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ON THE EXPECTATIONAL STABILITY OF RATIONAL EXPECTATIONS EQUILIBRIA IN NEWS-SHOCK DSGE MODELS

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The expectational stability (E-stability) property of rational expectations equilibria (REE) in linear macroeconomic dynamic stochastic general equilibrium (DSGE) models is known to be sensitive to the information available to decision makers as well as the structure of the economic environment considered. Models featuring news shocks as a source of macroeconomic fluctuations depart from traditional assumptions regarding both the structure of the economy and the information set of agents. This paper investigates whether E-stability of REE is affected by either the inclusion of news shocks by themselves or the complementary structural changes. The main results find that the E-stability property of REE is robust to the inclusion (or exclusion) of news shocks and that well-known news-shock DSGE models permit REE which are simultaneously E-stable and capable of producing qualitatively realistic expectationally driven business cycles.

Keywords: News shocks, Bounded rationality, Adaptive learning, Dynamic stochastic general equilibrium models

1. INTRODUCTION

Starting with Kydland and Prescott (1982), the economic literature seeking to explain stylized facts and comovements observed in postwar macroeconomic U.S. data through supply-side innovations has flourished. So-called real business cycle (RBC) theory maintains innovations to productivity are responsible for the booms and busts which comprise the business cycle. This implies a peculiar interpretation of cyclical activity: while expansions are driven by technological progress, recessions must be caused by technological regress.[1] Recently, a subset of RBC literature has begun exploring an information-based theory of business

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cycle activity in which booms and busts result from information received by decision makers about the future. By focusing on the potential role of information and expectations, the literature on so-called "news shocks"—defined to be exogenous changes to agents' information sets which are useful in predicting future economic fundamentals—has expanded the scope of plausible explanations for business cycle fluctuations.

As noted in Krusell and McKay (2010), business cycles are characterized by positive comovement in aggregate consumption, investment, employment, and output. While standard RBC models generate positive procyclical comovements in key variables in response to contemporaneous productivity shocks, they are unable to generate these positive comovements in response to anticipated productivity shocks. Such models typically imply news about future total factor productivity (TFP) will cause consumption to move opposite investment, employment, and output, that is, good news about the future state of the economy causes either a drop in consumption or a recession today.[2]

A variety of dynamic stochastic general equilibrium (DSGE) models from both the RBC and New Keynesian (NK) traditions have arisen in an effort to solve this "comovement problem" and produce qualitatively realistic expectationally driven business cycles in response to the receipt of news about future economic fundamentals. Beaudry and Portier (2004) leverage complementarity between durable and nondurable factors in final good production with short-term substitutability constraints on consumption and investment to induce positive comovement in response to news about future productivity in a flexible-price setting; in contrast, Jaimovich and Rebelo (2009) incorporate variable capital utilization, investment adjustment costs, and a novel preference specification, while Guo et al. (2015) introduce variable capital utilization and increasing returns to scale production into an otherwise standard RBC model to achieve the same positive comovement. Khan and Tsoukalas (2012) consider a sticky-price version of Jaimovich and Rebelo (2009) which maintains the positive comovement induced by anticipated productivity shocks, while Lorenzoni (2009) considers a sticky-price model in which firms receive a noisy public signal of future aggregate productivity and a private signal of their own productivity which is capable of generating empirically realistic comovements in response to anticipated demand shocks.

To this point, the news shock literature has relied exclusively on the assumption championed by Muth (1961) that agents have model consistent or rational expectations (RE). In other words, agents are perfectly informed as to the precise laws of motion governing the evolution of the economy, implying their forecasts will be correct on average. While RE addresses the econometric critique of Lucas (1976), it is a very strong assumption regarding the behavior of economic agents. Indeed, Lucas (1978) states

> "...[Rational Expectations] does not describe the way agents think about their environment, how they learn, process information, and so forth. It is rather a property likely to be (approximately) possessed by the outcome of this unspecified process of learning and adapting."

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The question of whether this property is approximately possessed or not by the resulting rational expectations equilibria (REE) of DSGE models has been forcefully taken up by the bounded rationality and adaptive learning literature assembled in Evans and Honkapohja (2001) and proposed by Marcet and Sargent (1989a,b,c). One way to gauge the strength of the RE assumption for a given model is to investigate whether the behavior of economic agents endowed with less sophisticated forecasting models can come to resemble that of agents endowed with rational expectations. REE possessing this quality are said to be Expectationally Stable (E-stable), a property which has emerged as a useful selection criterion in evaluating the plausibility of various REE.

The asymptotic behavior of such "boundedly rational" agents has been investigated within many well-known macroeconomic models, and the results have revealed E-stability of REE to be sensitive to both the particular economic environment considered and the specific assumptions regarding the information sets of decision makers.[3] That these are precisely the areas of existing models which must be modified to accommodate the information-based view of business cycles motivates the central questions of this paper: What effect (if any) does the inclusion of news shocks have on the E-stability of REE in general? What impact do the structural innovations used to generate qualitatively realistic expectationally driven business cycles have on the E-stability of REE within well-known news-shock DSGE models?

The answer to the first question comes in the form of two propositions which formally prove the E-stability property of a particular REE is robust to the inclusion (or exclusion) of news shocks themselves. These results are to an extent unsurprising, given that news shocks matter for forward-looking agents because they convey information about future exogenous variables, whereas E-stability is essentially a convergence property arising from backward-looking adaptive learning models. However, it is worth noting that E-stability can be quite sensitive to specific assumptions regarding the information set available to agents at the time of forecasting which makes a more formal approach worthwhile.[4]

The answer to the second question comes in the form of an E-stability analysis for the two canonical neoclassical news-shock DSGE models of Beaudry and Portier (2004) and Jaimovich and Rebelo (2009) which shows there is no inherent tradeoff between the REE of a given model possessing the E-stability property and permitting qualitatively realistic expectationally driven business cycles in response to news shocks. Indeed, the REE for both models are E-stable and capable of producing news-driven business cycles at the authors' original calibrations as well as a variety of alternate assumptions regarding the density of the information set and the calibration of parameters.

There are three main contributions this work makes to the literatures on news-driven business cycles and adaptive learning. First, it formally confirms the intuition that E-stability results should not be affected by the inclusion (or exclusion) of news shocks within a given model, thereby permitting the inclusion of richer informational assumptions in existing models without raising concerns for

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the plausibility of the resulting REE. Second, it clarifies that the known sensitivity of the E-stability property with respect to the timing of informational flows is related specifically to knowledge about endogenous rather than exogenous variables. Third, since E-stability is often used as a selection criterion for evaluating the plausibility of REE both within and between DSGE models, the finding that canonical news-shock models permit E-stable REE can be viewed as a confirmation of the viability and potential value for incorporating such models in applied and policy-centric macroeconomic research.

The paper proceeds as follows. Section 2 describes the E-stability analysis for a general class of models which may or may not feature news shocks and establishes the first result: the inclusion (or exclusion) of news shocks themselves has no bearing on whether a given REE is E-stable. Section 3 applies the analysis to the news-shock models of Beaudry and Portier (2004) and Jaimovich and Rebelo (2009) and establishes the second result: the REE for canonical news-shock models are simultaneously E-stable and capable of producing qualitatively realistic expectationally driven business cycles for a range of calibrations. Section 4 concludes and discusses avenues for future research.

2. ADAPTIVE LEARNING, E-STABILITY, AND NEWS SHOCKS

Consider an economy with temporary equilibria described in general by the stationary system of linear expectational difference equations

$$y_t = \alpha + \beta y_{t-1} + \chi w_t + \delta \hat{E}_t y_{t+1}, \tag{1a}$$

$$w_t = \varphi w_{t-1} + M\nu_t, \tag{1b}$$

where y_t is an $n_y \times 1$ vector of endogenous variables, ν_t is an $n_s \times 1$ vector of exogenous stochastic innovations, and w_t is an $n_w \times 1$ vector of auxiliary variables which filter the exogenous stochastic shocks into the model's driving variables. The matrices α, β, χ, and δ describe the correspondence between the contemporaneous realization of endogenous variables and observations of the past, expectations of the future, and exogenous stochastic shocks, while restrictions on the matrices φ and M describe the anticipation structure which determines the timing of information flows about current and future exogenous stochastic shocks as well as their actual impact on fundamentals. $\hat{E}_t$ denotes the (possibly subjective) expectations operator conditional on the time t information set $\mathcal{I}_t$ which, except in the cases of adaptive learning noted below, includes the history of all model variables dated time t or earlier.

