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Random vibration of floating ice tongues

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Abstract: A normal mode approach is used to model the behaviour of a linearly-damped, elastic, ice beam floating on a fluid foundation and subjected to a random distributed loading. As an example, two loading regimes are considered to act on the Erebus Glacier Tongue, McMurdo Sound: broad bandwidth ('white noise') loading, and an ocean wave-type pressure distribution beneath the tongue. For white noise input, the root mean square (rms) deflexion is found to good accuracy within the first few modes, but the rms bending moment increases with the number of modes included in the summation due to the unlimited frequency content of the forcing. Solutions for the rms deflexion and bending moments quickly converge to their mathematical limit after six modes when the forcing is due to ocean waves. A local maximum in rms bending moment near the end of the beam confirms that waves may be important as a mechanism for iceberg calving.

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Introduction

The work of Holdsworth (1969, 1974), Goodman & Holdsworth (1978), Holdsworth & Holdsworth (1978), Holdsworth & Glynn (1981) and Vinogradov & Holdsworth (1985), applied mainly to Erebus Glacier Tongue, McMurdo Sound, Antarctica has indicated a causal link between natural oscillations of ice tongues and iceberg calving. These oscillations can be set up by any forcing mechanism which can provide energy at the natural modal frequencies, e.g. ocean waves, the local passage of storms, or storm surges. A current long-term experiment on Erebus Glacier Tongue (W.H. Robinson, personal communication 1988) carried out by the Physics and Engineering Laboratory, Department of Scientific and Industrial Research, New Zealand, may provide a data set to test the hypothesized link between glacier oscillations and calving. Work concerned with random loading of structures and structural dynamics, such as reported in Newland (1984) or Lin (1967), enables various geophysical loading regimes to be considered in more detail, permitting the mean square deflexion, velocity and bending moment responses to be found for any continuous system subjected to a distributed random load. We apply these theories to a simple, linearly-damped, clamped-free beam configuration using as an example the flexural motion of the Erebus Glacier Tongue.

Theory

The beam equation and boundary conditions

We require to find the mean square deflexion and mean square bending moment along an ice tongue subjected to a stationary and homogeneous, randomly distributed loading $p(x,t)$. It is assumed that the two-dimensional spectral density $P(k, \omega)$ of the loading, where k is the wave number and ω is radian frequency, is known.

We begin by assuming that the motion of an ice tongue of thickness h may be modelled by a thin elastic beam equation of the form

$$\frac{\partial}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right)+\mu y+\rho A\frac{\partial^2 y}{\partial t^2}=p(x,t) \qquad (1)$$

(see Fung 1965, p. 319, for derivation and assumptions). In equation (1) $y(x,t)$ is the deflexion of the beam at time t, distance x from its end, E is Young's modulus, I is the second moment of area of the beam, μ is the foundation modulus for the fluid beneath the ice tongue, ρ is the mean density of the ice, A is the cross-sectional area and $p(x,t)$ is the loading per unit length on the beam. For generality, equation (1) is expressed in its derived form allowing variation of E and I with x, although in this paper we shall restrict our attention to beams of uniform cross-section so that E and I are constant. For uniform cross-section the first term on the left hand side of equation (1) becomes $EI\partial^4y/\partial x^4$. In modelling the fluid foundation contribution as buoyancy alone (i.e. by μy), we are neglecting any inertial effects in the fluid which would derive from the equations of fluid mechanics — in the inviscid case, Bernoulli's pressure equation. The Winkler foundation, as it is known, is used on the grounds that frequencies and amplitudes of oscillation are small.

There is no doubt that the use of an *elastic* beam equation to model the behaviour of ice in flexure is a gross approximation, as the material properties of ice are rate-dependent — ice is a non-linear viscoelastic fluid. The elastic approximation would, for example, be totally invalid if we were considering deformations of an ice tongue which were sufficiently slow to allow the ice to creep. A power 3 rate-dependent rheology (Glen 1955) would then be more appropriate. In the present paper, where small amplitude oscillations of an ice tongue rather than its long-term flow are being considered, the ice will also undoubtedly creep at the lower frequencies, leading to irrecoverable strains and energy dissipation. This energy dissipation will be included later, albeit in a very inadequate manner, with the introduction of a Newtonian viscosity term into the beam equation.

