7

Time Reversal Violation

Précis. *Time reversal symmetry violation in the electroweak interactions provides evidence for an asymmetry of time itself, given the Representation View.*

One common way to characterise the arrow of 'time itself' is using the concept of a temporal orientation on a relativistic spacetime.[1] A *temporal orientation* is an equivalence class $[\xi^a]$ of smooth timelike vector fields ξ^a that all 'point' in the same direction: at each spacetime point, the vectors are directed into the same lobe of the light cone, which can then be taken to define the 'future' direction. Thus, given a choice of temporal orientation $[\xi^a]$, we say that a vector field χ^a is *future-directed* if and only if $\chi^a \in [\xi^a]$. This provides a vivid image of time's arrow, illustrated in Figure 7.1.

There is some debate amongst philosophers about whether the arrow of time given by a temporal orientation is reducible to other kinds of facts.[2] My concern here will be with a related issue, of how we can possibly come to know that the structure of spacetime includes a temporal orientation. This problem was introduced in Section 5.1: the analogy of Price's table showed that, although most of our experiences involve asymmetries in the evolution of material systems, this is not the same as an asymmetry of 'time itself' – just

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[1] This standard practice in physics was formulated for the philosophy of time by Earman (1974). Relativistic spacetime was introduced in Section 2.5.3; see Malament (2012) for a systematic treatment of its foundations.

[2] Relationist approaches to time argue that the concept of time can be reduced to other variables: for example, causal theorists like Reichenbach (1928, 1956) and Grünbaum (1963) argue that time can be reduced to causation, while Brown (2005) proposes that spacetime more generally can be reduced to facts about matter-energy fields. In a discussion of such views, Earman (1974, p.20) proposes to consider the "heresy" that time is not reducible to anything else; see Maudlin (2002a) and Maudlin (2007, Chapter 4) for what he calls a "more aggressive" defence of heresy.

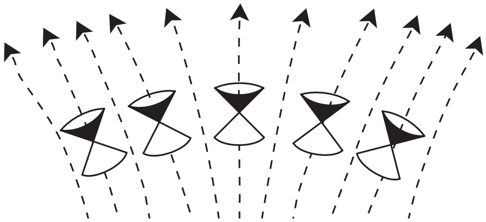

Figure 7.1 A temporal orientation determines a future direction.

as an asymmetry of the items on the table is not the same as an asymmetry of the table itself. I argued that what is missing is a *Spacetime–Evidential Link:* an account that establishes an asymmetry of time itself, together with some plausible empirical evidence in support of that asymmetry.

In this chapter, I will argue that evidence establishing this link is provided by a dynamical asymmetry that has been established for the laws of nature, which philosophers sometimes called a *lawlike* or *nomic* asymmetry.[3] The example I will discuss is the dynamical asymmetry arising in the electroweak interactions, which are what underpin the phenomenon of radioactivity.

Both philosophers and physicists have proposed that this phenomenon can be used to establish an asymmetry in spacetime structure. For example, Wald writes:

> Experiments demonstrating parity violation and (indirectly) demonstrating the failure of time reversal symmetry have shown that two further aspects of spacetime structure can appear in physical laws: the time orientation and space orientation of spacetime. (Wald 1984, p.60)

This idea was explained in more detail by Earman (2002b). Considering a collection of overlapping local neighbourhoods of spacetime, he suggests the following procedure:

> Use the failure of time reversal invariance of the laws to pick out the future direction of time in each of these neighbourhoods. (Earman 2002b, p.257)

He then proposes to use the temporal orientation in each neighbourhood to derive a global temporal orientation. Maudlin proposes a view of time asymmetry that is compatible with this, arguing that the discovery of time reversal symmetry violation in the weak interactions implies

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[3] Cf. Grünbaum (1973, p.273).

> that the laws of nature are not Time Reversal Invariant in any sense, and hence that the laws themselves require an intrinsic asymmetry in time directions, and hence that space-time itself, in order to support such laws, must come equipped with an orientation. (Maudlin 2007, p.120)

There is a logical leap in all of these statements: how does one go from a time asymmetry in a dynamical theory – even one confirmed by experimental evidence – to an asymmetry of "space-time itself"? A time orientation is, after all, just a timelike vector field. What would bring us to view it as part of the spacetime geometry, as opposed to its being some property of a matter or radiation field?[4]

One might argue that spacetime structure is entirely reducible to facts about matter and radiation, as in the dynamical approach to relativity proposed by Brown (2005) and Brown and Pooley (2006). But, as I argued in Section 4.5, this radical approach makes it difficult to formulate many modern dynamical theories, which inevitably make use of spacetime concepts in their foundation. Alternatively, one might postulate Earman's symmetry principle (SP2) as an axiom: that every dynamical asymmetry is also a spacetime asymmetry (Earman 1989, p.46). But, following our discussion in Section 4.5, we were left wondering what reason there is to believe this. I will try to offer some support for it in the next section.

What is missing is what I call a Spacetime–Evidential link (Section 5.1), and in particular an account of how symmetry violation in a dynamical theory can provide evidence of an asymmetry in time. So, the current dialectical situation is like squeezing opposite sides of a balloon: a temporal orientation provides a clear picture of time asymmetry but not of how one knows about it, while dynamical asymmetry appears to provide the opposite. Fortunately, a theme developed over the course of this book provides a solution: by adopting the Representation View, we can understand these two as connected through a representation.[5] My argument in this chapter will be that the Representation View, taken together with the experimental evidence for dynamical asymmetry in electroweak interactions, really does provide evidence for a robust time asymmetry. In short, the electroweak interactions establish an arrow of time itself.

Section 7.1 will give my argument that dynamical asymmetry can provide evidence for time asymmetry, as an application of the Representation View.

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[4] In contexts in which Einstein's equation is satisfied, one might say: when does a timelike vector field belong on the left side, and not the right?

[5] Section 5.1 argued that the Representation View provides an adequate Spacetime–Evidential link. The Representation View itself was introduced in Section 2.3 and described for the specific case of temporal symmetry in Section 2.7.

Section 7.2 will describe a limitation of this approach, associated with the fact that a representation may be 'incomplete'; I will argue that electroweak theory avoids this limitation thanks to evidence from renormalisation group theory. Section 7.3 will then review the discovery of time reversal symmetry violation in the dynamics of electroweak interactions, focusing in particular on how we can come to know such a fact. I emphasise that this is a robust discovery, which is much more general than our current approaches to particle physics. Finally, Section 7.4 will address a critique of Price (1996), which claims to establish that dynamical symmetry violation does not provide evidence for an arrow of time. I will argue that, at least in the case of weak interactions, Price's critique does not succeed.

7.1 Representing Time's Arrow

Let me begin with what it might mean to have a temporal asymmetry of spacetime structure. One would like to say that this expresses an asymmetry of 'time itself'. But what does this mean, if not a crude and qualitative expression of substantivalism? And, how can we come to know about it? In this section, I will show how the Representation View developed in Section 2.3 answers both these questions.