As demonstrated in Appendix A, the anticipation structure of exogenous stochastic innovations—which describes whether and how news shocks are included in the model—is in general captured by the specification of particular components of (1a) and (1b): the maximum length of the anticipation horizon (which may be zero) and the number of driving processes in the economy jointly determine the sizes of w_t, χ, φ, and M, while the non-zero elements of M explicitly characterize assumptions regarding the periodicity of information flows about

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the exogenous stochastic processes contained in ν_t. Notably, in this framework, decisions about the anticipation structure have no impact on y_t or the matrices α, β, and δ, thereby facilitating a direct comparison of a given model across different informational structures. In what follows, I will exploit this fact to assess what impact the inclusion (or exclusion) of news shocks from a given model has on the E-stability properties of corresponding REE.

The main results of this section are that what while the inclusion (or exclusion) of news shocks certainly affects the behavior of forward-looking agents, it leaves unaltered the E-stability of the associated REE under adaptive learning. The result is intuitive: news shocks affect the behavior of forward-looking agents in response to information about future exogenous variables, while adaptive learning is a backward-looking specification for how agents incorporate past forecast errors into their current forecasting models. Since news shocks convey information about variables which are beyond the control of agents, the informational feedback loops which may cause E-instability of REE should be unaffected by the anticipation structure. In what follows, I simply state these findings in two propositions: the interested reader is directed to Appendix B for technical details concerning the implementation of adaptive learning and E-stability analysis of REE for systems like (1a) and (1b) above, and Appendices C and D for proofs of each proposition.

Practically speaking, the analysis consists of evaluating conditions under which the forecasting model or "Perceived Law of Motion (PLM)" of boundedly rational economic agents can, over time, converge to that of fully rational agents. Actions taken by agents in response to the forecasts generated by the PLM yield the "Actual Law of Motion (ALM)" for the economy, and agents update their forecasting model in response to observed differences between the PLM and the ALM. An E-stable REE then has the interpretation as a dynamically stable fixed point in the mapping from the PLM to the ALM.

Formally, the dynamic stability of this mapping—called the "T-map"—is determined by the eigenvalues of its individual components. Propositions 10.1 and 10.3 of Evans and Honkapohja (2001) show that a given REE is E-Stable if all eigenvalues of the Jacobian of the vectorized T-map have real parts less than 1 when the values of contemporaneously determined endogenous variables are excluded or included in the information set of agents, respectively. Thus, to determine what effect the inclusion (or exclusion) of news shocks *per se* have on the E-stability of a given REE it is sufficient to study the behavior of these eigenvalues in response to changes in the matrices φ and M from the news-filtering system (1b) above, which jointly characterize the time t information set with respect to current and future exogenous stochastic shocks.

Proposition 1 establishes that making the anticipation structure longer or more dense does not affect the value of the eigenvalue with the largest real part when the value of contemporaneous endogenous variables is included in agents' information sets at the time they make their decisions; Proposition 2 establishes the same for the case when the values of contemporaneous endogenous variables are

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not included in agents' information sets and thus, under adaptive learning, must be forecast by agents.[5] As a result, the E-stability properties of REE from a given model are unaffected by the assumed structure for news shocks. Proofs for each can be found in Appendices C and D, respectively.

PROPOSITION 1. *If $y_t \in \mathcal{I}_t$, then the E-stability property of an REE for a given model is robust to changes to the anticipation structure for exogenous stochastic shocks.*

PROPOSITION 2. *If $y_t \notin \mathcal{I}_t$, then the E-stability property of an REE for a given model is robust to changes to the anticipation structure for exogenous stochastic shocks.*

Propositions 1 and 2 imply that the conditions under which a given REE is possessing of the E-stability property are the same regardless of whether and how news shocks are included in the model; that is, E-stability of REE is determined entirely by the economic structure characterized by y_t and the matrices α, β, and δ as opposed to the informational anticipation structure characterized by w_t, ν_t, and the matrices χ, φ, and M. One immediate implication of this is that any models with well-known E-stable REE can be modified to include news shocks if desired, while the plausibility of those which admit E-unstable REE will not be improved simply by changing the anticipation structure.[6]

The net effect of these results is to shift the focus from the news shocks themselves to the structural changes necessary to for DSGE models to generate qualitatively realistic expectationally driven business cycles in response to news. To explore this issue I turn to an E-stability analysis of the REE for the well-known news-shock DSGE models of Beaudry and Portier (2004) and Jaimovich and Rebelo (2009) to determine whether the modifications necessary to solve the comovement problem affect the E-stability of REE.

3. NEWS-SHOCK MODELS AND E-STABILITY

The major success of the news-shock literature has been to describe economic environments in which the anticipation and subsequent realization of such shocks can generate qualitatively and quantitatively realistic expectationally driven business cycles. Accomplishing this requires positive news about future fundamentals lead to increased private investment today which is financed by agents increasing their labor supply as opposed to decreasing their consumption. The news-shock models of Beaudry and Portier (2004) and Jaimovich and Rebelo (2009) take strikingly different approaches in modeling the economy so as to satisfy these criteria. In this section, I analyze the E-stability of the REE corresponding to baseline and alternate calibrations for the length and denseness of anticipation structures in each model. My results suggest that there is no inherent tradeoff between E-stability and a model's ability to generate qualitatively realistic expectationally driven business cycles: both admit E-stable REE which are capable of

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generating positive comovement in key macroeconomic variables in response to news about future economic fundamentals regardless of whether the values of contemporaneous endogenous variables are known to agents at the time they make their decisions or not.

3.1. E-Stability and Beaudry and Portier (2004)

Beaudry and Portier (2004) solve the comovement problem by constructing a three-sector RBC model featuring separate production processes for an investment or "durable" good used to produce capital and an intermediate or "nondurable" good which is combined with capital to produce a final consumption good. The keys to generating positive comovement in this model are twofold. Because the nondurable good and capital are assumed to be complements in production of the consumption good, positive news about the future technology used to produce the nondurable good causes the demand for investment to increase contemporaneously in order to build the capital stock. And because the consumption and investment decisions are decoupled from each other—the value of investment is related only to the loss of leisure by increasing labor in the durable good sector, as opposed to the loss of utility from reducing consumption—the increased investment is paid for by households increasing their labor supply. Thus, the model produces qualitatively realistic expectationally driven business cycles in which positive news about future fundamentals an increase in all key macroeconomic variables upon receipt of the news.

Formally, the composite consumption good C_t is produced from the nondurable good X_t and the (predetermined) capital stock K_t according to

$$C_t = \left(aX_t^v + (1-a)K_t^v\right)^{\frac{1}{v}}, \quad (2)$$

where $v \leq 0$ ensures the inputs are complements in production and $a \in [0, 1]$. The nondurable good is produced from household labor $l_{x,t}$, a fixed factor F_x, and technology $\theta_{x,t}$ according to the constant returns to scale (CRTS) production function

$$X_t = \theta_{x,t} l_{x,t}^{\alpha_x} F_x^{1-\alpha_x}, \quad (3)$$

where $\alpha_x \in (0, 1)$ captures labor's steady-state share of nondurable-good production. The law of motion for the aggregate capital stock is

$$K_{t+1} = (1-\delta)K_t + I_t, \quad (4)$$

where $\delta \in [0, 1]$ is the per-period rate of depreciation, I_t is gross private investment produced by the durable goods sector from household labor $l_{k,t}$, a fixed factor F_k, and technology $\theta_{k,t}$ according to the CRTS production function

$$I_t = \theta_{k,t} l_{k,t}^{\alpha_k} F_k^{1-\alpha_k}, \quad (5)$$

where $\alpha_k \in (0, 1)$ captures labor's steady-state share of durable-good production. The fixed factors are inelastically supplied by households and have the effect of