If the deflexion $y(x,t)$ is large we should include an additional term $-N\partial^2y/\partial x^2$ on the left hand side of equation (1), where N represents the axial force due to large deflexions. We do not include this term in the present analysis as it can introduce further complications which we do not believe are revelant in the context of this paper — an ice shelf is unlikely to experience deflexions of sufficient magnitude for elastic softening to be important. The assumption that the beam is thin is reasonable because the only oscillatory modes which can be excited by wind or wave pressure forces acting on the tongue will be long compared to the beam's thickness, and therefore rotary inertia and transverse shear contributions will be small.

The beam is assumed to float freely on the sea-water foundation, and to be in a clamped-free configuration, i.e. it is clamped at the hinge line and free to oscillate at its open water end. Thus, at the hinge line the deflexion and its derivative with respect to x vanish, and at the free end the bending moment and transverse shear vanish. Boundary conditions are therefore

$$y(x,t)\big|_{x=0}=0,\ \frac{\partial y}{\partial x}\bigg|_{x=0}=0,\ \frac{\partial^2 y}{\partial x^2}\bigg|_{x=l}=0,\ \frac{\partial^3 y}{\partial x^3}\bigg|_{x=l}=0 \qquad (2)$$

where l is the length of the beam.

Normal modes

The method of analysis initially follows that described by Timoshenko *et al.*(1974), and subsequently applied to an ice island by Goodman *et al.*(1980), whereby separable modal solutions of the form

$$y(x,t)=\sum_{j=1}^{\infty}\phi_j(x)\,y_j(t) \qquad (3)$$

are considered such that the functions $\phi_j(x)$ are orthogonal, i.e. they satisfy

$$\int_0^l \phi_j(x)\,\phi_k(x)\,dx = l\,\delta_{jk} \qquad (4)$$

Conditions for orthogonality are given by Lin (1967, p. 209). Substitution of expression (3) into equation (1), followed by multiplication by $\phi_j(x)$ and then integration over the length of the beam gives, using the orthogonality condition (4),

$$\ddot{y}_j+\omega_j^2\,y_j=\frac{1}{\rho A l}\,p_j(t) \qquad (5)$$

where $p_j(t)=\int_0^l \phi_j(x)\,p(x,t)\,dx$, and

$$\frac{d^4\phi_j(x)}{dx^4}-k_j^4\,\phi_j(x)=0 \qquad (6)$$

The quantity $p_j(t)$ is the modal exciting force per unit length. The j^{th} natural frequency, ω_j, is related to the j^{th} modal wave number, k_j, by

$$\omega_j^2=\frac{k_j^4}{\rho A}\left(EI+\frac{\mu}{k_j^4}\right) \qquad (7)$$

In equation (7) we note that the introduction of a Winkler foundation of modulus μ beneath the beam alters the j^{th} modal frequency. This is in fact the only effect the foundation has on the solution as pointed out by Timoshenko *et al.*(1974, p. 455).

Some *linear damping* is now introduced into equation (5). In reality, as discussed above, the ice responds non-linearly and with large deflexions an extra term $-N\partial^2y/\partial x^2$ should be included in equation (1). In this case an equivalent equation to (5) which includes an additional term proportional to y^3 (Duffing's equation) may be found for the first mode using Galerkin's method (Mei & Prasad 1976). It is well known that this non-linear equation can lead to solutions with the same period as the forcing, to sub-harmonic solutions, and to chaotic solutions. Various models of non-linear damping, e.g. terms proportional to $y^2\dot{y}$ and $y\dot{y}^2$ may also be included, and in the general case the equation for y becomes

$$\ddot{y}+f(y,\dot{y})=p(t)/m \qquad (8)$$

where m is the modal mass. The technique of stochastic linearization (Selcuk Atalik & Utku 1976) may then be used to reduce the equation to its optimal linear equivalent (Mei & Prasad 1976). This will be the subject of a later investigation, and for now we rest content with the sub-optimal linear model. Thus equation (5) becomes

$$\ddot{y}_j+\beta_j\dot{y}_j+\omega_j^2\,y_j=\frac{1}{\rho A l}\,p_j(t) \qquad (9)$$

which is the usual equation for a single degree of freedom damped oscillator. The modal damping coefficient β_j can be different for each mode as long as orthogonality is satisfied (Lin 1967, p. 209). The introduction of damping could have been done earlier, i.e. directly into the beam equation (1). In that case a term proportional to $\partial y/\partial t$ would be included in (1) to represent viscous dissipation.

