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Sensitiveness of light absorption for self-focusing at laser–plasma interaction with weakly relativistic and ponderomotive regime

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Abstract

In the present paper, we have examined the sensitiveness of light absorption for self-focusing of Gaussian laser beam in plasma. By introducing dielectric function of plasma under ponderomotive and weakly relativistic regime, we have established the differential equation for beam-width parameter by using parabolic equation approach under Wentzel–Kramers–Brillouin and paraxial approximations and solved it numerically. In order to incorporate the sensitiveness of light absorption for self-focusing, behavior of normalized beam-width parameter; plasma density distribution with dimensionless distance of propagation is presented graphically and discussed. Numerical analysis shows that light absorption plays a vital role in self-focusing of laser beam in plasma under weakly relativistic and ponderomotive regime and gives reasonably interesting results.

Keywords: Absorption; Gaussian beam; Self-focusing Relativistic

1. INTRODUCTION

The field of interaction of intense laser beams with plasmas has paved the way in different directions due to wide-ranging applications in laser-driven inertial confinement fusion (Hora, 2007; Winterberg, 2008; Atzeni, 2015), laser-based plasma acceleration (Jha *et al*., 2011, 2013; Rajeev *et al*., 2013) and ionospheric modification (Keskinen & Basu, 2003; Gondarenko *et al*., 2005; Sodha & Sharma, 2008), etc. For these applications, it is desirable that the optical beam should propagate for extended distances without divergence and without being absorbed. In the absence of an optical guiding mechanism, diffraction broadening of the laser beam negates the efficiency of laser–plasma coupling (Sprangle *et al*., 1987) and thus jeopardizes the feature of above mentioned applications. Therefore, there have been ongoing efforts to explore the methods or processes that may aid to increase the efficiency of laser–plasma coupling. Self-focusing is such a nonlinear phenomenon that averts the diffraction broadening of the laser beam. In plasmas, self-focusing of laser beams is mainly due to the change in dielectric function arising from two dominating contributions. One of them is the increasing relativistic mass of electrons arising from the quiver motion due to the laser electric field. This leads to transverse gradient of the refractive index, which results in decrease in the spot size of the beam. This is generally known as relativistic self-focusing (Brandi *et al*., 1993). The other contribution arises from the nonlinear electron density perturbations due to the ponderomotive force, which is known as ponderomotive self-focusing (Osman *et al*., 1999).

In the last few decades most of the investigations (Asthana *et al*., 1999, 2006; Khanna & Baheti, 2001; Hasson *et al*., 2010; Gill *et al*., 2011*a*, 2012; Kant *et al*., 2011; Patil *et al*., 2012; Nanda *et al*., 2013; Milani *et al*., 2014*b*; Nanda & Kant, 2014; Wani & Kant, 2016) have been directed toward separate studies on self-focusing of laser beams in plasma including relativistic or ponderomotive nonlinearities. However, together with relativistic nonlinearity, ponderomotive nonlinearity also becomes important (Liu *et al*., 2009). Therefore, their combination effect significantly influences the laser beam propagation in plasma. Gill *et al*. (2010*a*, *b*, 2011*b*) have presented the relativistic and ponderomotive effects on evolution of high-power laser beams in a plasma. We have also exploited the relativistic ponderomotive effects on self-focusing of high power laser beams in

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plasma (Patil & Takale, 2013*a*, *b*, Patil *et al*., 2013*c*) under different situations. Milani *et al*. (2014*a*) have highlighted the temperature effect on self-focusing and de-focusing of Gaussian laser beam propagation through plasma in weakly relativistic and ponderomotive regime. Bokaei *et al*. (2013) studied the turning point temperature in self-focusing of laser beam in plasma under relativistic and ponderomotive regime. Such regime of interaction has been exploited by Aggarwal *et al*. (2014) for propagation of laser beam in plasma with density ripple.

