Accuracy, Language Dependence, and Joyce's Argument for Probabilism*

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In this article, I explain how a variant of David Miller's argument concerning the language dependence of the accuracy of predictions can be applied to Joyce's notion of the accuracy of "estimates of numerical truth-values" (i.e., Joycean credences). This leads to a potential problem for Joyce's accuracy-dominance-based argument for the conclusion that credences (understood as "estimates of numerical truth-values" in Joyce's sense) should obey the probability calculus.

1. Miller on the Language Dependence of the Accuracy of Predictions. Suppose we have two numerical quantities ϕ and ψ. These might be, for instance, the velocities (in some common units) of two objects, at some time (or some other suitable physical quantity of two objects at a time). Suppose further that we have two sets of predictions concerning the values of ϕ and ψ, which are entailed by two hypotheses H_1 and H_2, and let us denote the truth about the values of ϕ and ψ (or, if you prefer, the true hypothesis about their values)—in our standard units—as T. Let the predictions of H_1 and H_2 and the true values T of ϕ and ψ be given by table 1. (Ignore the α/β columns of table 1, for now—I will explain the significance of those columns later.)

It seems clear that the predictions of H_2 are "closer to the truth T about ϕ and ψ" than the predictions of H_1 are. After all, the predicted values entailed by H_2 are strictly in between the values predicted by H_1 and the true values entailed by T. However, as Popper (1972, app. 2) showed

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TABLE 1. CANONICAL EXAMPLE OF THE LANGUAGE DEPENDENCE OF THE ACCURACY OF PREDICTIONS

	ϕ	ψ	α	β
H_1	.150	1.225	.925	2.000
H_2	.100	1.000	.800	1.700
T	.000	1.000	1.000	2.000

(using a recipe invented by David Miller [1975]), there exist quantities α and β (as in table 1) satisfying both of the following conditions:

1. The values α and β are symmetrically interdefinable with respect to ϕ and ψ in the following (linear) way:
$$\alpha = \psi - 2\phi \quad \beta = 2\psi - 3\phi,$$
$$\phi = \beta - 2\alpha \quad \psi = 2\beta - 3\alpha.$$
2. The values for α and β entailed by H_2 are strictly "farther from the truth T about α and β" than the values for α and β entailed by H_1.

As Miller (1975) explains (see Miller [2006], chap. 11, for a nice historical survey), there is a much more general result in the vicinity. It can be shown that for any pair of false theories H_1 and H_2 about parameters ϕ and ψ, many comparative relations of "closer to the truth" between H_1 and H_2 regarding ϕ and ψ can be reversed by looking at what the estimates provided by H_1 and H_2 for ϕ and ψ entail about quantities α and β, which are symmetrically interdefinable with respect to ϕ and ψ, via some (linear) intertranslation of the form
$$\alpha = a\psi + b\phi \quad \beta = c\psi + d\phi,$$
$$\phi = a\beta + b\alpha \quad \psi = c\beta + d\alpha.$$
That is, for many cases in which we judge that "H_2 is closer to the truth T about ϕ and ψ than H_1 is" (in many ways of comparing "closeness"), there will exist some member of the above family of symmetric intertranslations such that we will judge that "H_1 is closer to the truth T about α and β than H_2 is." In this way, we can often reverse accuracy comparisons of quantitative theories via such redescriptions of prediction problems. As such, many assessments of the accuracy of predictions are language dependent.[1]

1. Strictly speaking, this only becomes a *language dependence* problem if we adopt the language $\mathcal{L}_{\phi\psi}$ in which ϕ and ψ are *primitive parameters*, and we treat α and β as *defined parameters* in $\mathcal{L}_{\phi\psi}$—as opposed to adopting the language $\mathcal{L}_{\alpha\beta}$, and treating ϕ and ψ as defined in $\mathcal{L}_{\alpha\beta}$. Otherwise, we could characterize what is going on here as a dependence of "distances from the truth" on a choice of parameters within a single language $\mathcal{L}_{\phi\psi\alpha\beta}$. I intend this to be a problem of language dependence. So I assume we start with an adopted language + set of primitive parameters. I thank an anonymous referee for pressing this clarification.

2. Joyce on Probabilistic Coherence and the "Accuracy" of Credences. According to Joyce (1998), if we view credences (of rational agents) as numerical estimates of truth-values of propositions, then we can give an argument for probabilism that is based on considerations having to do with the "accuracy" of such estimates. I will not get into all the details of Joyce's various arguments here. Rather, I will focus on a simple, concrete example that illustrates a (potential) problem of language dependence.