Solution for the normal modes

We require to find the form of the functions $\phi_j(x)$ which satisfy equation (6), the modal boundary conditions for the clamped-free beam

$$\phi_j(x)\big|_{x=0}=0,\ \frac{\partial\phi_j(x)}{\partial x}\bigg|_{x=0}=0,\ \frac{\partial^2\phi_j(x)}{\partial x^2}\bigg|_{x=l}=0,$$

$$\frac{\partial^3\phi_j(x)}{\partial x^3}\bigg|_{x=l}=0 \qquad (10)$$

and the orthogonality condition (4). This is straight forward (see Timoshenko *et al.* 1974, p. 426). The general solution for each mode is

$$\phi_j(x) = C_1(\cos k_jx + \cosh k_jx) + C_2(\cos k_jx - \cosh k_jx) + C_3(\sin k_jx + \sinh k_jx) + C_4(\sin k_jx - \sinh k_jx) \qquad (11)$$

The boundary conditions require that $C_1 = C_3 = 0$ for each mode, and give a condition on each modal wave number as follows

$$\cos k_j l \cosh k_j l = -1 \qquad (12)$$

This has been solved to high precision (for reasons discussed below), and the results are presented in Table I. The boundary conditions give a further condition between C_2 and C_4, namely that

$$A_j = \frac{C_2}{C_4} = -\frac{\sin k_jl + \sinh k_jl}{\cos k_jl + \cosh k_jl} = \frac{\cos k_jl + \cosh k_jl}{\sin k_jl - \sinh k_jl} \qquad (13)$$

where C_2/C_4 is different for each mode. The orthogonality condition (4) is then used to obtain C_2. After much algebra we find

$$(C_2)^2 = 1 \qquad (14)$$

and can therefore write down the solution we require using equation (13) to determine $A_j = C_2/C_4$ for each mode (Table I). Some care has to be taken in finding the values of A_j, since it is particularly easy to introduce numerical instability as odd k_jl lay just above $\pi(i - ½)$ and even k_jl lay just below $\pi(i - ½)$. A small numerical error can flip k_jl to the wrong side of $\pi(i - ½)$ and lead to serious numerical instability because of sine or cosine sign changes in equation (13). As $i \rightarrow$ large, $k_jl \rightarrow \pi(i - ½)$, and $A_j \rightarrow -1$.

Mean square deflexion and mean square bending moment response

In common with other similar examples of random loading of structures, and following Newland (1984), the frequency response function for equation (9) is evaluated first. This is done by applying a distributed load per unit length of the

Table I. Non-dimensionalized modal wave number k_jl and A_j satisfying modal boundary conditions (10).

Mode	k_jl	$A_j = C_2/C_4$
1	1.8751040687	-1.3622205575
2	4.6940911330	-0.9818675392
3	7.8547574382	-1.0007761054
4	10.9955407349	-0.9999664479
5	14.1371683910	-1.0000014499
6	17.2787595321	-0.9999999373
7	20.4203522510	-1.0000000027
8	23.5619449018	-0.9999999999
9	26.7035375555	-1.0000000000
10	29.8451302091	-1.0000000000

form $p(x,t) = e^{i\omega t}\,\delta(x - s)$ where δ is the Dirac delta function and $i^2 = -1$. Then

$$p_j(t) = \phi_j(s)\,e^{i\omega t} \qquad (15)$$

and on substitution into equation (9) we obtain the frequency response function

$$H(x,s,\omega) = \sum_{j=1}^{\infty} \frac{\phi_j(x)\phi_j(s)}{\rho A l(\omega_j^2 - \omega^2 + i\beta_j\omega)} \qquad (16)$$

We assume a two-dimensional forcing spectral density which is correlated in time but is uncorrelated (white) in space, viz. of the form $P(k, \omega) = P(\omega)$ where we have suppressed a constant whose dimensions are metres. Then the force-per-unit-length cross-spectral density exciting the ice tongue will take the form $P(s_1, s_2, \omega) = 2\pi P(\omega)\delta(s_2 - s_1)$, i.e. it will be zero unless $s_1 = s_2$ when its value will be infinite. Its units are $N^2\,m^{-2}\,s$ as $\delta(s_2 - s_1)$ has units m^{-1}. The cross-spectral density of the response will be given by

$$P(x_1,x_2,\omega) = \int_0^l \int_0^l 2\pi H^*(x_1,s_1,\omega)\,H(x_2,s_2,\omega)\,P(\omega)\,\delta(s_2 - s_1)\,ds_2\,ds_1 \qquad (17)$$

