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A REVIEW OF WILKIE'S STOCHASTIC ASSET MODEL

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ABSTRACT

This paper reviews the United Kingdom stochastic asset model developed by Wilkie (1995b). Certain aspects of the methodology used to develop this model could be problematic. Moreover, Wilkie (1995b) did not provide a complete evaluation of his model; certain economic theories and the constancy of the model's parameter values did not appear to have been specifically considered. This paper attempts to provide a comprehensive review of Wilkie's model.

KEYWORDS

Stochastic Asset Models; Wilkie's Model; Financial Time Series

1. INTRODUCTION

1.1 *Background*

1.1.1 This paper aims to provide a comprehensive review of Wilkie's (1995b) United Kingdom stochastic asset model. The paper is introduced in this section by briefly discussing the literature on this model.

1.1.2 Wilkie's (1984, 1986a) original model was the first comprehensive U.K. actuarial stochastic asset model to be published. Its purpose "is to provide a realistic variance and covariance structure for many years ahead" (Wilkie, 1995b, 783); rather than to provide accurate short-term forecasts. This purpose reflects the long-term nature of its intended applications. Wilkie's model was initially used by a Faculty of Actuaries working party to investigate criteria for assessing the solvency of life offices (Limb *et al.,* 1986). Since then it has been extensively used in a range of applications, and appears to have become the standard U.K. stochastic asset model (see Wilkie, 1986b, 1987; Purchase *et al.,* 1989; Ross, 1989; Daykin & Hey, 1990; Hardy, 1993; Lewin *et al.,* 1995; Needleman & Roff, 1995).

1.1.3 The development of Wilkie's model started in the 'Report of the Maturity Guarantees Working Party' (Ford *et al.,* 1980), which proposed a model for simulating equity returns. Other preliminary work for the model included Wilkie (1981) and an unpublished report prepared by Gwilym Jenkins and Partners. The original model is made up of four interconnected models, namely: a price inflation model; a share dividend yield model; a share dividend model; and a long-term interest rate model. These models are essentially conventional ARIMA transfer function models that were developed from U.K. data over the interval 1919-82; earlier data were considered for the price inflation and long-term interest rate models. Wilkie (1992, 1995a, 1995b) updated the original model and extended it to include: alternative ARCH and VAR price inflation models; a wage inflation model; a short-term interest rate model; a property yield model; a property income model; and an index-linked yield model. These extensions left the original model's structure virtually unaltered. Furthermore, these models were fitted to data from numerous developed countries, and an exchange rate model was proposed.

1.1.4 Wilkie's model was first reviewed by Kitts (1988, 1990). Kitts only considered the empirical adequacy of the price inflation model, and reported that its residuals failed tests of normality and independence. In particular, these residuals were found to contain unusually long runs of the same sign. Furthermore, Kitts questioned the long-term validity of the model, because it does not accommodate structural changes, such as the apparent structural change in price inflation that occurred in the early 1900s (see Wilkie, 1981). Kitts' review prompted the Institute and the Faculty of Actuaries to establish a joint working party to review Wilkie's model and its applications (Geoghegan *et al.*, 1992). Geoghegan *et al.* (1992, 179) also focused on the price inflation model, and expressed concern that it was unable to account for:

> "(1) the existence of bursts of inflation, indicating that once an upward trend in inflation is established, there is a tendency for it to continue ...
> (2) the existence of large, irregular shocks, such as those of the mid-1970s ...
> (3) the possible non-normality of residuals, through asymmetry, etc. ..."

Nevertheless: "The Working Party agreed that there was little evidence to suggest that a better fitting parsimonious model could be estimated using standard Box-Jenkins methodology" (Geoghegan *et al.*, 1992, 179). Geoghegan *et al.* (1992) discussed a number of possible alternative models, but did not make any specific recommendations. A member of the working party, Clarkson (see Clarkson, 1991), and Wilkie suggested alternative inflation models. Geoghegan *et al.* (1992, 186) concluded that: "considerably more research is required in this area."

1.1.5 Other comments on Wilkie's model have been made by, amongst others, Daykin & Hey (1990), Ludvik (1993), Daykin *et al.* (1994) and Smith (1996). Daykin & Hey (1990) noted that the price inflation model generates a much higher proportion of years with negative inflation than has been observed over the post-war interval 1951-88. The price inflation model implies that the probability of negative inflation is roughly 20%; negative annual inflation has not occurred since the 1960s. Daykin & Hey (1990) suggested a few relatively minor modifications to the model's parameter values. Daykin *et al.* (1994) suggested that a skew distribution should be used to describe the price inflation model's residuals. Ludvik (1993) reported that the historical correlations, between the total returns on U.K. equity and fixed-interest securities, were far greater than the correlations implied by Wilkie's model. Smith (1996) noted that, by exploiting the market inefficiencies incorporated in Wilkie's model, investors should be able to achieve additional returns of roughly 3% for no extra risk (this was first reported in Wilkie, 1986b). Furthermore, the model implies that extremely large returns with very low levels of risk can be realised if short positions are permitted.

1.2 *Outline*

1.2.1 This paper reviews Wilkie's (1995b) U.K. stochastic asset model. Section 2 reviews the methodology used to develop this model and discusses the criteria by which asset models should be evaluated. This section argues that certain aspects of the methodology used by Wilkie (1995b) could be problematic, and that an assessment of a model should consider the constancy of the model's parameters.

1.2.2 Section 3 describes Wilkie's U.K. model. The models for other countries and the exchange rate model are beyond the scope of this paper. Section 4 reviews the theoretical consistency of Wilkie's model. This section shows that Wilkie's model is not consistent with the rational expectations hypothesis, the efficient market hypothesis and with aspects of portfolio theory.

1.2.3 Section 5 reviews the empirical adequacy of the model, and possible areas of weakness are reported. It is beyond the scope of this paper to develop an alternative asset model; the paper merely aims to highlight potential problems with Wilkie's model. Section 6 concludes.

1.2.4 The SAS and PcGive (Doornik & Hendry, 1994) computer packages were used to perform the calculations.

2. METHODOLOGY

2.1 *Wilkie's Methodology*

2.1.1 The objective of Wilkie's model is to describe parsimoniously the long-term behaviour of inflation and the returns on the major asset classes. Wilkie (1995b) assessed this objective using the criteria of empirical adequacy and theoretical consistency. The model appears to have been formulated initially using economic theory, and its detailed structure was largely determined on empirical grounds using the Box & Jenkins (1970) methodology. Hence, "a great variety of alternatives" (Wilkie, 1986a, 345) were considered, and the best fitting parsimonious model was chosen. The criterion of parsimony is dependent on the intended applications of the model; features that were not significant for actuarial applications were excluded. Moreover, models that could be rationalised in terms of economic theory were favoured (see Geoghegan *et al.*, 1992, 178).

2.1.2 Wilkie (1995b, 926) assessed the empirical adequacy of the models by testing whether their residuals were independent and normally distributed. If a model failed any of these diagnostic tests, then more elaborate models seem to have been considered. For example, Wilkie (1995b, 926) recommended that, if a model's residuals are found to be autocorrelated, then "a higher order AR(p) or MA(q) model should be tried."

2.1.3 The criteria used to assess the theoretical consistency of the models is less clear. Certain economic theories, such as the efficient market hypothesis, were rejected on empirical grounds (see Section 4.2), whereas other theories, such as the purchasing power parity hypothesis, were included regardless of the empirical evidence (see Wilkie, 1995b, 890).

2.2 *Data-Mining*

2.2.1 Model uncertainty is an important issue in econometric modelling (see Chatfield, 1995; Draper, 1995). Theoretical considerations are usually insufficient to completely specify an econometric model; in particular the time series characteristics of an econometric model are rarely theoretically determined. Hence, the problem facing the econometrician is to translate abstract theoretical models into well-defined statistical models about observable phenomena. This process, known as specification searching or data-mining, is inevitably data dependent (Leamer, 1978). However, specification searches invalidate traditional statistical inference because traditional inference generally assumes that the statistical model is known. This practice results in biased parameter estimates, termed model selection biases, and usually in overconfident assessments of the model's suitability or fit, known as the 'optimism principle' (Chatfield, 1995). The greater the range of the search, the greater the degree of optimism. Hence, these problems are especially relevant when the potential number of models considered is large. The importance of these problems is emphasised by Leamer's (1978, 13) comment that it has led to "a growing cynicism amongst economists toward empirical work."

