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SIMULTANEOUS EQUATIONS WITH INCOMPLETE PANELS

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This paper compares the performance of several single and system estimators of a two equation simultaneous model with unbalanced panel data. The Monte Carlo design varies the degree of unbalancedness in the data and the variance components ratio due to the individual effects. One useful result for applied researchers is that the feasible error components 2SLS and 3SLS procedures based on simple ANOVA type estimators of the variance components perform well with incomplete panels and are recommended in practice.

1. INTRODUCTION

The error component specification is popular in modeling the disturbance terms of panel data models, and one reason for its popularity is its parsimonious specification of heterogeneity across the cross-sectional units. As a result, simple estimators using 2SLS and 3SLS subroutines can be used to handle the endogeneity of the regressors (see Baltagi, 1995). Incomplete panels complicate the computations and the analytical derivations in that the panel data matrices now have missing observations in them. One purpose of this paper is to study how the incompleteness of the panel affects the small sample performance of various popular estimators used in the simultaneous equation error component literature. To achieve this purpose, Monte Carlo experiments are performed for a two equation simultaneous model with one-way error component disturbances based on randomly missing panel data. Previous Monte Carlo studies on incomplete panels are limited to a simple regression with exogenous regressors (see Wansbeek and Kapteyn, 1989; Mátyás and Lovrics, 1991; Baltagi and Chang, 1994). On the other hand, Monte Carlo studies for the simultaneous equations model with complete panel data include Baltagi (1984) and Mátyás and Lovrics (1990).

Our Monte Carlo study is the first to address the joint problem of simultaneity *and* incompleteness of the panel. Some of the important questions we ask are the following: (1) What results generalize from the complete to the incomplete panel

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simultaneous equations case? For example, can we distinguish among the various asymptotically efficient estimators of the structural coefficients according to small sample criteria? Will better estimates of the structural variance components necessarily imply better estimates of the structural parameters? (2) How does the degree of unbalancedness in the data affect the variance components estimates and in turn the structural coefficients estimates? (3) How much loss in efficiency is there in estimating the simultaneous equation model from a subbalanced panel relative to estimating it from the incomplete panel?

2. THE MODEL

Following Baltagi (1984), we consider the following two equations simultaneous model:

$$\Gamma \underset{\sim}{y}_{it} + \Lambda \underset{\sim}{x}_{it} = \underset{\sim}{u}_{it} \qquad i = 1, \ldots, N \qquad t = 1, \ldots, T_i, \tag{1}$$

where Γ is a 2×2 matrix of coefficients of current endogenous variables and Λ is a 2×4 matrix of coefficients of predetermined variables. Here $\underset{\sim}{y}_{it}$, $\underset{\sim}{x}_{it}$, and $\underset{\sim}{u}_{it}$ are column vectors of dimension 2, 4, and 2, respectively. The subscripts i and t denote the particular ith cross section and tth time period. The panel data are incomplete in the sense that there are N individuals observed over varying time period length (T_i for $i = 1, \ldots, N$). The error component structure for the jth equation is given by $u_{itj} = \mu_{ij} + \nu_{itj}$, where μ_{ij} denotes the ith cross-section effect and ν_{itj} the remainder error for the jth structural equation. Let $\underset{\sim}{\mu}_i' = (\mu_{i1}, \mu_{i2})$ and $\underset{\sim}{\nu}_{it}' = (\nu_{it1}, \nu_{it2})$; then $\underset{\sim}{u}_{it} = \underset{\sim}{\mu}_i + \underset{\sim}{\nu}_{it}$, where $\underset{\sim}{\mu}_i$ and $\underset{\sim}{\nu}_{it}$ are mutually independent normal random variables with zero means and covariance matrices $\Sigma_\mu = [\sigma_{\mu k \ell}]$ and $\Sigma_\nu = [\sigma_{\nu k \ell}]$ for $k, \ell = 1, 2$. Throughout the experiment

$$\Gamma = \begin{bmatrix} 1 & .5 \\ 4 & 1 \end{bmatrix} \quad \text{and} \quad \Lambda = \begin{bmatrix} 2 & -1.5 & 0 & 0 \\ 0 & 0 & 3 & -1.8 \end{bmatrix}.$$