We obtain, after substitution of the frequency response function and utilizing orthogonality and the properties of the delta function,

$$P(x_1,x_2,\omega) = \frac{2\pi}{\rho^2A^2l} \sum_{j=1}^{\infty} \frac{P(\omega)\phi_j(x_1)\phi_j(x_2)}{(\omega^2 - \omega_j^2)^2 + (\beta_j\omega)^2} \qquad (18)$$

Thus we find the mean square deflexion using

$$E\,[y^2(x,t)] = \int_{-\infty}^{\infty} P(x,x,\omega)d\omega \qquad (19)$$

This involves the evaluation of

$$I_j = \int_{-\infty}^{\infty} \frac{P(\omega)d\omega}{(\omega^2 - \omega_j^2)^2 + (\beta_j\omega)^2} \qquad (20)$$

for each mode. We have done this numerically for general $P(\omega)$ using a Romberg scheme to allow maximum flexibility in the choice of $P(\omega)$, and precision in evaluation of the integral. The final expression for mean square deflexion is therefore,

$$E[y^2(x,t)] = \frac{2\pi}{\rho^2A^2l} \sum_{j=1}^{\infty} I_j \left[(\cos k_jx - \cosh k_jx) + \frac{1}{A_j}(\sin k_jx - \sinh k_jx) \right]^2 \qquad (21)$$

The bending moment $M(x,t)$ is given by

$$M(x,t) = EI\frac{\partial^2 y(x,t)}{\partial x^2} \qquad (22)$$

Thus we require to define a new frequency response function $H_m(x,s,\omega)e^{i\omega t}$, where

$$H_m(x,s,\omega) = \frac{EI}{\rho A l} \sum_{j=1}^{\infty} \frac{\phi_j''(x)\phi_j(x)}{(\omega^2 - \omega_j^2 + i\beta_j\omega)} \qquad (23)$$

(Newland 1984). Then the mean square bending moment is given by

$$E[M^2(x,t)] = \frac{2\pi E^2 I^2}{\rho^2 A^2 l} \sum_{j=1}^{\infty} I_j k_j^4 \left[(\cos k_j x + \cosh k_j x) + \frac{1}{A_j} (\sin k_j x + \sinh k_j x) \right]^2 \qquad (24)$$

The forcing

Equations (21) and (24) allow us to calculate the rms deflexion and rms bending moment response for a two-dimensional spectral density which is uncorrelated in space and correlated in time, i.e. $P(k,\omega)$ is a function of ω alone. This is an approximation. However, it is a reasonable approximation for the types of pressure distribution we wish to study.

In the first instance we shall consider a further approximation, which will be relaxed later, namely that $P(\omega) = P_0 =$ constant. Then the forcing is uncorrelated in both space and time and is said to be white. In this case the integral I_j may be evaluated analytically to give

$$I_j = \frac{\pi}{\beta_j \omega_j^2} \qquad (25)$$

and the two expressions (21) and (24) become

$$E[y^2(x,t)] = \frac{2\pi^2}{lc} \sum_{j=1}^{\infty} \frac{1}{(EIk_j^4 + \mu)} \left[(\cos k_j x - \cosh k_j x) + \frac{1}{A_j} (\sin k_j x - \sinh k_j x) \right]^2 \qquad (26)$$

where $c = m\beta$ is assumed constant for all modes, and

$$E[M^2(x,t)] = \frac{2\pi^2 E^2 I^2}{lc} \sum_{j=1}^{\infty} \frac{k_j^4}{(EIk_j^4 + \mu)} \left[(\cos k_j x + \cosh k_j x) + \frac{1}{A_j} (\sin k_j x + \sinh k_j x) \right]^2 \qquad (27)$$

White noise may be a good model to use for wind-induced oscillations of the ice tongue, where the fluctuating applied pressure spectrum might be expected to be broad band, and uncorrelated in space and time (Newland 1984). In the simpler case of a beam which does not rest on a fluid (Winkler) foundation, these expressions can be further simplified as $\mu = 0$.