2.2.2 An example of the potential dangers associated with data-mining is the practice of 'correcting' residual autocorrelation (see Hendry, 1995, Chapter 6). This occurs when diagnostic tests reveal the presence of autocorrelation in the error terms and the response is, as recommended by Wilkie (1995b, 926), to add appropriately lagged variables to eliminate this autocorrelation. The problem with this response is that it is not obvious whether it leads to an improved model or simply conceals the real problems. For example, assume that the following model is the true model for some variable $X(t)$ (for $0 < t < T$):

$$X(t) = XMU(t) + XSD \cdot XZ(t) \tag{2.1}$$

where:

$$XMU(t) = \begin{cases} XMU1 & \text{for} \quad 0 < t \leq a \\ XMU2 & \text{for} \quad a < t \leq T. \end{cases} \tag{2.2}$$

2.2.3 If the following, constant mean, model was fitted: $X(t) = XMU + XSD \cdot XZ(t)$, then the autocovariance of its residuals is given by (see Hendry, 1995, 574):

$$\begin{aligned}
\operatorname{cov}(XZ(t),\, XZ(t-1)) &= \left(\frac{1}{T\cdot XSD^2}\right)\cdot \mathrm{E}\left[\sum_{k=1}^{T}(X(t)-\mu_1)\cdot(X(t-1)-\mu_2)\right] \\
&\approx \left(\frac{1}{T\cdot XSD^2}\right)\cdot \mathrm{E}\left[\sum_{k=1}^{T}(X(t)-\mu_1)\cdot(X(t-1)-\mu_1)\right] \\
&= \left(\frac{a\cdot(T-a-1)}{T^2}\right)\cdot\left(\frac{XMU1-XMU2}{XSD}\right)^2 \\
&> 0
\end{aligned} \tag{2.3}$$

where:

$$\mu_1 = \frac{1}{T}\cdot \mathrm{E}\left[\sum_{k=1}^{T} X(t)\right] = \frac{1}{T}\cdot(a\cdot XMU1+(T-a)\cdot XMU2 \tag{2.4}$$

$$\mu_2 = \frac{1}{T}\cdot \mathrm{E}\left[\sum_{k=0}^{T-1} X(t)\right] = \frac{1}{T}\cdot((a+1)\cdot XMU1+(T-a-1)\cdot XMU2) \tag{2.5}$$

2.2.4 Therefore, an unmodelled change in the mean results in positive autocorrelation. It is not appropriate to 'correct' this autocorrelation by fitting an autoregressive model.

2.2.5 In most circumstances, statistical tests only determine whether there is sufficient evidence to reject the null hypothesis; rejecting the null hypothesis does not necessarily imply that the alternative is appropriate. A misspecified model is likely to fail a number of diagnostic tests, and the failure of any one test generally invalidates an elementary interpretation of other tests. Therefore, test failures merely indicate the presence of a problem, and individual test results should not be used to recommend a specific alternative course of action. These considerations emphasise the problems associated with starting from a simple model and generalising it on the basis of statistical test results.

2.2.6 Furthermore, models can be, either deliberately or inadvertently, designed to satisfy most diagnostic tests (see Hendry, 1995, 554). Hence, it is important to distinguish between statistical tests employed as design or specification criteria and genuine misspecification tests (Mizon, 1977). It is meaningless to test a model using the same criteria that were used to design the model; "the good fit of a best fitting model should not be surprising!" (Chatfield, 1995, 427).

2.2.7 Two extreme responses to the problems of data-mining are: to ignore it; or, to assume that econometric models represent nothing more than a convenient summary of historical events until they have been tested against new data. To ignore the problem is likely to lead to overconfident predictions. To suspend the evaluation of econometric models is impractical, but possibly the only completely foolproof method of overcoming the problems of data-mining. Other notable methods for dealing with these problems include using a Bayesian approach (see Leamer, 1978), using a theory-directed rather than data-directed approach (see Darnell & Evans, 1990), and using a general-to-specific approach (see Hendry, 1995). Of these methods, general-to-specific modelling appears to be the most pragmatic methodology and has been the most influential econometric approach in the U.K.

2.2.8 Although parts of Wilkie's methodology appear to be subject to the problems associated with data-mining, these problems are likely to have been mitigated by his use of the criterion of theoretical consistency. Much of Wilkie's work appears to have been theory-directed.

2.3 *Model Evaluation Criteria*

2.3.1 Data-mining makes it difficult to evaluate econometric models, especially if *ad hoc* specification searches were used. Consequently, it is important that a wide range of criteria should be used to evaluate a model. Furthermore, emphasis should be given to criteria that were not used in the model development process, such as tests using data that were not available when the model was developed.

2.3.2 The main criteria by which econometric models should be evaluated include whether the model is consistent with prior economic theory, satisfies various goodness-of-fit tests, is parsimonious, and has constant parameters historically (see Spanos, 1986; Harvey, 1989; Hendry, 1995). Wilkie (1995b) considered all of these criteria, except parameter constancy, and reported that his model was broadly satisfactory. The following sections of this paper reconsider this evidence and, in addition, assess the parameter constancy of Wilkie's model. As parameter constancy does not appear to have been specifically considered in the development of the model, tests of parameter constancy are likely to constitute genuine misspecification tests. Parameter constancy is also crucial if the model is to be used to obtain an adequate representation of the modelled variables' long-term behaviour.

2.3.3 The criterion of theoretical consistency is not decisive. High level theories are often only promising conjectures that have not been shown to be completely false; they are not necessarily true. Some theories may even contradict other theories. Hence, it is not essential, and may not be possible, that asset models should be consistent with every economic theory. Nevertheless, theories encapsulate current knowledge and should not be lightly dismissed. Furthermore, the criterion of theoretical consistency limits the potential for the problems associated with data-mining and data sample dependency, especially when there are data shortages.

2.3.4 Lastly, as stressed by Wilkie (1995b), an important characteristic of actuarial asset models is that they only attempt to model the long-term features of the data. As a result, these models should be evaluated taking this into account. However, it is virtually impossible to evaluate empirically the long-term features of models because of data shortages, which are exacerbated by possible regime shifts or permanent structural changes. According to Keynes (1923, 65):

> "*In the long run* we are all dead. Economists set themselves too easy, too useless a task if in tempestuous seasons they can only tell us that when a storm is long past the ocean is flat again."

Consequently, tests of empirical adequacy inevitably consider the short-term forecasting ability of the model. The criteria of parsimony and theoretical consistency should be used to determine whether any short-term features can be legitimately excluded from the model for specific applications. For example, Smith (1996) appeared to use the criterion of parsimony to justify the claim that ARCH effects were not required in actuarial models because they did not appear to have a significant bearing on actuarial applications.

3. DESCRIPTION

3.1 *Inflation Models*

3.1.1 The price inflation models can be represented by the following equations (for $t > 0$):

$$\begin{aligned}\nabla \log_e Q(t) = {} & QMU + QA \cdot (\nabla \log_e Q(t-1) - QMU) \\ & + QW \cdot (\nabla \log_e W(t-1) - WMU) + QSD(t) \cdot QZ(t)\end{aligned} \tag{3.1}$$

where:

$$QSD(t)^2 = QSD^2 + QSB \cdot (\nabla \log_e Q(t-1) - QSC)^2. \tag{3.2}$$

3.1.2 The wage inflation models can be represented by the following equation (for $t > 0$):

$$\begin{aligned}\nabla \log_e W(t) = {} & WMU + WW1 \cdot \nabla \log_e Q(t) + WW2 \cdot \nabla \log_e Q(t-1) \\ & + WA \cdot (\nabla \log_e W(t-1) - WMU - WW1 \cdot \nabla \log_e Q(t-1) \\ & - WW2 \cdot \nabla \log_e Q(t-2)) + WQ \cdot (\nabla \log_e Q(t-1) - QMU) \\ & + WSD \cdot (\rho \cdot QZ(t) + \sqrt{1-\rho^2} \cdot WZ(t))\end{aligned} \tag{3.3}$$

where $Q(t)$ is the consumer price index, $W(t)$ is the wage, or earnings, index, ∇ represents the backwards difference operator, and $QZ(t)$ and $WZ(t)$ are sequences of independently distributed unit normal random variables.

3.1.3 Recommended parameter values are given in Table 3.1. The ARCH price inflation models have been reparameterised as follows: $QSD^2 \equiv QSA$. The VAR model (see Wilkie, 1995b) has been reparameterised as follows: $QA \equiv A_{11}$, $QW \equiv A_{12}$, $WQ \equiv A_{21}$, and $WA \equiv A_{22}$. The full and reduced standard bases represent the original parameter values recommended by Wilkie (1984, 1986a). These values were based on estimates obtained using data over the interval 1919-82. The reduced basis appeared to have been initially recommended for most applications. The Wilkie (1995b), ARCH, and the VAR values were based on estimates obtained using data over the interval 1923-94. The interval 1919-22 was excluded from the latter investigations because it was considered to be an exceptional post-war period and was found to have an unduly large influence on the parameter estimates (see Wilkie, 1995a, 256).