In recent studies, Kant and Wani (2015) have reported the effect of linear absorption on density transition-based self-focusing of laser beam in a plasma and noticed that an impact of absorption coefficient on plasma density ramp is found to affect the nature of self-focusing/defocusing of laser beam significantly. The propagation of quadruple Gaussian laser beam in a plasma characterized by axial inhomogeneity and nonlinearity due to ponderomotive force has been studied by Aggarwal *et al*. (2015). They have considered the effect of linear absorption on the propagation characters of laser beam in a plasma. We have also studied the effect of light absorption on relativistic self-focusing of Gaussian laser beam in plasma and found that depth of self-focusing gets reduced while the periodicity gets elongated because of absorption (Patil *et al*., 2015). Present paper deals with the influence of light absorption on temperature-based self-focusing and defocusing of Gaussian laser beam in plasma under relativistic and ponderomotive regime. As usual, the present analysis employs the parabolic equation approach (Sodha *et al*., 1976) under the Wentzel–Kramers–Brillouin (WKB) and paraxial approximations. In Section 2, intensity distribution of input laser beam and dielectric function of plasma under relativistic and ponderomotive regime is presented. The second order differential equation governing the nature of self-focusing of the beam in plasma is obtained in Section 3. The discussion of results is presented in Section 4 and finally a brief conclusion is added in Section 5.

2. THEORETICAL CONSIDERATIONS

We start by considering the propagation of a Gaussian laser beam along the z-direction through underdense, unmagnetized, and collisionless plasma. Initial intensity distribution of the beam in this situation is expressed as

$$EE^* = E_0^2 \exp\left(-\frac{r^2}{r_0^2}\right), \tag{1}$$

where r is the radial coordinate of cylindrical coordinate system and r_0 is the initial beam width. The Gaussian laser beam exerts a ponderomotive force on plasma electrons due to the transverse intensity gradient of the beam. In a steady-state condition, the modified plasma density can be written as (Gill *et al*., 2011*b*), $n_e = n_0 \exp[-\beta_0(\gamma - 1)]$, where n_0 is the unperturbed plasma electron density, $\beta_0 = m_0c^2/T_e$ and $\gamma = (1 + \alpha EE^*)^{1/2}$ is the Lorentz relativistic factor with $\alpha = e^2/m_0^2\omega^2c^2$. Here e and m_0 are the charge and rest mass of electron; respectively, ω is the frequency of laser beam and c is the speed of light in free space. This perturbed plasma electron density leads to a modified dielectric function of the plasma as

$$\varepsilon = 1 - \frac{n_e}{n_0\gamma}. \tag{2}$$

One can formally express the dielectric function of plasma as (Sodha *et al*., 1976)

$$\varepsilon = \varepsilon_0 + \Phi(EE^*) - i\varepsilon_i, \tag{3}$$

where ε_0 and Φ are the linear and non-linear parts of the dielectric function respectively, ε_i takes care of absorption.

3. SELF-FOCUSING

The wave equation governing the electric field E of the beam in plasmas with the dielectric function given by Eq. (3) can be written as

$$\nabla^2 E + \frac{\omega^2}{c^2}\varepsilon E = 0. \tag{4}$$

Keeping in mind that $(c^2/\omega^2)|(1/\varepsilon)\nabla^2 \ln \varepsilon| \ll 1$, within the WKB approximation, we neglect the term $\nabla(\nabla.E)$ while writing Eq. (4). Now, $E = A(r, z)\exp(-ik_0z)$ is introduced, where $A(r, z)$ is the complex function of its argument and can be described by the parabolic equation in the WKB approximation as

$$2ik_0\frac{\partial A}{\partial z} + \nabla_\perp^2 A + \frac{\omega^2}{c^2}[\Phi(EE^*) - i\varepsilon_i]A = 0. \tag{5}$$

Expressing $A(r, z) = A_0(r, z)\exp(-ik_0S)$, where A_0 and S are real functions of r and z (S being the eikonal of the beam) and substituting in Eq. (5), one can obtain

$$2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k_0^2A_0}\nabla_\perp^2 A_0 + \frac{\omega_p^2}{\omega^2\varepsilon_0}\left\{1 - \frac{1}{\gamma}\exp[-\beta_0(\gamma - 1)]\right\}, \tag{6}$$

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial A_0^2}{\partial r}\frac{\partial S}{\partial r} + A_0^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r} - k_0\frac{\varepsilon_i}{\varepsilon_0}\right) = 0. \tag{7}$$

The solution of Eqs (6) and (7) satisfying the initial condition for intensity distribution of a Gaussian beam can be expressed as

$$S = \frac{r^2}{2f}\frac{df}{dz} + \varphi(z), \tag{8}$$

$$A_0^2 = \frac{E_0^2}{f^2}\exp\left(-\frac{r^2}{r_0^2f^2} - 2K_iz\right), \tag{9}$$

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where K_i is the absorption coefficient and $\phi(z)$ is the axial phase.