Consider an agent *S* facing a very simple situation, involving only one atomic sentence *P*. Suppose that *S* is logically omniscient (i.e., *S* assigns the same credences to logically equivalent statements, and he also assigns zero credence to all contradictions and credence one to all tautologies in his toy language). Thus, all that matters concerning *S*'s coherence (in Joyce's sense) is whether *S*'s credences *b* in *P* and ¬*P* sum to 1 (and are nonnegative). Now, following Joyce, we will associate the truth-value *True* with the number 1 and the truth-value *False* with the number 0. Let ϕ be the *numerical* value associated with *P*'s truth-value, and let ψ be the numerical value associated with ¬*P*'s truth-value (of course, ϕ and ψ will vary in the obvious ways across the two salient possible worlds: w_1, in which *P* is false, and w_2, in which *P* is true). We can now state (informally) the sort of theorem(s) that Joyce has been writing about for a number of years.

> **Theorem** (Joyce). If *S*'s credence function *b*—construed as providing estimates of ϕ and ψ—fails to be probabilistic, then there exists a probabilistic b' that is more accurate than *b* (according to a suitable "scoring rule") regarding ϕ and ψ—in all possible worlds. And no coherent (probabilistic) credence function is accuracy-dominated in this sense by any incoherent credence function (a key asymmetry).

Joyce makes various assumptions about how to measure "the accuracy of estimates of ϕ and ψ—in a possible world." The various choices of "scoring rule" that one might make in order to render such "accuracy measurements" will not be important for the issue that I am going to raise here. The phenomenon will arise for any such instantiation of Joyce's framework. Rather than describing my "reversal theorem" in such general terms, I will illustrate it via a very simple concrete example, regarding our toy agent *S*, and assuming the Brier score as our "accuracy measure." Suppose that *S*'s credence function (*b*) assigns the following values *P* and ¬*P* (i.e., *b* entails the following numerical "estimates" of the quantities ϕ and ψ; see table 2).

Joyce's theorem entails the existence of a coherent set of estimates (b') of ϕ and ψ, which is more accurate than *b* (under the Brier score) in both of the salient possible worlds. I will say that such a b' Brier dominates *b*

TABLE 2.
THE CREDENCE FUNCTION (B) OF OUR SIMPLE INCOHERENT AGENT (S)

	ϕ	ψ
b	1/2	1/4

with respect to ϕ and ψ. To make things very concrete, let us look at an example of such a b' in this case. Table 3 depicts the *Euclidean-closest* such b', relative to Joyce's $\{0, 1\}$-representation of the truth-values (i.e., ϕ and ψ). (Ignore the α/β columns of table 3 for now—I will explain their significance later.)

The estimates entailed by b' are more accurate—with respect to ϕ and ψ—in both w_1 and w_2, according to the Brier score. A natural question to ask (in light of sec. 1, above) is whether there is a Miller-style symmetric intertranslation that can reverse this Brier-dominance relation. Interestingly, it can be shown (proof omitted) that there is no linear Miller-style symmetric intertranslation (of the simple form above) that will do the trick. But there is a slightly more complex (nonlinear) symmetric intertranslation that will yield the desired reversal (as depicted in table 3). Furthermore, it can be shown that this very same numerical intertranslation will yield such a reversal for any coherent function b' that Brier dominates b for this incoherent agent S (with respect to ϕ and ψ). To be more precise, we have the following theorem about our (particular) agent S:

Theorem. For any coherent function b' that Brier dominates S's credence function b with respect to ϕ and ψ, there exist quantities α and β that are symmetrically interdefinable with respect to ϕ and ψ, via the following specific symmetric intertranslations.[2]

$$\alpha = \frac{1}{2}\phi + \frac{1}{2}\psi + \frac{1}{16}\left(\frac{\phi + \psi}{\phi - \psi}\right) \quad \beta = \frac{1}{2}\phi + \frac{1}{2}\psi - \frac{1}{16}\left(\frac{\phi + \psi}{\phi - \psi}\right),$$

$$\phi = \frac{1}{2}\alpha + \frac{1}{2}\beta + \frac{1}{16}\left(\frac{\alpha + \beta}{\alpha - \beta}\right) \quad \psi = \frac{1}{2}\alpha + \frac{1}{2}\beta - \frac{1}{16}\left(\frac{\alpha + \beta}{\alpha - \beta}\right),$$

where b Brier dominates b' with respect to α and β. It is also noteworthy that the true values of α and β "behave like truth-values,"

2. Although our translations are more complex than the very simple, linear Miller-style translations above, our translations can be rendered *dimensionally homogeneous* by replacing "1/16" in the statement of the translations with "c/16," where c is in the units of ϕ and ψ, and c takes the value 1. So amended, our translations would be appropriate for quantities with an associated physical dimension (e.g., velocities). But because we are dealing with dimensionless quantities here (e.g., *probabilities*), dimensional homogeneity is not even a pressing issue for us. See Szirtes (2007), chap. 6, for a useful discussion concerning dimensional homogeneity.