Table 3.1. Parameter values for the inflation models

Parameter	Full standard	Reduced standard	Wilkie (1995b)	ARCH	VAR
QMU	0.05	0.05	0.047	0.04	0.0359
QA	0.6	0.6	0.58	0.62	0.1817
QW	-	-	-	-	0.5927
QSD	0.05	0.05	0.0425	0.0256	0.0408
QSB	-	-	-	0.55	-
QSC	-	-	-	0.04	-
WMU	-	-	0.021	0.021	0.0509
WA	-	-	0.0	0.0	0.5618
$WW1$	-	-	0.6	0.6	-
$WW2$	-	-	0.27	0.27	-
WQ	-	-	-	-	0.2315
ρ	-	-	-	-	0.7139
WSD	-	-	0.0233	0.0233	0.0335

3.1.4 Wilkie (1995b) did not specifically recommend parameter values for the VAR model. The VAR model's parameter values, reported in Table 3.1, were obtained from the full model estimated over the interval 1923-94 (see Wilkie, 1995b, 813).

3.1.5 'Neutral' initial conditions are:

$$\nabla \log_e Q(0) = \nabla \log_e Q(-1) = QMU$$

$$\nabla \log_e W(0) = WMU + (WW1 + WW2) \cdot QMU.$$

3.2 *Equity Models*

3.2.1 The share dividend yield model is defined by the following equations (for $t > 0$):

$$\log_e Y(t) = YW \cdot \nabla \log_e Q(t) + YN(t) \tag{3.4}$$

where:

$$(1 - YA \cdot \mathrm{B}) \cdot (YN(t) - \log_e YMU) = YE(t) = YSD \cdot YZ(t). \tag{3.5}$$

3.2.2 The share dividend model is defined by the following equations (for $t > 0$):

$$\begin{aligned} \nabla \log_e D(t) = \; & DM(t) + DX \cdot \nabla \log_e Q(t) + DMU + DY \cdot YE(t-1) \\ & + (1 + DB \cdot \mathrm{B}) \cdot DSD \cdot DZ(t) \end{aligned} \tag{3.6}$$

where:

$$DM(t) = DW \cdot \left(\frac{DD}{1 - (1 - DD) \cdot \mathrm{B}} \right) \cdot \nabla \log_e Q(t) \tag{3.7}$$

and $Y(t)$ is the share dividend yield, $D(t)$ is the share dividend index, B represents the lag operator, and $YZ(t)$ and $DZ(t)$ are sequences of independently distributed unit normal random variables.

3.2.3 The term $DM(t) + DX \cdot \nabla \log_e Q(t)$ represents the inflationary component of equity dividend growth, and the remainder of equation (3.6) represents the real growth component. If $DW + DX$ is set to equal one, then dividends will grow in line with inflation, so that a 1% increase in inflation will eventually result in a 1% increase in dividends. Wilkie (1995b, 840) suggested that DB could be justified by a tendency for boards of directors to smooth dividend payments over time.

3.2.4 Recommended parameter values are given in Table 3.2.

Table 3.2. Parameter values for the equity models

Parameter	Full standard	Reduced standard	Wilkie (1995b)
YMU	0.04	0.04	0.0375
YA	0.6	0.6	0.55
YW	1.35	1.35	1.8
YSD	0.175	0.175	0.155
DMU	0.0	0.0	0.016
DB	0.375	0.0	0.57
DD	0.2	0.2	0.13
DW	0.8	0.8	0.58
DX	0.2	0.2	$1 - DW$
DY	−0.2	−0.3	−0.175
DSD	0.075	0.1	0.07

3.2.5 'Neutral' initial conditions are: $\nabla \log_e Q(0) = QMU$, $Y(0) = YMU \cdot e^{YW \cdot QMU}$, $YE(0) = 0$, $DM(0) = DW \cdot QMU$, $DZ(0) = 0$.

3.3 *Interest Rate Models*

3.3.1 The long-term interest rate model is made up of a real component $CR(t)$ and a component representing investors' inflationary expectations $CM(t)$. The model is defined by the following equations (for $t > 0$):

$$C(t) = CM(t) + CR(t) \tag{3.8}$$

where:

$$CM(t) = CW \cdot \left(\frac{CD}{1-(1-CD) \cdot \mathrm{B}} \right) \cdot \nabla \log_e Q(t) \tag{3.9}$$

$$\begin{aligned} \log_e CR(t) = {} & (CA1 \cdot \mathrm{B} + CA2 \cdot \mathrm{B}^2 + CA3 \cdot \mathrm{B}^3) \cdot (\log_e CR(t) - \log_e CMU) \\ & + \log_e CMU + CY \cdot YE(t) + CSD \cdot CZ(t). \end{aligned} \tag{3.10}$$

3.3.2 The short-term interest rate model is defined by the following equation (for $t > 0$):

$$\begin{aligned} \log_e B(t) = {} & \log_e C(t) - BMU - BA \cdot (\log_e C(t-1) - \log_e B(t-1) - BMU) \\ & - BSD \cdot BZ(t). \end{aligned} \tag{3.11}$$

3.3.3 The index-linked yield model is defined by the following equation (for $t > 0$):

$$\begin{aligned} \log_e R(t) = {} & \log_e RMU + RA \cdot (\log_e R(t-1) - \log_e RMU) + RBC \cdot CSD \cdot CZ(t) \\ & + RSD \cdot RZ(t) \end{aligned} \tag{3.12}$$

where $C(t)$ is the long-term interest rate, $B(t)$ is the short-term interest rate, $R(t)$ is the real yield on index-linked securities, and $CZ(t)$, $BZ(t)$ and $RZ(t)$ are sequences of independently distributed unit normal random variables.

3.3.4 The transformations used in the long-term interest rate model were selected so that the model satisfies the Fisher relation (see Section 4.1). However, these transformations make it difficult to estimate the model's parameter values because the logarithm transformation used in equation (3.10) implies that CSD can be reduced by simply increasing *CMU* and reducing *CW* and *CD* to compensate (see Wilkie, 1984, 98). This is because the variance of the logarithm of a series of numbers tends to reduce if the mean of that series increases. Hence, the model's parameters cannot be determined using the method of maximum likelihood or least squares. Wilkie (1984, 1995b) did not estimate *CD* and *CW*; these parameters were merely set to 'plausible' values. Consequently, the model assumes that the chosen representation of the Fisher relation is valid. It does not provide a test of this hypothesis.

3.3.5 Recommended parameter values are given in Table 3.3. To prevent negative nominal yields, a minimum value of 0.005 is postulated for $C(t)$. 'Neutral' initial conditions are: $CM(0) = CW \cdot QMU$, $CR(0) = CR(-1) = CR(-2) = CMU$, $\log_e C(0) - \log_e B(0) = BMU$, $R(0) = RMU$.

Table 3.3. Parameter values for the interest rate models

Parameter	Full standard	Reduced standard	Wilkie (1995b)
CMU	0.035	0.035	0.0305
*CA*1	1.2	0.91	0.9
*CA*2	-0.48	0.0	-
*CA*3	0.2	0.0	-
CD	0.045	0.05	0.045
CW	1.0	1.0	1.0
CY	0.06	0.0	0.34
CSD	0.14	0.165	0.185
BMU	-	-	0.23
BA	-	-	0.74
BSD	-	-	0.18
RMU	-	-	0.04
RA	-	-	0.55
RBC	-	-	0.22
RSD	-	-	0.05

3.4 *Property Models*

3.4.1 The property models are similar to the equity models. The property yield model is defined by the following equation (for $t > 0$):

$$\log_e Z(t) = ZMU + ZA \cdot (\log_e Z(t-1) - ZMU) + ZSD \cdot ZZ(t). \tag{3.13}$$

3.4.2 The property income model is defined by the following equations (for $t > 0$):

$$\nabla \log_e E(t) = EM(t) + EX \cdot \nabla \log_e Q(t) + EMU + EBZ \cdot ZSD \cdot ZZ(t) + ESD \cdot EZ(t) \tag{3.14}$$

where:

$$EM(t) = EW \cdot \left(\frac{ED}{1-(1-ED)\cdot \mathrm{B}} \right) \cdot \nabla \log_e Q(t) \tag{3.15}$$

and $Z(t)$ is the property yield, $E(t)$ is the property income index, and $ZZ(t)$ and $EZ(t)$ are sequences of independently distributed unit normal random variables.

3.4.3 Recommended parameter values and 'neutral' initial conditions are:

$ZMU = 0.074$, $ZA = 0.91$, $ZSD = 0.12$, $EMU = 0.003$, $EW = 1$, $EX = 1 - EW$, $ED = 0.11$, $EBZ = 0.24$, $ESD = 0.06$, $\nabla\log_e Q(0) = QMU$, $Z(0) = ZMU$, $EM(0) = EW \cdot QMU$.