The second forcing spectrum $P(\omega)$ we shall consider is that due to ocean waves passing beneath the ice tongue. In this case we will use numerical integration, as for an arbitrary ocean wave energy spectrum (possibly obtained from a waverider or pitch-roll buoy or from a pressure transducer) it is unlikely that I_j will be analytically integrable. If the spectrum were found from data the numerical scheme we are using (Romberg) is definitely overly precise, as one could argue that a confidence interval, defined according to the amount of smoothing done on the original data record, should be associated with each value in the energy spectrum. However, since we will be using a functionally defined spectrum in this instance, and we wish to retain as much generality as possible, we accept the additional precision and subsequent increase in computing time.

The spectrum we shall use is the Pierson-Moskowitz spectrum. This has the form

$$\Phi(\omega) = \alpha g^2 \omega^{-5} \exp\left[-\frac{5}{4} \left(\frac{\omega}{\omega_{peak}} \right)^{-4} \right] \qquad (28)$$

where $\alpha \sim 1.2 \times 10^{-3}$ is Phillip's constant, g is the acceleration due to gravity, and ω_{peak} is the frequency at which the spectral peak occurs. This spectrum is a simplification of the more general *JONSWAP* spectrum (Phillips 1977, p. 139). We require to find the forcing beneath the ice tongue and so we must calculate the effect of the Pierson-Moskowitz spectrum at depth according to

$$P(\omega) = \left[\mu \frac{\cosh k(D-d)}{\cosh kD} \right] \Phi(\omega) \qquad (29)$$

where D is the depth to the sea bed, d is the depth to the underside of the beam and k, the wave number, is determined interactively from the usual dispersion relation $\omega^2 = gk \tanh kD$. It is important to note that the effect of (29) is to bias the Pierson-Moskowitz spectrum towards long periods, as these periods are felt most at depth. This effect is very significant since it will prevent the 'high-frequency tail' of the surface Pierson-Moskowitz spectrum inducing modal oscillation in the tongue. We are now in a position to consider solutions for the white noise and Pierson-Moskowitz forcing cases.

Results

The above theory is now applied to the Erebus Glacier Tongue. This is an obvious choice as much work has already been carried out on this ice tongue, notably by Holdsworth (1969, 1974), Holdsworth & Glynn (1981) and Vinogradov & Holdsworth (1985), and its dimensions and properties are reasonably well known. Furthermore, an experimental study is presently underway to monitor flexural motions of the tongue until calving occurs (W.H. Robinson, personal communication 1988). The values used in the model are as follows: $E = 8.0 \times 10^9$ Pa, $I = 1.1 \times 10^9$ m^4, $\rho = 867$ kg m^{-3}, $l = 13\,000$ m, $A = 3.0 \times 10^5$ m^2, $\mu = 1.5 \times 10^7$ kg m^{-1} s^{-2}, $D = 400$ m, and $d = 170$ m. The damping coefficient β has been set at 3.84×10^{-3} s^{-1}, which corresponds to $c = 1.0 \times 10^6$ N m^{-2} s. This is an unknown quantity, but simulations suggest that the solutions are not greatly dependent on its value within reasonable limits. When $\beta = 0$ the solutions will be unbounded if the forcing frequency coincides with a natural mode.

