as follows:

1. Assume *O* alters from *a* at t_1 to *b* at t_2 ($a \neq b$, $t_1 \neq t_2$).
2. Since no alteration happens without a cause, there must be some cause of *O*'s alteration from *a* to *b*.
3. In any infinitesimal quantity of time during the interval between t_1 and t_2 , this cause can exert only an infinitesimal degree of efficacy (this is PGC).
4. Infinitesimally efficacious causes can bring about only infinitesimal effects.
5. Consequently, in any infinitesimal quantity of time during the interval between t_1 and t_2 , *O*'s alteration has only infinitesimal magnitude.
6. If in every infinitesimal quantity of time during an alteration the change is never greater than an infinitesimal magnitude, then the entire alteration is continuous.
7. Therefore, *O*'s alteration from *a* to *b* is continuous.

This argument is valid and the continuity it describes is the real thing (see note 17), so if Kant is justified in appealing to PGC, then we'd have a successful argument. The problem is that Kant offers no real justification for this key premise. Nothing in the argument as he presents it rules out the possibility that in an infinitesimally brief moment a cause could exert a *finite* (i.e., noninfinitesimal) degree of efficacy (we could call this "abrupt" causation). If this happened, then there would be a "leap" in the alteration, with intermediate values getting skipped over, violating LCA. Rather, in the italicized line from the argument quoted above, it seems that Kant is *defining* the cause of an alteration as something that proceeds gradually rather than abruptly. But such a claim is hardly analytic. If PGC were true, it would be a synthetic a priori law, hence one in need of its own transcendental justification. But Kant offers no such argument in the *Critique*, leaving the reader in the dark as to why he asserts it dogmatically.²¹

²⁰Although Kant does not explicitly mention infinitesimal quantities in the passage quoted above, he makes it clear that he has this in mind in an elaboration in the next paragraph (A209/B254). Moreover, Kant had earlier gone on the record declaring himself to be a realist about infinitesimals, in contrast to the Leibnizians and Wolffians (see *New Doctrine*, 2:22 and *Negative Magnitudes*, 2:169). Friedman (2013, 384–86) also reads this argument in terms of infinitesimal quantities of causal efficacy (however, he had earlier (1992, 73–77) made the case that Kant does not need to appeal to infinitesimals in his treatment of continuous structures because an appeal to Newtonian fluxions would have worked just as well). See also Cohen (1885, 277; 425–29; 462; 596–97).

²¹Although he doesn't justify it in the *Critique*, one might look to the "General Remark to Mechanics" from *MFNS* for an argument for PGC. There Kant argues that, "the moment of acceleration must therefore contain only an infinitely small speed, because otherwise the body would thereby attain an infinite speed in a given time, which is impossible" (4:551). Stated more generally, Kant's point is that if a cause could exert a finite amount of efficacy in an infinitesimal amount of time, then when infinitely many infinitesimal moments of finite efficacy are summed across a finite duration, the total quantity of efficacy, and thus the corresponding effect, would be infinite. This reasoning can be understood as an attempt at a *reductio* against the possibility of a counterexample to PGC. But if this is Kant's intent, it's not a successful *reductio*. For Kant is still assuming here that causes don't exert their full efficacy all in one instant, but rather do so continuously over a finite duration. That is, he is

There is however one obvious way that Kant could have justified PGC: LCA, if true, would entail PGC (since then all alterations would happen through infinitesimal increments, each of which would depend on its own infinitesimally efficacious causal ground, as described by PGC). Thus, by entailing PGC, the truth of LCA could have interesting *consequences* for Kant's metaphysics of causation. But in that case, PGC could play no role in an argument for LCA without begging the question.²² At the end of the day, it seems that PGC is really just LCA as seen through the lens of the Second Analogy's claim that all alterations require a cause. Given all this, I think we should conclude that Kant's appeal to the metaphysics of causation doesn't yield a satisfactory demonstration of LCA.