The Representation View advises that for a spacetime symmetry to have meaning in the context of a dynamical theory, we must have a representation or 'homomorphic copy' of it amongst the symmetries of that dynamical theory. Otherwise, the dynamical theory would not deserve the name 'dynamical'. This led to my argument in Section 5.1 that Price's table is not quite the right analogy, since it seems to suggest that the structure of time is entirely independent of its contents. On the contrary, according to the Representation View, these two are not independent: a representation 'projects' the structure of time down onto each dynamical theory, just as a shadow is a projection of the structure of a table down onto the floor. This link can be exploited to establish a robust arrow of time, drawing together some threads from earlier chapters. Here is a summary of how this argument goes.

We begin by shifting focus from the concept of a 'time coordinate' to the more structural concept of a 'time translation', motivated by the discussion of Section 2.4. There, I argued that reversing the time translations provides the appropriate definition of time reversal and showed that this generalises the notion of 'reversing temporal orientation'. Namely: given a smooth timelike vector field defining a temporal orientation, one can express time translations as a collection of diffeomorphisms φ_t that 'flow' along the integral curves of that vector field, in either direction. If the vector field is complete, then by definition those diffeomorphisms are isomorphic to the Lie group $\mathbb{T} = (\mathbb{R}, +)$. If it is not complete, then the time translations can still be viewed as a *local* Lie group, associated with some neighbourhood of the identity 0 of $(\mathbb{R}, +)$. As I argued in Section 4.1.2, one can then view temporal symmetry as the statement that time reversal $\tau : t \mapsto -t$ is an automorphism of the time translations, which correspondingly induces a reversal of temporal orientation.

We can use this thinking to encode what it means for 'time itself' to have an asymmetry, similar to the kind afforded by a temporal orientation: it means that $\tau : t \mapsto -t$ is not an automorphism of the time translations. There are various structures with this property that can be used to describe asymmetric time translations.[6] For example, one can add a partial order to our Lie group, replacing $(\mathbb{R}, +)$ with a structure $(\mathbb{R}, +, <)$, called an *ordered Lie group*. Or, one can simply remove all the time translations that flow in one 'direction'. That is, we can choose the time translations $\varphi_t \in (\mathbb{R}, +)$ such that $t \geq 0$, and drop all of their inverses. This turns the time translations into a structure called a *semigroup*, which is roughly speaking a 'group without inverses'. This structure has no time reversing automorphism $t \mapsto -t$, since the inverses have all been removed.[7] Of course, the 'opposite' set of time translations $\varphi_t \in (\mathbb{R}, +)$ such that $t \leq 0$ can be chosen as well. But, this produces the same structure, since it is isomorphic to the first semigroup.

Formulating time asymmetry in terms of the time translations provides a direct, structural perspective on the arrow of 'time itself': namely, as the statement that $t \mapsto -t$ is not an automorphism of the time translations. This is not quite the same as adopting a temporal orientation, for both temporal orientations produce the same asymmetric time translation group; I will discuss this more in Section 7.4.2. But I see this as an advantage: I will argue that it provides an accurate description of the kind of time asymmetry we find in electroweak theory, without requiring any commitment to 'instants', or any other perspective on the metaphysics of what time translations relate.[8]

Given a dynamical theory on a state space M, the Representation View ensures that, since this theory is 'dynamical', it must admit a representation

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[6] The representation theory of both ordered Lie groups and of semigroups is well-studied and so is perfectly amenable to the Representation View advocated in this book; see Neeb (1993) for an introduction.

[7] More precisely, this structure has no non-trivial automorphisms at all; this is a consequence of Proposition 2.1.

[8] This structuralist-functionalist perspective on spacetime was developed in more detail in Section 2.4.1.

of some time translations $\mathbb{T}$ amongst the automorphisms of state space, given by a homomorphism,

$$\varphi : \mathbb{T} \to \mathrm{Aut}(M), \tag{7.1}$$

When those time translations form a one-parameter set indexed by t, the trajectories of the theory are given by a space of curves $\psi(t)$, often expressed as solutions to a differential equation. What makes these trajectories 'dynamical' is the fact that they are determined by a representation of time translations. As I argued in Chapter 3, virtually all dynamical theories can be understood in this way, from Newtonian particle mechanics to quantum theory.

This representation immediately establishes the Spacetime–Evidential link. First, we have an account of what it means for 'time itself' to have an asymmetry: it means that the time translations $\mathbb{T}$ do not admit a time reversal automorphism $t \mapsto -t$. Second, we have an explanation of how one could produce evidence for this time asymmetry: by studying the asymmetries of a dynamical theory, the representation allows us to draw inferences about the asymmetries of time itself, and in particular of the time translations. Like learning about the symmetries of a table by studying its shadow, we can learn about the symmetries of time by studying its representation in a dynamical theory, as I argued in Section 5.1.

There remains a subtlety in establishing such an asymmetry: at least formally speaking, it is possible for the time translations $\mathbb{T}$ to admit a time reversal symmetry $t \mapsto -t$ and for the dynamical theory that represents $\mathbb{T}$ to still be time reversal violating. As I argued in Section 4.5, this happens whenever the structure of state space prevents a representation of time translations from being extended to include time reversal; indeed, we saw that both symplectic mechanics and spontaneous symmetry breaking provide examples of this.[9]

However, *if* a dynamical theory violates time reversal symmetry, while also providing a 'complete' description of the relevant physical information – in a sense to be discussed in Section 7.2 – then this is still a powerful signal. The completeness of the dynamical theory implies that no empirical evidence could ever be produced according to which 'time itself' is temporally symmetric, since no transformation in the dynamical theory can ever represent that symmetry. In other words, if time reversal is not a dynamical

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[9] The absence of a representation of the time reversal operator is just what it means for a dynamical theory to be time reversal symmetry violating, by what I have called the 'Symmetry Existence Criterion'; see Section 4.2.1.

symmetry, then no empirical information could ever support the statement that it is a symmetry of time translations.

Thus, even the slightest hint of empiricism leads to the conclusion proposed by Earman (1989, p.46), that "conditions of adequacy on theories of motion" require us to infer that if a 'complete' dynamical theory violates time reversal symmetry, then spacetime itself must violate this symmetry too. Demanding an empirical basis for temporal symmetry thus leads to the result that *time reversal symmetry violation establishes an arrow of time itself*.

Let me not keep you waiting: in Section 7.3, I will argue that an arrow of time of this kind has indeed been established, through the study of weak interactions and neutral kaon decay. But first, since so much depends on what it means to be a 'complete' theory, let me first discuss what I mean by this in more detail.

7.2 Complete Enough Representations

Using time reversal symmetry violation to establish an arrow of time requires the dynamical theory to be 'complete' in a certain sense. I will argue that the Standard Model is indeed complete in the required sense, thanks to insights from renormalisation group theory. However, let me first illustrate why this kind of completeness is needed.