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introducing diminishing marginal returns in labor supply while maintaining CRTS in overall production.[7]

Households are infinitely lived and receive utility from consumption and disutility from supplying labor to the durable and nondurable sectors. The lifetime utility function is assumed separable in consumption and labor, and is given by

$$U = \hat{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t) - v_0 \left(\bar{l} - l_{x,t} - l_{k,t} \right) \right\} \right], \quad (6)$$

where $\beta \in (0, 1)$ is the discount factor, $v_0 > 0$ scales the disutility of supplying labor, and $\bar{l}$ is total disposable time. $\hat{E}_t$ denotes the subjective expectations of the household given their time t information set. The flow budget constraint is

$$C_t + P_t I_t = W_{x,t} l_{x,t} + W_{k,t} l_{k,t} + R_t K_t + \Pi_{x,t} + \Pi_{k,t}, \quad (7)$$

where P_t is the price of the investment good in terms of the consumption good. Markets are competitive, and $W_{x,t}$ and $W_{k,t}$ are the wage rates paid to labor in the nondurable and durable goods sectors, R_t is the gross rental rate of capital, and $\Pi_{x,t}$ and $\Pi_{k,t}$ are the returns to renting the fixed factor in the nondurable and durable goods sectors, respectively. Returns to factors are all expressed in terms of the consumption good. The aggregate resource constraint is

$$Y_t = C_t + P_t I_t, \quad (8)$$

where Y_t is the total output of the economy. Technology of the durable good exhibits a deterministic trend such that

$$\log \theta_{k,t} = g_{0,k} + g_1 t, \quad (9)$$

while the technology of the nondurable grows stochastically according to

$$\log \theta_{x,t} = g_{0,x} + g_1 t + \log \hat{\theta}_{x,t}, \quad (10)$$

$$\log \hat{\theta}_{x,t} = \lambda \log \hat{\theta}_{x,t-1} + w^0_{\hat{\theta}_x,t}, \quad (11)$$

where $\lambda \in (0, 1)$. One may interpret innovations to nondurable good-specific technology as capturing the process of product differentiation, for example, the creation of higher quality or entirely new products. New goods will require a higher stock of infrastructure, and this complementarity between nondurable TFP and the capital stock is key to the model's ability to generate qualitatively realistic expectationally driven business cycles. News itself is assumed to take the form of a shock to the growth of nondurable technology which is anticipated by agents $N > 0$ periods in advance; that is, we have

$$w^0_{\hat{\theta}_x,t} = \sigma^0_{\hat{\theta}_x} \nu^0_{\hat{\theta}_x,t} + \sigma^N_{\hat{\theta}_x} \nu^N_{\hat{\theta}_x,t-N},$$

where $\sigma^n_{\hat{\theta}_x} \geq 0$ for all n.

The temporary equilibrium is nonlinear and nonstationary; detrending the variables and log-linearizing the system of equations around its nonstochastic steady state results in a linear system of first-order expectational difference equa-

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TABLE 1. Calibrated parameters from Beaudry and Portier (2004)

Baseline parameterization	
Parameter	**Values**
β	0.98
δ	0.05
α_x	0.60
α_k	0.97
$\bar{l}$	2
v_0	1
F_x	1
F_k	1
λ	0.999
v	−3.78

tions which takes the form of (1a) and (1b) above. Given calibrated values for parameters it is straightforward to apply the results from Appendix B to explore E-stability of the REE by calculating the eigenvalues of the vectorized Jacobian of the T-map. Calibrated values from Beaudry and Portier (2004) are determined using a combination of previous research, steady-state targets, and the method of simulated moments (MSM). I utilize their baseline calibrations, which are summarized in Table 1.

I begin by setting $N = 0$ which corresponds to the case in which all innovations to the process describing the growth of nondurable goods are completely unanticipated. The Jacobian of the vectorized T-map can be shown to be composed of three independent elements; under adaptive learning, the largest real roots of each component are given by 0.5199, 0.5195, and 0 when the value of contemporaneous endogenous variables are included in the information set and 0.1547, 0.1538, and 0 when they are instead forecast by boundedly rational agents.

Because these are all less than 1 the REE of this news-shock model is E-Stable under both informational assumptions; Propositions 1 and 2 suggest the REE will remain E-stable under alternate specifications with longer or denser anticipation structures, that is, cases where $N > 0$ so that news shocks are included as well as cases where multiple innovations are anticipated across different periods. Indeed, replicating the E-stability analysis for $N = 1, 2, ..., 10$ so that households receive information about the technological innovation between one and ten periods ahead under the baseline parameterization, as well as anticipation structures featuring a multitude of shocks which differ by their anticipation horizon as in Appendix A.3 causes no change to the size of any eigenvalues.[8] Thus, the REE for the model of Beaudry and Portier (2004), which is capable of generating realistic expectationally driven business cycles, is E-stable at the baseline parameterization used in the original paper and a variety of alternate anticipation structures.

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TABLE 2. Robustness in Beaudry and Portier (2004)

	Robustness checks	
Parameter	**Smallest value**	**Largest value**
β	0.001	1
δ	0	1
α_x	0.001	0.999
α_k	0.001	0.999
v	−100	−0.001
a	0.001	0.999

The key parameter for generating qualitatively realistic expectationally driven business cycles in this model is v, which characterizes the elasticity of substitution between the nondurable and capital goods in producing the consumption good. As long as this is negative, the inputs are complements and hence positive news about the future state of nondurable technology will generate higher contemporaneous investment to build the capital stock. To explore whether alternate values for this and other parameters may impact the E-stability results, I adopt a strategy of incrementally changing the value of structural parameters one at a time while holding all others at their baseline calibration, searching only within the parameter space consistent with the model generating qualitatively realistic expectationally driven business cycles. Table 2 displays the smallest and largest values considered for each parameter. The REE is E-stable for all parameterizations considered whether or not the values of contemporaneous endogenous variables are included in the information set of agents.[9]

3.2. E-Stability and Jaimovich and Rebelo (2009)

Jaimovich and Rebelo (2009) construct a one-sector RBC model featuring variable capacity utilization, costs to adjusting gross investment, and a novel preference structure which disciplines the wealth elasticity of labor supply. Because adjusting gross investment is costly, households optimally choose to gradually increase their investment flows upon receiving positive news about future capital returns; they simultaneously increase their utilization of the (now larger) capital stock since the actual state of the economy—and hence the returns on the existing capital stock—has not yet changed. The net effect is to lift the marginal product of labor and hence the wage rate, and preferences are calibrated to ensure that the substitution effect dominates the wealth effect of labor supply. As a result, consumption, gross investment, hours worked, and output all simultaneously rise upon receipt of positive news about future fundamentals.

Formally, the representative household chooses a sequence for consumption C_t and hours worked h_t to maximize the lifetime utility function

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$$U = \hat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - \psi h_t^{\theta} S_t\right)^{1-\sigma} - 1}{1-\sigma}, \quad (12)$$

where $\beta \in (0, 1)$ characterizes the household's discount factor, $\sigma > 0$ is the (inverse) intertemporal elasticity of substitution, $\psi > 0$ scales the disutility of labor supply, and $\theta > 0$ governs the Frisch elasticity of labor supply. S_t is a geometric average of current and past habit-adjusted consumption and takes the form

$$S_t = C_t^{\gamma} S_{t-1}^{1-\gamma}, \quad (13)$$

where $\gamma \in (0, 1]$ governs the magnitude of the wealth elasticity of labor supply. This preference specification, often referred to as "JR preferences," allows the modeler to calibrate the wealth elasticity of labor supply to be small in magnitude relative to the substitution elasticity of labor supply while permitting a balanced growth path.[10] Small values for γ imply a relatively weak wealth effect of labor supply which permits labor supply to rise in response to higher wages.