4. THEORETICAL REVIEW

4.1 *Fisher Relation*

4.1.1 The Fisher relation broadly states that expected inflation is fully reflected in nominal interest rates (Fisher, 1930). As a result, this relation assumes that investors' expectations of average future inflation can be approximately determined by subtracting the average future real return required by investors from nominal interest rates.

4.1.2 The Fisher relation was explicitly included in the long-term interest rate model. This model assumes that the average future real return required by investors is given by $CR(t)$ and that investors' expectation of average future inflation is given by $CM(t)$. Figure 4.1 shows the values of these two components, over the interval 1923-1994, calculated using Wilkie's (1995b) long-term interest rate model. Thus, Wilkie's model implies that investors required average future real returns of over 10% in 1974, and returns of over 5% during most of the interval 1969-82. These returns appear to be high by historical standards. Moreover, this appears to contradict Wilkie's (1995a, 267) assertion that over the interval 1968-78 "the concept of 'negative real returns' was widely spoken of."

4.1.3 Furthermore, $CM(t)$ can be compared with the optimal estimate of average future inflation, which is given by:

$$\lim_{n\to\infty} \left\{ \mathrm{E}_t \left[\frac{1}{n} \cdot \sum_{r=1}^{n} \nabla \log_e Q(t+r) \right] \right\} = QMU \tag{4.1}$$

where E_t is the expectations operator conditional on all information available at time t.

4.1.4 This comparison is illustrated in Figure 4.2, which shows that Wilkie's model implies that investors consistently underestimated average future inflation

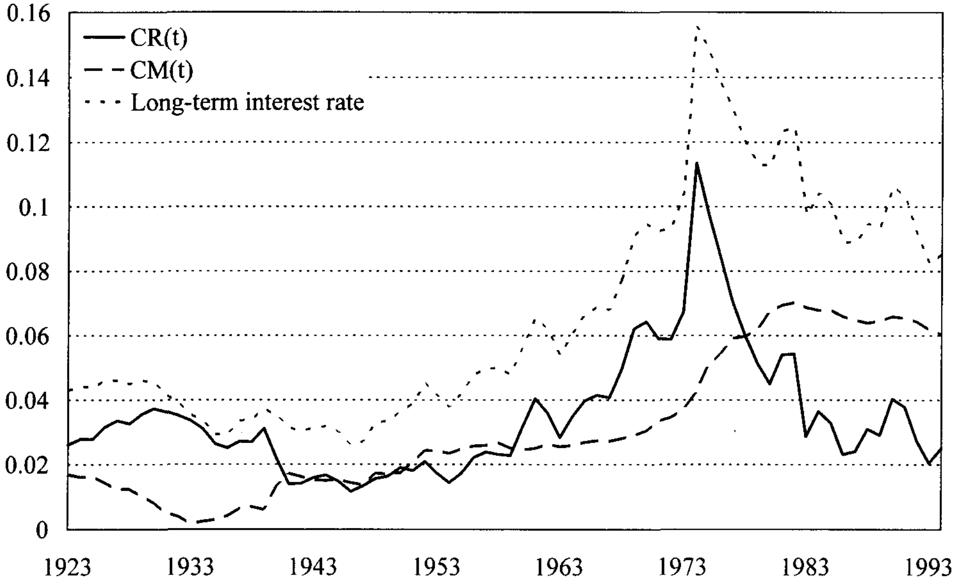

Figure 4.1. Expected price inflation and real returns, 1923-1994

over the interval 1923-75 and overestimated average future inflation since 1975. Furthermore, if Wilkie's model is true, then investors will continue to overestimate average future inflation by at least 0.5% until 2012. This contradicts the rational expectations hypothesis, which states that investors do not knowingly make systematic *ex ante* forecasting errors (see Huang & Litzenberger, 1988). It is not essential that Wilkie's model should be consistent with the rational expectations hypothesis (see Section 2.3). However, it is not obvious from Wilkie (1984, 1986a, 1995b) that this hypothesis was given detailed consideration before being rejected (see Booth's discussion of Wilkie, 1995b, 960). It is possible that investors misjudged future inflation in the past, but it seems less likely that Wilkie's model provides better *ex ante* forecasts than those made by investors (see Dyson & Exley, 1995).

4.1.5 If a stochastic asset model were to incorporate the rational expectations hypothesis, then the inflation expectation implicit in the long-term interest rate model would need to be constrained to be consistent with the price inflation model. This can be accomplished in a number of ways. For example, if Wilkie's long-term interest rate model were left unchanged, then the following price inflation model would incorporate the rational expectations hypothesis:

$$\nabla \log_e Q(t) = CM(t-1) + QSD(t) \cdot QZ(t). \tag{4.2}$$

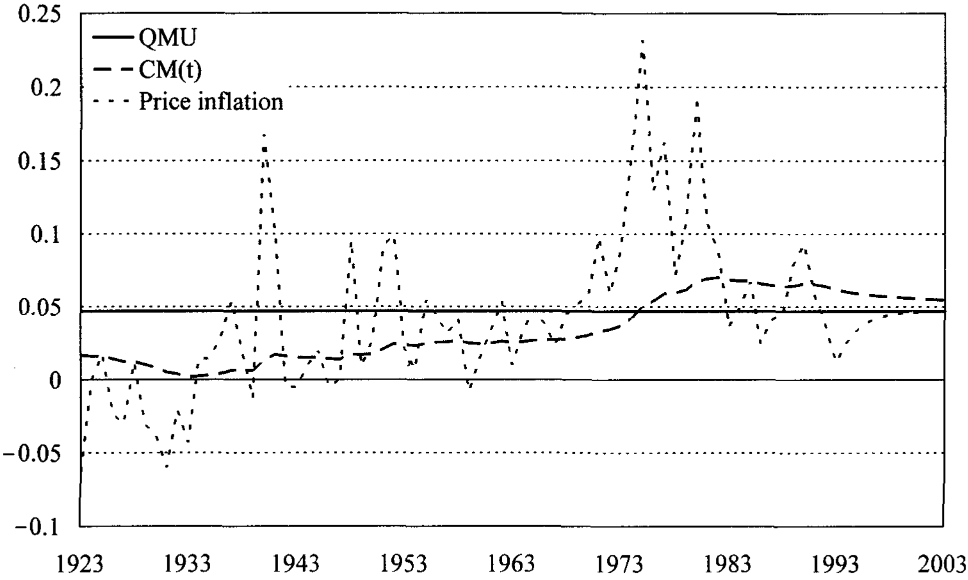

Figure 4.2. Expected price inflation, 1923-2003

4.1.6 Multiplying both sides of equation (4.2) by (1 - (1 - *CD*)·B) and rearranging gives (see equation 3.9):

$$\begin{aligned}\nabla \log_e Q(t) &= (1 - CD + CW \cdot CD) \cdot \nabla \log_e Q(t-1) \\ &\quad + QSD(t) \cdot QZ(t) - (1 - CD) \cdot QSD(t-1) \cdot QZ(t-1)\end{aligned} \qquad (4.3)$$

4.1.7 If *CW* = 1, as assumed by Wilkie, then equation (4.3) implies that price inflation is a non-stationary (integrated of order one) variable. This accords with the view that price inflation is largely determined by individuals' expectations, and that these expectations are not necessarily consistent over time (Black, 1986, 540). Numerous other authors have also suggested integrated price inflation models, including Osborn (1990), Franses & Paap (1994) and Clare *et al.* (1994). These suggestions were supported by the results of unit-root tests on the U.K. General Index of Retail Prices, which was first published in 1956. Moreover, Wilkie (1995a) argued that long-term average price inflation has increased from roughly 0% over the interval 1660-1900 to roughly 5% since the 1950s. However, an integrated model implies that the variance of the predicted force of price inflation in year *t* + *k* tends to infinity as *k* tends to infinity (see Wilkie, 1995b, 933). Consequently, the use of an integrated model implies that (Wilkie, 1986a, 361):

> "at term 100 the average mean rate of inflation is 7.69% with a standard deviation of 32.39%. The range is enormous, and includes both 'hyper-inflations' and 'hyper-deflations'. These latter are a consequence of the model, but, unlike the former, which have in fact occurred, they seem to me to be economically unrealistic."

Furthermore, using Wilkie's data over the extended interval 1923-94, the Dickey-Fuller unit-root test (see Doornik & Hendry, 1994) suggests that price inflation is not integrated. Hence, a univariate integrated ARMA price inflation model is probably not a reasonable long-term model.