We construct the predetermined matrix X such that $X'X = I_4$. This can be done using the Gram–Schmidt orthogonalization procedure. This orthogonalization was suggested by Rhodes and Westbrook (1981) because it precludes multicollinearity and enhances the computational accuracy of the estimation procedure. For each fixed N, we followed the suggestion given by Swallow and Searle (1978) of selecting T-patterns that range from slightly to badly unbalanced. Let $5(15)$ denote the T-pattern with 15 individuals, each observed over five periods. For $N = 30$, the following unbalanced T-patterns are used: $P_1 = 5(15), 9(15)$; $P_2 = 5(10), 7(10), 9(10)$; $P_3 = 3(6), 5(6), 7(6), 9(6), 11(6)$; $P_4 = 3(9), 5(6), 9(6), 11(9)$; $P_5 = 3(24), 23(6)$; $P_6 = 2(15), 12(15)$. The total number of observations is held fixed at 210 for all these unbalanced patterns. A measure of unbalancedness as given by Ahrens and Pincus (1981) is defined as $\omega = N/\overline{T} \sum (1/T_i)$, where $\overline{T} = \sum T_i / N$, with $0 < \omega \leq 1$. Note that ω takes the value of one when the pattern is balanced but it takes smaller values as the pattern gets more severely unbalanced.

Denoting $\omega(i)$ as the measure of unbalancedness for the pattern P_i and denoting ω as the vector of $\omega(i)$'s, then $\omega = (0.918, 0.841, 0.813, 0.754, 0.519, 0.490)$. That is, the degree of unbalancedness increases as the subscript of P gets large. One thousand replications were performed for every model specification considered. Models considered are distinguished from each other by the values of the variance components and the pattern of unbalancedness.[1] The variance-covariance matrix between the jth and ℓth reduced form equations (see Baltagi, 1984) can be written as

$$\Sigma_{j\ell}^{*} = \sigma_{j\ell}^{*}[\rho_{j\ell}^{*}\,\mathrm{diag}(J_{T_i}) + (1-\rho_{j\ell}^{*})I_n] \qquad j,\ell = 1,2, \tag{2}$$

where $\sigma_{j\ell}^{*} = \sigma_{\mu j\ell}^{*} + \sigma_{\nu j\ell}^{*}$ and $\rho_{j\ell}^{*} = \sigma_{\mu j\ell}^{*}/\sigma_{j\ell}^{*}$. Here I_n is an identity matrix of dimension n, and J_{T_i} is a matrix of ones of dimension T_i. Throughout the experiments, $\sigma_{11}^{*} = \sigma_{22}^{*} = 20$ and $\sigma_{12}^{*} = 10$. However, $\rho_{j\ell}^{*}$ was varied over the set $(0, 0.2, 0.5, 0.8)$ such that Σ_{μ}^{*} and Σ_{ν}^{*} were all positive definite.[2] In total, 42 experiments were performed corresponding to seven variance-components patterns times six unbalanced patterns.

3. ESTIMATION METHODS

The jth structural equation can be written in vector form as

$$y_j = Y_j\gamma_j + X_j\lambda_j + u_j = Z_j\delta_j + u_j \qquad j = 1,2, \tag{3}$$

where y_j is $n \times 1$, Y_j is $n \times 1$, X_j is $n \times 2$, and $n = \sum_{i=1}^{N} T_i$. Also,