Households are assumed to rent their predetermined stock of physical capital K_t to firms in a competitive factor market. Each household's stock of capital evolves according to the law of motion

$$K_{t+1} = (1 - \delta(u_t)) K_t + I_t \left[1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right], \quad (14)$$

where I_t denotes gross private investment. The function $\Phi\left(\frac{I_t}{I_{t-1}}\right)$ imposes an increasing cost to adjusting investment from its previous level, while $\delta(u_t)$ implies capital depreciation is an increasing function of its utilization rate u_t. I follow Schmitt-Grohe and Uribe (2012) in assuming quadratic functional forms for each and calibrate the model to imply, the depreciation rate is constant and there are no investment-adjustment costs on the balanced growth path.

Each period the household receives labor income from working h_t hours at rate W_t, rental income from renting $u_t K_t$ units of effective capital at gross rental rate R_t, and lump sum firm-profits of Π_t. The household uses this income to purchase consumption and investment goods. The flow budget constraint is given by

$$C_t + A_t I_t = W_t h_t + R_t (u_t K_t) + \Pi_t, \quad (15)$$

where A_t is a stationary process representing the current state of technology for producing investment goods from consumption goods which is subject to a combination of anticipated and unanticipated exogenous stochastic innovations.[11]

Finally, the representative firm rents h_t worker-hours and rents $u_t K_t$ units of effective capital to produce output Y_t using CRTS technology according to the production function

$$Y_t = z_t (u_t K_t)^{1-\alpha} h_t^{\alpha}, \quad (16)$$

where $\alpha \in (0, 1)$ governs labor's steady state share of output and z_t is a stationary process representing the level of TFP which is subject to a combination of

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TABLE 3. Calibrated parameters from Jaimovich and Rebelo (2009)

Parameter	Value	Description
σ	1	Intertemporal Elasticity of Substitution
θ	1.4	Frisch-labor Supply Elasticity
γ	0.001	Wealth Elasticity of Labor Supply
β	0.985	Subjective Discount Factor
α	0.64	Steady-state Labor Share
δ_0	0.025	Steady-state Depreciation Rate
u	1	Steady-state Capacity Utilization Rate
h	0.2	Steady-state Labor Supply
κ	1.3	Adjustment Cost Acceleration
$\frac{\varphi''(\bar{u})\bar{u}}{\varphi'(\bar{u})}$	0.15	Elasticity of Depreciation
ρ_A	0.5	Persistence of Investment-specific Growth
ρ_z	0.9	Persistence of TFP Growth

anticipated and unanticipated exogenous stochastic innovations. All markets are competitive and hence the gross rental rate and wage rates equal the value of the marginal product of effective capital and labor, respectively, while firms earn zero profits. The aggregate resource constraint is thus

$$Y_t = C_t + A_t I_t, \tag{17}$$

Finally, investment-specific technology and TFP are assumed to evolve according to

$$\log z_t = \rho_z \log z_{t-1} + w^0_{z,t},$$
$$\log A_t = \rho_A \log A_{t-1} + w^0_{A,t},$$

where the auxiliary variables $w^0_{j,t}$ for $j = \{z, A\}$ are constructed as in Appendix A.4 to allow for both anticipated and unanticipated exogenous stochastic shocks. The model is calibrated using a combination of commonly used values in the literature, estimates obtained from Schmitt-Grohe and Uribe (2012), and steady-state targets for some endogenous variables. This is summarized in Table 3 below.

The system of equations describing the log-linearized temporary equilibrium can again be put in the standard form of (1a) and (1b) above and the E-stability analysis again proceeds as in section Appendix B. If the longest anticipation horizon is $\bar{N}_j = 0$ so that the innovations to both driving processes are entirely unanticipated, the largest real roots of the three elements which comprise the Jacobian of the vectorized T-map under adaptive learning are 0.9822, 0.9794, and 0 when the values of contemporaneous endogenous variables are included in the information set and 0.9820, 0.9792, and 0.3040 when they are instead forecast by boundedly rational agents. Since these are all less than 1, the REE of this news-shock model is E-Stable under both informational assumptions and, as suggested by Propositions 1 and 2, this property is robust to alternate anticipation structures which include anticipated shocks to the driving processes.

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Jaimovich and Rebelo (2009) conduct robustness tests on the parameters governing the key features of the model and determine there are large regions of parameter spaces in which the model can continue producing qualitatively realistic expectationally driven business cycles. In particular, the wealth elasticity of labor supply and the cost of increasing utilization rates must be sufficiently low while the cost of investment adjustment and the substitution elasticity of labor supply must be sufficiently high.[12] Unsurprisingly, fixing all other parameters at their baseline levels and varying each of these key parameters reveals parameter constellations which fail to generate positive comovement in response to news about future economic fundamentals. However, the REE remains E-stable in all cases whether or not the values of contemporaneous endogenous variables are included in agents' information sets.

4. CONCLUSION

The plethora of macroeconomic DSGE models invites methods to evaluate the plausibility of predictions both within and between models. The E-stability property of REE provides a simple selection criteria based on the sensitivity of the model's predictions to the modern paradigm of assuming rational expectations. This paper has shown that modifying information structures to include anticipated shocks does not impact the learnability of REE—that is, including anticipated shocks to agents' information sets will not by itself cause E-stability properties to change—and suggests the key to analyzing whether a given news-shock model will permit an E-stable REE is to focus on the economic structures required to generate positive comovement between key macroeconomic variables in response to news about future economic fundamentals. That the models of Beaudry and Portier (2004) and Jaimovich and Rebelo (2009) produce E-stable REE suggests news shock models are no less reliable for use in applied work than more traditional models which feature only unanticipated innovations to driving processes.

This is particularly important as one of the most promising uses of news shocks is as an additional tool in models featuring various nominal and real frictions for explaining the dynamics observed in macroeconomic data. Assessments of the quantitative importance of news shocks using estimated DSGE models can be found in a variety of papers including Schmitt-Grohe and Uribe (2012), Khan and Tsoukalas (2012), Beaudry and Portier (2014), and Sims (2016), yet determining the empirical relevance of anticipated shocks remains an active area of research. Furthermore, the scope of shocks considered has been expanded beyond traditional supply-side shocks: Ramey (2011) and Born, Peter and Pfeifer (2013) reveal the anticipation of fiscal policy to be an important source of macroeconomic volatility , news about monetary policy as in Milani and Treadwell (2012) and Keen, Richter and Throckmorton (2017) and Gunn and Johri (2018) show news about future returns on assets held by financial institutions can generate business cycle fluctuations via a credit channel.

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This paper sits at the intersection of literatures which consider the importance of news shocks and bounded rationality separately in driving business cycles, and is the first to specifically explore the interaction of news shocks and E-stability in DSGE models. A handful of other papers exist at this intersection, such as Slobodyan and Wouters (2016) and Dombeck (2019), which both explore the quantitative effects of interactions between news shocks and bounded rationality in macroeconomic models. This seems to be a natural direction of exploration given that the literature on news shocks is essentially focused on the role of what decision makers know at any given point in time, while the literature on bounded rationality focuses on how decision makers respond to a given information set. Information and expectation formation are fundamental to the study of economics in general and macroeconomics in particular, and this research contributes to our understanding of how the economic patterns we observe may be influenced by the assumptions we make. Indeed, recent and forthcoming papers such as Mitra, Evans and Honkapohja (2019) and Milani (in press) show that introducing adaptive learning can by itself produce estimates for fiscal multipliers and the Fed's inflation targets which are superior to those generated under RE.

Several other promising avenues for future research emerge from this study. For instance, the E-stability property is an asymptotic convergence result which operates on the extensive margin; that is, in the limit REE are either E-stable or they are not. But the property itself says nothing about the speed of convergence to the REE which can broadly be thought of as an intrinsic margin. A natural question is whether either the modifications necessary to produce qualitatively realistic expectationally driven business cycles or the news shocks themselves affect the speed of convergence for E-stable REE. Furthermore, E-stability has frequently been used in irregular models to select between a menu of possible REE. It would be interesting to consider whether the structural modifications of news-shock DSGE models have any impact on the existence and stability of the sunspot equilibria from such models. This latter line of research would help speak to whether and in which ways news shocks and sunspots are related.