A familiar example of an 'incomplete' dynamical system is the classical damped oscillator, which slows to a stop under the force of friction. To first order approximation, the oscillator follows a curve $x(t)$ satisfying Newton's equation, with

$$m\frac{dx^2}{dt^2} = F(x,\dot{x}) = -kx - c\dot{x}, \tag{7.2}$$

for some constants $c,k > 0$. The first term $-kx$ describes a 'Hooke's law' force, proportional to the oscillator's displacement from some equilibrium position. The second term $-c\dot{x}$ describes a force of damping, such as through friction in a spring or in the air, which is proportional to the system's speed.

This equation manifestly fails to be time reversal invariant: a damped oscillator can slow to a stop, but it cannot spontaneously start oscillating. One can check this by noting that time reversal is not a dynamical symmetry, in the sense of Section 4.1.2: although it is common to justify this by substituting $t \mapsto -t$, a more rigorous check is to observe that there is no transformation of the form $(x,\dot{x}) \mapsto (x,-\dot{x})$ that preserves the solution space. Thus, a representation of time reversal symmetry is impossible for the damped harmonic oscillator.

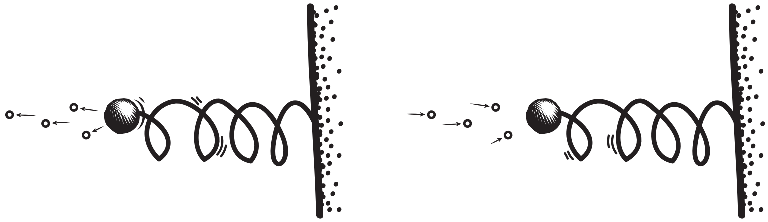

Figure 7.2 A damped oscillator interacts with its microscopic medium (left) in a way that is symmetric under time reversal (right).

However, this is no evidence for an arrow of time, because the description omits relevant degrees of freedom. This includes the energy of the oscillator's motion that is converted into internal energy in the spring, and to a lesser extent into the surrounding medium, jiggling the roughly 10^{26} molecules of the spring and in the air. When those missing degrees of freedom are restored, we find a representation of time reversal symmetry: reversing the 'jiggling' produces just the right kind of collision with a rest oscillator so as to start it bouncing, as in Figure 7.2. The time asymmetry of a damped harmonic oscillator is an illusion due to 'losing' information into the environment, of the kind that we saw many times in Chapter 5.

We will soon discuss an example of a more serious time asymmetry in electroweak theory. Unlike the examples above, this theory aims to give a more or less complete description of the interactions involved in electroweak phenomena. But, how can we can be sure that this will not turn out to be another case of missing information? There is an enormous amount of information that is not captured by the Standard Model, especially concerning phenomena occurring at energy scales so high that we have not yet probed them, and perhaps never will. Given the preponderance of false asymmetries that arise from missing information, this should give one pause: Is symmetry violation in electroweak a similar 'illusion' that arises merely from missing information?

Part of the wonder of the human condition is to be continually learning, and so no evidence is perfectly safe. However, in the special case of particle physics, we do have a remarkable framework of ideas that assures us that its dynamics will not be overturned by missing information. In theories that aim to describe the fundamental interactions of matter and energy, the missing information is of a particular kind, corresponding to facts about energy scales that are large enough – or equivalently, about energy variations on distance scales that are small enough – so as to make them inaccessible to any available experiments. However, unlike the dynamics of the damped harmonic oscillator above, the dynamics of modern particle physics is formulated in such a way that this missing information is irrelevant: that is, the dynamics of the theory is decoupled from the details of the missing information. This happy situation is the result of what is known as *renormalisation theory*.

One of the central results about the quantum field theories of the Standard Model is that they are renormalisable. This means, roughly speaking, that their observable predictions remain the same even as one scales the system down to arbitrarily small distances.[10] In particular, the renormalisability of massive Yang–Mills theories, like the electroweak theory that concerns us, was established by 't Hooft (1971). As a result, predictions about accessible energy scales in the these theories *decouple* from the physics of higher energy scales: the fact that we are missing information about the latter is irrelevant. This leaves us in a particularly fortunate situation, characterised here by Huang (1982):

> It is fortunate that, at any given stage, we were able to regard certain particles as provisionally fundamental, without jeopardizing the right to change our mind. The reason is that, according to quantum mechanics, it is a good approximation to ignore those quantum states of a system whose excitation energies lie far above the energy range being studied. (Huang 1982, p.1)

Butterfield and Bouatta (2015) have argued that this prediction of renormalisation theory, that high energy scales decouple from accessible scales, helps to explain why a renormalisable theory like the Standard Model is so empirically successful.[11] But, the same result can also be viewed as providing a kind of scientific *confirmation*: the Standard Model implies that high energy scales decouple from accessible energies; so, the spectacular empirical success of the Standard Model is strong evidence *confirming* that this decoupling is true of our world.

That said, the Standard Model does contain conceptual problems related to its very definition. In particular, we have no mathematically rigorous interacting quantum field theory in $3 + 1$ dimensions, let alone a rigorous

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[10] The development of renormalisation theory following Wilson (1971) showed a very broad sense in which this holds; see Duncan (2012, Chapters 16–19) for its conceptual foundations and Rivat (2021) for a historical analysis. Recent discussions of its philosophical significance in the context of scientific realism have been given by Butterfield and Bouatta (2015); Fraser (2018); Rivat (2019); Rivat and Grinbaum (2020); Ruetsche (2018); Williams (2020).

[11] Butterfield and Bouatta (2015, p.439) argue that the generic results following Wilson (1971) explain why quantum field theories formulated at accessible energies are renormalisable, concluding: "It is by analyzing this flow that one deduces that what seemed 'manna from heaven' (that some renormalisable theories are empirically successful) is to be expected: the good fortune we have had is generic".

formulation of electroweak theory.[12] But, these problems are of a different kind. The framework, and in particular electroweak theory, is still 'complete enough' to avoid making mistakes by neglecting higher-energy phenomena. Although it is of course not a complete description of reality, its empirical successes, together with the results of renormalisation theory, provide a remarkable assurance that missing information will not interfere with its predictions. As a result, the discovery of time reversal symmetry violation in this theory is not just a result about an incomplete theory: it holds independently of the missing information.

With this assurance in place, the Representation View now provides a reliable way to test whether the structure of time itself is asymmetric. Suppose a system can be described by a quantum theory whose state space provides a representation of time translations amongst its automorphisms and which is 'complete enough' for the description of the phenomena in the sense described above. Then the test is simple: a dynamical system in which time reversal symmetry is violated provides evidence for an arrow of time itself.

7.3 The Discovery

The 'great shock' of electroweak theory is that time reversal is not a dynamical symmetry. I gave a brief history of that discovery in Section 1.5. Let me now discuss in more detail how we came to know about it. I will try to keep the detailed physics relatively light, so as to make the main ideas accessible.