$$u_j = Z_\mu \mu_j + \nu_j \qquad j = 1,2, \tag{4}$$

where $Z_\mu = \mathrm{diag}(\iota_{T_i})$, ι_{T_i} is a vector of ones of dimension T_i, $\mu_j = (\mu_{j1}, \mu_{j2},\ldots,\mu_{jN})'$, and $\nu_j = (\nu_{j11},\ldots,\nu_{j1T_1},\ldots,\nu_{jN1},\ldots,\nu_{jNT_N})'$ for $j = 1,2$. The variance-covariance matrix between the jth and ℓth structural equations is given by

$$\Sigma_{j\ell} = E(u_j u_\ell') = \sigma_{\nu j\ell}\,\mathrm{diag}[E_{T_i}] + \mathrm{diag}[w_{ij\ell}\bar{J}_{T_i}], \tag{5}$$

where $E_{T_i} = I_{T_i} - \bar{J}_{T_i}$, with $\bar{J}_{T_i} = J_{T_i}/T_i$, and $w_{ij\ell} = T_i\sigma_{\mu j\ell} + \sigma_{\nu j\ell}$. Therefore,

$$\sqrt{\sigma_{\nu j\ell}}\,\Sigma_{j\ell}^{-1/2} = \mathrm{diag}(E_{T_i}) + \mathrm{diag}\left[\sqrt{(\sigma_{\nu j\ell}/w_{ij\ell})}\,\bar{J}_{T_i}\right] \tag{6}$$

(see Baltagi, 1995). The typical element of $\sqrt{\sigma_{\nu j\ell}}\,\Sigma_{j\ell}^{-1/2}y_j$ is $y_{jit} - \theta_{ij\ell}\bar{y}_{ji.}$, where $\bar{y}_{ji.} = \sum_{t=1}^{T_i} y_{jit}/T_i$ and $\theta_{ij\ell} = 1 - \sqrt{\sigma_{\nu j\ell}/w_{ij\ell}}$. We consider the following single equation estimation methods.

(a) Two stage least squares (2SLS). This completely ignores the error component structure of the disturbances: $\hat{\hat{\delta}}_{j,\mathrm{2SLS}} = (Z_j'P_X Z_j)^{-1}Z_j'P_X y_j$ for $j = 1,2$ where $P_X = X(X'X)^{-1}X'$.

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(b) Within two stage least squares (W2SLS). This assumes that the μ_i's are fixed parameters to be estimated: $\hat{\delta}_{j,\mathrm{W2SLS}} = [Z_j' QX(X'QX)^{-1}X'QZ_j]^{-1} \times Z_j' QX(X'QX)^{-1}X'Qy_j$, where $Q = \mathrm{diag}(E_{T_i})$.

(c) Error components two stage least squares (EC2SLS). This accounts for the random error component structure on the disturbances. Premultiplying the jth structural equation by $\sqrt{\sigma_{\nu jj}}\Sigma_{jj}^{-1/2}$ given in (6), one gets

$$y_j^* = Z_j^* \delta_j^* + u_j^* \tag{7}$$

with $y_j^* = \sqrt{\sigma_{\nu jj}}\Sigma_{jj}^{-1/2} y_j$, $Z_j^* = \sqrt{\sigma_{\nu jj}}\Sigma_{jj}^{-1/2} Z_j$ and $u_j^* = \sqrt{\sigma_{\nu jj}}\Sigma_{jj}^{-1/2} u_j$. EC2SLS based on the true values of the variance components is asymptotically equivalent to running 2SLS on (7) with the matrix of instruments $X^* = \sqrt{\sigma_{\nu jj}}\Sigma_{jj}^{-1/2} X$ (see Baltagi, 1995):

$$\begin{aligned}\hat{\delta}_{j,\mathrm{EC2SLS}} &= (Z_j^{*\prime} P_{X^*} Z_j^*)^{-1} Z_j^{*\prime} P_{X^*} y_j^* \\ &= [Z_j' \Sigma_{jj}^{-1} X(X'\Sigma_{jj}^{-1}X)^{-1} X'\Sigma_{jj}^{-1} Z_j]^{-1} Z_j' \Sigma_{jj}^{-1} X(X'\Sigma_{jj}^{-1}X)^{-1} X'\Sigma_{jj}^{-1} y_j.\end{aligned}$$