NOTES

1. Stiglitz (2014) points out that "... by implication, the Great Depression was marked by an episode of acute amnesia, where in large parts of the world, people got less productive!"

2. For example, the baseline calibration of Kydland and Prescott's original model suggests households will substitute investment for consumption in anticipation of higher future marginal factor productivity stemming from anticipated future technological growth. Because leisure is a normal good, households also reduce employment, causing a drop in the level of output.

3. See, for example, Bullard and Mitra (2002) for a discussion of E-stability in a simple NK model in which determinate REE become E-unstable when the monetary authority sets the contemporaneous nominal interest rate as a function of expected contemporaneous values for inflation and the output gap.

4. Bullard and Eusepi (2014), for example, find that the well-known "determinacy implies E-stability" result from McCallum (2007) does not necessarily apply when the information set of agents does not include the value of contemporaneously determined endogenous variables in a standard

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NK-type model. They are able to document cases of determinant REE which are no longer E-stable under this informational delay. The present paper concludes that there is no similar sensitivity to changes in the anticipation structure resulting from the inclusion (or exclusion) of anticipated shocks.

5. The separate propositions are necessary because the informational assumptions change the implied decision-forecast feedback loop, and hence the structure of the T-map. Because model consistent expectations imply forecasts are correct on average, and the solution under rational expectations is unaffected by timing assumptions regarding the information available. When considering bounded rationality, however, this is no longer the case; differences between the PLM and the ALM will drive a wedge between expected and realized values and these residuals may be serially correlated. This wedge can in general significantly impact the E-stability property of REE. The lagged information assumption is akin to assuming agents must nowcast the variables relevant to their decisions rules and is therefore often regarded as a better representation of the forecasting environment for real-world agents. For example, policy makers typically rely on estimates of current inflation and the output gap in formulating an interest rate rule; likewise, households make consumptions/savings decisions without knowing what the aggregate behavior of the economy will be in that period.

6. For example, the sunspot REE considered in Ji and Xiao (2018) would continue to be unstable under learning if information sets were to be augmented to include news about the stochastic processes.

7. These fixed factors can be thought of as any scarce resource that constrains production such as privately held land or managerial capital.

8. As detailed in Appendix B, the inclusion of news shocks results in a duplication of the eigenvalues of the Jacobian of the vectorized T-map for a given model.

9. In general, the largest real parts of eigenvalues are larger when households discount the future less (smaller β), when the depreciation rate of capital is smaller (smaller δ), when the decreasing returns to producing the nondurable good are smaller (larger α_x), when the decreasing returns to producing the durable good are larger (smaller α_k), when the complementarity between capital and nondurable goods in producing the consumption good is weaker (larger ν), and as the relative importance of the nondurable good to capital in producing consumption decreases (smaller a). Even so, setting these parameters jointly to values which on their own would tend to imply relatively large eigenvalues does not change the E-stability of the REE whatsoever.

10. This specification nests two well-known and important preference specifications. $\gamma \to 0$ corresponds to the preferences of Greenwood et al. (1988) in which labor supply depends only on current real wages and is independent of the marginal utility of income, while $\gamma = 1$ corresponds to the preferences of King et al. (1988) which are compatible with a balanced growth path at the optimal steady state of the economy.

11. A_t may be interpreted as the relative price of investment goods in terms of consumption goods, that is A_t units of consumption may be traded for a single unit of investment (equivalently, $1/A_t$ units of the investment good are required to purchase a single unit of the consumption good).

12. Given the assumed functional forms, these dynamics are tuned through γ, δ_2, κ, and θ, respectively.

13. Alternatively one may simply set $N_x = 0$, which directly implies

$$w^0_{x,t} = \sigma^0_x \nu^0_{x,t}$$

and thus

$$x_t = \rho_x x_{t-1} + \sigma^0_x \nu^0_{x,t}$$

since we then have $w_{x,t} = w^0_{x,t}$, $\varphi_x = 0$, and $M_x = \sigma^0_x$. The main drawback to this approach is that it masks the fact that models which feature no anticipated shocks can be easily written as special nested cases of a more general framework in which anticipated shocks are allowed, a situation explored in A.3

14. For example, since the system governing the evolution of b is independent of a and c we may start with the equation

$$T_b(b) = (I - \delta b)^{-1} \beta$$

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the matrix differential of this equation with respect to b is obtained by applying the rule $dF^{-1} = -F^{-1}(dF)F^{-1}$, and hence

$$dT_b(b) = (I - \delta b)^{-1}\,\delta(db)\,(I - \delta b)^{-1}\,\beta$$

Since $d\,(\text{vec}\,x) = \text{vec}\,(dx)$ and $\text{vec}\,(ABC) = (C' \otimes A) \cdot \text{vec}\,(B)$, the vectorized Jacobian $DT_b = \partial\,\text{vec}\,T_b/\partial(\text{vec}\,b)'$ determining the local stability of this system evaluated at a particular b is given by

$$DT_b(b) = \left[(I - \delta b)^{-1}\,\beta\right]' \otimes \left[(I - \delta b)^{-1}\,\delta\right]$$

Similar operations can be employed to obtain the expressions for DT_a and DT_c which are described by (B.6a) and (B.6c).

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APPENDIX A: ADJUSTING THE ANTICIPATION STRUCTURE

This paper is concerned with economies possessing temporary equilibria described by the stationary system of linear expectational difference equations

$$y_t = \alpha + \beta y_{t-1} + \chi w_t + \delta \hat{E}_t y_{t+1}, \tag{A.1}$$

$$w_t = \varphi w_{t-1} + M\nu_t, \tag{A.2}$$

where y_t is an $n_y \times 1$ vector of endogenous variables, ν_t is an $n_s \times 1$ vector of exogenous stochastic innovations, and w_t is an $n_w \times 1$ vector of auxiliary variables which filter the exogenous stochastic shocks into the model's driving variables according to the informational assumptions of the particular model. $\hat{E}_t$ denotes the (possibly subjective) expectations operator conditional on the time t information set $\mathcal{I}_t$. The anticipation structure of exogenous stochastic innovations is characterized by the matrices φ and M, which partially determine the information contained in $\mathcal{I}_t$, while the matrices α, β, χ, and δ—which are independent of φ and M—describe the evolution of the endogenous variables.

Suppose the economy is driven in part by some stationary exogenous process $x_t \in y_t$ which is subject to exogenous stochastic disturbances which may be anticipated or unanticipated by agents. Assume without loss of generality that the law of motion for this process is given by

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$$x_t = \rho_x x_{t-1} + w^0_{x,t}, \tag{A.3}$$

where $-1 < \rho_x < 1$ and $w^0_{x,t} \in w_t$ is an auxiliary state variable which maps the anticipated and unanticipated exogenous stochastic innovations contained in ν_t to the law of motion for x_t in equation (A.3) according to the anticipation structure specified by the filter in the system (A.2).

Without further loss of generality, suppose that the auxiliary state variables can be represented recursively as

$$w^0_{x,t} = w^1_{x,t-1} + \sigma^0_x \nu^0_{x,t} \tag{A.4a}$$
$$w^1_{x,t} = w^2_{x,t-1} + \sigma^1_x \nu^1_{x,t} \tag{A.4b}$$
$$\vdots = \vdots \tag{A.4c}$$
$$w^{N_x-1}_{x,t} = w^{N_x}_{x,t-1} + \sigma^{N_x-1}_x \nu^{N_x-1}_{x,t} \tag{A.4d}$$
$$w^{N_x}_{x,t} = \sigma^{N_x}_x \nu^{N_x}_{x,t} \tag{A.4e}$$

where $N_x \geq 0$ is the maximum length of the anticipation horizon. $\sigma^n_{x,t} \geq 0$ is the standard deviation for the exogenous stochastic innovation $\nu^n_{x,t}$ which is anticipated n periods in advance; that is, $\nu^n_{x,t-n} \in \mathcal{I}_t$ for $n \in \{0, 1, 2, \cdots, N_x\}$. The innovations are assumed to be i.i.d normal with mean zero and variance equal to 1.