Following the paraxial approach given by Sodha *et al.* (1976) the dimensionless beam-width parameter f is obtained as

$$\frac{d^2f}{d\eta^2} = \frac{1}{f^3} - \rho_0^2 \frac{P_\eta}{2Q_\eta^3 f^3}\left[1 + (\beta_0 Q_\eta)\right] \times \exp\{\beta_0[1 - Q_\eta]\}, \quad (10)$$

where $K_i' = K_i R_d$ is the normalized absorption coefficient, $R_d = k_0 r_0^2$ is the Rayleigh length, $P_\eta = P_0 \exp(-2K_i'\eta)$ with $P_0 = \alpha E_0^2$, $Q_\eta = [1 + (P_\eta / f^2)]^{1/2}$, $\eta = z/R_d$ is the dimensionless distance of propagation and $\rho_0 = r_0\omega_p/c$ is the normalized equilibrium beam radius. Equation (10) can be solved numerically with appropriate boundary conditions. For an initially plane wavefront at $\eta = 0$, $f = 1$ and $df / d\eta = 0$, the condition $d^2f/d\eta^2 = 0$ leads to the propagation of Gaussian laser beam in the self-trapped mode.

4. NUMERICAL RESULTS AND DISCUSSION

Equation (10) is the second-order non-linear differential equation governing beam-width parameter f as a function of dimensionless distance of propagation η to study the effect of light absorption on temperature-based self-focusing and defocusing of Gaussian laser beam in a plasma. In order to present the behavior of beam-width parameter, Eq. (10) is solved by using fourth-order Runge–Kutta method for an initially plane wavefront ($f = 1$, $df/d\eta = 0$ at $\eta = 0$). The dependence of beam-width parameter and electron density distribution is studied for laser–plasma parameters; $\lambda = 800$ nm, $n_0 = 2.6 \times 10^{17}$ cm^{-3}, $T_e = 20 - 100$ KeV, $r_0 = 20$ μm, $P_0 = 0.15$. When $d^2f/d\eta^2 = 0$, the initial beam-width does not change along the propagation in the plasma and the so-called waveguide/self-trapped mode. Herein, when $f < 1$ or $f > 1$, the laser beam will be focused (converged) or defocused (diverged) during the propagation of laser beam in the plasma. It is to be noted that in absence of light absorption effect ($K_i' = 0$), Eq. (10) reduces to Eq. (12) of Milani *et al.* (2014*a*) Also, on setting $\beta_0 = 0$ in Eq. (10), one can cast Eq. (10) of our earlier result (Patil *et al.*, 2015) for influence of light absorption under relativistic case only. Further by choosing suitable temperature and absorption level, we can investigate the self-focusing of Gaussian laser beam in a plasma under weakly relativistic and ponderomotive regime.

Figure 1 presents the variation of f with η under weakly relativistic and ponderomotive regime of interaction and only relativistic case of reference (solid line). In the numerical parameters as given above, we have chosen the normalized absorption coefficient $K_i' = 0.005$. It is obvious from Figure 1 that by increasing the electron temperature from $T_e = 20$ KeV (dashed curve) to $T_e = 100$ KeV (dotted curve), self-focusing loses its power and beam is diverged for $T_e = 100$ KeV in relativistic and ponderomotive regime as

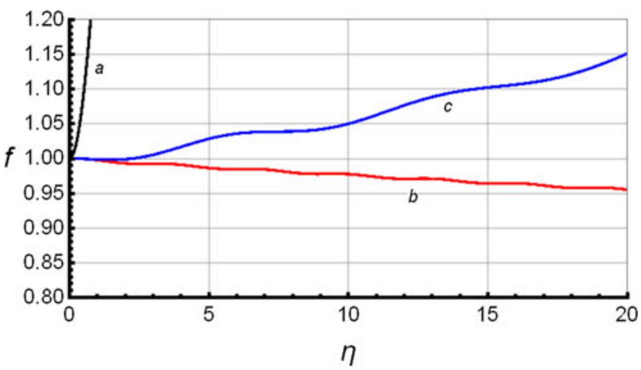

Fig. 1. Variation of beam-width parameter f with dimensionless distance of propagation η in plasma. (a) Relativistic case only, (b) $T_e = 20$ KeV, and (c) $T_e = 100$ KeV. The other laser–plasma parameters are: $\lambda = 800$ nm $r_0 = 20$ μm, $n_0 = 2.6 \times 10^{17}$ cm^{-3}, $P_0 = 0.15$, and $K_i' = 0.005$.

reported earlier by Bokaei *et al.* (2013) in the absence of light absorption ($K_i' = 0$). It is important to note from Figure 1 that light absorption causes further to improve/obstruct the behavior of self-focusing/defocusing under weakly relativistic and ponderomotive regime of interaction. As obvious, occurrence of self-focusing/defocusing depends not only on the range/limit of requisite numerical parameters such as initial plasma density n_0, electron temperature T_e, intensity parameter P_0 etc., but also on the light absorption.