5. Argument from perceptual continuity

There is one final passage that demands our attention. Three paragraphs after the Argument from Gradual Causation, in the second to last paragraph of the Second Analogy, Kant introduces a new argument regarding the continuity of alterations. For reasons that will shortly become apparent, I'll call it the "Argument from Perceptual Continuity." His reasoning rests on the claim that the changes in the intensities of sensations in perceptions of alterations will themselves be continuous alterations. That is, I can represent an object's alteration from *a* to *b* only through a "progress" or "advance in perception" (A210/B255) that involves a continuous change from a sensation with intensity *a** to one with intensity *b** (where *a** and *b** are the intensities of the sensations of *a* and *b*, respectively). This necessary feature of our representations of alterations (their continuity) somehow supports or leads to LCA's claim that the perceived alterations themselves must be continuous as well. Here is the passage in question, quoted in full:

[1] All growth of empirical cognitions and every advance in perception is nothing but an amplification of the determination of inner sense, i.e., a progress in time, whatever the objects may be, either appearances or pure intuitions. [2] This progress in time determines everything, and is not itself determined by anything further: i.e., its parts are only in time, and given through the synthesis of it, but are not given before it. [3] For this reason every transition in perception to something that follows in time is a determination of time through the generation of this perception and, since that is always and in all its parts a magnitude, the generation of a perception as a magnitude through all degrees, of which none is the smallest, from zero to its determinate degree. [4] It is from this that the possibility of cognizing a priori a law concerning the form of alterations becomes obvious. [5] We anticipate only our own apprehension, the formal condition of which, since it is present in us prior to all given appearance, must surely be able to be cognized a priori. (A210/B255–56; numerical markers added)

Before attempting to reconstruct this murky passage, it's worth asking: what is its relation to the Argument from Gradual Causation from a few paragraphs earlier? The answer is not obvious, and there seems to be at least two different ways to read it. On the one hand, this passage could be

assuming the Second Analogy's claim that "All alteration is therefore possible only through a continuous action of causality" (A208/B254). This assumption is required for the absurd conclusion that the total effect would be the infinitely great sum of infinitely many finite effects. The LCA skeptic, who accepts the possibility of "abrupt" causation, will insist that a cause could exert a finite effect in an infinitesimal amount of time, but then just stop with that and not have to continuously exert finite effects over and over again *ad infinitum*. (I am grateful to an anonymous referee for drawing my attention to the relevance of this passage to PGC.)

²²Warren also indicates that the argument reconstructed here appears to be question-begging in its assumption of what I've labeled PGC (2001a, 101). His concern, however, is with Kant's rejection of the mechanistic conception of solidity, rather than with LCA itself.

intended as an entirely new argument meant to provide its own independent justification for LCA. On the other, it could be intended as a sort of supplement to the previous one, which attempts to buttress the case for LCA by elucidating its transcendental basis. I'll consider both possibilities in turn.

One motivation for reading this passage as a standalone argument for LCA lies in the fact that it has nothing to do with causation, which was the central concept of the previous argument. Furthermore, in the paragraph just preceding this one, Kant suggests that we haven't yet seen a transcendental justification for LCA, which would reveal how such a principle "is possible completely a priori" (A209/B254). While he certainly does not go so far as to repudiate the Argument from Gradual Causation, he does warn that that argument might appear "dogmatic [...] without documents that could provide a well-grounded deduction" (A209/B255).²³ Kant could be taken to mean that a transcendental deduction, i.e., an "explanation of the way in which concepts can relate to objects a priori" (A85/B117), of LCA is required.²⁴ For whatever the (supposed) merits of the Argument from Gradual Causation, it didn't have much to say about transcendental conditions on the possibility of experience of alterations. This new argument, by contrast, turns on considerations regarding the possibility of apprehending an alteration in perception and, for this reason, might perhaps provide the missing "documents."

Assuming for the moment that the paragraph in question should be read as an independent transcendental deduction of LCA, how exactly should we understand its inference? While the passage is certainly not Kant's clearest, it is nevertheless possible to discern the outline of the type of argument he might have had in mind. The first two sentences make a point about the flow of perceptual states in inner sense. The "growth" (*Zuwachs*) or "amplification" (*Erweiterung*) he describes is the change in the intensive magnitudes of the sensations constituting the perception. As a perception changes over time, the intensity of the sensations that make up the perception will change as well. Kant next connects this point about temporally extended perceptions with the structure of time itself: since time is continuous, a perception altering in time must be a "generation [*Erzeugung* ...] through all degrees" (A210/B255), i.e., be continuous as well.²⁵ Call this claim that all changes in perceptions are continuous alterations the "Law of the Continuity of Alterations of Perceptions" (LCAP). LCAP is of course just LCA applied to the specific case of altering perceptions. This principle is presented as the most important claim of the paragraph, as he immediately goes on to say that "from this [*hieraus*]" we can discern the "possibility of cognizing a priori a law concerning the form of alterations," which clearly refers to LCA (A210/B255). Nothing further is said to explain the inference, and thus if Kant really is attempting to prove LCA in this paragraph, he seems to be inferring directly from the continuity of perceptions of alterations to the continuity of the perceived alterations themselves (i.e., from LCAP to LCA).