Collecting the auxiliary state variables and innovations specific to the process x_t into vectors $w_{x,t} = \left(w^0_{x,t}, w^1_{x,t}, \cdots, w^N_{x,t}\right)'$ and $\nu_{x,t} = \left(\nu^0_{x,t}, \nu^1_{x,t}, \cdots, \nu^N_{x,t}\right)'$ permits a compact representation of equations (A.4a)–(A.4e) as

$$w_{x,t} = \varphi_x w_{x,t-1} + M_x \nu_{x,t}, \tag{A.5}$$

where the matrices φ_x and M_x govern the anticipation structure for agents. Hence, N_x determines the length of the anticipation structure for the process x_t, while the nonzero elements of M_x determine how dense this structure is. In what follows, I provide several examples showing how this structure can accommodate various anticipation structures commonly found in the literature.

A.1. No Anticipated Shocks

Suppose none of the components which determine the evolution of x_t are anticipated by agents so that $\sigma^n_{x,t} = 0$ for all $n > 0$. The filter given by (A.5) takes the compact form

$$\begin{pmatrix} w^0_{x,t} \\ w^1_{x,t} \\ \vdots \\ w^{N_x-1}_{x,t} \\ w^{N_x}_{x,t} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} w^0_{x,t-1} \\ w^1_{x,t-1} \\ \vdots \\ w^{N_x-1}_{x,t-1} \\ w^{N_x}_{x,t-1} \end{pmatrix} + \begin{pmatrix} \sigma^0_x & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \nu^0_{x,t} \\ \nu^1_{x,t} \\ \vdots \\ \nu^{N_x-1}_{x,t} \\ \nu^{N_x}_{x,t} \end{pmatrix},$$

and hence x_t evolves according to the system

$$x_t = \rho_x x_{t-1} + w^0_{x,t}$$
$$w^0_{x,t} = w^1_{x,t-1} + \sigma^0_x \nu^0_{x,t}$$
$$w^1_{x,t} = w^2_{x,t-1}$$
$$\vdots = \vdots$$

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$$w_{x,t}^{N_x-1} = w_{x,t-1}^{N_x}$$
$$w_{x,t}^{N_x} = 0$$

Repeated substitution of the auxiliary state variables yields

$$x_t = \rho_x x_{t-1} + \sigma_x^0 \nu_{x,t}^0,$$

which demonstrates the idea that the driving process x_t is subjected to shocks which are observed by agents only in period t.[13]

A.2. Shocks Anticipated $N_x > 0$ Periods Ahead, Dense Anticipation Structure

Suppose the evolution of x_t is determined by shocks which are anticipated and unanticipated and that $\sigma_x^n > 0$ for all $n \in \{0, 1, 2, \cdots, N_x\}$. Then the filter given by by (A.5) takes the form

$$\begin{pmatrix} w_{x,t}^0 \\ w_{x,t}^1 \\ \vdots \\ w_{x,t}^{N_x-1} \\ w_{x,t}^{N_x} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} w_{x,t-1}^0 \\ w_{x,t-1}^1 \\ \vdots \\ w_{x,t-1}^{N_x-1} \\ w_{x,t-1}^{N_x} \end{pmatrix} + \begin{pmatrix} \sigma_x^0 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_x^1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \sigma_x^{N_x} \end{pmatrix} \begin{pmatrix} \nu_{x,t}^0 \\ \nu_{x,t}^1 \\ \vdots \\ \nu_{x,t}^{N_x-1} \\ \nu_{x,t}^{N_x} \end{pmatrix},$$

and hence x_t evolves according to the system

$$\begin{aligned} x_t &= \rho_x x_{t-1} + w_{x,t}^0 \\ w_{x,t}^0 &= w_{x,t-1}^1 + \sigma_x^0 \nu_{x,t}^0 \\ w_{x,t}^1 &= w_{x,t-1}^2 + \sigma_x^1 \nu_{x,t}^1 \\ \vdots &= \vdots \\ w_{x,t}^{N_x-1} &= w_{x,t-1}^{N_x} + \sigma_x^{N_x-1} \nu_{x,t}^{N_x-1} \\ w_{x,t}^{N_x} &= \sigma_x^{N_x} \nu_{x,t}^{N_x} \end{aligned}$$

Repeated substitution of the auxiliary state variables yields

$$\begin{aligned} x_t &= \rho_x x_{t-1} + \sigma_x^0 \nu_{x,t}^0 + \sigma_x^1 \nu_{x,t-1}^1 + \sigma_x^2 \nu_{x,t-2}^2 \cdots + \sigma_x^{N_x} \nu_{x,t-N_x}^{N_x} \\ &= \rho_x x_{t-1} + \sum_{n=0}^{N_x} \sigma_x^n \nu_{x,t-n}^n. \end{aligned}$$

Since $\nu_{x,t-n}^n \in \mathcal{I}_t$ for all $n \in \{0, 1, 2, \cdots, N_x\}$, this demonstrates the innovations to the driving process x_t are composed of shocks which are observed by agents not only in period t but also in each of the N_x periods preceding it. These news shocks imply agents receive information about future economic fundamentals prior to the innovations actually arriving.

A.3. Shocks Anticipated $N_x > 0$ Periods Ahead, General Anticipation Structure

Suppose the evolution of x_t is determined by some combination of anticipated or unanticipated shocks such that there is some subset of time periods given by $\tilde{n} \subseteq \{0, 1, 2, \cdots, N_x\}$

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for which $\sigma_x^n > 0$ if $n \in \tilde{n}$ and $\sigma_x^n = 0$ if $n \notin \tilde{n}$. Then the filter given by (A.5) implies x_t evolves according to

$$x_t = \rho_x x_{t-1} + \sum_{n \in \tilde{n}} \sigma_x^n \nu_{x,t-n}^n.$$

Since $\nu_{x,t-n}^n \in \mathcal{I}_t$ for all $n \in \tilde{n}$, this demonstrates the innovations to driving process x_t are composed of some combination of shocks which may be anticipated or unanticipated. Note that this specification fully nests each of the previous cases considered: if there are no news shocks (as in Appendix A.1), we have $\tilde{n} = \{0\}$, while in the presence of a dense anticipation structure (as in Appendix A.3), we have instead $\tilde{n} = \{0, 1, 2, \cdots, N_x\}$.

A.4. News Shocks with Multiple Driving Processes

The flexible framework of Appendix A.3 can be easily extended to consider environments featuring multiple driving processes. Indeed, suppose there are n_j driving processes indexed by $j = 1, 2, \cdots, J$. Then the system describing the anticipation structure for all j takes the form implied by (A.5) so that

$$w_{j,t} = \varphi_j w_{j,t-1} + M_j \nu_{j,t}. \tag{A.6}$$

Collecting the auxiliary state variables and innovations for all process into vectors $w_t = (w_{1,t}, w_{2,t}, \cdots, w_{J,t})'$ and $\nu_t = (\nu_{1,t}, \nu_{2,t}, \cdots, \nu_{J,t})'$, the entire anticipation structure can be written as

$$\begin{pmatrix} w_{1,t} \\ w_{2,t} \\ \vdots \\ w_{J,t} \end{pmatrix} = \begin{pmatrix} \varphi_1 & 0 & \cdots & 0 \\ 0 & \varphi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \varphi_J \end{pmatrix} \begin{pmatrix} w_{1,t-1} \\ w_{2,t-1} \\ \vdots \\ w_{J,t-1} \end{pmatrix} + \begin{pmatrix} M_1 & 0 & \cdots & 0 \\ 0 & M_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M_J \end{pmatrix} \begin{pmatrix} \nu_{1,t} \\ \nu_{2,t} \\ \vdots \\ \nu_{J,t} \end{pmatrix},$$

or compactly as

$$w_t = \varphi w_{t-1} + M \nu_t,$$

which is identical to (A.2). This illustrates that heterogeneous anticipation structures such as Beaudry and Portier (2004) in which agents anticipate shocks to a single driving process two periods in advance, Jaimovich and Rebelo (2009) in which agents receive anticipate shocks to each of two driving processes two periods in advance, and Schmitt-Grohe and Uribe (2012) in which agents anticipate shocks to each of seven driving processes four and eight periods in advance can all be easily accommodated.