Two feasible versions of EC2SLS are compared based on two ANOVA type estimators of the variance components.[3] Knowing the true disturbances, it is easy to show that

$$\hat{\sigma}_{\nu j\ell} = u_j' Q u_\ell \Big/ \sum_{i=1}^{N} (T_i - 1) \quad \text{and} \quad \hat{\sigma}_{\mu j\ell} = (u_j' P u_\ell - N\hat{\sigma}_{\nu j\ell}) \Big/ \sum_{i=1}^{N} T_i \tag{8}$$

are unbiased estimators of $\sigma_{\nu j\ell}$ and $\sigma_{\mu j\ell}$, respectively. Here $P = \mathrm{diag}(\bar{J}_{T_i})$ and $Q = I_n - P = \mathrm{diag}(E_{T_i})$. To make the EC2SLS estimators feasible, we substitute 2SLS residuals for the true residuals in (8). This is denoted by EC2SLS1. This is an extension of the Wallace and Hussain (1969) estimator, which uses ordinary least squares (OLS) residuals in the single equation with exogenous regressors. Next, we substitute within 2SLS residuals in (8). This is denoted by EC2SLS2. This is an extension of the estimator of Amemiya (1971), who uses within residuals in the single equation with exogenous regressors.

For system estimation, we stack the two structural equations as follows:

$$y = Z\delta + u, \tag{9}$$

where $y' = (y_1', y_2')$, $Z = \mathrm{diag}[Z_j]$, $\delta' = (\delta_1', \delta_2')$, and $u' = (u_1', u_2')$. In this case, $\Sigma = E(uu')$ has a typical subblock $\Sigma_{j\ell} = E(u_j u_\ell')$ given by (5). It can be easily shown that Σ^{-1} has a typical subblock

$$\Sigma_{j\ell}^{-1} = (\sigma_\nu^{j\ell})\mathrm{diag}(E_{T_i}) + \mathrm{diag}[(w_i^{j\ell})\bar{J}_{T_i}], \tag{10}$$

where $\sigma_\nu^{j\ell}$ is the typical $(j\ell)$th element of Σ_ν^{-1} and $w_i^{j\ell}$ is the typical $(j\ell)$th element of $(\Sigma_\nu + T_i\Sigma_\mu)^{-1}$. Therefore, Σ^{-1} is more involved than the balanced panel data case described in Baltagi (1984). Note that one can still apply the Cholesky decomposition on Σ_ν and $(\Sigma_\nu + T_i\Sigma_\mu)$ to obtain the typical subblock of

$\Sigma^{-1/2}$, as suggested by Kinal and Lahiri (1990). However, for incomplete panels, $(\Sigma_\nu + T_i\Sigma_\mu)$ now varies with T_i. The system estimators considered are as follows.

(d) Three stage least squares (3SLS). This ignores the error component structure in estimating the system: $\hat{\delta}_{3SLS} = [Z'(S^{-1} \otimes P_X)Z]^{-1}Z'(S^{-1} \otimes P_X)y$, where S is the classical estimator of the system covariance matrix Σ that is based on 2SLS residuals.

(e) Within three stage least squares (W3SLS). This assumes that the individual effects are fixed parameters to be estimated: $\tilde{\delta}_{W3SLS} = [\tilde{Z}'(S_\nu^{-1} \otimes P_{\tilde{X}})\tilde{Z}]^{-1}\tilde{Z}'(S_\nu^{-1} \otimes P_{\tilde{X}})\tilde{y}$, where $\tilde{Z} = (I_2 \otimes Q)Z$, $\tilde{X} = (I_2 \otimes Q)X$, and $\tilde{y} = (I_2 \otimes Q)y$. Also, S_ν is the estimator of the covariance matrix Σ_ν based on within 2SLS residuals.