4.1.8 Another method of incorporating both the Fisher relation and the rational expectations hypothesis into the price inflation model is to set:

$$\mathrm{E}_t[\nabla \log_e Q(t+1)] = C^*(t) - CMU + CA \cdot (\nabla \log_e Q(t) - C^*(t-1) + CMU) \quad (4.4)$$

where $C^*(t)$ is the long-term force of interest, CMU is the mean real return, and CA represents a possible real return autoregressive effect.

4.1.9 Alternatively, the following relationship would make the long-term interest rate model consistent with the inflation expectations implied by the price inflation model:

$$\mathrm{E}_t[C^*(t+1)] = QMU + CMU + CA \cdot (C^*(t) - QMU - CMU). \quad (4.5)$$

4.1.10 The above relationships illustrate the types of asset models that are consistent with the rational expectations hypothesis (see also Dyson & Exley, 1995; Smith, 1996). They do not represent complete alternative models. The empirical adequacy of these relationships would need to be examined before they are included in a stochastic asset model.

4.2 *Efficient Market Hypothesis*

4.2.1 The efficient market hypothesis was considered in the development of the equity models. However, the main implication of this hypothesis, that security returns cannot be forecast, was rejected on empirical grounds (see Ford *et al.*, 1980). Thus, Wilkie's model assumes that security returns can be partially forecast; but, over short time horizons, Wilkie's model is virtually identical to a random walk model, which is consistent with the efficient market hypothesis (Wilkie, 1995b, 826).

4.2.2 The efficient market hypothesis has been the subject of much controversy (see LeRoy, 1989). Wilkie's proposed resolution of this controversy, that security returns should be modelled so that they are virtually unpredictable over short time horizons, but can be partially forecast over longer time horizons, was also made by Shiller (1981, 294). This suggestion was supported by Fama & French (1988a,b). However, other studies have also found considerable evidence of predictability of short horizon returns, which contradicts Wilkie's suggestion (Lo & MacKinlay, 1988; Hodrick, 1992). Thus, while there is evidence to support Wilkie's suggestion, this issue appears to be unresolved.

4.2.3 Nevertheless, whereas, in practice, it has been difficult for individual investors to consistently achieve above average returns (Malkiel, 1990), Wilkie's model assumes that it is possible to achieve excess returns of roughly 3% for virtually no extra risk by switching between equity and fixed-interest securities (Wilkie, 1986b; Smith, 1996). This seems to be an unrealistic assumption. Consequently, Wilkie (1986b, 41) warned that his model should not be used to develop dynamic trading strategies. If Wilkie's (1986b) strategy of investing in the asset class with the highest expected return in the following year, calculated using Wilkie's (1986a) model, had been followed over the interval 1983-95, then an average return of 2% less than the return on equities would have been achieved.

4.2.4 This consequence of the model could be avoided if the expected real returns for each asset class are assumed to be constant over time. Alternatively, it would at least be necessary to assume that the expected risk premiums are consistent over time. These risk premiums could be assumed to be constant or a positive increasing function of the expected real returns. The latter assumption reflects the view that a payment when the economy is in a relatively prosperous state has a lower utility than a payment when the economy is in a poor state (Breeden, 1979). Hence, a higher risk premium is assumed to be required when real returns are expected to be relatively high. This assumes that the relative riskiness of securities is independent of the level of real returns. This view is implicit in the short-term interest rate model, which implies that the expected term premium or spread, $C(t) - B(t)$, is an increasing function of interest rates.

4.2.5 Another implication of the efficient market hypothesis is that prices respond to information about events when this information becomes known rather than when the events occur. As a result, equity price changes are likely to anticipate future changes in equity dividends because information affecting equity dividends is often available before the dividends are declared. Thus Wilkie's model, by incorporating the term $DY \cdot YE(t - 1)$ in equation (3.6), assumes that equity prices anticipate future changes in equity dividend growth rates.

4.3 *Unit Gain*

4.3.1 Another hypothesis that was considered and included in the share dividend and property income models is that dividends and property income respond to inflation with 'unit gain'; so that a 1% increase in prices will eventually lead to a 1% increase in dividends and property income. This hypothesis is intuitively appealing, but it is not essential, because "there is also a case for arguing that dividends 'in real terms' do better in times of stable prices than in periods of high and uncertain inflation" (Wilkie, 1995b, 840).

4.3.2 Moreover, doubts have been expressed about whether it is sensible to model dividends using linear time series models (Campbell & Ammer, 1993, 13). These doubts relate to the Modigliani & Miller (1958) propositions, which state that the dividend policy of a company is largely irrelevant to its value. Thus managers may not necessarily use a consistent dividend policy over time. However, this argument is only partially valid because dividend policies are relevant for tax and other reasons.

4.4 *Portfolio Theory*

4.4.1 Portfolio theory broadly states that investors select investments on the basis of their expected risk and return; investors will only include a security in their portfolio if its inclusion either increases the expected return or decreases the expected risk of the portfolio (see Huang & Litzenberger, 1988). A common, though controversial, measure of risk is the standard deviation of return. Hence, assets with a lower expected return should either have a lower expected standard deviation of return or be negatively correlated with other assets.

4.4.2 However, Wilkie's (1995b, 904) model assumes that the property asset class has a higher expected total return, a lower standard deviation of return, and a similar covariance structure compared to the equity asset class. Furthermore, the index-linked asset class has a higher expected real return, a lower standard deviation, and a roughly similar covariance structure compared to the long-term fixed-interest asset class. Hence, there appears to be little incentive to invest in either the equity or the long-term fixed-interest asset classes (Smith, 1996).

4.4.3 This possible weakness can be overcome by changing the mean or standard deviation parameters of the model. This accords with Wilkie's (1995b, 785) recommendation that users of his model "should form their own opinions about the choice of appropriate mean values." Specific values for these parameters could be obtained using an 'equilibrium' method (see Smith, 1996).

5. EMPIRICAL REVIEW

5.1 *Price Inflation Model*

5.1.1 Wilkie (1995b, 781) reported that the mean of the residuals from his original price inflation model over the out-of-sample interval 1983-94 was not significantly different from its expected value, but that the variance of these residuals was significantly less than its expected value. In addition, Wilkie (1995b, 785) reported that the residuals from his updated original model appeared to be independent, but that they did not appear to be normally distributed. He suggested that the significantly low variance of the residuals over the out-of-sample interval and the apparent non-normality of the updated model's residuals could have been due to an unmodelled ARCH effect. Consequently, he fitted an ARCH model and indicated that it appeared to be empirically adequate.

5.1.2 In addition to the above tests, the parameter constancy of the original price inflation model can be examined by recursively estimating its parameters on

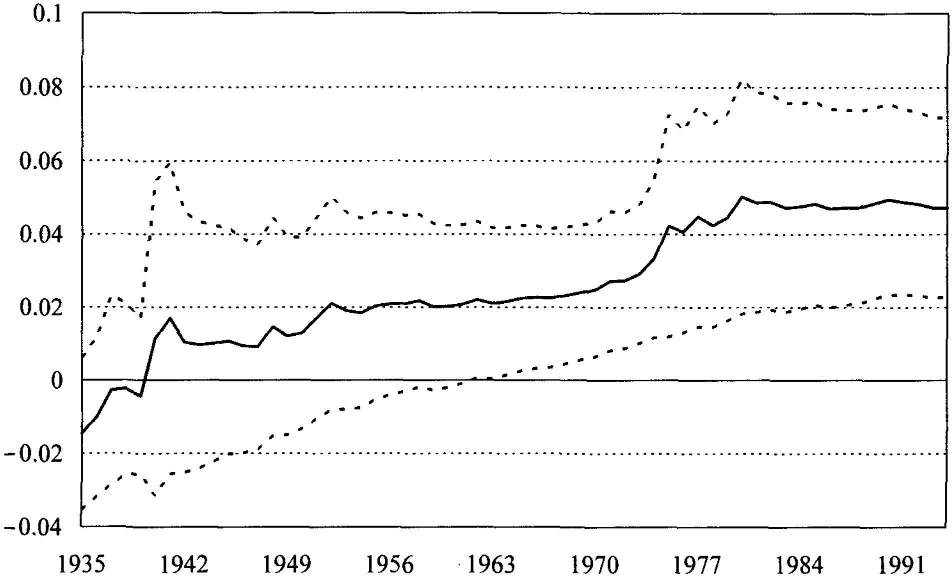

Figure 5.1. Recursive estimates of *QMU* with approximate 95% confidence intervals, from 1923

incrementally larger data sets (see Spanos, 1986; Hendry, 1995). Figures 5.1 and 5.2 present recursive estimates and approximate 95% confidence intervals of *QA* and *QMU*, respectively, calculated using data sets from 1923 to the years on the x-axes. These graphs suggest that *QA* and *QMU* may not be constant over the interval 1923-94. In Figure 5.1 the recursive estimates of *QMU* tend to increase over most of the period and only become significantly different from zero after 1960. The recursive estimates of *QMU* after 1980 are also significantly larger than those calculated over earlier intervals. This supports Wilkie's (1986a, 346) comment that there is "considerable uncertainty about the value to use for *QMU*." In Figure 5.2 the recursive estimates of *QA* jump, in the mid-1970s, from a value of approximately 0.37 to a value of approximately 0.58. This contradicts Wilkie's (1986a, 346) remark that: "There is fairly little uncertainty about the appropriate value[s] for *QA*."