Section 7.3.1 will briefly review some elements of electroweak theory. Section 7.3.2 will then discuss the ideas that led to the 1964 discovery of symmetry violation in this theory. Part of what is so interesting about this story is that the discovery of time reversal symmetry violation was made around a decade before the Standard Model was formulated. So, one naturally wonders: how can we know the symmetries of a theory we don't have? We will soon see that it is thanks to the incredible generality of these experimental discoveries, made possible through the application of symmetry principles that have in any case been of some independent interest to philosophers.

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[12] For example, Streater (1988, p.144) writes, "To test a model, we must study its predictions. The trouble is that Yukawa theory (and QED when one looked hard at it) is not well defined, and different approximations give quite different predictions, rather than very similar predictions. One cannot tell whether one is testing the theory or the method". See Wallace (2006) for a defence of Standard Model methods.

My discussion will include both the discovery of ‘indirect’ and ‘direct’ evidence for time reversal symmetry violation. Taken together, they provide powerful reason to believe that any ‘complete enough’ dynamical theory of the phenomena at issue will violate time reversal symmetry. By the argument presented in Sections 7.1 and 7.2, this cannot be condemned as just a property of the matter-energy fields involved in the phenomena: it is the robust evidence that time itself has an arrow.

7.3.1 The Standard Model of Electroweak Interactions

In this discussion, I will make frequent use of three discrete symmetry transformations: parity or ‘spatial reversal’, described by a unitary transformation P; matter–antimatter exchange, described by a unitary transformation C; and time reversal, described by an antiunitary transformation T.[13] I will also be interested in the combined application of these transformations, such as CP, PT, and CPT. My discussion here will also adopt some standard language from particle physics: when a dynamical theory does not admit an appropriate representation of some transformation X, this is referred to as ‘X violation’. Thus, the discussion that concerns us now is T violation in electroweak theory.

The theory of electroweak interactions is a gauge theory: its mathematical formulation in the Standard Model begins with a classical Yang–Mills gauge theory, which is a cousin of electromagnetism when formulated with a $U(1)$ gauge potential. But, we instead choose the non-abelian gauge group $SU(2) \times U(1)$. The classical theory can then be quantised to give a renormalisable quantum field theory, by appeal to a variety of techniques. The result is a theory capable of dealing in a relatively complete and unified way with both the phenomena of electromagnetism and with weak forces, the combination of which explain radioactive decay.

In honour of its founders, electroweak theory is sometimes called the Glashow–Cabibbo–Weinberg–Salam model.[14] The four generators of $SU(2) \times U(1)$ in a Hilbert space representation were associated with four conserved quantities: the first three associated with $SU(2)$ are called ‘weak isospin’, while the fourth associated with $U(1)$ is called ‘weak hypercharge’. Although this model enjoyed some early predictive successes, in the 1960s

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[13] I have explained the origin of the time reversal transformation T on the Representation View in Chapters 2 and 3. The other transformations have a similar origin in representation theory, which I will introduce in Chapter 8 when we turn to CPT symmetry.

[14] Its early formulation was given by by Cabibbo (1963), Glashow (1961), Salam (1968), and Weinberg (1967) and was later shown to be renormalisable by ’t Hooft (1971).

little was understood about the degrees of freedom associated with it and consequently about the mechanism that could give rise to T violation or CP violation. At the time, only three quark fields were well accepted, with some hints about a fourth 'charm' quark field. At a 1966 conference in Berkeley, soon after the 1964 discovery of CP violation by Cronin and Fitch, Cabibbo is reported to have said that,

> At that time, physicists thought that the theory of the weak interactions was in good shape since they 'understood' how to compute radiative correction to μ decay. In comparison, CP violation was like an atomic bomb blowing up in the background. (Bigi and Sanda 2009, p.145)

A breakthrough came with an application of the Representation View, as presented in Section 2.3. When applied to a global gauge symmetry group such as $SU(2) \times U(1)$, it implies that the gauge transformations only acquire meaning in the context of a state space representation of the group amongst the symmetries of that state space. In an 'elementary' quantum system that cannot be decomposed into further component parts, this would be an irreducible unitary representation.[15]

This led Kobayashi and Maskawa (1973) to analyse the irreducible unitary representations of $SU(2) \times U(1)$. They found that no plausible representations with only four quark fields allowed for the CP and T violation that had been discovered in the previous decade. This led them to propose the six-quark field model, which naturally incorporates CP and T violation: it was this six-quark model that was quickly incorporated into the Standard Model. Kobayashi and Maskawa shared half of the 2008 Nobel Prize in physics for their discovery.

The electroweak contribution to the Lagrangian of the Standard Model does not commute with the 'canonical form' of the antiunitary time reversal operator T, in the sense discussed in Section 3.4.3. I will not go into the details of that Lagrangian here, which are not needed for the level of generality of my discussion.[16] What I would like to discuss is the consequence that, in the electroweak theory, since T does not commute with the Lagrangian, it does not commute with the Hamiltonian generator H of the unitary time translations $t \mapsto U_t = e^{-itH}$. This implies[17] that T is not a representation of

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[15] This approach, made famous by Wigner (1939) in the analysis of the Lorentz group, had recently been successfully applied by Gell-Mann (1961) to the group $SU(3)$ in the study of strong interactions.

[16] Introductions can be found in Bigi and Sanda (2009, Chapter 8) and Huang (1982, §6.2).

[17] It is easiest to argue for the contrapositive statement: if $U_{-t} = e^{itH} = Te^{-itH}T^{-1}$, then $e^{itH} = e^{T(-itH)T^{-1}} = e^{itTHT^{-1}}$, where the last equality applies the fact that T is antiunitary and thus conjugates complex constants. Since the generator of a continuous one-parameter unitary

time reversal, in that $TU_tT^{-1} \neq U_{-t}$. So the representation of time translations $t \mapsto U_t = e^{itH}$ in electroweak theory cannot be extended to include time reversal. For at the group level, time reversal satisfies $\tau t \tau^{-1} = -t$, and so any representation of time reversal $\tau \mapsto T$ must satisfy $TU_tT^{-1} = U_{-t}$. This is the sense in which the electroweak interactions violate time reversal symmetry.

Thus, in electroweak theory, time reversal symmetry is violated. Indeed, this same Lagrangian requires the violation of many other discrete symmetries as well. But, before we get too carried away, let me return to the curious puzzle in this story: the representation of time translations that we are considering did not appear in explicit form until after the publication of Kobayashi and Maskawa (1973), long after the discovery of CP violation in 1964. So, the discovery of symmetry violation was not a simple matter of checking whether the dynamics fails to admit a representation of time reversal invariance, as I have sketched above, for the dynamics of electroweak theory had not yet been discovered!