(f) Error components three stage least squares (EC3SLS). This accounts for the random error component structure. Premultiplying the system of equations given in (9) by $\Sigma^{-1/2}$ one gets

$$y^* = Z^*\delta + u^*, \tag{11}$$

where $y^* = \Sigma^{-1/2}y$, $Z^* = \Sigma^{-1/2}Z$, and $u^* = \Sigma^{-1/2}u$. EC3SLS based on the true values of the variance components turns out to be asymptotically equivalent to running 3SLS on (11) with the matrix of instruments $X^* = \Sigma^{-1/2}X$ (see Baltagi, 1995):

$$\begin{aligned}\hat{\delta}_{EC3SLS} &= (Z^{*\prime}P_{X^*}Z^*)^{-1}Z^{*\prime}P_{X^*}y^* \\ &= (Z'\Sigma^{-1}X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Z)^{-1}Z'\Sigma^{-1}X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}y.\end{aligned}$$

Two feasible versions of EC3SLS are considered, corresponding to the two ANOVA type estimators of the variance components described previously. These are denoted by EC3SLS1 and EC3SLS2, respectively.

4. RESULTS

4.1. The Structural Parameters

Table 1 gives the bias, standard deviation, and root mean squared error (RMSE) of the structural parameters for a typical experiment (pattern P_2 with $\rho_{11}^* = \rho_{12}^* = 0.5$). These are normalized by the corresponding values of the true parameters. Tables for other experiments are available upon request from the authors. The relative bias in Table 1 varies between 0% and 18% depending on the parameter and the estimation method considered. It is evident that there is gain according to RMSE in performing EC2SLS, rather than 2SLS, for all structural parameters. There is also gain according to RMSE in performing EC3SLS rather than EC2SLS for all structural parameters.[4] From the experiments performed, we also conclude that better estimates of the structural variance components (according to the RMSE criteria) did not necessarily imply better estimates of the structural coefficients. A

TABLE 1. The bias, standard deviation, and RMSE of the structural parameters:[a] Unbalanced pattern P_2 ($\rho^*_{11} = \rho^*_{12} = 0.5$)

Method	γ_{12}	λ_{11}	λ_{12}	γ_{21}	λ_{23}	λ_{24}
True value	−0.5	−2	1.5	−4	−3	1.8
			Single equation estimators			
2SLS	0.034	0.071	−0.044	−0.005	0.051	−0.031
	(0.544)	(1.167)	(1.260)	(0.393)	(1.397)	(2.016)
	0.545	1.169	1.260	0.393	1.398	2.016
W2SLS	0.000	0.006	−0.003	0.006	0.027	0.001
	(0.352)	(0.801)	(0.854)	(0.241)	(0.864)	(1.332)
	0.352	0.801	0.854	0.241	0.865	1.332
EC2SLS1	0.008	0.020	−0.009	0.001	0.029	−0.000
	(0.330)	(0.742)	(0.791)	(0.224)	(0.805)	(1.244)
	0.330	0.742	0.791	0.224	0.806	1.244
EC2SLS2	0.006	0.017	−0.008	0.001	0.028	−0.001
	(0.329)	(0.740)	(0.788)	(0.223)	(0.803)	(1.235)
	0.329	0.741	0.788	0.223	0.804	1.235
EC2SLS	0.006	0.016	−0.008	0.001	0.025	0.003
	(0.326)	(0.732)	(0.783)	(0.222)	(0.798)	(1.233)
	0.326	0.733	0.783	0.222	0.798	1.233
			System equation estimators			
3SLS	0.034	0.164	−0.159	−0.005	0.184	−0.170
	(0.544)	(1.26)	(1.141)	(0.393)	(1.209)	(1.269)
	0.545	1.137	1.152	0.393	1.223	1.280
W3SLS	0.004	0.064	−0.067	0.008	0.075	−0.084
	(0.351)	(0.775)	(0.772)	(0.240)	(0.788)	(0.810)
	0.351	0.778	0.775	0.240	0.792	0.814
EC3SLS1	0.012	0.068	−0.066	−0.001	0.078	−0.057
	(0.331)	(0.722)	(0.726)	(0.230)	(0.747)	(0.842)
	0.332	0.725	0.729	0.230	0.751	0.844
EC3SLS2	0.010	0.065	−0.067	0.003	0.072	−0.075
	(0.330)	(0.721)	(0.718)	(0.224)	(0.737)	(0.758)
	0.330	0.723	0.721	0.224	0.740	0.762
EC3SLS	0.008	0.060	−0.060	0.002	0.070	−0.064
	(0.326)	(0.710)	(0.714)	(0.221)	(0.725)	(0.767)
	0.326	0.713	0.716	0.221	0.728	0.770