5.1.3 The Chow test (Spanos, 1986, 483-5) can be used to test whether a model's parameters are constant. This test is made up of two parts. The first part tests the null hypothesis that the variances of the residuals are equal over both sub-periods against the alternative that they are different. If the model satisfies the first test, the second part tests the null hypothesis of parameter constancy against the alternative of non-constancy. Table 5.1 shows the parameter estimates

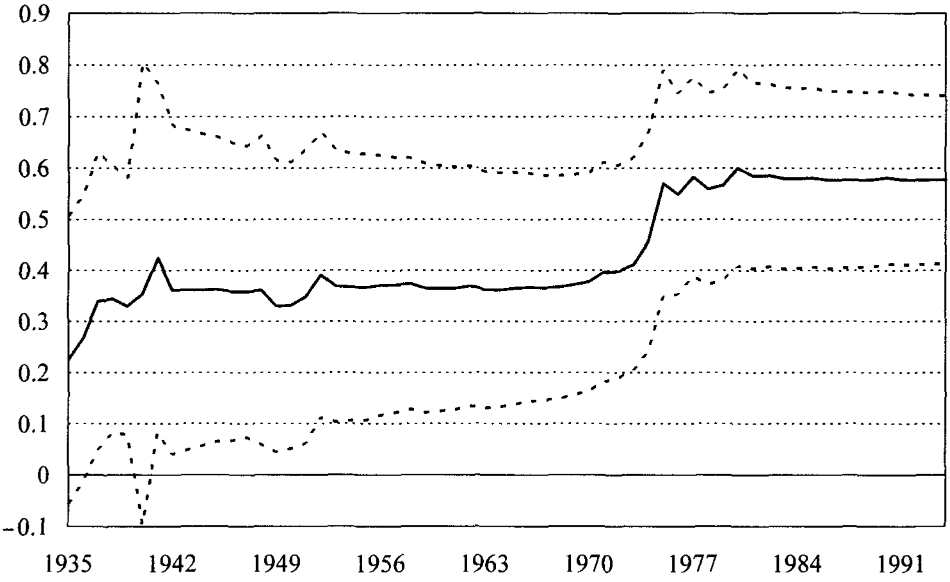

Figure 5.2. Recursive estimates of *QA* with approximate 95% confidence intervals, from 1923

obtained from fitting the model over the two equally sized, and arbitrarily chosen, intervals 1923-58 and 1959-94. Over these sub-periods, the variance of the residuals appears to be unchanged (F(34,34) = 1.17, p = 0.3238), but the null hypothesis of parameter constancy is rejected at the 5% level (F(2,68) = 3.16, p = 0.0487).

5.1.4 As discussed in Section 2.2, it is difficult to interpret these results. They may simply be due to the non-normality of the residuals or they could be due to the change in the calculation of the official U.K. price index. Over the interval 1923-47 a cost-of-living index was calculated, whereas over the interval 1947-94 a general index of retail prices was calculated (see Wilkie, 1995b, 942). Alternatively, as suggested by the theoretical review (see Section 4.1), the parameter non-constancy could be due to changes in the mean rate of inflation (see Figure 4.2). Moreover, a change in the mean rate of inflation may have biased the recursive estimates of *QA* and caused them to increase in the mid-1970s (see Section 2.2 and Figure 5.2).

5.1.5 A non-parametric test that can be used to assess whether the mean rate of inflation was not constant is the rank-sum test. This test has a broadly similar intention to the test used by Kitts (1990). If the mean rate of inflation is not constant, then the sums of the ranks of the model's residuals are likely to be lower

Table 5.1. Estimated parameters for the original price inflation model

Interval	*QMU*	*QA*	*QSD*
1923-94[†]	0.0473 (0.0120)	0.5773 (0.0798)	0.0427
1923-58	0.0218 (0.0118)	0.3744 (0.1230)	0.0424
1959-94	0.0674 (0.0197)	0.6584 (0.1304)	0.0392

[†] Source: Table 2.3 of Wilkie (1995b)

(higher) than expected over intervals where the mean is lower (higher) than average. After ranking the residuals from the model fitted over the interval 1923-94, the sums of the ranks over the intervals 1923-58 and 1959-94 are 1,487 and 1,141, respectively. These are marginally not significantly different from the expected sum 1,314 at the 5% level ($z = 1.95$, $p = 0.0514$). This result is inconclusive, but it suggests a potential area of weakness in the price inflation model.

5.1.6 Wilkie's ARCH model is able to deal effectively with the problem of non-normality and heteroskedasticity. However, it is unlikely to be able to accommodate possible changes in the mean rate of inflation. This is supported by a rank-sum test. The sums of the ranks of the ARCH model's residuals over the intervals 1923-58 and 1959-94 are 1,496 and 1,132, respectively. These are significantly different from the expected sum 1,314 at the 5% level ($z = 2.05$, $p = 0.0404$). This result suggests that the ARCH model's residuals may not be independent and identically distributed. Nevertheless, on the whole, the ARCH model appears to describe the data better than the original model. Thus, it should generally be used in applications of the model, unless the ARCH effect is not significant for those particular applications.

5.2 *Wage Inflation Model*

5.2.1 Wilkie (1995b, 810) reported that the transfer function wage inflation model's residuals over the interval 1923-94 appeared to be normally distributed, but were significantly correlated with the price inflation model's residuals. Consequently, he fitted a VAR model with price and wage inflation as input variables (see Section 5.3). Out-of-sample residuals are not yet available for the wage inflation model, because it was first reported in Wilkie (1995b).

5.2.2 Further tests on the wage inflation model reveal that their residuals do not appear to be independent and identically distributed. The model's residuals only have 25 runs, which is significantly low at the 5% level ($z = -2.83$, $p = 0.0046$). Moreover, the sums of the ranks of the model's residuals over the intervals 1923-58 and 1959-94 are 1,518 and 1,110, respectively. These are significantly different from the expected sum 1,314 at the 5% level ($z = 2.30$, $p = 0.0216$). These results suggest that the transfer function wage inflation model is not empirically adequate.

5.3 *VAR Inflation Model*

5.3.1 Wilkie (1995b, 813) reported that the VAR inflation model's residuals failed tests of normality at the 5% level. No other test results were reported.

5.3.2 In fitting the VAR model Wilkie did not use lagged data for the initial values, as he did in fitting all the other models. The reason for this inconsistency appears to be the unusually low values of price and wage inflation in 1922 of −20% and −31% respectively. These starting values would have had an undue influence on the estimates of the model's parameter values. However, it is not possible to determine the model's residuals, and to conduct further tests, without knowing the actual starting values that were used. Hence, to further evaluate the VAR model, it has been refitted over the interval 1925-94 using lagged data for the initial values (see Table 5.2). These estimates are similar to Wilkie's (1995b, 813), except that the mean values are larger; Wilkie's estimates of *QMU* and *WMU* were 0.0359 and 0.0509 for the full model and 0.0205 and 0.0344 for the reduced model. The mean values reported in Table 5.2 are more consistent with the other inflation model's mean values (see Table 3.1). Table 5.2 suggests that the parameters *QA* and *WQ* are not significantly different from zero because the log likelihood of the reduced model is not significantly greater than the log likelihood of the full model ($\chi^2_2 = 2.15$, $p = 0.3405$). Hence the reduced model appears to be the most suitable VAR model and will be evaluated further.

5.3.3 The reduced model's residuals satisfy the runs test (for prices $z = -0.42$, $p = 0.3362$, for wages $z = -0.91$, $p = 0.1831$). The sums of the ranks of the model's residuals over the intervals 1925-59 and 1960-94 are 1,371 and 1,114 for prices, and 1,406 and 1,079 for wages. These are not significantly different from the expected sum 1,242.5 at the 5% level (for prices $z = 1.51$, $p = 0.1312$, for wages $z = 1.92$, $p = 0.0548$). Note that the result for wages is only marginally not significant, which suggests that the VAR model may not have been able to fully overcome the problems associated with the other inflation models. The model's residuals fail tests of normality at the 5% level (for prices skewness = 1.52, $z = 5.20$, $p = 0.0000$ and kurtosis = 6.74, $z = 6.39$, $p = 0.0000$, for wages skewness = 1.35, $z = 4.60$, $p = 0.0000$ and kurtosis = 5.04, $z = 3.48$, $p = 0.0005$). Nevertheless, despite these results, this model appears to provide the most promising price and wage inflation models.