If the above does reflect Kant's intent, there would be at least two serious problems with the argument. The first is that the argument for LCAP is structurally identical to the Argument from the Continuity of Time discussed earlier. There Kant had argued that since time is continuous, it follows that alterations happening in time are continuous as well. Now he seems to be making the same move, albeit in a more limited context: since time is continuous, it follows that alterations of

²³I take Kant to be saying that his own Argument from Gradual Causation might appear dogmatic without the transcendental backing provided by a "deduction." An anonymous reviewer points out an alternative reading of this passage: Kant could be referring to the arguments of his rationalist predecessors (i.e., members of the Leibnizian-Wolffian lineage) who, because they didn't recognize the need for a deduction, defended LCA only dogmatically.

²⁴This connection is bolstered by the fact that Kant speaks here of needing "*Documente*" for LCA, while he spoke similarly of needing a "*Geburtsbrief*" for the categories in the Deduction (A86/B119).

²⁵The reference to apprehension later in the passage gives some hint as to what Kant meant here. Kant had earlier insisted that since all apprehension takes place in time (as a synthesis of successive sensations) it follows that "all of our cognitions are in the end subjected to the formal conditions of inner sense, namely time, as that in which they must all be ordered, connected, and brought into relations" (A99). That is, the structure of time determines the structure of representations occurring in time.

perception are continuous as well. However, if the continuity of time was not sufficient to establish the continuity of alterations in general, there's no discernible reason why it should be sufficient to establish the continuity of this particular kind of alteration. The continuity of time (LCEM) shows at most that LCAP is metaphysically possible, but not that it is indeed true (much less that it is true a priori).²⁶

The second problem is that (supposing we grant LCAP to Kant) the inference from LCAP to LCA is highly implausible. The inference moves from a (supposedly) necessary feature of our representations of alterations (their continuity) to a necessary feature of the alterations themselves (*their* continuity). On its face, this is a familiar Kantian move, and it sounds like an application of the principle that, "the conditions on the *possibility of experience* in general are at the same time conditions on the *possibility of objects of experience*" (A158/B197, emphasis in original). However, one must be careful about what exactly can count as the relevant sort of "condition on the possibility of experience." Note that LCAP only states that perceptions themselves (i.e., considered as mental episodes) have a certain property, *viz.*, continuity in their alteration. In order to move from LCAP to LCA, one would need to appeal to the notion that the properties of a representation mirror the properties of the object it represents. But only a hardcore Berkeleyan phenomenalist would accept this, and Kant certainly would not. For instance, all representations are fleeting and impermanent, but this does not entail that the substances they represent are fleeting and impermanent as well. Accordingly, even if it were a necessary feature of our psychology that our representations of alterations involve continuously growing or diminishing sensations, this psychological fact would not entail anything about the objects of our representations.²⁷ In fact, it wouldn't even entail anything about the necessary *content* of our representations. For again, even though my representations of substances are fleeting and impermanent, it does not follow that I must represent substances as impermanent. Thus LCAP doesn't even entail that I must *represent* alterations as continuous.

These significant problems give us reason to consider the other interpretation of the Argument from Perceptual Continuity. Perhaps the argument does not aim at establishing LCA directly, but rather aims to supplement the reasoning in the Argument from Gradual Causation (which, again, appears just a few short paragraphs earlier).²⁸ One reason to read the passage this way is that Kant never explicitly states that it proves LCA. Rather, he says that through this reasoning, the possibility of cognizing an a priori principle such as LCA "becomes obvious [*erhell*]" (A210/B255). Perhaps then the missing "documents" demanded in the previous paragraph were needed to justify the pretensions of the Argument from Gradual Causation, rather than LCA itself. More specifically, he'd be trying to show how LCA is the kind of principle that could be an object of a priori cognition in the first place. On this reading, Kant would simply be trying to show that the concept of continuous alteration is in conformity with the a priori form of sensibility, which conditions possible experience. Since time is continuous (LCEM), and since perceptions changing in time are continuous (LCAP), it follows that the understanding's concept of continuous alteration is possible as an object of experience.²⁹ Thus, on this reading, Kant is simply trying to show that LCA is the kind of principle that could receive an a priori defense, because what it describes (continuous alteration) is in conformity with the conditions on the possibility of experience.