[a]In each cell in this table, the upper number denotes the bias, the middle one in parentheses denotes the standard deviation, and the bottom number denotes the RMSE, all normalized by the corresponding value of the true parameter.

similar result was obtained by Baltagi (1984) for the balanced simultaneous equations case.

To summarize the results of all experiments, two measures of the overall performance of each estimator were obtained. The first is the normalized root mean

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square deviation (NORMSQD), and the second is the normalized mean absolute deviation (NOMAD). These are defined in Sasser (1973).[5] Table 2 gives the NORMSQD and NOMAD for the structural parameter estimates relative to that of true EC3SLS by method of estimation for the unbalanced designs P_1, P_3, and P_5. Results for other unbalanced designs are available upon request from the authors. The following conclusions can be drawn from Table 2. For a specific pattern of unbalancedness: (1) 2SLS deteriorates in NORMSQD and NOMAD when the variance components increase as a percentage of the total variance, i.e., with increasing ρ_{11}^* and ρ_{12}^*. (2) W2SLS improves in NORMSQD and NOMAD performance as ρ_{11}^* and ρ_{12}^* increase. (3) Feasible EC2SLS methods always do better than 2SLS (except when $\Sigma_\mu = 0$). These methods also do better than W2SLS. (4) There is not much difference among the two feasible EC2SLS methods in NORMSQD and NOMAD performance. In fact, these are always close to NORMSQD and NOMAD of EC2SLS using the true values of the variance components. (5) The performance of 3SLS deteriorates as ρ_{11}^* and ρ_{12}^* increase. However, 3SLS always does better than 2SLS in NORMSQD and NOMAD. (6) The performance of W3SLS improves as ρ_{11}^* and ρ_{12}^* increase. Also, W3SLS always does better than W2SLS in NORMSQD and NOMAD. (7) Feasible EC3SLS methods always do better than 3SLS (except when ρ_{11}^* and ρ_{12}^* are small and close to zero). These methods also do better than W3SLS (except when ρ_{11}^* and ρ_{12}^* are large and close to 0.8). (8) There is not much difference among the two feasible EC3SLS methods in NORMSQD and NOMAD. In fact, these are always close to NORMSQD and NOMAD of EC3SLS using the true variance components. (9) The EC3SLS estimators are always better in NORMSQD and NOMAD than the corresponding EC2SLS estimators. (10) What is interesting in this setup is that for large ρ_{11}^* and ρ_{12}^*, EC2SLS does better than 3SLS. This says that for our limited experiments with only two equations, ignoring the presence of large variance components can be more dangerous than ignoring the estimation of two equations simultaneously.