5.4 *Share Dividend Yield Model*

5.4.1 Wilkie (1995b, 822) reported that the dividend yield model was 'satisfactory'. Over the out-of-sample interval 1983-94, the mean and variance of the model's residuals were not significantly different from their expected values. Over the interval 1923-94, the model's residuals were found to be independent and normally distributed. All the model's parameters were found to be significant.

5.4.2 However, Wilkie (1984, 58) reported that: "The values of *YW* vary considerably according to the period chosen." Over the intervals 1919-82, 1933-82 and 1946-82 Wilkie (1984) estimated *YW* as 1.35, 2.41, and 1.77 respectively.

Table 5.2. Estimated parameters for the VAR inflation model

Parameter	1925-94	1925-94
QMU	0.0457 (0.0133)	0.0455 (0.0135)
QA	0.1484 (0.1682)	-
QW	0.6134 (0.1863)	0.7533 (0.0981)
WMU	0.0609 (0.0146)	0.0607 (0.0166)
WA	0.5906 (0.1459)	0.7692 (0.0770)
WQ	0.1896 (0.1318)	-
QSD	0.0399	0.0402
WSD	0.0313	0.0318
ρ	0.6919	0.6961

Furthermore, Wilkie (1995b, 831) found that the estimates of *YW* for various other countries were noticeably variable and ranged from 0.5 for the United States of America to 1.8 for the U.K. This suggests that *YW* may not be constant over time.

5.4.3 The suitability of *YW* can be examined by re-expressing the share dividend yield model as follows (for $t > 0$) and plotting the resulting regression (see Figure 5.3):

$$(1 - YA \cdot \mathrm{B}) \cdot (\log_e Y(t) - \log_e YMU) = YW \cdot (1 - YA \cdot \mathrm{B}) \cdot \nabla \log_e Q(t) + YE(t). \quad (5.1)$$

5.4.4 Figure 5.3 illustrates the sensitivity of *YW* to the years 1940 and 1974. These years correspond to the years in which the greatest increases in prices and yields occurred. If they are excluded from the regression, then *YW* becomes insignificantly different from zero (t-value of *YW* with intervention variables in 1940 and 1974 is 1.67, $p = 0.0990$). Wilkie (1995b, 822) appeared to acknowledge this finding, but nevertheless concluded that *YW* was justified because its estimate was significantly greater than zero. The problem with including *YW* is that it results in a general tendency for changes in yields to be correlated with changes in inflation, but this correlation only seems to be appropriate for large increases in yields and inflation. An alternative method for determining the appropriate value of *YW*, which was suggested by a referee of this paper, is to use robust estimation techniques. This is beyond the scope of this paper. Other than this possible weakness, the share dividend yield model appears to fit the data reasonably well.

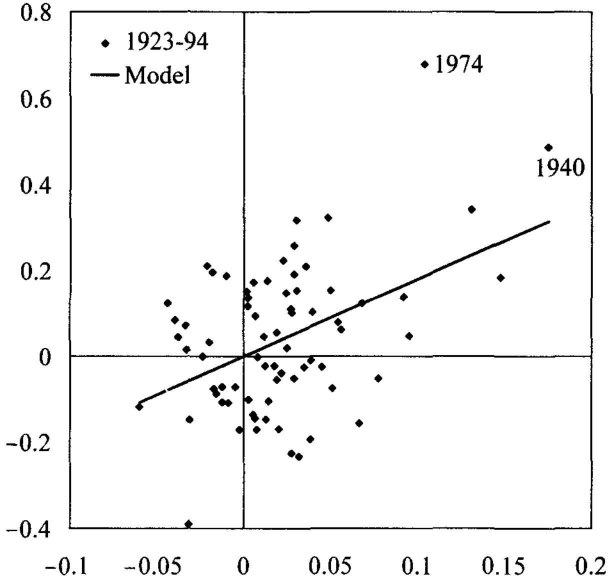

Figure 5.3. $(1 - YA \cdot \mathrm{B}) \cdot (\log_e Y(t) - \log_e YMU)$ plotted against $(1 - YA \cdot \mathrm{B}) \cdot \nabla \log_e Q(t)$

5.5 *Share Dividend Model*

5.5.1 Wilkie (1995b, 840) reported that the mean and variance of the share dividend model's residuals over the out-of-sample interval 1983-94 were not significantly different from their expected values. However, Wilkie reported that the model's residuals over the interval 1923-94 failed tests of independence and normality at the 5% level. These residuals had too many runs of the same sign, they were negatively skewed, and they were leptokurtic. Wilkie (1995b, 844) suggested that the latter results may have been due to the large falls in dividends in 1925, 1928, 1931, 1932, and 1941. Furthermore, the estimate of *DD* appeared to be insignificantly different from zero. These findings suggest that the share dividend model is not empirically adequate.

5.5.2 In addition to the above tests, the correlation between the out-of-sample residuals from this model and the price inflation model is 0.76, which is significant at the 5% level (standard error of the correlation coefficient ≈0.29). The model was also fitted over the intervals 1923-58 and 1959-94 (see Table 5.3). The Chow test suggests that the variance of the model's residuals is significantly lower over the latter interval (F(31,31) = 2.52, p = 0.0060). Hence, the share dividend model does not appear to have had constant parameters historically. This result may have been caused by the change in the dividend index used. Wilkie (1995b, 943) mainly used the Actuaries Indices up to 1962 and the FTSEA All-Share Index thereafter. The Actuaries Indices were based on far fewer securities than the FTSEA All-Share Index, which suggests that the Actuaries Indices are likely to have been more variable than the FTSEA All-Share Index.

5.5.3 In addition, *DD* and *DW* are far less significant over the latter interval 1959-94. The significance of *DD* can be examined by refitting the model with *DX* included and *DD* and *DW* excluded (see Table 5.3). This shows that the model may have been over-parameterised, because the variance of the model's residuals does not increase significantly after replacing *DD* and *DW* with *DX* (F(1,67) = 3.57, p = 0.0631). However, this result is not decisive. Excluding *DD* and *DW* results in an optimal value of *DX* that is less than one, which indicates that the 'unit gain' effect may not be appropriate (see Section 4.3).

Table 5.3. Estimated parameters for the dividend model

Interval	*DMU*	*DD*	*DW*	*DX*	*DY*	*DB*	*DSD*
1923-94[†]	0.0157	0.1344	0.5793	1 –*DW*	–0.1761	0.5733	0.0671
	(0.0124)	(0.0800)	(0.2157)		(0.0439)	(0.1295)	
1923-58	0.0001	0.1492	1.0343	1 –*DW*	–0.3404	0.6075	0.0751
	(0.0219)	(0.1747)	(0.4207)		(0.0927)	(0.1579)	
1959-94	0.0251	0.0818	0.3540	1 –*DW*	–0.1307	0.4712	0.0473
	(0.0185)	(0.2260)	(0.2542)		(0.0464)	(0.1647)	
1923-94	0.0296	-	-	0.6513	–0.1711	0.6000	0.0689
	(0.0152)			(0.1786)	(0.0433)	(0.0985)	

[†]Source: Table 5.3 of Wilkie (1995b)

5.6 *Long-Term Interest Rate Model*

5.6.1 Wilkie (1995b, 858) reported that the mean of the long-term interest rate model's residuals over the out-of-sample interval 1983-94 was not significantly different from its expected value, but that the variance of these residuals was highly significantly greater than its expected value. Wilkie (1995b, 861) also reported that the model's residuals over the interval 1923-94 appeared to be normally distributed and uncorrelated with one another. However, these residuals were found to be significantly correlated with the residuals from the price inflation model and the share dividend yield model. Wilkie suggested that it may be appropriate to consider alternative values of *CD* or *CW* to alleviate this model's empirical problems.

5.6.2 Further tests emphasise the empirical inadequacy of this model. The correlation between the out-of-sample residuals from this model and the price inflation model is 0.59, which is significant at the 5% level (standard error of the correlation coefficient $\approx$0.29). The Chow test rejects the hypothesis that variances of the residuals are equal over the intervals 1923-58 and 1959-94 at the 5% level (see Table 5.4. F(33,33)=2.15, p=0.0156). Therefore, there is considerable evidence to suggest that the parameter *CSD* is not constant. An important event that may have influenced this result is that during and after World War II the government set minimum prices for government fixed-interest securities.