The solution to this puzzle lies in an extraordinary feature of the experimental evidence for symmetry violation. That evidence turns out to be so general that, in almost any reasonable theory that one could produce, the dynamics would have to violate time reversal symmetry, and also all those other symmetries I just alluded to. This is of course what led Kobayashi and Maskawa to search for models that violate time reversal symmetry. I will argue that there is a further lesson from this: that our evidence for time reversal symmetry violation is much more general than the Standard Model alone. These instances of symmetry violation are here to stay, and so they provide remarkably robust evidence for an arrow of time.

This result may still seem a little mysterious: how could one show in such a robust way that the 'reversal of time translations' is not a dynamical symmetry? Understanding this requires a more careful look into the discoveries themselves, which turn out to have some interesting philosophy underpinning them.

7.3.2 *The Discovery of CP and T Violation*

Recall that, as recounted in Section 1.5, the discovery of parity violation in 1956 by Chien-Shiung Wu and her collaborators led to the identification of a new class of particle called a K meson or *kaon*. The kaon of their experiment was positively charged. In contrast, the kaon that led to the discovery of CP violation is neutral, called a *neutral kaon*.

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group is unique, it follows that $THT^{-1} = H$. Equivalently, if $THT^{-1} \neq H$, then $TU_tT^{-1} \neq U_{-t}$, and so T is not a representation of time reversal.

At the time, most models of neutral kaons required both CP invariance and T invariance, such as the model of Weinberg (1958). Indeed, CP symmetry is equivalent to T symmetry for a very general class of relativistic quantum theories, by a collection of results known as the CPT theorem, which I will discuss in Chapter 8. So, evidence of CP violation is also good evidence of T violation. When the evidence for T violation relies on the detection of CP violation together with CPT symmetry, it is called *indirect*. In contrast, evidence for T violation that does not rely on CPT symmetry is called *direct*.

The first indirect evidence for T violation was discovered for what is called the 'long-lived' neutral kaon state K_L. At the time, these states were best known for their decay into three pions, $K_L \rightarrow \pi^0\pi^+\pi^-$. The neutral pion π^0 does not ionise in a spark chamber, and so it is invisible in such a device, but its trajectory can be calculated from the trajectories of the other two charged pions, using the conservation of momentum.

Although the dynamics for these decay events was not yet known, practitioners could still make inferences about the symmetries of whatever the dynamics might turn out to be. These inferences made use of a symmetry principle, which is commonly attributed to Pierre Curie (1894): that every asymmetric effect must originate from an asymmetric cause.[18] More specifically, the form of this principle that is appropriate for dynamical theories was formulated by Belot (2003, §4.2) and Earman (2004): if a final state has an asymmetry, and the initial state lacks that asymmetry, then its only possible origin is in the dynamics.

In fact, this principle does not need to be postulated as an assumption about quantum theory. It is a mathematical fact, which can be formulated for any unitary symmetry and for any unitary representation of time translations.[19] Let me express it here in a form that is particularly relevant for its use in particle physics, using a unitary S-matrix on a Hilbert space.

Recall that an *S-matrix* is a unitary operator, which can be viewed as the 'infinite-time limit' of a unitary group $U_t = e^{-itH}$. What is particularly relevant for our purposes is that if an S-matrix fails to commute with the CP, meaning that $[CP, S] \neq 0$, then the unitary dynamics is CP violating.[20] The possible decay events, like $K_L \rightarrow \pi^0\pi^+\pi^-$, called *decay*

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[18] See Belot (2003); Chalmers (1970); Earman (2004); Ismael (1997), and the volume edited by Brading and Castellani (2003), for some classic discussions of Curie's principle; my own view on these matters can be found in Roberts (2013a, 2015a); see also the responses by Castellani and Ismael (2016); Kinney (2021); Norton (2016s).

[19] See Roberts (2015b, p.10, Facts 1 and 2) for two versions and their proofs.

[20] This is the case for most other unitary symmetries as well. For example, suppose the Hamiltonian can be expressed in the form $H = H_0 + V$, with H_0 the free Hamiltonian, which satisfies $[CP, H_0] = 0$. Then, writing the S-matrix as a Dyson series $S = \mathcal{T}\exp\left(\int_{\mathbb{R}} V(t)dt\right)$ for some time-ordering operator $\mathcal{T}$, one finds that $[H, CP] = 0$ implies that $[S, CP] = 0$ by the linearity of the integral. Equivalently, if $[S, CP] \neq 0$, then $[H, CP] \neq 0$, which means that the unitary dynamics is CP violating.

modes, are associated with non-zero amplitudes of an S-matrix, written $\langle \pi^0\pi^+\pi^-, SK_L\rangle \neq 0$. Adopting this general formalism, which applies to an extremely broad class of quantum theories, one can now observe the following fact.

> **Proposition 7.1** *Let* $CP : \mathcal{H} \to \mathcal{H}$ *be any unitary on a Hilbert space (the 'symmetry'), and let* S *be any bijection (the 'S-matrix') such that* $\langle \psi^{out}, S\psi^{in}\rangle \neq 0$. *Suppose either of the following conditions hold:*
>
> 1. *(in but not out)* $CP\psi^{in} = \psi^{in}$ *but* $CP\psi^{out} = -\psi^{out}$*; or*
> 2. *(out but not in)* $CP\psi^{out} = \psi^{out}$ *but* $CP\psi^{in} = -\psi^{in}$.
>
> *Then we have CP violation, in the sense that,* $(CP)S(CP)^{-1} \neq S$.

Proof We argue the contrapositive: let $(CP)S = S(CP)$. We will show that $\langle \psi^{out}, S\psi^{in}\rangle = 0$. Since CP is unitary, $\langle \psi^{out}, S\psi^{in}\rangle = \langle (CP)\psi^{out}, (CP)S\psi^{in}\rangle = \langle (CP)\psi^{out}, S(CP)\psi^{in}\rangle$. So, if either the 'in but not out' or the 'out but not in' conditions hold, then,

$$\langle \psi^{out}, S\psi^{in}\rangle = \langle (CP)\psi^{out}, S(CP)\psi^{in}\rangle = -\langle \psi^{out}, S\psi^{in}\rangle, \tag{7.3}$$

which implies that $\langle \psi^{out}, S\psi^{in}\rangle = 0$. ■

This means that if a unitary symmetry like CP reverses the sign of an ingoing state but not the outgoing state, or vice versa, then – regardless of what the dynamics describing this process turns out to be – *the dynamics must be CP violating*. Let me illustrate how this was used to show that neutral kaons are CP violating and thus (indirectly) that they are T violating.

When the neutral kaon decays into three pions, $K_L \to \pi^0\pi^+\pi^-$, both the ingoing and outgoing states can be viewed as reversing sign under CP:

$$(CP)K_L = -K_L \qquad\qquad (CP)\pi^0\pi^+\pi^- = -\pi^0\pi^+\pi^-. \tag{7.4}$$

This decay is compatible with CP symmetry, because Proposition 7.1 does not have any implications for such ingoing and outgoing states. This was the most well-known decay mode associated with the long-lived neutral kaon prior to 1964.