4.2. A Comparison of Some Unbalanced Patterns with Their Corresponding Subbalanced Counterparts

Let $P_1^A = 5(30)$ be the subbalanced pattern obtained from P_1 by dropping the four extra observations on the second set of 15 individuals to make the panel balanced.[6] Alternatively, let $P_1^B = 9(15)$ maximize the time-series lengths by dropping the first 15 individuals observed over only five periods. Finally, let $P_3^A = 3(30)$ maximize the number of individuals observed in constructing a balanced panel from P_3. Pattern P_1^A retains $(\frac{5}{7})$, whereas P_1^B retains $(\frac{9}{14})$ and P_3^A retains $(\frac{3}{7})$ of the original sample. Our Monte Carlo results demonstrate that using these subbalanced patterns is costly. For example, for $\rho_{11}^* = \rho_{12}^* = 0.8$, the ratio of NORMSQD of true EC3SLS for P_1^A relative to that of P_1 is 1.188. For P_1^B, this ratio is 1.224, and for P_3^A it is 1.596. Similar ratios are obtained for NOMAD. For example, for $\rho_{11}^* = 0.5$ and $\rho_{12}^* = 0.8$, the ratio of NOMAD of true EC2SLS for P_1^A relative to that of P_1 is 1.146. For P_1^B, this ratio is 1.286, and for P_3^A it is 1.313.

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Table 2. Normalized root mean square deviation and normalized absolute deviation of the structural parameters with unbalanced panels[a]

	ρ^*_{11}	ρ^*_{12}	2SLS	W2SLS	EC2SLS1	EC2SLS2	EC2SLS	3SLS	W3SLS	EC3SLS1	EC3SLS2
P_1	0	0	1.224	1.983	1.230	1.242	1.224	0.985	1.623	0.992	1.026
			1.171	1.772	1.176	1.186	1.171	0.988	1.481	0.993	1.025
	0.2	0	1.343	1.599	1.250	1.251	1.229	1.086	1.287	1.089	1.024
			1.278	1.482	1.195	1.197	1.175	1.086	1.241	1.033	1.029
	0.2	0.2	1.402	1.542	1.248	1.243	1.230	1.133	1.235	1.012	1.018
			1.334	1.435	1.196	1.192	1.178	1.134	1.195	1.016	1.017
	0.5	0.2	1.830	1.372	1.233	1.233	1.223	1.487	1.101	1.013	0.991
			1.705	1.313	1.194	1.196	1.184	1.454	1.103	1.026	1.006
	0.5	0.5	2.048	1.356	1.254	1.248	1.243	1.666	1.076	1.034	1.003
			1.878	1.288	1.204	1.200	1.193	1.600	1.071	1.031	1.008
	0.5	0.8	2.185	1.284	1.217	1.203	1.200	1.772	1.012	1.023	0.968
			2.055	1.251	1.190	1.180	1.176	1.745	1.031	1.035	0.987
	0.8	0.8	3.695	1.294	1.270	1.264	1.262	2.972	1.021	1.072	0.999
			3.301	1.232	1.215	1.209	1.206	2.818	1.021	1.056	1.001
P_3	0	0	1.224	2.020	1.230	1.240	1.224	0.985	1.654	0.989	1.029
			1.171	1.826	1.175	1.184	1.171	0.987	1.532	0.991	1.027
	0.2	0	1.330	1.656	1.246	1.246	1.229	1.074	1.329	1.004	1.012
			1.269	1.542	1.191	1.192	1.174	1.080	1.291	1.012	1.021
	0.2	0.2	1.389	1.597	1.249	1.243	1.229	1.124	1.283	1.012	1.016
			1.326	1.491	1.195	1.190	1.176	1.128	1.247	1.015	1.016
	0.5	0.2	1.812	1.442	1.253	1.254	1.244	1.470	1.140	1.026	0.999
			1.679	1.370	1.202	1.203	1.192	1.435	1.144	1.034	1.015