Table 5.4. Estimated parameters for the long-term interest rate model

Interval	*CMU*	*CA*1	*CD*	*CW*	*CY*	*CSD*
1923-94†	0.0305 (0.0065)	0.8974 (0.0442)	0.0450	1.0000	0.3371 (0.1436)	0.1853
1923-58	0.0237 (0.0056)	0.8918 (0.0742)	0.0450	1.0000	0.1862 (0.1680)	0.1421
1959-94	0.0392 (0.0082)	0.8200 (0.0870)	0.0450	1.0000	0.4406 (0.2375)	0.2083

†Source: Table 6.3 of Wilkie (1995b)

5.6.3 This model can be further examined by considering the estimated values of the parameters *CA*1 and *CY*. Table 5.4 shows that the estimates of *CA*1 are relatively close to one. This suggests that an integrated model could be appropriate. Moreover, the Dickey-Fuller unit-root test (see Doornik & Hendry, 1994) suggests that the long-term interest rate series is integrated of order one. However, as stressed by Wilkie (1995b, 779), an integrated model is probably inappropriate for real rates of return.

5.6.4 In addition, Wilkie (1995b: 860) noted that *CY* becomes insignificantly different from zero when an intervention variable for 1974 was included. Hence, *CY* appears to have a similar problem to *YW* (see Section 5.4). The parameter *CY* seems to describe mainly the event that the largest increase in interest rates coincided with the largest residual from the share dividend yield model. Note that *CY* is not significant over the interval 1923-58 (see Table 5.4). However, if *CY* is set to zero, then Wilkie's model implies that there is no relationship between equity returns and real interest rates. As this does not appear to be a reasonable assumption, it may explain why Wilkie (1995b) included *CY* in the model. Nevertheless, the relationship described by *CY* does not appear to be particularly robust.

5.7 *Short-Term Interest Rate Model*

5.7.1 Wilkie (1995b) reported that the short-term interest rate model's residuals over the interval 1923-94 appeared to be independent and normally distributed.

5.7.2 These findings are confirmed by additional tests. The recursive estimates of *BMU* and *BA* do not change significantly over the interval 1923-94. The parameter estimates do not appear to have been significantly affected by outliers. The runs test is also satisfactory. Hence the short-term interest rate model appears to be empirically adequate.

5.8 *Index-Linked Yield Model*

The index-linked yield model was only fitted over the interval 1981-94. There are insufficient data to carry out a full empirical appraisal. Consequently, this model should be used with caution in long-term studies. Nevertheless, certain tests can be conducted to obtain a broad view of the model's suitability. These tests should use a higher level of significance, 10% say, to reflect the relative shortage of data (see Hendry, 1995, 490). Wilkie (1995b) conducted these tests and reported that the index-linked model's residuals appeared to be independent and normally distributed. Thus, based on the limited evidence available, this model appears to be satisfactory.

5.9 *Property Yield Model*

5.9.1 Data for the property models were only available over the interval 1967-94. Hence, only a limited empirical appraisal of these models can be conducted and a higher level of significance, 10% say, should be used in empirical tests (see Section 5.8). Wilkie (1995b) reported that this model's residuals appeared to be independent and normally distributed.

5.9.2 Further evidence suggests that ZA may not be constant over time. Figure 5.4 presents the property yield data used by Wilkie (1995b), and shows that property yields changed substantially in the late 1960s and in the 1990s. As in the original price inflation model, these changes may have biased the estimate of ZA (see Section 2.2). Over the interval 1970-90 the estimate of ZA is 0.3435 with a standard error of 0.2133, which compares with Wilkie's (1995b, 877) estimate of 0.9115 with a standard error of 0.1007. However, there are insufficient data to draw any definitive conclusions and, given the available evidence, Wilkie's estimates are optimal.

5.10 *Property Income Model*

5.10.1 Wilkie (1995b) reported that the property income model's residuals over the interval 1967-94 appeared to be independent and normally distributed. However, as in the share dividend model, ED was found to be not significantly different from zero.

5.10.2 The significance of ED can be examined by refitting the model with ED and EW excluded (see Table 5.5). This shows that the model may have been over-parameterised, because the increase in the variance of the model's residuals after excluding ED and EW is only just significant ($F(1,24) = 3.19$, $p = 0.0865$). This finding is not decisive and is similar to that for the share dividend model (Section 5.5).

Table 5.5. Estimated parameters for the property income model

Interval	*ED*	*EW*	*EMU*	*EBZ*	*ESD*
1968-94†	0.1289 (0.0689)	1.0000	0.0032 (0.0132)	0.2363 (0.0974)	0.0599
1968-94	-	-	0.0797 (0.0129)	0.2695 (0.1069)	0.0637

† Source: Table 8.2 of Wilkie (1995b)

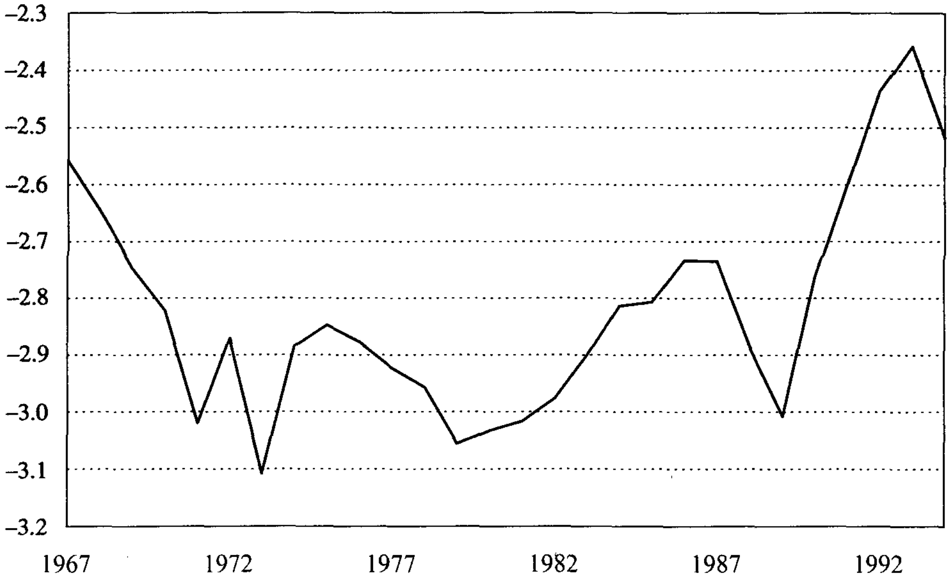

Figure 5.4. The logarithm of the property yield, 1967-94

6. SUMMARY

6.1 Adequate stochastic asset models are extremely difficult to construct because of the complicated nature of the economy. These difficulties are compounded by data shortages, the absence of a reliable and detailed theoretical foundation, and possible regime shifts caused by a number of factors including changes in government policy and technological innovations. A further consequence of these difficulties is that econometric investigations are especially susceptible to the problems of data-mining. These considerations make it difficult to evaluate models and make it important that a range of evaluation criteria is used.

6.2 Economic theory was considered in the development of Wilkie's model, but it is inconsistent with certain orthodox financial economic theories. The long-term interest rate model implies that investors' expectations of average future price inflation are not consistent with the price inflation models. Hence, the model does not incorporate the rational expectations hypothesis. Furthermore, the model is not consistent with the efficient market hypothesis nor does it appear to be consistent with certain aspects of portfolio theory.

6.3 Nevertheless, asset models do not need to be consistent with every economic theory, because theories may not provide a sufficiently accurate description of reality. A more decisive test of a theory or model is whether it is supported by observable events or is empirically adequate. Wilkie's model appears to describe the historical data reasonably well, given the problems associated with econometric investigations. The model's possible empirical weaknesses include that the inflation models do not appear to represent adequately the apparent changes in the mean rate of inflation. The parameters *YW* and *CY* seem to have been unduly affected by outliers. The 'unit gain' effect may not be appropriate in both the share dividend and property income models.

6.4 A more significant empirical problem with Wilkie's original model is that it did not provide an adequate variance and covariance structure for the out-of-sample residuals. The variances of the out-of-sample residuals from the price inflation model and the long-term interest rate model were significantly less than, and significantly greater than, the respective values implied by the original model. There was also a significant cross-correlation between the out-of-sample residuals from the price inflation model and the share dividend and long-term interest rate models.

6.5 Due to the problems associated with data-mining, these empirical weaknesses should not generally be used to suggest an alternative model structure. A complete re-evaluation of economic theory and the data is required before an alternative can be suggested. In particular, detailed consideration should be given to incorporating theories, such as the rational expectations hypothesis and the efficient market hypothesis, in an alternative stochastic asset model.

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