APPENDIX B: IMPLEMENTING ADAPTIVE LEARNING

In what follows I focus on the minimum state variable (MSV) solutions to the system given by (1a) and (1b) in Section 2 of the main text which I recreate here

$$y_t = \alpha + \beta y_{t-1} + \chi w_t + \delta \hat{E}_t y_{t+1}, \tag{B.1a}$$
$$w_t = \varphi w_{t-1} + M \nu_t, \tag{B.1b}$$

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where y_t is an $n_y \times 1$ vector of endogenous variables, ν_t is an $n_s \times 1$ vector of exogenous stochastic innovations distributed i.i.d normal with mean zero and variance-covariance matrix equal to the identity matrix, and w_t is an $n_w \times 1$ vector of auxiliary variables which filter the exogenous stochastic shocks into the model's driving variables according to the informational assumptions of the particular model. $\hat{E}_t$ denotes the (possibly subjective) expectations operator conditional on the time t information set $\mathcal{I}_t$. The anticipation structure of exogenous stochastic innovations is characterized by the matrices φ and M, which partially determine the information contained in $\mathcal{I}_t$, while the matrices α, β, χ, and δ describe the evolution of the endogenous variables.

I focus here on the Minimal State Variable (MSV) solutions to systems like (B.1a) and (B.1b) which contain the same set of variables obtained by solving the model under rational expectations, which facilitates a direct comparison of solutions obtained under different expectation formation assumptions. These take the form

$$y_t = a + by_{t-1} + cw_t, \tag{B.2a}$$

$$w_t = \varphi w_{t-1} + M\nu_t, \tag{B.2b}$$

where the matrices a, b, and c are functions of deep parameters and expectations. An MSV solution obtained from the rational expectations assumption is called a rational expectations equilibrium (REE).

B.1. No Informational Delay

Suppose the forecasting model of agents (their so-called "Perceived Law of Motion (PLM)") takes the form of the solution given in equation (B.2a). If the value of contemporaneous endogenous variables are included in agents' information set so that $y_t \in \mathcal{I}_t$, then expectations are given by

$$\begin{aligned}\hat{E}_t y_t &= y_t,\\ \hat{E}_t y_{t+1} &= a + b\hat{E}_t y_t + c\hat{E}_t w_{t+1},\\ &= a + by_t + c\varphi w_t.\end{aligned}$$

This forecast can be substituted into (B.1a) to yield the "Actual Law of Motion (ALM)" for the economy: conditional on their forecast, decision makers take actions which cause the economy to evolve according to

$$(I - \delta b)\, y_t = (\alpha + \delta a) + \beta y_{t-1} + (\chi + \delta c\varphi)\, w_t. \tag{B.3}$$

The system described in (B.3) provides a mapping from beliefs (characterized by the PLM) to the ALM. For a given set of beliefs (a, b, c), this "T-map" is given by

$$T(a, b, c) = \left\{(I - \delta b)^{-1}(\alpha + \delta a), (I - \delta b)^{-1}\beta, (I - \delta b)^{-1}(\chi + \delta c\varphi)\right\}.$$

The individual elements of the T-map incorporate the beliefs contained in a, b, and c into agents' decision rules. E-stability of a solution is determined by the matrix differential equation

$$\frac{d}{d\tau}(a, b, c) = T(a, b, c) - (a, b, c), \tag{B.4}$$

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which characterizes the forecast errors of agents. It is worth noting that a REE, denoted as $(\bar{a}, \bar{b}, \bar{c})$, can be interpreted as the solution to the fixed point problem

$$0 = T\left(\bar{a}, \bar{b}, \bar{c}\right) - \left(\bar{a}, \bar{b}, \bar{c}\right).$$

That is, a set of beliefs which are self-fulfilling and coincide on average with realizations such that forecast errors are zero and

$$\left(I - \delta\left(I + \bar{b}\right)\right)\bar{a} = \alpha, \tag{B.5a}$$

$$\delta\bar{b}^2 - \bar{b} + \beta = 0, \tag{B.5b}$$

$$\left(I - \delta\bar{b}\right)\bar{c} - \delta\bar{c}\varphi = \chi. \tag{B.5c}$$

Inspection of equation (B.5b) shows the coefficient matrix $\bar{b}$ is independent of the assumed structure for anticipated and unanticipated shocks; that is, the coefficient matrix $\bar{b}$ is independent of φ and M from (B.1a). Furthermore, for a given $\bar{b}$ equations (B.5a) and (B.5c) uniquely determine the coefficient matrices $\bar{a}$ and $\bar{c}$. However, $\bar{b}$ is the solution of a matrix-quadratic which is not in general unique, and hence more sophisticated solution techniques must in general be employed to obtain the REE.

The sensitivity of a particular model's solution to departures from RE can be studied by analyzing the dynamic properties of the Jacobian of the vectorized matrix differential equation (B.4) evaluated at the REE. This can be shown to be composed of three blocks[14]

$$DT_a(\bar{a}, \bar{b}) = I \otimes \left(I - \delta\bar{b}\right)^{-1}\delta, \tag{B.6a}$$

$$DT_b(\bar{b}) = \left[\left(I - \delta\bar{b}\right)^{-1}\beta\right]' \otimes \left[\left(I - \delta\bar{b}\right)^{-1}\delta\right], \tag{B.6b}$$

$$DT_c(\bar{b}, \bar{c}) = \varphi' \otimes \left(I - \delta\bar{b}\right)^{-1}\delta. \tag{B.6c}$$

Proposition 10.3 of Evans and Honkapohja (2001) states that if $y_t \in \mathcal{I}_t$, then an REE is E-Stable if all eigenvalues of the matrices $DT_a(\bar{a}, \bar{b})$, $DT_b(\bar{b})$, and $DT_C(\bar{b}, \bar{c})$ have real parts less than 1. Thus, to determine what effect news shocks *per se* have on the E-stability of a given REE, it is sufficient to study the behavior of eigenvalues of these matrices in response to changes in the matrices φ and M which jointly characterize the anticipation structure. This is established by Proposition 1 in the main text, repeated here for convenience

PROPOSITION 1. *If $y_t \in \mathcal{I}_t$, then the E-stability property of an REE for a given model is robust to changes to the anticipation structure for exogenous stochastic shocks.*

The proof of this proposition is presented in Appendix C. Intuitively, the equations (B.6a) and (B.6b) are independent of the factors which characterize the anticipation structure, while the structure of φ implies the eigenvalues of (B.6c) will always equal zero. Thus, the size of the largest real eigenvalues of these components will be unaffected by changes in the anticipation structure.

B.2. Informational Delay

Suppose now that the PLM is still given by (B.2a), but the values of contemporaneous endogenous variables are not included in agents' time t information set and are instead forecast forecast by agents using this PLM. Expectations are now given by

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$$\hat{E}_t y_t = a + by_{t-1} + cw_t,$$
$$\text{and } \hat{E}_t y_{t+1} = a + b\hat{E}_t y_t + c\hat{E}_t w_{t+1},$$
$$= (I+b)a + b^2 y_{t-1} + (bc + c\varphi)w_t.$$

As before, these forecasts can be substituted into (B.1a) to obtain the ALM

$$y_t = [\alpha + \delta\,(I+b)\,a] + \left[\beta + \delta b^2\right] y_{t-1} + [\chi + \delta\,(bc + c\varphi)]\, w_t, \tag{B.7}$$

and thus the mapping of beliefs from the PLM to the ALM is given by

$$T(a,b,c) = \left\{\alpha + \delta\,(I+b)\,a, \beta + \delta b^2, \chi + \delta\,(bc + c\varphi)\right\}, \tag{B.8}$$

and expectational stability of a solution is determined by the matrix differential equation

$$\frac{d}{d\tau}(a,b,c) = T(a,b,c) - (a,b,c). \tag{B.9}$$

where again an REE can be seen as the solution to the fixed point problem

$$0 = T\left(\bar{a},\bar{b},\bar{c}\right) - \left(\bar{a},\bar{b},\bar{c}\right),$$

which implies

$$\left(I - \delta\left(I + \bar{b}\right)\right)\bar{a} = \alpha, \tag{B.10a}$$
$$\delta\bar{b}^2 - \bar{b} + \beta = 0, \tag{B.10b}$$
$$\left(I - \delta\bar{b}\right)\bar{c} - \delta\bar{c}\varphi = \chi. \tag{B.10c}$$

Inspection of equations (B.10a), (B.10b), and (B.10c) reveals them to be identical to equations (B.5a), (B.5b), and (B.5c) from Appendix B.1 which determine REE when $y_t \in \mathcal{I}_t$, thereby establishing that when agents are perfectly informed to the true ALM for the economy (as is the case under rational expectations) the model solution does not depend on whether the values of contemporaneous endogenous variables are included in the information set as their forecasts will by assumption be correct on average.