The variation of f with η for different values of normalized absorption coefficients K_i' is displayed in Figure 2. It is observed from this figure that in absence of the absorption effect, for $T_e = 20$ KeV, laser beam is self-focused in stationary oscillatory mode during propagation through plasma under weakly relativistic and ponderomotive regime. Such stationary mode has been reported by Bokaei *et al.* (2013) for same laser–plasma parameters and their regime of interaction. However, such stationary oscillatory character of f gets destroyed due to absorption effect. It is interesting to note that by increasing the absorption level, self-focusing of beam is improved and takes place for earlier values of η.

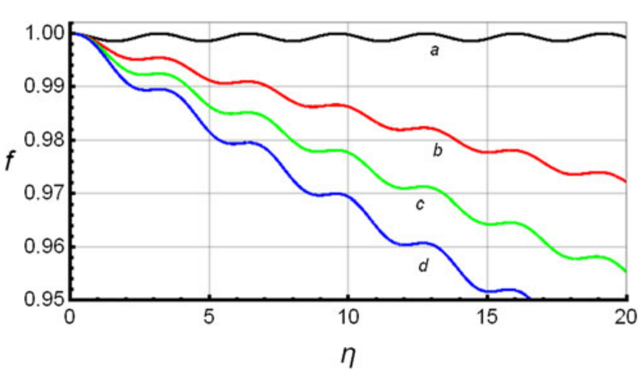

Fig. 2. Variation of beam-width parameter f with dimensionless distance of propagation η in plasma under weakly relativistic and ponderomotive regime with $T_e = 20$ KeV for different absorption levels, (a) $K_i' = 0.00$, (b) $K_i' = 0.003$, (c) $K_i' = 0.005$, (d) $K_i' = 0.007$. The other parameters are same as in Figure 1.

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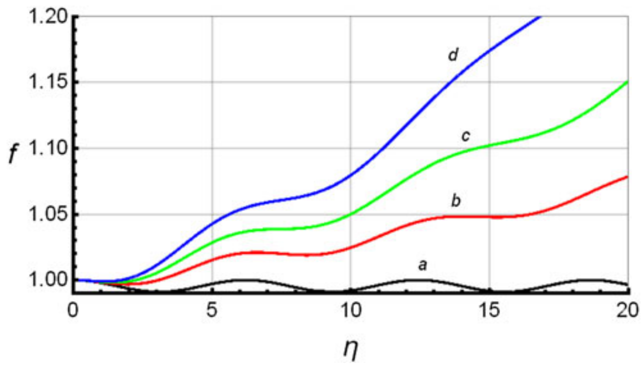

Fig. 3. Variation of beam-width parameter f with dimensionless distance of propagation η in plasma under weakly relativistic and ponderomotive regime with $T_e = 100$ KeV for different absorption levels, (a) $K_i' = 0.00$, (b) $K_i' = 0.003$, (c) $K_i' = 0.005$, (d) $K_i' = 0.007$. The other parameters are same as in Figure 1.

As obvious, penetration decreases as beam propagates through plasma on account of absorption.

In Figure 3, we have presented the effect of same absorption levels on dependence of f with η for $T_e = 100$ KeV. In contrast to Figure 2, it is observed from Figure 3 that by increasing the absorption for $T_e = 100$ KeV, result is reversed. This is due to the existence of temperature interval in which self-focusing can occur, while the beam diverges outside this region as reported (Bokaei *et al.*, 2013) earlier.