²⁶And if anything, if LCAP is true at all, it is probably best understood as an empirical psychological observation about how sensations happen to change, rather than as an a priori condition on how they must change. Bennett (1966, 172) and Guyer (1987, 204) both press Kant on this point.

²⁷Some scholars reject Kant's reasoning in the Anticipations of Perception (A165/B207ff.) because they see Kant making the same sort of fallacious move there: sensations have intensive magnitudes, therefore the corresponding realities in objects do too (see Guyer 1987, 200).

²⁸I'm grateful to an anonymous referee for encouraging me to take seriously this interpretive option. I also note that most scholars who have noticed the Argument from Perceptual Continuity treat it as a continuation of the reasoning begun in the Argument from Gradual Causation. See Kemp Smith (1918, 380), Paton (1936, 288) and Friedman (2013, 398ff.).

²⁹I will return to the question of whether continuous alterations are possible objects of experience in the next section.

Assuming that Kant intends his remarks about continuity in apprehension to play such a supporting role, has he strengthened his case for LCA in any substantive way? Once again, I think we must unfortunately answer in the negative. For one, this reading still requires that Kant be able to establish LCAP. But as I argued above, his reasoning here mirrors the fallacious reasoning from the Argument from the Continuity of Time. More to the point, even if we accepted LCAP as an a priori law, Kant's reasoning in the Argument from Perceptual Continuity would only show how a *successful* argument for LCA could have a transcendental basis. It would not, however, be able to strengthen the argument that Kant had given (the Argument from Gradual Causation), which Kant seems to take to be sufficient on its own merits and not in need of further support. More specifically it would not be able to fix the main problem that I identified with that argument, viz., that its central premise, PGC, was never justified, and it seems to be just a question-begging assumption of LCA dressed up as a causal principle. Accordingly, on this reading of the Argument from Perceptual Continuity, Kant still would not have a successful a priori demonstration of LCA.

For the above reasons, I think that the Argument from Perceptual Continuity fails to establish LCA as a transcendental law of understanding. And as far as I am aware, Kant offers no other arguments for LCA. Thus I conclude that he does not have a successful argument for the principle, not at least if it's considered an a priori law of understanding governing all possible objects of experience.

6. Rehabilitating LCA

Defending a metaphysical principle as a transcendental law of the understanding requires meeting a very high standard. If my analyses in the previous three sections are on the mark, then Kant has not shown that LCA can meet this standard. But this leaves open the possibility that LCA might justifiably be rehabilitated within a different part of Kant's system. I conclude with three different ways this might go (though I won't be able to go into them in detail). I list them in order of increasing philosophical interest and complexity.

6.a. LCA as empirical law

Guyer says that it is "the *Critique's* unqualified assertion that the continuity of change is only an empirical matter" (1987, 205). I think it's clear that he was wrong about what Kant intended, but perhaps he was right about what Kant should have said. Perhaps LCA is a true generalization about natural processes, but not a regularity that could have been known a priori. After all, LCA does seem to describe the world as it was portrayed by the best physical theories of Kant's day.³⁰ Thus LCA could still have a place in a broadly scientific picture of the world. That being said, giving up on *any* a priori status for LCA would amount to denying it a place in philosophy proper, since Kant takes a principle to be philosophical only if it has a basis in "pure rational cognition" (2002b, 4:469).

6.b. A merely mechanical LCA

In *MFNS*, Kant distinguishes between a "metaphysical" law of continuity and a "mechanical" law of continuity (4:552–3). The metaphysical law is the LCA discussed in this paper. The mechanical law is LCA applied to alterations in moving forces (resulting in change of velocity or direction). The mechanical law is thus narrower in scope than LCA, because it does not apply to changes in mental states, and it also does not apply to changes in realities in physical objects other than their moving forces. But for this reason, defending the a priori status of the mechanical law on its own might pose a less onerous challenge.