	0.5	0.5	1.987	1.391	1.255	1.250	1.242	1.618	1.099	1.041	1.004
			1.831	1.325	1.201	1.197	1.189	1.564	1.104	1.033	1.008
	0.5	0.8	2.131	1.310	1.219	1.205	1.197	1.733	1.032	1.034	0.979
			2.003	1.273	1.187	1.176	1.170	1.704	1.055	1.035	0.995
	0.8	0.8	3.532	1.313	1.274	1.267	1.265	2.849	1.028	1.075	1.000
			3.169	1.249	1.213	1.208	1.205	2.707	1.037	1.055	1.003
P_5	0	0	1.225	2.792	1.231	1.253	1.225	0.984	1.967	0.986	1.144
			1.167	2.298	1.172	1.191	1.167	0.988	1.753	0.989	1.068
	0.2	0	1.286	2.250	1.221	1.235	1.216	1.043	1.531	1.025	1.157
			1.239	1.937	1.169	1.183	1.163	1.060	1.468	1.009	1.062
	0.2	0.2	1.347	2.162	1.248	1.250	1.240	1.095	1.467	1.009	1.076
			1.298	1.882	1.194	1.197	1.185	1.111	1.423	1.012	1.052
	0.5	0.2	1.617	1.813	1.226	1.269	1.251	1.324	1.232	1.007	1.018
			1.528	1.621	1.203	1.210	1.193	1.313	1.214	1.020	1.020
	0.5	0.5	1.754	1.754	1.307	1.302	1.286	1.444	1.191	1.038	1.025
			1.645	1.564	1.230	1.229	1.215	1.414	1.169	1.026	1.023
	0.5	0.8	1.917	1.696	1.327	1.311	1.300	1.581	1.156	1.055	1.024
			1.788	1.515	1.250	1.239	1.229	1.530	1.138	1.041	1.022
	0.8	0.8	2.884	1.548	1.394	1.382	1.373	2.375	1.049	1.060	1.003
			2.651	1.403	1.297	1.286	1.280	2.276	1.041	1.047	1.007

[a]In each cell of this table, the first number denotes NORMSQD, and the second number denotes NOMAD. Both measures are relative to that of true EC3SLS.

These performance measures get worse as the number of deleted observations increases.

5. SUMMARY AND CONCLUSIONS

Many of the results obtained for the simultaneous equation error component model with balanced data carry over to the unbalanced case. For example, both feasible EC2SLS estimators considered performed reasonably well, and it is hard to choose between them. Simple ANOVA methods can still be used to obtain good estimates of the structural and reduced form parameters even in the unbalanced panel data case. Replacing negative estimates of the variance components by zero did not seriously affect the performance of the corresponding structural or reduced form estimates. Better estimates of the structural variance components do not necessarily imply better estimates of the structural coefficients. Finally, do not make the data balanced to simplify the computations. The loss in RMSE can be huge.

NOTES

1. Our Monte Carlo results are limited in that they do not vary the degree of overidentification, the number of simultaneous equations, the X matrix, or key parameters that could influence the small sample properties of the simultaneous equations (see Phillips, 1983).

2. The following pairs of variance components ratios were used: $(\rho_{11}^*, \rho_{12}^*) = \{(0,0),(0.2,0),(0.2,0.2),(0.5,0.2),(0.5,0.5),(0.5,0.8),(0.8,0.8)\}$ with $\rho_{11}^* = \rho_{22}^*$.

3. ANOVA estimators are minimum variance unbiased under normality of the disturbances and, in general, best quadratic unbiased (BQU) estimators of the variance components whenever the panel is balanced. However, for unbalanced panels, the BQU estimators are a function of the variance components themselves (see Townsend and Searle, 1971).

4. This dominance has some exceptions depending on the structural parameter and experiment considered.

5. NOMAD computes the absolute deviation of each parameter estimate from the true parameter, normalizing it by the true parameter and averaging it over all parameters and replications considered. NOMSQD computes the mean square error for each parameter, normalizing it by the square of the true parameter and averaging it over all parameters considered in the model. NORMSQD is the square root of NOMSQD.

6. Nobody advocates dropping these observations, but it is the intent of this study to emphasize the dangers from such practice.

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