An effect of initial electron temperature T_e on profile of normalized electron density has been shown in Figures 4 and 5 for $K_i' = 0.005$. Figure 4 depicts the normalized electron density at $T_e = 20$ KeV for self-focusing regime. It is

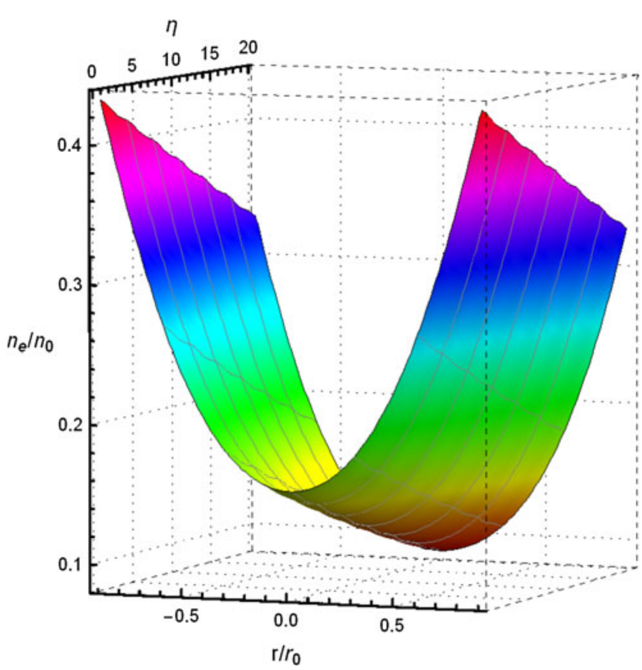

Fig. 4. Dependence of normalized plasma electron density n_e/n_0 on r/r_0 and η at $T_e = 20$ KeV. The other parameters are same as in Figure 1.

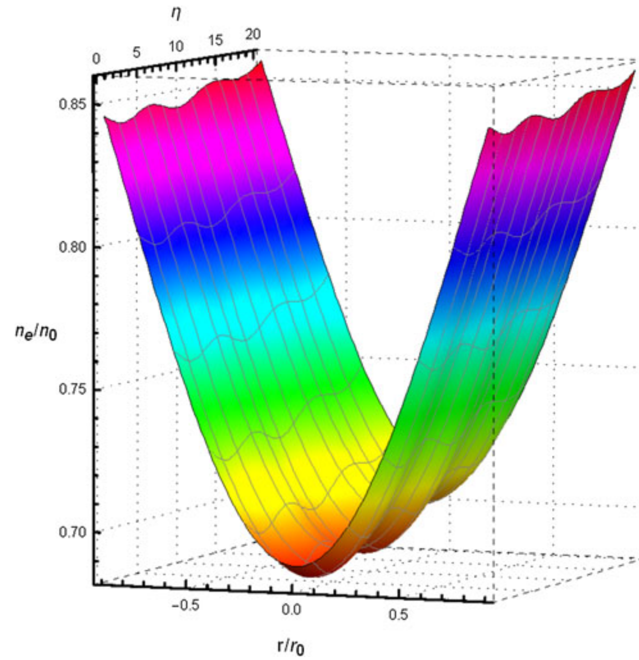

Fig. 5. Dependence of normalized plasma electron density n_e/n_0 on r/r_0 and η at $T_e = 100$ KeV. The other parameters are same as in Figure 1.

observed from this figure that the slight depressed oscillatory trend of normalized electron density along the distance of propagation occurs due to absorption. This is in accordance with behavior of beam-width parameter f with dimensionless distance of propagation η (Fig. 2). Figure 5 presents the normalized electron density at $T_e = 100$ KeV for defocusing regime. From Figure 5 it is observed that the normalized electron density at $T_e = 100$ KeV shows slight increased oscillatory behavior due to absorption in accord with Figure 3.

5. CONCLUSION

In the present work, we have studied the sensitiveness of light absorption for self-focusing of Gaussian laser beam in plasma under weakly relativistic and ponderomotive regime by using parabolic equation approach through paraxial approximation. Following important conclusions are drawn from the present analysis.

- Self-focusing of laser beam under weakly relativistic and ponderomotive regime is strongly sensitive to the light absorption. As obvious, penetration of laser beam decreases during its propagation through plasma due to light absorption. Thus larger absorption level prevents the longer propagation of laser beam through plasma.
- If plasma electron temperature is selected such that oscillatory self-focusing (defocusing) of the beam takes place, absorption enhances (disrupts) focusing behavior accordingly under weak relativistic ponderomotive regime.

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- Normalized electron density distribution is sensitive to the light absorption and has different behavior in self-focusing/defocusing modes corresponding to appropriate absorption level.

The present study may serve as a supplement for various kinds of experimental investigations in which the self-guided propagation of laser beam over a long distance without being absorbed is required, such as in laser-driven fusion and laser wakefield acceleration.

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