However, the additional complication for boundedly rational households can in general have significant effects on the conditions necessary to ensure asymptotic convergence to a REE under adaptive learning. In particular, Proposition 10.1 in Evans and Honkapohja (2001) states that if the value of contemporaneous endogenous variables is not included in the information set and thus, under adaptive learning, must be forecast, then an REE is E-Stable if all eigenvalues of the Jacobian of the vectorized T-map, which again is composed of three blocks, have real parts less than 1. Under this alternate timing regime, the blocks can be shown to be

$$DT_a(\bar{a},\bar{b}) = I \otimes \delta + I \otimes \delta\bar{b}, \tag{B.11a}$$
$$DT_b(\bar{b}) = \bar{b}' \otimes \delta + I \otimes \delta\bar{b}, \tag{B.11b}$$
$$DT_c(\bar{b},\bar{c}) = \varphi' \otimes \delta + I \otimes \delta\bar{b}. \tag{B.11c}$$

As in the previous section, the matrices DT_a and DT_b are unaffected by the modifications to φ and M necessary to include news shocks, and hence their eigenvalues are similarly unaffected. In general, very little can be said of the eigenvalues of the sum of matrices as in DT_c; however, the structure of the news-filtering mechanism implies a particular form for φ which leads to Proposition 2 in the main text, repeated here for convenience

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PROPOSITION 2. *If $y_t \notin \mathcal{I}_t$, then the E-stability property of an REE for a given model is robust to changes to the anticipation structure for exogenous stochastic shocks.*

The proof is presented in D. As in Appendix B.1, the equations (B.11a) and (B.11b) are independent of the factors which characterize the anticipation structure, but unlike before the eigenvalues of DT_c implied by (B.11c) are no longer necessarily equal to zero. However, the structure of φ can be used to show the modulus of these eigenvalues is independent of the characterization of the anticipation structure and hence again the largest real eigenvalues of these components will be unaffected by changes in the anticipation structure.

APPENDIX C: PROOF OF PROPOSITION 1

The proposition states: *If $y_t \in \mathcal{I}_t$ then the E-stability property of an REE for a given model is robust to changes to the anticipation structure for exogenous stochastic shocks.*

Proof. Suppose a REE given by $\left(\bar{a}, \bar{b}, \bar{c}\right)$ is E-stable for some specified anticipation structure characterized by $\bar{c}$, φ, and M. Then since (B.5b) shows the value of $\bar{b}$ is independent of $\bar{c}$, φ, and M, and since (B.5a) shows $\bar{a}$ is uniquely determined for given $\bar{b}$, it follows from inspection of (B.6a) and (B.6b) that changing the length or denseness of the anticipation structure to include (or exclude) news shocks will have no effect on the matrices DT_a and DT_b or the eigenvalues thereof.

Furthermore, as shown in Appendix A, φ is a nilpotent matrix containing 1's along its super-diagonal and zeros elsewhere; as a result the eigenvalues of φ are all equal to zero. Since (B.6c) shows the eigenvalues of DT_c are the set of pairwise-products of eigenvalues of φ' and $\left(I - \delta\bar{b}\right)^{-1}\delta$, and since φ is nilpotent, the largest eigenvalue of DT_c will always be equal to zero.

As a result, the inclusion (or exclusion) of news shocks to the anticipation structure will leave unchanged the E-stability property of a given REE when the value of contemporaneous endogenous variables is included in agents' time t information set. ■

APPENDIX D: PROOF OF PROPOSITION 2

The proposition states: *If $y_t \notin \mathcal{I}_t$ then the E-stability property of an REE for a given model is robust to changes to the anticipation structure for exogenous stochastic shocks.*

Proof. Suppose a REE given by $\left(\bar{a}, \bar{b}, \bar{c}\right)$ is E-stable for some specified anticipation structure characterized by $\bar{c}$, φ, and M. Then since (B.11b) shows the value of $\bar{b}$ is independent of $\bar{c}$, φ, and M, and since (B.11a) shows $\bar{a}$ is uniquely determined for given $\bar{b}$, it follows that changing the length or denseness of the anticipation structure to include (or exclude) news shocks will have no effect on the matrices DT_a and DT_b or the eigenvalues thereof.

Furthermore, as shown in Appendix A, φ is a nilpotent matrix containing 1's along its super-diagonal and zeros elsewhere. Denoting the number of driving processes as n_j and the longest anticipation horizon across all such processes as $\bar{N}_j = \max\left(\{N_j\}\right)$, φ can be written as

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$$\varphi = I_{n_j} \otimes U_{\bar{N}_j+1},$$

where $U_{\bar{N}_j+1}$ is an $(\bar{N}_j + 1) \times (\bar{N}_j + 1)$ upper-shift matrix with 1's along the super-diagonal and zeros elsewhere. Substituting this expression for φ into (B.11c) yields

$$\begin{aligned} DT_c(\bar{b}, \bar{c}) &= \varphi' \otimes \delta + I \otimes \delta\bar{b}, \\ &= \left(\left(I_{n_j} \otimes U_{\bar{N}_j+1}\right)' \otimes \delta\right) + \left(I_{n_j} \otimes I_{\bar{N}_j+1} \otimes \delta\bar{b}\right), \\ &= \left(I_{n_j} \otimes L_{\bar{N}_j+1} \otimes \delta\right) + \left(I_{n_j} \otimes I_{\bar{N}_j+1} \otimes \delta\bar{b}\right), \end{aligned}$$

where $L_{\bar{N}_j+1}$ is an $(\bar{N}_j + 1) \times (\bar{N}_j + 1)$ lower-shift matrix with 1's along the sub-diagonal and zeros elsewhere. The Kronecker product is a bilinear map and hence

$$\begin{aligned} DT_c(\bar{b}, \bar{c}) &= \left(I_{n_j} \otimes L_{\bar{N}_j+1} \otimes \delta\right) + \left(I_{n_j} \otimes I_{\bar{N}_j+1} \otimes \delta\bar{b}\right), \\ &= I_{n_j} \otimes \left[\left(L_{\bar{N}_j+1} \otimes \delta\right) + \left(I_{\bar{N}_j+1} \otimes \delta\bar{b}\right)\right]. \end{aligned}$$

It is straightforward to show that the matrix $[(L_{\bar{N}_j+1} \otimes \delta) + (I_{\bar{N}_j+1} \otimes \delta\bar{b})]$ is a lower-block triangular matrix with $\bar{N}_j$ blocks of $\delta\bar{b}$ along its diagonal. Since the eigenvalues of a block-triangular matrix are the eigenvalues of each of its block-diagonal matrices, it follows that the eigenvalues of DT_c are given by $n_j * (\bar{N}_j + 1)$ copies of the eigenvalues of the matrix $\delta\bar{b}$. Since $\bar{b}$ and δ are independent from $\bar{c}$, φ, and M which govern the anticipation structure it follows that the largest real parts of the eigenvalues of $\delta\bar{b}$—and hence the largest real parts of the eigenvalues of DT_c—are also independent of the assumed anticipation structure.

As a result, the inclusion (or exclusion) of news shocks to the anticipation structure will leave unchanged the E-stability property of a given REE even when the value of contemporaneous endogenous variables is not included in agents' time t information set and, under adaptive learning, must be forecast. ∎

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