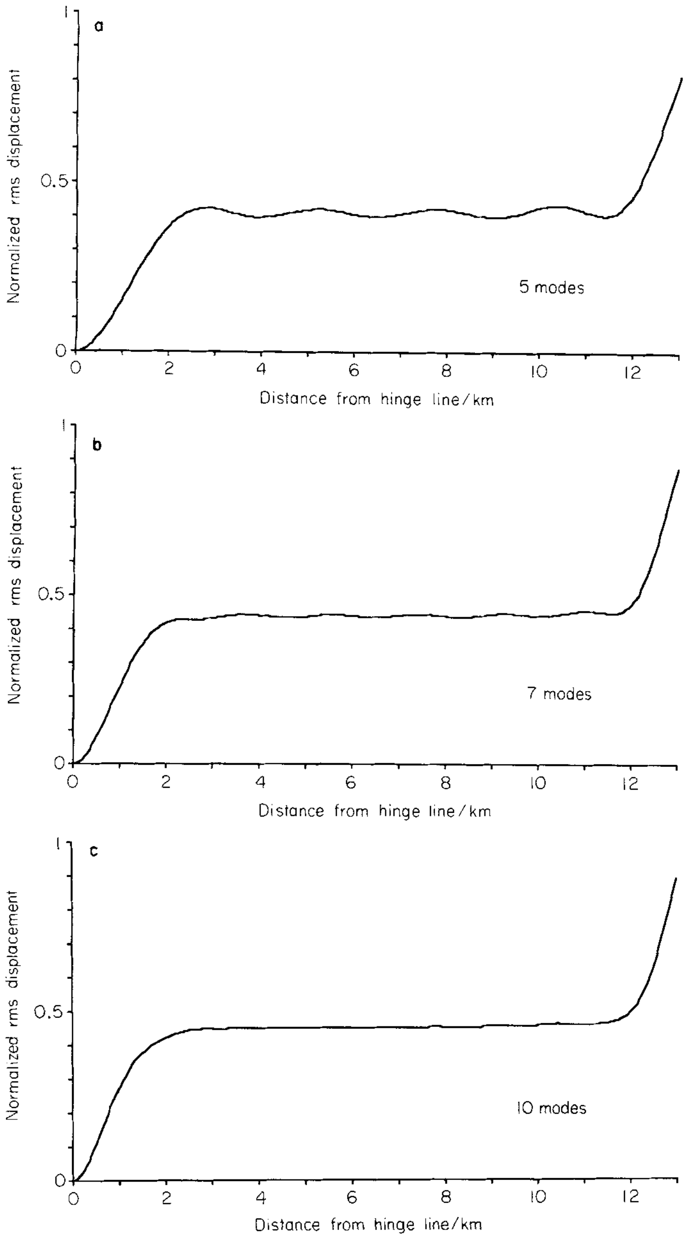

Fig. 1. Normalized rms deflexion induced in ice tongue summed to: **a.** 5 modes, **b.** 7 modes and **c.** 10 modes due to white noise forcing.

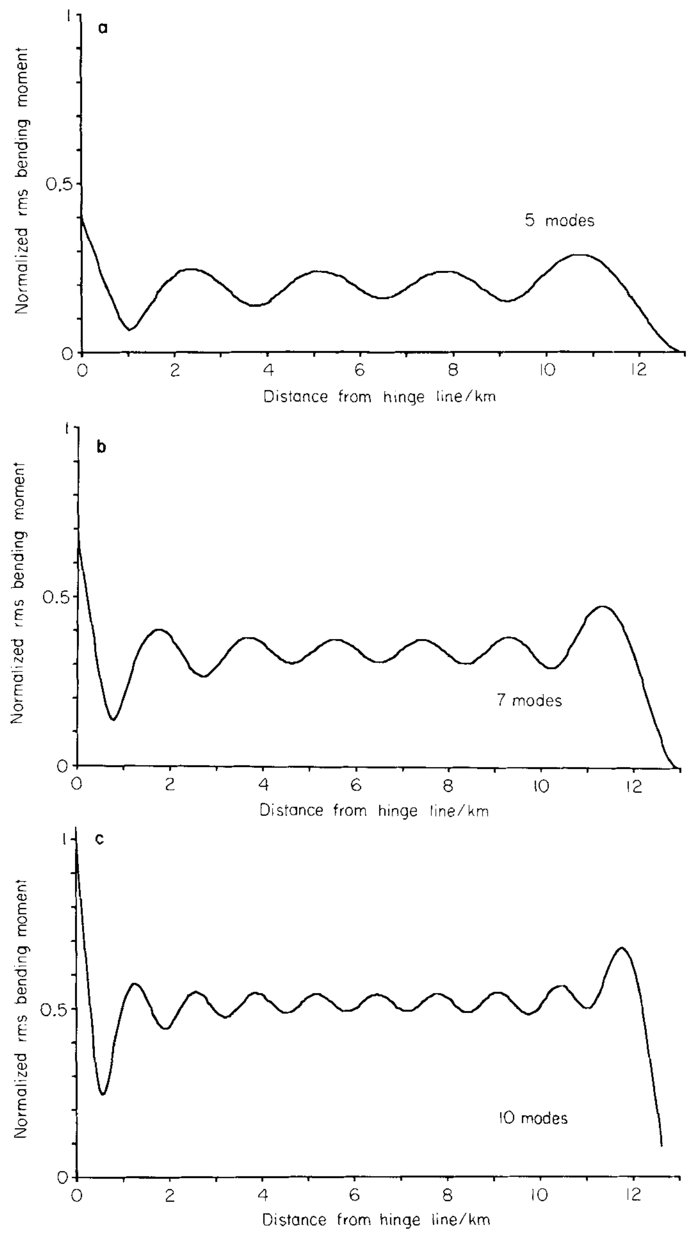

Fig. 2. Normalized rms bending moment induced in ice tongue summed to: **a.** 5 modes, **b.** 7 modes and **c.** 10 modes due to white noise forcing.