However, a *two*-pion state $\pi^+\pi^-$ does not reverse sign under CP but is rather left unchanged:

$$(CP)\pi^+\pi^- = \pi^+\pi^-. \tag{7.5}$$

This means that, if one could be sure that a neutral kaon K_L can decay into a two pion state – even once! – then the 'in but not out' criterion would be satisfied, and then Proposition 7.1 guarantees CP violation.

This is exactly what Cronin and Fitch discovered in 1964.[21] In fact, their original intention was to try to show that, to a high degree of accuracy, no two-pion decay events could be found, which would have helped to *confirm* CP violation. As fate would have it, they instead found a small but unmistakable number of CP violating instances in which a neutral kaon decayed into two pions, in about one of every 500 decay events. By the CPT theorem, this also provided the first (indirect) evidence for T violation.

More recently, *direct* evidence for T violation has been produced as well, without appeal to CPT symmetry. This turns out to make use of a different symmetry principle, which is less commonly discussed by philosophers. Curie's principle does not apply to time reversal, or any antiunitary symmetry, due to a subtlety in the proof of Proposition 7.1.[22] However, one can still apply a version of the 'principle of detailed balance', which I call 'Kabir's principle' in this context after its proposal by Kabir (1968, 1970). Here we use the fact that, in just the same way that time reversal symmetry is expressed by $TU_tT^{-1} = U_{-t} = U_t^*$ with $U_t = e^{-itH}$, so also, time reversal symmetry of an S-matrix is expressed by $TST^{-1} = S^*$. Let me first state the principle and then interpret it.

Proposition 7.2 *Let $S : \mathcal{H} \to \mathcal{H}$ be a unitary operator on a Hilbert space (the 'S-matrix'), and let T be an antiunitary bijection ('time reversal'). If there exist any $\psi^{in}, \psi^{out} \in \mathcal{H}$ such that*

$$\langle \psi^{out}, S\psi^{in} \rangle \neq \langle T\psi^{in}, ST\psi^{out} \rangle, \tag{7.6}$$

then S exhibits time reversal violation, $TST^{-1} \neq S^*$.

Proof We prove the contrapositive: let $TST^{-1} = S^*$, and thus $TS = S^*T$. Since T is antiunitary, $\langle \psi^{out}, S\psi^{in} \rangle = \langle T\psi^{out}, TS\psi^{in} \rangle^* = \langle TS\psi^{in}, T\psi^{out} \rangle$. Therefore,

$$\langle \psi^{out}, S\psi^{in} \rangle = \langle TS\psi^{in}, T\psi^{out} \rangle = \langle S^*T\psi^{in}, T\psi^{out} \rangle = \langle T\psi^{in}, ST\psi^{out} \rangle. \tag{7.7}$$

Equivalently, if $\langle \psi^{out}, S\psi^{in} \rangle \neq \langle T\psi^{in}, ST\psi^{out} \rangle$, then $TST^{-1} \neq S^*$. ■

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[21] See Christenson et al. (1964); Cronin and Greenwood (1982).

[22] This leads to an argument that, in spite of its popularity, Curie's principle is false (Roberts 2013a, 2015b).

The assumption of Proposition 7.2, Kabir's principle, is that the amplitude determining the probability of a certain decay event $\psi^{in} \rightarrow \psi^{out}$ is different from the amplitude for the time reverse of that decay, $T\psi^{out} \rightarrow T\psi^{in}$. If these two amplitudes are different, then Proposition 7.2 implies that, whatever the dynamics turns out to be, it must be T violating. No appeal to CPT symmetry is needed for this inference, and so such a difference in amplitudes provides direct evidence for T violation.

This kind of evidence has recently been found. Oscillating kaon-antikaon states turned out to be a fruitful place to test for this: a kaon state K^0 is known to decay into an 'antikaon' state $\bar{K}^0$, which can then decay back again to K^0, creating an oscillation. This is a particularly convenient phenomenon for the application of Kabir's principle, since time reversal T can be viewed as leaving both of these states unchanged: $TK^0 = K^0$ and $T\bar{K}^0 = \bar{K}^0$. So, by checking whether the amplitude for the decay $K^0 \rightarrow \bar{K}^0$ is different from its time reverse $\bar{K}^0 \rightarrow K^0$, we can check whether this system is T violating. Indeed it is: direct T violation of this kind was discovered by Angelopoulos et al. (1998). Related arguments later showed that T violation also occurs in the B meson sector as well (Lees et al. 2012). Some preliminary recent evidence even suggests a small amount of CP and T violation in the lepton sector through neutrino oscillations (T2K Collaboration 2020). Thus, the experimental evidence for T violation in the Standard Model is becoming ever more secure.

I have formulated the symmetry principles above in the context of S-matrix theory, because this formalism makes the most direct contact with the experimental evidence. However, both have recently been shown to be much more general than that, relying on only a tiny fragment of the structure of quantum theory. In particular, Ashtekar (2015) has shown that both are facts about any dynamical theory that can be formulated in *generalised mechanics*, a framework allowing one to capture a wide variety of theories, from classical and quantum mechanics to gravitation. I will not go into the details of that formalism here; my point is just to say that, from many different perspectives, the experimental evidence for T violation is extremely robust.

In summary, the experimental evidence for T violation is here to stay. The evidence was produced using robust symmetry principles that apply to a wide variety of quantum theories, and even beyond quantum theory. And, when formulated as a renormalisable quantum field theory, the resulting electroweak theory captures this phenomenon in a way that makes missing information from higher energy scales irrelevant.

Let me finally note that, as a result of all this experimental evidence for discrete symmetry violation over the last 60 years, the Standard Model does not admit a representation of any of the following discrete symmetries:[23]

$$C, P, T, CP, CT, \text{ or } PT.$$

This leaves only one combination of these transformations that can be represented in the Standard Model: the combination of all of them, CPT. I will return to the discussion of this transformation in Chapter 8.

7.4 The Price Critique

In this chapter, I have argued that time reversal symmetry violation can provide evidence for an arrow of time itself and that the evidence from electroweak theory for such an arrow is extraordinarily strong. In this section, I would like to respond to an important critique of this perspective due to Huw Price (1996, 2011).

Nature is under no obligation to provide us with a physical asymmetry in the structure of time itself. Our world might have had the character that virtually all temporal asymmetries were only apparent. One would just need to explain how this great illusion could happen in a way that is compatible with our experience – and with the results of modern physics. Some of that is explained by our discussion of 'arrows that misfire' in Chapter 5 and Chapter 6. But, another important part of this undertaking has been carried out by Huw Price (1996). Price calls his view the "perspectival view" of time's arrow: that all facts about time asymmetry depend crucially on the standpoint of an observer and thus do not provide evidence for an arrow of time itself.[24] This is a remarkable analysis, and I agree with Price's efforts to dispose of 'misfiring arrows', many of which align with my analysis in previous chapters.