TABLE 3. An Example of the Language Dependence of Joycean Brier Domination

	ϕ	ψ	α	β
b	1/2	1/4	9/16	3/16
b'	5/8	3/8	3/4	1/4
w_1	0	1	7/16	9/16
w_2	1	0	9/16	7/16

in the sense that (1) the true value of α (β) in w_1 (w_2) is identical to the true value of β (α) in w_2 (w_1), and (2) the true values of α and β always sum to one. Indeed, these transformations are guaranteed to preserve coherence of all dominating b''s, and the "truth-vectors."[3]

So while it is true that there are some aspects of "the truth" with respect to which S's credence function b is bound to be less accurate than (various) coherent b''s, it also seems to be the case that (for any such b') there will be specifiable, symmetrically interdefinable aspects of "the truth" on which the opposite is true (i.e., with respect to which b' is bound to be less accurate).

In the next section, I consider several possible reactions to this Milleresque "language dependence of the accuracy of credences" phenomenon. In the end, I think the upshot will be that Joyce needs to tell us more about (precisely) what he means when he says that "credences are (numerical) estimates of (numerical) truth-values." Specifically, I think the present phenomenon challenges us to get clearer on the precise content of the *accuracy norm*(*s*) that are applicable to (or constitutive of) the Joycean cognitive act of "estimation of the (numerical) truth-value of a proposition."

3. Some Possible Reactions.

3.1. Naturalness/Privileged Language. One might try to maintain that (in some sense) the quantities ϕ and ψ are "more natural" (in this context) than α and β or that the "estimation problem" involving ϕ and ψ is somehow "privileged" (in comparison to the α/β "estimation problem"). I do not really see how such an argument would go. First, from the point of view of the α/β-language, the quantities ϕ and ψ seem just as "ger-

3. A *Mathematica* notebook that contains verifications of all of the technical claims made in this article is available from the author. The notebook can be downloaded from http://fitelson.org/joyce.nb. More general results can be proven (and further constraints can be accommodated on the desired translation scheme). But all I need (dialectically) is one incoherent agent S for which I can ensure reversals of all such Brier-dominance relations via a single, symmetric intertranslation to/from the ϕ/ψ representation and the α/β representation. See the last section for further discussion.

rymandered" as the quantities α and β might appear from the point of view of Joyce's preferred numerical representation of the truth-values. Moreover, there is a disanalogy to the case of physical magnitudes like velocity, since truth-values do not seem to have numerical properties (per se). That is, there is already something a little artificial about thinking of truth-values as the sort of things that can be "numerically estimated" (where the "estimates" are numerically scored for "accuracy").

3.2. "Asymmetries" in Accuracy-Dominance in the α/β-Language. One might try to find some (new) accuracy-dominance *asymmetry* between coherent and incoherent credences in the α/β-*language*. I see two problems with this strategy. First, in the α/β-language (as opposed to the ϕ/ψ-language), some coherent vectors (in Joyce's sense) are Brier dominated by an incoherent vector (witness the example above). Having said that, it is also true that there do exist other coherent credence functions $b^{\star}$ that Brier dominate b with respect to α and β. As such, we do not have an "utter reversal" of the (full) asymmetry between coherent and incoherent vectors in the ϕ/ψ-language. I am not sure that is required here (for our purposes). We have certainly broken the (full) asymmetry between coherent and incoherent vectors. Moreover, we could define up another pair of quantities γ and η (symmetrically interdefinable with respect to α and β—perhaps relative to a new family of intertranslations), which reverses these new ($b^{\star}$ vs. b) relations of Brier dominance, and so on. So, this response just seems to reiterate the initial problem.

3.3. Disanalogies between "Estimation" and "Prediction." I think the most promising (and useful) response to the phenomenon is to argue (i) that there are crucial *disanalogies* between "estimation" (in Joyce's sense) and "prediction" (in the sense presupposed by Popper and Miller), and (ii) these disanalogies imply that my "reversal argument" is presupposing something incorrect regarding the norms appropriate to "estimation."