Figs 1 and 2 show rms deflexion and bending moment curves under white noise forcing conditions. The curves are normalized as the forcing value P_0 is unknown; only the form of the curves is important. Curves for deflexion have been checked for consistency against those for bending moment. In the case of deflexion the series in expression (26) converges to very close to its mathematical limit by about 10 modes. The rms bending moment on the other hand converges less rapidly. Moreover, it exhibits the effect described by Newland (1984) for the simpler free-free beam, whereby the wavelengths and wave amplitudes at the centre of the tongue decrease, but the level of rms bending moment increases with the number of modes included in the summation. This is due to the white noise input which is uncorrelated in space and time *at all wave numbers and frequencies*. The behaviour is analogous to Gibb's phenomenon of Fourier series, where the sum of Fourier components near a step contains oscillations whose frequency increases with the number of modes summed but whose amplitude remains constant (Newland 1984). If a large enough number of modes are included in the summation the rms bending moment becomes approximately constant for most of the tongue expect near the clamped and free ends.

Figs 3 and 4 show the induced rms deflexion and bending

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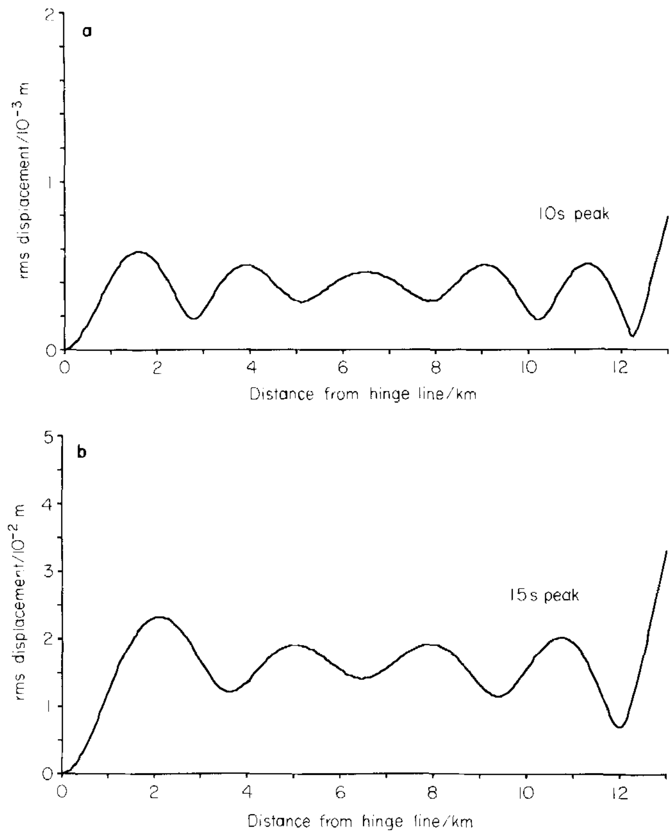

Fig. 3. rms deflexion induced in ice tongue due to Pierson-Moskowitz forcing with peak at: **a.** 10 s; and **b.** 15 s.