However, I part ways with Price regarding electroweak interactions. About this topic, he writes:

> It is true, of course, that the T-asymmetry of the neutral kaon could provide the basis for a universal convention for *labelling* lightcones as 'past' and 'future'. But

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[23] Both C and P were violated in the original parity violating experiments of Chien-Shiung Wu (Garwin, Lederman, and Weinrich 1957; Wu, Wang, et al. 2015). Given CPT symmetry, this implies that both PT and CT are violated as well. The Cronin and Fitch experiments then settled that both CP and T are violated as well.

[24] Rovelli (2017) similarly identifies a 'standpoint' with the selection of a statistical mechanical coarse-graining, in a related argument that the arrow of time is perspectival; see Section 5.3.2.

> this no more requires that time even be anisotropic – let alone objectively oriented – than does our universal signpost for space. (Price 2011, p.294)

This ‘universal signpost’ will be discussed in detail in the next section. To support his conclusion, Price raises three objections to viewing time asymmetry in a dynamical theory as evidence for a physical arrow of time. They are: that it is independent of the question of time asymmetry; that even if it were not, the time asymmetry can always be dissolved due to a ‘gauge’ symmetry in its interpretation; and that even without these two objections, existing evidence provides too weak a signal for an arrow of time.

Each of these concerns is important, and I will discuss each of them in turn. As in previous chapters, the response that I will offer draws essentially on the Representation View of Section 2.3. My conclusion, *pace* Price, is that T violation really does imply that time itself is asymmetric – and even ‘objectively oriented’, if one likes – in a way that avoids his objections.

7.4.1 The Independence Objection

Price’s table analogy provides an indication of his first concern: that the behaviour of matter-energy seems to be independent of the structure of spacetime. If this were strictly true, then an asymmetry of the former would be no evidence for an asymmetry of the latter. Price later makes this concern explicit using a different analogy:

> Suppose everything in the universe were to vanish, except a single giant signpost, pointing forlornly to a particular corner of the sky. Or suppose that a universe had always been like this. This spatial asymmetry would not require that space itself be anisotropic, presumably, or that the direction in question be distinguished by anything other than the fact that it happened to be the orientation of the signpost. Similarly in the case of time. The contents of time – that is, the arrangement of physical stuff – might be temporally asymmetric, without time itself having any asymmetry. Accordingly, we need to be cautious in making inferences from observed temporal asymmetries to the anisotropy of time itself. (Price 2011, p.292)

Price is right that an asymmetry in space itself is conceptually different from an asymmetry of a signpost; similarly, an asymmetry in time itself is conceptually different from an asymmetry in the dynamics of its contents. But, like the table analogy, Price’s signpost argument only works if those contents are independent of the structure of space and time. Here I disagree.

The very meaning of a dynamical theory requires that it be intimately connected to the structure of time itself: otherwise, it does not deserve the name ‘dynamical’. In order to capture this fact, we must postulate the existence of a representation, or a homomorphism from the abstract group

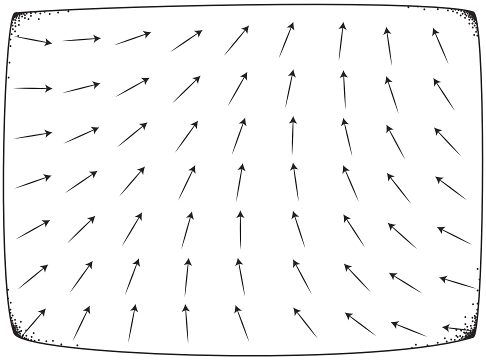

Figure 7.3 A collection of signposts at every spacetime point: we assume that every rigid spatial translation produces a possible collection of signposts as well.

of time translations to the symmetries of the state space that describes the "physical stuff". This is what I called the Representation View in Section 2.3. A better metaphor for this view was given in Section 5.1, of learning about a table from its shadow on the floor. In a similar way, the asymmetries of time are projected down onto a dynamical theory by a representation. This is the fundamental problem with Price's signpost analogy: once the link is adequately made between the symmetries of spacetime and the symmetries of a dynamical theory, Price's objection about their independence is dissolved.

A second problem with Price's signpost example is that it suggests that T violation in electroweak theory is much weaker than it actually is. Price describes a system with a single instance of an asymmetric signpost. But, the evidence from T violation is much stronger and more striking than that because it is much more pervasive. We should rather imagine encountering a great set of signposts at every point in spacetime, as in Figure 7.3 – and also knowing that a rigid translation of the entire set by a spatial translation in any direction will produce another possible set of signposts – but noting that 'spatial reversal' produces a set of signposts that are physically impossible. In other words, we should imagine a system in which there is no way to represent the reflection of spatial transformations $s \mapsto -s$ on the state space of the system. According to the Representation View, the structure of spatial translations on state space is a 'homomorphic copy' or 'shadow' of the structure of space itself. As a result, this discovery would indeed provide evidence that the structure of space has an asymmetry.

The discovery of T violation in electroweak theory is much more like this than it is like Price's solitary signpost. The dynamics of electroweak theory provides a representation of time translations, describing all the possible time evolutions on the state space of electroweak theory. The discovery of T violation shows that on that state space, it is impossible to represent the reversal of those time translations, $t \mapsto -t$. So, since the structure of time translations on state space is a homomorphic copy of the structure of time itself, we have evidence that the time translations lack this symmetry. Thus, although Price's concern is well-motivated, it is avoided once we adopt the Representation View and recognise the full strength of the discovery of T violation.

7.4.2 The 'Gauge' Objection

Price's second objection concerns whether T violation in a dynamical theory allows the freedom to adopt either of the two temporal orientations on a (temporally orientable) spacetime. Price expresses this in a critique of Maudlin (2002a, 2007), who takes T violation to be evidence for a temporal orientation. Price responds:[25]

> What would it mean to say the laws exhibited a specific temporal asymmetry, unless one could orient a choice of temporal coordinate consistently across spacetime, in the manner guaranteed by orientability? This isn't what Maudlin has in mind, however. Orientability is a much weaker condition than existence of an objective distinction between earlier and later – it doesn't imply even that time is anisotropic, much less that it is objectively directed. . . . [W]e should read Maudlin as claiming that the T-violation exhibited by the neutral kaon 'requires' such an orientation. But . . . this is simply not true: a lawlike time asymmetry does not even require temporal anisotropy, let alone the true directionality that Maudlin is after. (Price 2011, p.294)

Price is right that, if T violation in electroweak theory only implied that spacetime admits a smooth timelike vector field, which is to say a temporal orientation, then by definition this would mean that spacetime is *temporally orientable*. Such a vector field does not by itself imply that time has an asymmetry. It does, however, mean that we can choose one temporal orientation and call it 'the future' if we wish. But, says Price, this would only establish a descriptive redundancy, analogous to the descriptive redundancy of gauge theory, which always allows us to call the other temporal orientation 'the future' as well.