³⁰On the other hand, contemporary quantum mechanical descriptions of the smallest things in terms of discrete, quantized states would seem to undermine LCIM and thus LCA along with it.

Michael Friedman offers an outline of how this might work. He argues that even though the mechanical law “is an instantiation or realization of the metaphysical law” (2013, 402), Kant’s argument for the former is independent of the argument for the latter.³¹ He argues that the argument for the mechanical law depends on “the de facto continuity of the action of the two fundamental forces” (398), “on all three of Kant’s Law of Mechanics” (400), and in general on “a consideration of space and its geometry” (401). These grounds of proof are quite distinct from those Kant appealed to in his argument for the general LCA.

I won’t consider his reconstruction in detail here. But if Friedman is right that the argument for the mechanical law is independent of the argument for the general LCA,³² and if the former is successful, then Kant would still have achieved a great deal. For, as I remarked in the introduction, one reason why the question about an a priori basis for the continuity of alterations is important is because of Kant’s goal of giving physics a philosophical foundation. The basic laws of Newtonian mechanics describe changes of motion in terms of continuous functions (e.g., the inverse-square law describing universal gravitation), and these changes of motion would all fall under the scope of the mechanical law of continuity.

6.c. LCA as regulative principle of reason

Finally, I conclude with what I take to be the most promising and interesting way in which LCA could be rehabilitated. As I understand him, Kant wants to treat LCA as a transcendental law of the understanding. That is, he considers it a “constitutive” principle that was necessarily true of all possible objects of experience. Perhaps he’s correct to ascribe LCA a transcendental status, but wrong to locate its a priori basis in the understanding. Perhaps it’s best understood not as a constitutive law of understanding but, instead, as a regulative principle of reason.

There would be some precedent for treating LCA as a regulative principle, as Kant had already assigned this status to other laws of continuity. First, matter is described as a “quantum continuum” (A527/B555) because it is infinitely divisible. But this infinite divisibility must be construed merely regulatively insofar as there can never be a cognition of a completed infinite division (A523/B551ff.). The principle asserting matter’s infinite divisibility amounts to the instruction “never to take the empirical regress in the composition of what is extended [...] to be absolutely complete” (A527/B555). Second, according to the principle that “*datur continuum formarum*,” between any two “species” (i.e., forms, or natural kinds), “intervening species are always possible” (A659/B687). This principle is merely regulative because “we are given nothing more than a general indication” (A661/B689) that we should seek out intermediate species even though there is no guarantee that such intermediate species are there to be found.

Now, there is ongoing interpretive debate about how exactly regulative principles of reason should be understood and I don’t want to beg any interpretive questions here.³³ But I take it to be uncontroversial to say that regulative principles are laws that reason prescribes to understanding in order to guide and encourage a systematic investigation of nature (A644/B671–72, A647/B675, and A832/B861). This is in contrast to the principles of understanding, which understanding prescribes

³¹In fact, Friedman even argues that “the proof of the metaphysical law formulated in the second analogy does not lead by any natural or direct continuation to a corresponding proof of the mechanical law” (2013, 402).

³²McNulty disagrees with Friedman on this point: “Kant’s argument for dynamism hinges on his denial of mechanical philosophy, which, in turn, rests on his endorsement of Leibniz’s law of continuity [i.e., LCA]” (2019, 1597). If McNulty is right, then even if the dynamic theory of matter is sufficient to entail the mechanical law of continuity (as Friedman argues), the dynamical theory itself depends on LCA, and so in this way the mechanical law would depend on the general LCA after all. I will not attempt to resolve this disagreement here.