Here, it is important to note that Joyce does not tell us very much about what he means by "estimation." He does say a few things that are suggestive about what "estimation" is not. Specifically, Joyce clearly thinks:

1. Estimates are not *guesses*. Joyce (1998, 587) explicitly distinguishes estimation and guessing. Presumably, guessing (as a cognitive act) does not have the appropriate normative structure to ground the sorts of accuracy norms (for credences) that Joyce has in mind.
2. Estimates are not *expectations*. Joyce (1998, 587–88) explicitly disavows thinking of estimates as expectations. Indeed, this is supposed

to be one of the novel and distinctive features of Joyce's approach. In fact, it is supposed to be one of the advantages of his argument (over previous, similar arguments). Here, Joyce seems to think that expectations have two sorts of (dialectical) shortcomings, in the present context. First, he seems to think that they have a pragmatic element, which is not suitable for a nonpragmatic vindication of probabilism. Second, expectations seem to build in a nontrivial amount of *probabilistic structure* (via the definition of expectation, which presupposes that $b(\neg p) = 1 - b(p)$), and this makes the assumption that estimates are expectations question-begging in the present context.

3. Estimates are not *assertions* that the values of the parameters are such-and-such. This is clear (just from the nature of these "estimation problems"), since it is not a good idea to assert things that you know (a priori) must be false. And, whenever you offer "estimates" of credences that are nonextreme, you know (a priori) that the parameters (ϕ and ψ) do not take the values you are offering as estimates (an analogous point can be made with respect to what b and b' "assert" about α and β).

These are the only (definite, precise) commitments about "estimates" that I have been able to extract from Joyce's work (apart from the implicit assumption concerning the appropriateness of "scoring" them in terms of "accuracy" using the Brier score). Unfortunately, these negative claims about what Joyce means by "estimation" do not settle whether my "reversal argument" poses a problem for Joyce. Allow me to briefly explain why.

Let $\ulcorner \mathcal{E}(x, y) = \langle p, q\rangle \urcorner$ be the claim that $\ulcorner S$ is committed to the values $\langle p, q\rangle$ as their "estimates" (in Joyce's sense) of the quantities $\langle x, y\rangle \urcorner$. What we need to know are the conditions under which the following principle (which is implicit in my "reversal argument") is acceptable, relative to Joyce's notion of "estimation" $\mathcal{E}$:

(†) If $\mathcal{E}(\phi, \psi) = \langle p, q\rangle$, then $\mathcal{E}(\alpha, \beta) = f(p, q)$, where f is a symmetric intertranslation function that maps values of $\langle \phi, \psi\rangle$ to/from values of $\langle \alpha, \beta\rangle$.

Presumably, there will be some symmetric intertranslation functions f (in some contexts) such that † is acceptable to Joyce. The question is, which translation functions f are acceptable—in my example above?

Since Joyce does not give us a (sufficiently precise) theory of $\mathcal{E}$, it is difficult to answer this question definitively. But, if my "reversal argument" is going to be blocked, then I presume that Joyce would want to reject

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† for our intertranslation function $f^{\star}$ above. It is natural to ask precisely what grounds Joyce might have for such a rejection of our $f^{\star}$.

It is useful to note that † is clearly implausible, under certain interpretations of $\mathcal{E}$. Presumably, if $\mathcal{E}$ involves guessing, then one could argue that † should not hold (in general). Perhaps it is just fine for S's guesses about the values of $\langle \phi, \psi \rangle$ to be utterly independent of S's guesses about $\langle \alpha, \beta \rangle$ (at least, to the extent that I understand the "norms of guessing"). Similarly, if $\mathcal{E}$ involves expectation, then † will demonstrably fail (in general) for nonlinear functions like our $f^{\star}$. Although, on an expectation reading of "estimate," † will demonstrably hold for all linear intertranslations. Unfortunately for Joyce, neither of these interpretations of $\mathcal{E}$ is available to him. So, this yields no concrete reasons to reject † in our example.

On the other hand, if $\mathcal{E}$ involved *assertion* (as in item 3 above), then † would be eminently plausible. On an assertion reading of $\mathcal{E}$, † is tantamount to a simple form of deductive closure for assertoric commitments (in the traditional sense). And this would be very similar to the way Popper (1972) and Miller (1975) were thinking about the predictions of (deterministic, quantitative) scientific theories.[4] It seems clear that $\mathcal{E}$ is not exactly like that (in this context), but this (alone) does not give us any concrete reasons to reject † in this case.

I submit that what we need here is a (sufficiently precise) theory of $\mathcal{E}$, which satisfies Joyce's explicit commitments 1–3 above and which is also precise enough to explain why † should fail for $f^{\star}$ (in our example above). At the very least, this article serves as an invitation to Joyce to provide such an (independent) philosophical explication of his $\mathcal{E}$.

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4. See Hacking (1965) for an interesting discussion of different senses of "estimation" and "prediction" in the context of statistical theory.