However, T violation in electroweak theory again implies a stronger sense of time asymmetry than Price suggests here. T violation establishes that

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[25] Price (2011, p.293) gives a similar analysis of a toy time asymmetric model due to Horwich, as does Farr (2020, §2.2.2) for a toy model using monotonic functions.

time reversal $t \mapsto -t$ is not an automorphism of the time translations t in spacetime. So, the local structure of spacetime cannot be Minkowski spacetime alone! Nor can it be any temporally orientable spacetime whose time translations admit a time reversal automorphism. This can be captured in the language of orientations by the statement that, in order to accurately describe the symmetries of spacetime, we must choose *some* temporal orientation, although it may not matter which one we choose. One might refer to such a description as a *temporally oriented spacetime*.

In a temporally oriented spacetime, Price is right to say that there remains a gauge redundancy as to which temporal orientation we choose. For example, if we say that a decay event like $K^0 \to \bar{K}^0$ is future-directed with respect to the temporal orientation $[\xi^a]$, then we can always say that the opposite decay event $\bar{K}^0 \to K^0$ is future-directed with respect to $[-\xi^a]$. This really is a descriptive redundancy in the theory. However, it is not the same as saying that a spacetime is 'merely orientable'. The symmetries of spacetime are still restricted by an orientation, although it may not matter which.

To see this, let me write $\text{Aut}(M, \eta_{ab})$ to refer to the automorphisms of Minkowski spacetime, called the *complete Poincaré group*. That group is *not* isomorphic to the symmetries of Minkowski spacetime with an orientation, $\text{Aut}(M, \eta_{ab}, [\xi^a])$, which is sometimes called the *orthochronous* Poincaré group and which does *not* include time reversal. However, both temporal orientations $[\xi^a]$ and $[-\xi^a]$ equally give rise to the same orthochronous Poincaré group, in the sense that $\text{Aut}(M, g_{ab}, [\xi^a])$ is isomorphic to $\text{Aut}(M, g_{ab}, [-\xi^a])$. So, both equally capture the temporal asymmetry implied by T violation. It is just an awkwardness of representing asymmetries using a temporal orientation that we must choose one. But, this is avoided when we shift focus to the symmetries of time translations directly, as I have done in the analysis of T violation above.

So, there is a sense in which the comments at the outset of this chapter by Wald, Earman, and Maudlin are correct: the time asymmetry of electroweak theory implies that the spacetime symmetries are characterised by a temporal orientation. And, Price is also right, that we have equal reason to choose either one. But, Price's valid point is no objection to the fact that electroweak theory provides evidence that time itself has an asymmetry, in the sense that I have argued for in the earlier sections of this chapter.

7.4.3 The Weak Signal Objection

Price's final objection to the claim that T violation establishes an asymmetry of time itself is that it is too rare to matter:

> It is true that there appears to be one exception to this general principle [of temporal symmetry] . . . the case of the decay of the neutral kaon. Even here the departure from perfect symmetry is tiny, however, and the puzzling character of the existence of this tiny exception serves to highlight the intuitive appeal of the prevailing rule. To a very large extent, then, the laws of physics seem to be blind to the direction of time – they satisfy T-symmetry, as we may say. (Price 1996, p.116)

I agree with Price that the size of time reversal symmetry violation is an important issue for modern physics. For example, it is essential in determining to what extent symmetry violation is responsible for the cosmological asymmetry, and for the baryon asymmetry, if one adopts the Sakharov (1967) conditions discussed in Section 5.4.3. However, one should not conflate concerns about the relationship between different asymmetries with the question of whether time itself has an asymmetry. Although this 'weak signal' objection is relevant to the former, it does not provide any argument against the latter.

It may help to recall what is meant by 'size' in the context of the neutral kaon experiments that Price refers to. Recall from our discussion in Section 7.3.2 that the first evidence for T violation was the decay of the long-lived neutral kaon into two pions, $K_L \to \pi^+\pi^-$. Since K_L and $\pi^+\pi^-$ transform in opposite ways under CP, the discovery of this decay event implies, by our version of Curie's Principle (Proposition 7.1), that the dynamics must be CP violating and therefore by the CPT theorem also T violating. This remarkable decay only occurs in roughly one out of every 500 decays of the neutral kaon. In the modern gauge theory of electroweak interactions, this turns out to be near the maximum amount of T violation that is compatible with the theory (cf. Witten 2018). But it is still a rare occurrence.

The fact that an occurrence is rare is no reason to ignore its implications. T violation implies that a representation of time reversal symmetry does not exist: this rare event is just a witness to the conclusion! Our formulation of Curie's principle (Proposition 7.1) shows that if the decay $K_L \to \pi^+\pi^-$ occurs even once, then time reversal symmetry is violated. Similarly, Kabir's Principle (Proposition 7.2) shows directly that if the probabilities in kaon oscillation differ by even the smallest amount, then time reversal symmetry is violated. Just as one smoking gun out of 500 is still enough to prove a murder, so these rare kaon decays are enough to prove that in our world, time is not symmetric. The time asymmetric dynamics applies to *all* experimental runs, despite the fact that only a small number of them are used to prove the asymmetry itself.

7.5 Summary

At last, we have arrived at an asymmetry of 'time itself'. In this chapter I have argued that the time asymmetry established by electroweak theory really does deserve to be called 'the arrow of time'. Perhaps one might like to refer to it more conservatively, as 'one arrow' of time. However, the other asymmetries that are usually called 'arrows' are currently all misfires, as I have argued in Chapters 5 and 6. In this sense, the arrow of time arising from electroweak theory really is unique.

Some subtle argumentation has been needed to establish this. My first step was to move to a more structural perspective, viewing t as referring to time translations, rather than to a time coordinate. The Representation View then provides the connection between the structure of time and the trajectories of a dynamical theory: the symmetries of time are 'projected' down onto the state space of a dynamical theory by a representation, just as the symmetries of a table are projected by its shadow down onto the floor. As a result, T violation on the state space of electroweak theory teaches us not only about state space but about the structure of time itself.

Of course, there were a number of subtleties to this story. The evidence for an arrow of time drew in part on the robustness of electroweak theory as a renormalisable quantum field theory as well as on the generality of the experimental evidence for T violation. And, the sense in which 'time itself' has an arrow is not exactly in the sense of a temporal orientation but rather as an asymmetry of time translations. However, I hope now to have shown that, with all these subtleties considered, the arrow of time is secure.

My discussion in this chapter has concerned the 'canonical form' of the time reversal operator, as distinct from other time reversing transformations like CPT. However, there is a remaining doubt about time asymmetry, that there is a sense in which the transformation CPT might be what 'plays the role' of time reversal in relativistic quantum field theory. So, since all such theories are thought to be CPT symmetric, perhaps there is still some sense in which time has a symmetry too. Making sense of this idea requires a careful examination of the meaning of CPT symmetry, which is the subject of the final chapter.