³³For instance, there is debate regarding how much of a positive epistemological status Kant means to give to reason (see Rush 2000 and McNulty 2015), whether Kant’s view changes by the time he writes the third *Critique* (see Guyer 1990 and McLaughland 2014), and even whether his position is coherent at all (see Bennett 1974, 270–79 and Bird 2006, 730–33).

to nature itself (A664/B692).³⁴ Moreover, regulative principles receive their normative force from “ideas,” which are concepts of reason to which no object of experience could correspond (A645/B673), and which are generated by reason’s “interest” (A616/B644) in completeness and systematic unity among understanding’s cognitions.³⁵ Because reason’s interests in systematicity and completeness is nonnegotiable (“here reason does not beg but commands” [A653/B681]), it follows that the regulative principles that are based on these ideas carry their own kind of transcendental necessity. This is not the objective necessity of understanding’s laws, but a subjective necessity governing the way that we must attempt to organize our cognitions (A651/B679).³⁶

To begin to motivate the idea that LCA ought properly to be construed as a regulative principle, let’s consider the epistemological situation one would face in trying to determine whether any given alteration is continuous. Obviously, if LCA were an a priori law of understanding, we’d have an easy answer. But barring that, one would need to look closely at the alteration itself. For instance, in the alteration from *a* to *b*, one might distinguish a great many intermediate states (*c*, *d*, *e*, etc.). And looking more closely, one might make even finer-grained distinctions (*f* between *a* and *c*, *g* between *a* and *f*, etc.), which might strongly suggest continuity. However, continuous alterations involve a transition through *infinitely* many states, and Kant insists (quite plausibly) that there cannot be cognitions with infinite conceptual content (see B40). Accordingly, it seems that no experience of an alteration, however precise and detailed, could justify more than a probabilistic inductive inference that the alteration was continuous, since one could never rule out completely the possibility that a “leap” had occurred in some interval too brief for our eyes or instruments to notice. Now, I make this observation not to raise further skepticism regarding LCA. Rather, my point is that even if LCA is true (whether empirically or a priori), a cognition (conceptually) representing the full range of transitional states would nevertheless still not be possible. Rather, because we can determinately cognize only finitely many transitional states, the most LCA could say regarding our experience of alterations is that any time we look for intermediate states in an alteration, we should expect to be able to find them, and that ever finer-grained descriptions of the alteration will always be possible.

Three remarks are in order regarding this observation. First, it shows that when LCA is given cash value in terms of possible experience, it functions as a rule that tells us what we should expect to find when we investigate alterations in nature. This should call to mind the function of regulative principles, which are rules that instruct understanding in its systematic investigation of nature.

Second, if fully continuous alterations can never be cognized with full specificity in experience but can only be approximated with increasingly fine-grained determinations of the intermediate states in the alteration, then it seems to follow that the very notion of continuous alteration would be an *idea* (in Kant’s technical sense). Kant argues that the concepts of pure water, air, earth, etc. are never encountered in experience, yet nevertheless function as ideas that guide understanding (A646/B674).³⁷ Kant says that “such concepts of reason are not created by nature, rather we question nature according to these ideas, and we take our cognition to be defective as long as it is not adequate to them” (A645/B673). If the infinity of states in a fully continuous alteration can only ever be approximated in experience, never fully determined, then perhaps the concept of continuous alteration is best construed as an idea that regulatively guides the way we “question nature.”

³⁴Kant also makes a regulative/constitutive distinction within the principles of pure understanding (see A179/B222 and A236/B296). I am not concerned with this distinction here.

³⁵Kant emphasizes completeness as a regulative ideal in the Antinomies (A508/B536, A515/B543ff.), and systematic unity in the Appendix to the Transcendental Dialectic (A668/B696, A682/B710, A692–3/B720–21) and the Discipline of Pure Reason (A771/B799, A786/B814).

³⁶I take no stand here on how exactly this transcendental subjective necessity ought to be understood, as such an analysis lies beyond the scope of this paper. Some think this necessity amounts to no more than mere methodological heuristic, while others think it amounts to something much stronger. That being said, I take my remarks in what follows about a regulative LCA to be consistent with both weaker and stronger interpretations of the regulative function of reason.

³⁷See McNulty (2015, 3–7) for a discussion of the regulative status of these ideas.

Third, the above analysis shows that the similarity between LCA and the other two regulative laws of continuity (regarding the composition of matter and the continuity of species) is more than superficial. For the regulative status of those principles consists in the fact that finer-grained distinctions can always be made (whether in the structure of the composition of matter, or in the structure of possible natural kinds), together with the caveat that a complete determination of these structures is beyond the reach of possible experience. We can see now that if LCA is true, it would be making the same kind of claim.