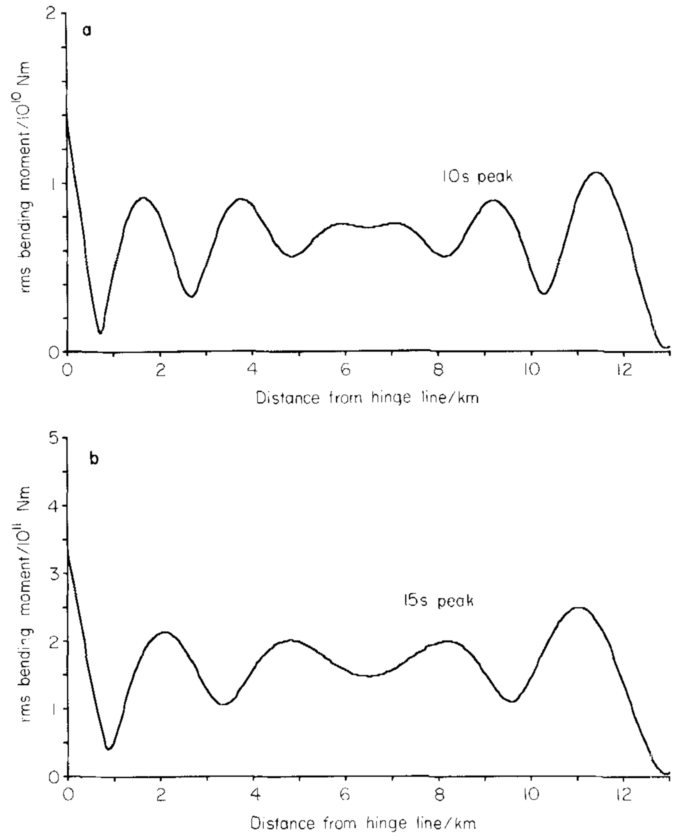

Fig. 4. rms bending moment induced in ice tongue due to Pierson-Moskowitz forcing with peak at: **a.** 10 s; and **b.** 15 s.

moment curves for the input Pierson-Moskowitz spectrum. Peak frequencies corresponding to periods of 10 s and 15 s are chosen to bracket the range of wave periods observed by one of the authors (V.A.S.) near Erebus Glacier Tongue in the austral spring of 1985. At the peak frequency the spectral density is ~2.6 m and ~0.3 m respectively, so the induced deflexions and bending moments will differ in magnitude. Summations (including all modes) up to 10 have been carried out, corresponding to modal oscillation periods of 26.2, 26.0, 25.2, 22.9, 19.4, 15.6, 12.2, 9.7, 7.7 and 6.3 seconds for modes 1 to 10 respectively. It is found that the solutions for deflexion and bending moment for both input spectra reach their limiting form by six modes. The viscosity in the original beam equations serves to damp out unrealistically-dominant modal resonances which originate from negligible energy contributions, although it is not suggested that this is effectively modelling the viscoelastic nature of glacial ice. Higher frequency natural modes in the ice tongue–fluid foundation system are not excited because the pressure field, derived from the Pierson-Moskowitz spectrum modulated by the depth relation (29), has negligible energy at these frequencies. The depth relation also tends to favour longer period wave energy and make it more effective in exciting lower modes (see Table I).

The rms bending moment curves of Fig. 4 have a local maximum about 1.5 km from the free end of the 10-s spectrum, and at about 2 km for the 15-s spectrum. Furthermore, in each case the maximum is greater than other local maxima except for that at the hinge line. This would suggest that fracture would occur first at the hinge line. However, the Erebus Glacier Tongue is not of constant cross-section as we have assumed; it tapers from 340 m near the hinge to about 70 m at the free end. This thinning would have the effect of decreasing the stress at the hinge line and increasing the stress further towards the free end, as stress is proportional to bending moment/thickness^2. Thus we tentatively suggest that the first maximum from the free end shown in Fig. 4a, b could dominate stresses induced at the hinge line. These maxima lead to strains at the upper and lower surfaces of the order of 1.2×10^{-7} and 2.8×10^{-6} for each spectrum respectively. It would therefore seem reasonable to conclude that incoming ocean waves could initiate the calving of a tabular iceberg if the wave energy were of sufficient magnitude. The distance of the bending moment peak from the free end of the ice tongue (1.5–2.0 km) roughly matches the typical size for newly calved icebergs in the area. The fact that the Erebus Glacier Tongue has not calved an iceberg recently, although rather longer than usual, may be due to the attenuating influence of the canopy of shore fast ice in McMurdo Sound which until 1986 had not broken out for several years.

Conclusions

A simple randomly-excited elastic beam resting on a fluid foundation has been used to model the deflexion and bending moment response of an ice tongue in terms of rms values. Vibrations are induced by a distributed force-per-unit-length spectrum which is correlated in time but not in space. The model has been applied to the Erebus Glacier Tongue in McMurdo Sound, although its geometry is such that a thinning from the hinge line to the free end should really be taken into account. As an example of the method, however, the Erebus Glacier Tongue is an obvious choice because of the large amount of data available. Results suggest that the random vibration normal mode theory reported in Newland (1984) may be an effective tool in modelling ice tongue deflexion and stresses, and in iceberg calving prediction. One of the authors (V.A.S.) is presently working on the removal of some of the approximations employed in this simplest of models with a view to including elastic softening and a non-linear (power 3) damping rheology (although it is not clear *a priori* how important these effects might be), and a more satisfactory forcing spectrum which allows for space correlation as well as time.

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