I take all of this to provide strong circumstantial evidence that if LCA is true at all, it would most naturally find a home within reason's system of principles (at least after its eviction from its intended place within understanding's system of principles). That is, it's the *kind* of principle that we'd expect to have a regulative status. Note, however, that this is not yet an argument that LCA is indeed true as a regulative principle. And an argument would, of course, be required since we cannot just wantonly bestow the title of "regulative principle" on any scrap we might want to salvage from the wreckage of precritical rationalism. The sort of justification that would be needed is a demonstration that LCA (together with the idea of continuous alteration) helps satisfy one of reason's speculative interests, viz., the ideals of completeness and systematicity.

A full construction of such an argument lies beyond the scope of this paper (and after all, there's only so much value in speculating about an argument that a historical philosopher didn't make, for a position they didn't endorse). That being said, it's not too difficult to see how Kant might have justified the notion that LCA could serve reason's interest in systematicity. In order for a set of cognitions to be ordered systematically, there must be an "idea ... which precedes the determinate cognition of the parts and contains the conditions for determining a priori the place of each part and its relation to the others" (A645/B673). For instance, the idea of "pure air" functions as a standard against which the impurities of empirical samples can be measured; multiple such samples can then be ordered in relation to each other in terms of this standard, and the cognitions of the samples thereby become systematic (A646/B674). Consider then what would be required for an experience of an alteration to count as systematic. Since only finitely many of the intermediate states in an alteration can be cognized in any given experience, if there were no idea of continuous alteration to guide and structure our investigations into natural processes, these "snapshots" of moments within the alteration will be only so many piecemeal cognitions. However, with the regulative idea of continuous alteration as an a priori presupposition, understanding will be able to situate these disparate cognitions within a representation of the fully continuous alteration of which they were a part.³⁸ In other words, the idea of continuous alteration allows understanding to determine "the place of each part in its relation to the others" (A646/B674), and thereby satisfy reason's demand for systematic unity.

While I don't claim to have given a full defense of the above reasoning here, I do take these considerations to make the following two claims highly plausible: (1) LCA does the kind of work that we would expect of a regulative principle of reason, viz., it prods understanding in its investigations of nature by way of a concept ("continuous alteration") that experience can hope only to approximate, and (2) LCA is indeed such a regulative principle, since a regulative presupposition of the idea of continuous alteration allows for systematic unity among cognitions of natural processes.

³⁸A quick example: a researcher studying acceleration due to gravity on earth might measure an object's velocity to be 0 m/s at one instant, 18.6 m/s two seconds later, and 49 m/s three seconds after that, but wouldn't be motivated to seek out the continuous function underlying these observations ($v = 9.8 \text{ m/s}^2 * t\text{s}$) without a regulative presupposition that such a function was there to be discovered. Discovering such a function brings systematic unity to the observations at $t = 0, 2, 5$ by showing how they are related to each other (and to other unobserved moments during the alteration) as so many points along the curve of that function.

7. Conclusion

I stated at the beginning of this paper that a consideration of Kant's justifications for LCA was important because it clarifies both the specific connections between transcendental philosophy and Kant's philosophical physics, and the general question of the range and scope of Kant's metaphysics of nature. The foregoing analyses have hopefully brought some clarity to both of these questions. Kant intended to establish LCA as a law of understanding governing all possible objects of experience. However, none of his arguments were successful, and thus in this instance at least an expansion of understanding's laws beyond its core principles is not available. Perhaps this result is not so surprising, given Kant's critical appraisal of so many other aspects of precritical rationalist metaphysics. Kant is able to demonstrate a great deal by appealing only to the categories and the forms of intuition. But, at the end of the day, these resources are very minimal, and so it was perhaps too optimistic to think that they could go so far as to reestablish cherished principles from the Leibnizian-Wolffian era, such as LCA.

Either way, there remain at least two avenues for preserving LCA as a component of the philosophical foundation of natural science. If Friedman is correct, then the mechanical LCA retains a place within Kant's philosophical physics. Separately, if my remarks about LCA as a regulative principle are on the right track, then LCA can be construed as a law demanded by reason as it guides understanding's systematic investigations of nature. In this respect, LCA would retain a privileged position within Kant's philosophy of science. More generally, this strategy shows that an investigation of reason's principles, not just understanding's, might be necessary for determining the full scope of Kant's metaphysics.

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